

Exercise Solutions

Question 1: Assume that the mass of a nucleus is approximately given by $M = Am_p$ where A is the mass number. Estimate the density of matter in kgm^{-3} inside a nucleus. What is the specific gravity of nuclear matter?

Solution:

$$\text{Density} = \text{Mass/Volume} \dots(1)$$

$$\text{Mass of nucleus}(M) = Am_p \text{ (Given)}$$

$$\text{Mass of proton} = m_p = 1.007276 \mu$$

$$\mu = \text{atomic mass unit} = 1.6605 \times 10^{-27} \text{ kg}$$

Now,

$$\text{Radius of nucleus} = R = R_0 A^{(1/3)}$$

$$= 1.1 \times 10^{-15} A^{(1/3)} \text{ m}$$

$$\text{Volume} = (4/3) \pi R^3 = (4/3) \pi \times R_0^3 A^{(1/3)^3} = (4/3)\pi R_0^3 A$$

$$(1) \Rightarrow \text{Density} = [A \times 1.007276 \times 1.6605 \times 10^{-27}] / [(4/3)\pi R_0^3 A]$$

$$= 3.00006 \times 10^{17} \text{ kg m}^{-3}$$

$$\text{Specific gravity of nuclear matter} = \text{Density}/1000 = = 3.00006 \times 10^{14}.$$

Question 2: A neutron star has a density equal to that of the nuclear matter. Assuming the star to be spherical, find the radius of a neutron star whose mass is $4.0 \times 10^{30} \text{ kg}$ (twice the mass of the sun).

Solution:

$$\text{Mass of star}(M) = 4 \times 10^{30} \text{ kg (Given)}$$

$$\text{Density of nuclear matter}(D) = 2.3 \times 10^{17} \text{ kg/m}^3$$

$$\text{Volume} = \text{Mass/Density} = [4 \times 10^{30}] / [2.3 \times 10^{17}] \text{ m}^3$$

$$= [4 \times 10^{13}] / 2.3 \text{ m}^3$$

$$\text{Again, Volume} = (4/3)\pi R^3$$

Where, R = Radius of the neutron star

$$\Rightarrow (4/3)\pi R^3 = 4 \times 10^{13} / 2.3$$

$$\Rightarrow R^3 = [3 \times 10^{13}] / [2.3 \pi]$$

$$\Rightarrow R = 16.07 \text{ km}$$

Question 3: Calculate the mass of an α -particle. Its binding energy is 28.2 MeV.

Solution:

Binding energy = 28.2 MeV (given)

Mass of proton = 1.007276 u

Mass of neutron = 1.008665 u

Where, Atomic mass unit = u = $1.6605402 \times 10^{-27}$ kg

Now,

$\Delta M = (\text{number of proton} \times \text{mass of proton} + \text{number of neutron} \times \text{mass of neutron}) - M$

Where, M = mass of an alpha particle

$$\Delta M = (2 \times 1.007276 \text{ u} + 2 \times 1.008665 \text{ u}) - M \dots(1)$$

Also, Binding energy = $\Delta M c^2$

$$\Rightarrow \Delta M c^2 = 28.2$$

$$\Rightarrow \Delta M = 28.2 / [931.5]$$

[Here $c = 931.5 \text{ MeV/u}$]

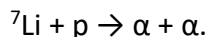
$$\Rightarrow \Delta M = 0.030273 \text{ u} \dots(2)$$

From (1) and (2)

$$(2 \times 1.007276 \text{ u} + 2 \times 1.008665 \text{ u}) - M = 0.030273 \text{ u}$$

$$M = 4.0016 \text{ u}$$

Question 4: How much energy is released in the following reaction:



Atomic mass of ${}^7\text{Li} = 7.0160 \text{ u}$ and that of ${}^4\text{He} = 4.0026 \text{ u}$.

