Exercise Solutions

Question 1: Calculate the number of states per cubic metre of sodium is 3s band. The density of sodium is 1013 kg m^{-3}. How many of them are empty?

Solution:

Given: Mass of the sodium in 1 m^3 = 1013 kgm^{-3}

Atomic mass of sodium = 23

Total number of atoms = N = [mass x avogadro's number]/[atomic mass]

= $[1013 \times 10^3 \times 6.022 \times 10^{23}]/[23]$

= 265.22×10^{26} atoms

In N atoms of sodium there will be 2N states possible states and N states will be empty.

Total number of states = 2 x N = $2 \times 265.22 \times 10^{26} = 5.30 \times 10^{28}$ states

As atomic number of sodium = 11

Its electronic configuration = $1s^2, 2s^2, 2p^6, 3s^1$

=> The 3s band is half-filed in case of sodium, the total number of unoccupied states is 2.65×10^{28}.

Question 2: In a pure semiconductor, the number of conduction electrons is 6×10^{19} per cubic metre. How many holes are there in a sample of size 1 cm × 1cm × 1 mm?

Solution:

Since in pure semiconductor the number of holes = number of conduction electron

=> Number of holes = $6 \times 10^{19}\ m^{-3}$

Therefore, Number of holes in 1 cm × 1cm × 1 mm = $6 \times 10^{19} \times 10^{-7} = 6 \times 10^{12}$ holes

[Using 1mm = $10^{-7}\ m^3$]

Question 3: Indium antimonide has a band gap of 0.23 eV between the valence and the conduction band. Find the temperature at which kT equals the band gap.

Solution:

We know, kT = 0.23 eV

Where, k = Boltzmann constant and t = temperature

=> T = 0.23 eV/k = 0.23 eV/[8.62×10^{-5} eVK^{-1}] = 2670 K (approx)

Question 4: The band gap for silicon is 1.1 eV.
(a) Find the ratio of the band gap to kT for silicon at room temperature 300 K.
(b) At what temperature does this ratio become one tenth of the value of 300 K? (Silicon will not retain its structure at these high temperatures)

Solution:
The band gap for silicon is 1.1 eV.

kT at room temp = 0.026 eV

(a) Ratio of band gap to kT = 1.1/0.026 = 43 (approx)

(b) Consider new ratio is $1/10^{th}$ of the Part (a) result.

Now, Band gap/kT = 4.3

=> 1.1/[$8.62 \times 10^{-5} \times T$] = 4.3

=> T = 3000 K(approx)

Question 5: When a semiconducting material is doped with an impurity, new acceptor levels are created. In a particular thermal collision, a valence electron revives an energy equal to 2kT and just reaches one of the acceptor levels. Assuming that the energy of the electron was at the top of edge of the valence band and that the temperature T is equal to 300 K, find the energy of the acceptor levels above the valence band.

Solution:
The electron was at the top of the edge of valance band and reaches acceptor level on receiving the energy given.

Energy gap between the acceptor level and valence band must be 2kT.

Where, k = Boltzmann constant and t = temperature

=> E = $2 \times 8.62 \times 10^{-5} \times 300$ = 51.72 meV

Question 6: The band gap between the valence and the conduction bands in zinc oxide (ZnO) is 3.2 eV. Suppose an electron in the conduction band combines with a hole in the valence band and the excess energy is released in the form of electromagnetic radiation. Find the maximum wavelength that can be emitted in this process.

Solution:
For the electron to move from the conduction band to valance band will have to lose energy same as the energy band gap of ZnO OR The max energy released in this process will be equal to the band gap of the material.

=> E = 3.2 eV

=> 3.2 = [1242eV-nm]/λ

=> λ = 388.1 nm

Question 7: Suppose the energy liberated in the recombination of a hole-electron pair is converted into electromagnetic radiation. If the maximum wavelength emitted is 820 nm, what is the band gap?

Solution:
The min energy release in the recombination of the conduction band electron with valence bond hole = band gap of the material

We know, E = hc/λ

=> E = [1242eV-nm]/820nm

[given λ = 820nm]

=> E = 1.5 eV

Question 8: Find the maximum wavelength of electromagnetic radiation which can create a hole-electron pair in germanium. The band gap in germanium is 0.65 eV.

Solution:
The band gap in germanium is 0.65 eV.