Solution:

Atomic mass of ${}^7\text{Li} = 7.0160 \text{ u}$

Atomic mass of ${}^4\text{He} = \text{Mass of alpha particle} = 4.0026 \text{ u}$

Mass of proton = 1.007276 u

Mass defect (ΔM) = Mass of reactants - Mass of products

$$= [7.0160 + 1.007276] - 2 \times 4.0026$$

$$\Rightarrow \Delta M = 0.018076 \text{ u}$$

Now, energy release, $E = \Delta M c^2 = 0.018076 \times 931.5 = 16.83 \text{ MeV}$

[Here $c = 931.5 \text{ MeV/u}$]

Question 5: Find the binding energy per nucleon of ${}^{197}_{79}\text{Au}$ if its atomic mass is 196.96 u .

Solution:

Atomic mass of Au = 196.96 u

Atomic Number of Au = $Z = 79$

Number of nucleons = $A = 197$

Number of neutrons = $N = A - Z = 118$

Now, Binding energy is

$$B = (Zm_p + Nm_n - M)c^2$$

Where, $M = \text{Atomic Mass}$

$m_n = \text{mass of neutron}$

$m_p = \text{mass of proton}$

$c = \text{Speed of light} = 931.5 \text{ MeV/u}$

$$\Rightarrow B = [(79 \times 1.007276 + 118 \times 1.008665) - 196.96] \times 931.5 = 1525.12 \text{ MeV}$$

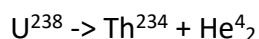
Therefore, Binding energy per nucleon = $1525.12/197 = 7.741$

Question 6: (a) Calculate the energy released if ${}^{238}\text{U}$ emits an α -particle.

(b) Calculate the energy to be supplied to ${}^{238}\text{U}$ if two protons and two neutrons are to be emitted one by one. The atomic masses of ${}^{238}\text{U}$, ${}^{234}\text{Th}$ and ${}^4\text{He}$ are 238.0508 u , 234.04363 u and 4.00260 u respectively.

Solution:

When ^{238}U emits an α -particle, the reaction is as follow,



So, energy released is the product of c^2 and subtraction of mass of reactant and mass of products.

Here $c^2 = 931.5 \text{ MeV/u}$

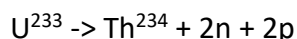
Now,

$$\Rightarrow E = \Delta m c^2$$

$$\Rightarrow E = [238.0508 - (234.04363 + 4.00260)] \times 931.5 = 4.2569 \text{ MeV}$$

(b) When two protons and two neutrons are to be emitted one by one. The atomic masses of ^{238}U , ^{234}Th and ^4He are 238.0508 u, 234.04363 u and 4.00260 u respectively.

Reaction will be,



$$\text{So, mass defect} = \Delta m = m(\text{U}^{233}) - m(\text{Th}^{234}) + 2(m_n) + 2(m_p)$$

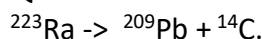
Where, m_n = mass of neutron = 1.008665 u and m_p = mass of proton = 1.007276 u

$$\Delta m = 238.0508 - [234.04363 + 2 \times 1.008665 + 2 \times 1.007276]$$

$$\Delta m = 0.024712 \text{ u}$$

$$\text{So, energy released} = E = \Delta m c^2 = 0.024712 \times 931.5 = 23.02 \text{ MeV}$$

Question 7: Find the energy liberated in the reaction



The atomic masses needed are as follows.

^{223}Ra	^{209}Pb	^{14}C
223.018u	208.981u	14.003u

Solution:

Given reaction is: $^{223}\text{Ra} \rightarrow ^{209}\text{Pb} + ^{14}\text{C}.$

$$\text{Energy} = E = [223.018u - (208.981u + 14.003u)]c^2$$

$$= 0.034 \times 931$$

$$= 31.65 \text{ MeV}$$

Question 8: Show that the minimum energy needed to separate a proton from a nucleus with Z protons and N neutrons is

$$\Delta E = (M_{Z-1, N} + M_H - M_{Z, N}) c^2$$

Where $M_{Z, N}$ = mass of an atom with Z protons and N neutrons in the nucleus and M_H = mass of a hydrogen atom. This energy is known as proton-separation energy.

Solution:

As hydrogen contains only protons, so we can write energy equation as follows:

$$E_{(Z,N)} \rightarrow E_{(Z-1,N)} + p_1$$

$$\Rightarrow E_{(Z,N)} \rightarrow E_{(Z-1,N)} + {}^1\text{H}_1$$

So, minimum energy needed to separate a proton from a nucleus with Z protons and N neutrons is

$$\Delta E = (M_{Z-1, N} + M_H - M_{Z, N}) c^2$$

Question 9: Calculate the minimum energy needed to separate a neutron from a nucleus with Z protons and N neutrons in terms of the masses $M_{Z, N}$, $M_{Z, N-1}$ and the mass of the neutron.

Solution:

We know, energy released = Mass difference $\times c^2$

The reaction is,

$$E_{Z,N} = E_{Z,N-1} + {}^1n_0$$

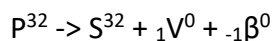
The minimum energy needed to separate the neutron will be,

$$\Delta E = (M_{Z,N-1} + M_N - M_{Z,N}) \times c^2$$

Question 10: ${}^{32}\text{P}$ beta-decays to ${}^{32}\text{S}$. Find the sum of the energy of the antineutrino and the kinetic energy of the β -particle. Neglect the recoil of the daughter nucleus. Atomic mass of ${}^{32}\text{P} = 31.974 \text{ u}$ and that of ${}^{32}\text{S} = 31.972 \text{ u}$.

Solution:

The reaction will be



The sum of the energy of the antineutrino and the kinetic energy of the β -particle:

$$E = m(P^{32}) - m(S^{32}) \times c^2$$

$$= (31.974 - 31.972) \times 931.5$$

$$= 1.863 \text{ MeV}$$

Question 11: A free neutron beta-decays to a proton with a half-life of 14 minutes.

(a) What is the decay constant?

(b) Find the energy liberated in the process.

Solution:

(a) Decay constant:

Half-life of 14 minutes = 840 sec (Given)

We know, half life = $\ln(2)/\lambda$

Where λ = decay constant

$$\Rightarrow \ln(2)/\lambda = 840$$

$$= 8.25 \times 10^{-4} \text{ s}^{-1}$$

(b) Find the energy liberated in the process:

$$E = [m_n - (m_p + m_\beta)] \times c^2$$

Where,

M_n = mass of Neutron = 1.008665 u

M_p = mass of proton = 1.007276 u

M_β = mass of β -particle = 0.0005486 u

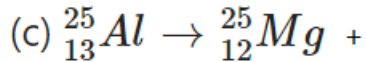
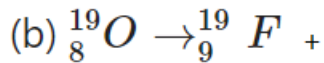
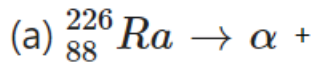
c = Speed of light = 931.5 MeV/u

$$\Rightarrow E = [1.008665 - (1.007276 + 0.0005486)] \times 931.5$$

$$= 0.78283 \text{ MeV}$$

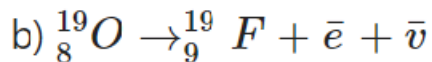
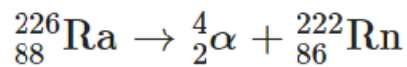
$$= 782.83 \text{ KeV}$$

Question 12: Complete the following decay schemes.



Solution:

(a) One α particle is produced so atomic number will decrease by 2 and the mass by 2,
Resultant reaction:



Question 13: In the decay ${}_{29}^{64}\text{Cu} \rightarrow {}_{28}^{64}\text{Ni} + e^{+} + \nu$, the maximum kinetic energy carried by the positron is found to be 0.650 MeV.