The energy required to produce the hole-electron pair = energy band gap
We know, E = hc/λ

=> $\lambda = hc/E = [4.14 \times 10^{-15} \times 3 \times 10^{8}]/0.65$

= 1.9×10^{-6} m

Which is required wavelength.

Question 9: In a photodiode, the conductivity increases when the material is exposed to light. It is found that the conductivity changes only if the wavelength is less than 620 nm. What is the band gap?

Solution:
Maximum wavelength at which the conduction starts = 620 nm
and this wavelength provides minimum energy to the electron to move to conduction band.

we know, Energy of this radiation = Energy band gap

=> E = $[4.14 \times 10^{-15} \times 3 \times 10^{8}]/[620 \times 10^{-9}]$ = 2.003 eV = 2 eV (approx)
Which is the required band gap.

Question 10: Let ΔE denote the energy gap between the valence band and the conduction band. The population of conduction electrons (and of the holes) is roughly proportional to $e^{-\Delta E/2kT}$. Find the ratio of the concentration of conduction electrons in diamond to that in silicon at room temperature 300 K. ΔE for silicon is 1.1 eV and for diamond is 6.0 eV. How many conduction electrons are likely to be in one cubic meter of diamond?

Solution:
Let ΔE_1 and ΔE_2 be the energy band gap for silicon and diamond.

=> ΔE_1 = 1.1 eV and ΔE_2 = 6.0 eV (Given)

Also, we are given that, population of conduction electrons (and of the holes) is roughly proportional to $e^{-\Delta E/2kT}$.

Therefore,

$$n = ke^{-\frac{\Delta E}{2kT}}$$

For silicon,

$$n_1 = ke^{-\frac{\Delta E_1}{2kT}} = ke^{-\frac{1.1}{2\times 8.62\times 10^{-5}\times 300}}$$

$$= k \times 5.84 \times 10^{-10}$$

For Diamond,

$$n_2 = ke^{-\frac{\Delta E_2}{2kT}} = e^{-\frac{6.0}{2\times 8.62\times 10^{-5}\times 300}}$$

$$= k \times 4.18 \times 10^{-51}$$

Now,

$$\frac{n_2}{n_1} = \frac{k\times 4.18\times 10^{-51}}{k\times 5.84\times 10^{-10}} = 7.157 \times 10^{-42}$$

Because of more band gap, there will be almost zero conduction electrons will be there in the diamond.

Question 11: The conductivity of a pure semiconductor is roughly proportional to $T^{3/2} e^{-\Delta E/2kT}$ where ΔE is the band gap. The band gap for germanium is 0.74 eV at 4 K and 0.67 eV at 300 K. By what factor does the conductivity of pure germanium increase as the temperature is raised from 4 K to 300 K.

Solution:

Let E_1 = 0.74 eV and E_2 = 0.67 eV

Let σ_1 and σ_2 be the conductivity at 4 K and 300K.

Therefore,

$$\frac{\sigma 1}{\sigma 2} = \left(\frac{4}{300}\right)^{\frac{3}{2}} \times \left(e^{-\frac{\Delta E_1}{2kT}}\right)/\left(e^{-\frac{\Delta E_2}{2kT}}\right)$$

$$= 0.0015 \times e^{\frac{\frac{\Delta E_2}{300} - \frac{\Delta E_1}{4}}{2\times 8.62\times 10^{-5}}}$$

$$= 0.0015 \times e^{\frac{\frac{0.67}{300} - \frac{0.74}{4}}{2\times 8.62\times 10^{-5}}}$$

$$= 0.0015 \times 4.5 \times 10^{-461}$$

or $\sigma_2 = 0.148 \times 10^{463} \times \sigma_1 = 10^{463}\ \sigma_1$ (approx)

Which shows that conductivity increases by the factor 10^{463}.

Question 12: Estimate the proportion of boron impurity which will increase the conductivity of a pure silicon sample by a factor of 100. Assume that each boron atom creates a hole and the concentration of holes in pure silicon at the same temperature is 7×10^{15} holes per cubic meter. Density of silicon is 5×10^{28} atoms per cubic meter.

Solution:
Initially, the number of charge carriers = $7 \times 10^{15} + 7 \times 10^{15} = 14 \times 10^{15}$
[As number conduction electron and holes are the same]

Total charge carriers after adding the impurity to the pure silicon = $14 \times 10^{15} \times 100 = 14 \times 10^{17}$

let "y" be holes after adding the impurity and $(14 \times 10^{17} - y)$ will be the electrons.