(a) What is the energy of the neutrino which was emitted together with a positron of kinetic energy 0.150 MeV?

(b) What is the momentum of this neutrino in kg ms^{-1} ? Use the formula applicable to a photon.

Solution:

(a) Energy the neutrino = 0.650 - K.E. of given positron

Maximum kinetic energy carried by the positron is found to be 0.650 MeV. (Given)

$$= 0.650 - 0.150$$

$$= 0.5 \text{ MeV}$$

$$= 500 \text{ keV}$$

(b) Momentum of this neutrino is

$$P = E/c = [500 \times 1.6 \times 10^{-19}] / 3 \times 10^8 \times 10^3$$

$$= 2.67 \times 10^{22} \text{ Kg m/s}$$

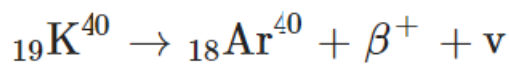
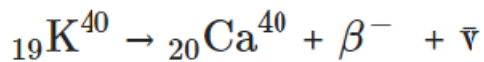
Question 14: Potassium -40 can decay in three modes. It can decay by β^- -emission, β^+ -emission or electron capture.

(a) Write the equations showing the end products.

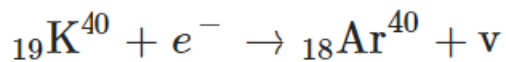
(b) Find the Q-value in each of the three cases. Atomic masses of $^{40}\text{Ar}_{18}$, $^{40}\text{K}_{19}$ and $^{40}\text{Ca}_{20}$ are 39.9624 u, 39.9640 u and 39.9626 u respectively.

Solution:

(a) Decay of Potassium -40 by β^- -emission, β^+ -emission or electron capture:



And,



(b) We know, Q- value = [mass of reactants - Mass of products] c^2

Where $c = 931.5 \text{ MeV/u}$

Now,

Q-value in β^- decay = $[39.9640 - 39.9626] \times 931.5 = 1.30141 \text{ MeV}$

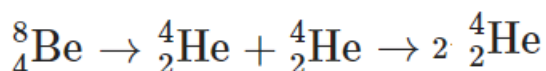
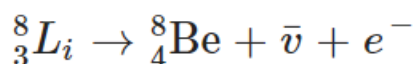
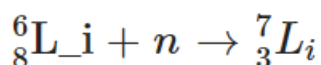
Q-value in β^+ decay = $[39.9640 - (39.9626 + 2 \times 0.005486)] \times 931.5 = 0.4683 \text{ MeV}$

Q-value in electron capture = $[39.9640 - 39.9624] \times 931.5 = 1.490 \text{ MeV}$

Question 15: Lithium ($Z = 3$) has two stable isotopes ${}^6\text{Li}$ and ${}^7\text{Li}$. When neutrons are bombarded on lithium sample, electrons and α -particles are ejected. Write down the nuclear processes taking place.

Solution:

Given: Lithium ($Z = 3$) has two stable isotopes ${}^6\text{Li}$ and ${}^7\text{Li}$.



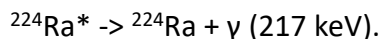
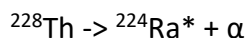
Question 16: The masses of ^{11}C and ^{11}B are respectively 11.0114 u and 11.0093 u. Find the maximum energy a positron can have in the β^+ -decay of ^{11}C to ^{11}B .

Solution:

The maximum energy for the positron in the β^+ -decay = energy due to the mass defect (ΔM)

$$\Rightarrow \Delta M = [11.0114 - 11.0093] \times 931 = 1.955 \text{ MeV}$$

Question 17: ^{228}Th emits an alpha particle to reduce to ^{224}Ra . Calculate the kinetic energy of the alpha particle emitted in the following decay:



Atomic mass of ^{228}Th is 228.028726u, that of ^{224}Ra is 224.020196 u and that of ^4He is 4.00260 u.

Solution:

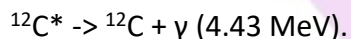
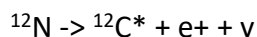
$$\text{Mass of } ^{224}\text{Ra} = 224.020196 \times 931 + 0.217 = 208563.0195 \text{ MeV}$$

$$\text{K.E. of } \alpha \text{ particle} = (\text{mass of Th} - \text{mass of Ra} - \text{mass of alpha particle}) \times (\text{speed of light})^2$$

$$= (228.028726 \times 931) - (208563.0195 + 4.00260 \times 931)$$

$$= 5.304 \text{ MeV}$$

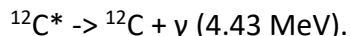
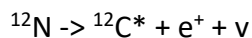
Question 18: Calculate the maximum kinetic energy of the beta particle emitted in the following decay scheme:



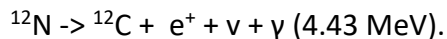
The atomic mass of ^{12}N is 12.018613 u

Solution:

Given reaction are:



Adding both the reactions, we get



Now, find $K.E._{\text{max}}$

$$\text{Max K.E of } \beta\text{-particle} = [(\text{mass of } ^{12}\text{N} - \text{mass of } ^{12}\text{C}) \times c^2] - 4.43\text{Mev}$$

$$= [12.018613 - 12] \times 931 - 4.43$$

$$= 12.89 \text{ MeV}$$

Question 19: The decay constant of $^{197}\text{Hg}_{80}$ (electron capture to $^{197}\text{Au}_{79}$) is $1.8 \times 10^{-4} \text{ s}^{-1}$.

(a) What is the half-life? (b) What is the average-life? (c) How much time will it take to convert 25% of this isotope of mercury into gold?

Solution:

(a) Half life = $\ln(2)/[\text{decay constant}]$

$$= \ln(2)/[1.8 \times 10^{-4}]$$

$$= 3850.81 \text{ sec}$$

$$= 64 \text{ minutes (approx)}$$

(b) Average life = $1/[\text{decay constant}]$

$$= 1/[1.8 \times 10^{-4}]$$

$$= 5555.56 \text{ sec}$$

$$= 92 \text{ minutes (approx.)}$$

(c) Using relation,

$$A/A_0 = (1/2)^N$$

where N = Number of half lives

A = Activity of the substance

A_0 = Initial activity

As per question,

$$\text{Present activity} = (1 - 0.25) A_0 = 0.75 A_0$$

$$\Rightarrow (0.75A_0)/A_0 = (1/2)^N$$

Taking “ln” both the sides,

$$\ln 0.75 = N \times \ln(1/2)$$

$$\Rightarrow N = 1598.23 \text{ sec}$$

Question 20: The half-life of ^{198}Au is 2.7 days.

(a) Find the activity of a sample containing 1.00 μg of ^{198}Au .

(b) What will be the activity after 7 days? Take the atomic weight of ^{198}Au to 198 g mol^{-1} .

Solution:

The half-life of ^{198}Au is 2.7 days.

(a) Disintegration constant = $\lambda = 0.693/T_{1/2}$

$$= 0.693/[2.7 \times 24 \times 60 \times 60]$$

$$= 2.97 \times 10^{-6} \text{ s}^{-1}$$

Let N be the number of atoms left undecayed. then

$$N = [1 \times 10^{-6} \times 6.023 \times 10^{23}] / 198$$

Activity, $A = \lambda N$

$$= 2.97 \times 10^{-6} \times [1 \times 10^{-6} \times 6.023 \times 10^{23}] / 198 \text{ Ci}$$

$$= 0.244 \text{ Ci}$$

(b) Using relation,

$$A/A_0 = (1/2)^N$$

where N = Number of half lives

A = Activity of the substance

A_0 = Initial activity

Here, $N = 2.592$

$$A = (1/2)^{2.592} \times 0.244 \text{ Ci}$$

$$\Rightarrow A = 0.0404 \text{ Ci}$$