Now,
Product of the total conduction electrons and holes remains nearly constant.

=> $7 \times 10^{15} \times 7 \times 10^{15} = y \times (14 \times 10^{17} - y)$

Solving above equation for y, we get

$y = 13.982 \times 10^{15}$ or $y = 1.8 \times 10^{15}$

As, initial number of holes must be smaller than the number of holes after doping, so neglecting 2nd value.

Thus, the number of boron atoms added = $13.982 \times 10^{17} - 7 \times 10^{15}$

$= 1398.2 \times 10^{15} - 7 \times 10^{15}$

$= 13.912 \times 10^{17}$

13.912×10^{17} atoms are added per 5×10^{28} atoms of Silicon per cubic meter.

So, 1 atom of boron is added per $[5 \times 10^{28}]/[13.912 \times 10^{17}] = 3.59 \times 10^{10}$ atoms of Silicon per cubic meter.

Therefore, 3.59×10^{10} is the proportion of born impurity.

Question 13: The product of the hole concentration and the conduction electron concentration turns out to be independent of the amount of any impurity doped. The concentration of conduction electrons in germanium is 6×10^{19} per cubic meter. When some phosphorus impurity is doped into a germanium sample, the concentration of conduction electrons increases to 2×10^{23} per cubic meter. Find the concentration of the holes in the doped germanium.

Solution:

Concentration of holes = 6×10^{19}

Product of concentration of conduction electron and that of holes remains constant.

Let n_h be the Concentration of holes.

=>$(6 \times 10^{19}) \times (6 \times 10^{19}) = (2 \times 10^{23}) \times n_h$

As per question, 2×10^{23} be the concentration of conduction electron after doping. And 6×10^{19} be the concentration of conduction electron before doping.

=> $n_h = [36 \times 10^{38}]/[2 \times 10^{23}]$

=> $n_h = 18 \times 10^{15}$

Thus, number of holes after doping is 18×10^{15} per cubic metre.

Question 14: The conductivity of an intrinsic semiconductor depends on temperature as $\sigma = \sigma_0 e^{-\Delta E/2kT}$, where σ_0 is a constant. Find the temperature at which the conductivity of an intrinsic germanium semiconductor will be double of its value at T = 300 K. Assume that the gap for germanium is 0.650 eV and remains constant as the temperature is increased.

Solution:

ΔE (Band gap) = 0.650 eV

Given: $\sigma = \sigma_0 e^{-\Delta E/2kT}$ ...(1)

Let the conductivities at temperatures T_1 and T_2 be σ_1 and σ_2 respectively.

Given T_1 = 300K and T_2 = temperature at which the conductivity is double of its value at T_1.

=> $2\sigma_1 = \sigma_2$

(1)=>

$$2\sigma_0 e^{\frac{-\Delta E}{2kT_1}} = \sigma_0 e^{\frac{-\Delta E}{2kT_2}}$$

$$2e^{\frac{-0.650}{2\times 8.62\times 10^{-5}\times 300}} = e^{\frac{-0.650}{2\times 8.62\times 10^{-5}\times T_2}}$$

$$e^{-\frac{0.650}{2\times 8.63\times 10^{-5}\times T_2}} = 6.9656\times 10^{-6}$$

Taking natural log both sides, we get

$$\frac{-0.650}{2\times 8.68\times 10^{-5}\times T_2} = -11.874525$$

or T_2 = 317.512 K

Question 15: A semiconducting material has a band gap of 1 eV. Acceptor impurities are doped into it which create acceptor levels 1 meV above the valence band. Assumed that the transition from one energy level to the other is almost forbidden if kT is less than 1/50 of the energy gap. Also, if kT is more than twice the gap, the upper levels have maximum population. The temperature of the semiconductor is increased from 0K. The concentration of the holes increases with temperature and after a certain temperature it becomes approximately constant. As the temperature is further increased, the hole concentration again starts increasing at a certain temperature. Find the order of the temperature range in which the hole concentration remains approximately constant.

Solution:

A semiconducting material has a band gap of 1 eV. Acceptor impurities are doped into it which create acceptor levels 1 meV above the valence band.

Band gap after doping = 1eV -1 meV = (1 - 0.001) = 0.999 eV

Given Condition 1: Any transition from one energy level to the other is almost forbidden if kT is less than 1/50 of the energy gap.

=> kT_1 = 0.999/50

Where, k = Boltzmann constant

T_1 be the upper limit for temperature over which transition becomes a forbidden transition.

=> $T_1 = 0.999/[50\times 8.62\times 10^{-5}]$ = 232.8 K (approx)

Given Condition 2: If kT is more than twice the gap, the upper levels have maximum population.

For max limit

$kT_2 = 2 \times 10^{-3}$

=> $T_2 = [2 \times 10^{-3}]/[8.62 \times 10^{-5}] = 23.2$ K

Required temperature range = (23.2K - 232.8K)

Question 16: In a p-n junction, the depletion region is 400 nm wide and an electric field of 5×10^5 V m^{-1} exists in it.
(a) Find the height of the potential barrier (b) What should be the minimum kinetic energy of a conduction electron which can diffuse from the n-side to the p-side.

Solution:

(a) We know the relation between electric field and potential,

E = V/d

or $V = Ed = 5 \times 10^5 \times 4 \times 10^{-7} = 0.2$ volts

Given $E = 5 \times 10^5$ Vm^{-1} and $d = 4 \times 10^{-7}$ m

(b) Minimum K.E. = Potential barrier x charge on an electron

$= 0.2 \times 1.6 \times 10^{-19}$

= 0.2 eV

Question 17: The potential barrier existing across an unbiased p-n junction is 0.2 volt. What minimum kinetic energy a hole should have to diffuse from the p-side to the n-side if
(a) the junction is unbiased,
(b) the junction is forward biased at 0.1 volt and
(c) the junction is reverse biased at 0.1 volt?

Solution:
Minimum K.E. = [Potential barrier - biasing voltage] x charge on hole

(a) Biasing voltage = 0 V

Minimum K.E. = 0.2V x e = 0.2 eV

(b) Biasing Voltage for forward biased = 0.1V

Minimum K.E. = (0.2-0.1) × e = 0.1

(c) Biasing Voltage for reverse biased = -0.1

Minimum K.E. = (0.2 + 0.1) × e = 0.3 eV

Question 18: In a p-n junction, a potential barrier of 250 meV exists across the junction. A hole with a kinetic energy of 300 meV approaches the junction. Find the kinetic energy of the hole when it crosses the junction if the hole approached the junction
(a) from the p-side and (b) from the n-side

Solution:
(a) A junction will act like forward biasing when the hole approaches the junction from p-side

K.E. of the hole will decrease.

Final kinetic energy of the hole = (300-250) = 50 meV

(b) A junction will act like reverse biasing, when the hole approaches the junction from n-side

K.E. of the hole in this case will increase.

Final kinetic energy of the hole = (300+250)= 550 meV

Question 19: When a p-n junction is reverse biased, the current becomes almost constant at 25 μA. When it is forward biased at 200 mV, a current of 75 μA is obtained. Find the magnitude of diffusion current when the diode is
(a) unbiased (b) reverse biased at 200 mV (c) forward biased at 200 mV.

Solution:
When a p-n junction is reverse biased, the current becomes almost constant at 25 μA. When it is forward biased at 200 mV, a current of 75 μA is obtained.

(a) When the diode is unbiased,
Diffusion current = drift current = 25μA

(b) When the diode is reverse biased, then diffusion current = 0

(c) When the diode is forward biased at 200 mV, then

Diffusion current - Drift current = Forward biasing current

Diffusion current = (75 + 25) μA = 100 μA

Question 20: The drift current in a p-n junction is 20.0 μA. Estimate the number of electrons crossing a cross section per second in the depletion region.

Solution:
The drift current (i_d) in a p-n junction is 20.0 μA = $20.0\text{x}10^{-6}$A

we know, $i_d = (N_e + N_h) \text{ x } e$

where, N_h = Number of holes crossing a cross section per second
N_e= Number of electrons crossing a cross section per second
e = magnitude of charge on a hole = 1.6×10^{-19} C

Without any biasing applied on the junction, $N_h = N_e = N$

=> $i_d = 2N \text{ x } e$

or $N = i_d/2e = [20\text{x}10^{-6}]/[2\text{x}1.6\text{x}10^{-19}] = 6.25 \text{ x } 10^{13}$.