Exercise Solutions

Question 1: Assume that the mass of a nucleus is approximately given by $M = Am_p$ where A is the mass number. Estimate the density of matter in kgm^{-3} inside a nucleus. What is the specific gravity of nuclear matter?

Solution:

Density = Mass/Volume ...(1)

Mass of nucleus(M) = Am_p (Given)

Mass of proton = m_p = 1.007276 μ

μ = atomic mass unit = 1.6605×10^{-27} kg

Now,
Radius of nucleus = $R = R_0 A^{(1/3)}$

$= 1.1 \times 10^{-15} A^{(1/3)}$ m

Volume = $(4/3)\pi R^3 = (4/3)\pi \times R_0^3 A^{(1/3)}]^3 = (4/3)\pi R_0^3 A$

(1)=> Density = $[A \times 1.007276 \times 1.6605 \times 10^{-27}]/[(4/3)\pi R_0^3 A]$

$= 3.00006 \times 10^{17}$ kg m^{-3}

Specific gravity of nuclear matter = Density/1000 = = 3.00006×10^{14}.

Question 2: A neutron star has a density equal to that of the nuclear matter. Assuming the star to be spherical, find the radius of a neutron star whose mass is 4.0×10^{30} kg (twice the mass of the sun).

Solution:
Mass of star(M) = 4×10^{30} kg (Given)
Density of nuclear matter(D) = 2.3×10^{17} kg/m^3

Volume = Mass/Density = $[4 \times 10^{30}]/[2.3 \times 10^{17}]$ m^3

$= [4 \times 10^{13}]/2.3$ m^3

Again, Volume = $(4/3)\pi R^3$

Where, R = Radius of the neutron star

=> $(4/3)\pi R^3 = 4x10^{13}]/2.3$

=> $R^3 = [3x10^{13}]/[2.3\ \pi]$

=> R = 16.07 km

Question 3: Calculate the mass of an α-particle. Its binding energy is 28.2 MeV.

Solution:
Binding energy = 28.2 MeV (given)

Mass of proton = 1.007276 u
Mass of neutron = 1.008665 u
Where, Atomic mass unit = u = $1.6605402 \times 10^{-27}$ kg

Now,
ΔM = (number of proton x mass of proton + number of neutron x mass of neutron) - M

Where, M = mass of an alpha particle

ΔM = (2 x 1.007276 u + 2 x 1.008665 u) - M ...(1)

Also, Binding energy = ΔMc^2

=> $\Delta Mc^2 = 28.2$

=> ΔM = 28.2/[931.5]

[Here c = 931.5 MeV/u]

=> ΔM = 0.030273 u ...(2)

From (1) and (2)

(2 x 1.007276 u + 2 x 1.008665 u) - M = 0.030273 u

M = 4.0016 u

Question 4: How much energy is released in the following reaction:
$^7Li + p \rightarrow \alpha + \alpha$.
Atomic mass of ^7Li = 7.0160 u and that of ^4He = 4.0026 u.

Solution:
Atomic mass of ^{7}Li = 7.0160 u
Atomic mass of ^{4}He = Mass of alpha particle = 4.0026 u
Mass of proton = 1.007276 u

Mass defect (ΔM) = Mass of reactants - Mass of products

= [7.0160 + 1.007276] - 2 x 4.0026

=> ΔM = 0.018076 u

Now, energy release, E = ΔMc^2 = 0.018076 x 931.5 = 16.83 MeV

[Here c = 931.5 MeV/u]

Question 5: Find the binding energy per nucleon of $^{197}_{79}Au$ if its atomic mass is 196.96 u.

Solution:
Atomic mass of Au = 196.96 u
Atomic Number of Au = Z = 79
Number of nucleons = A = 197
Number of neutrons = N = A - Z = 118

Now, Binding energy is

$$B = (Zm_p + Nm_n - M)c^2$$

Where, M = Atomic Mass
m_n = mass of neutron
m_p = mass of proton
c = Speed of light = 931.5 MeV/u

=> B = [(79 x 1.007276+118x1.008665)-196.96] x 931.5 = 1525.12 MeV

Therefore, Binding energy per nucleon = 1525.12/197 = 7.741

Question 6: (a) Calculate the energy released if ^{238}U emits an α-particle.
(b) Calculate the energy to be supplied to ^{238}U if two protons and two neutrons are to be emitted one by one. The atomic masses of ^{238}U, ^{234}Th and ^{4}He are 238.0508 u, 234.04363 u and 4.00260 u respectively.

Solution:
When ^{238}U emits an α-particle, the reaction is as follow,

$U^{238} \rightarrow Th^{234} + He^4_2$

So, energy released is the product of c^2 and subtraction of mass of reactant and mass of products.

Here c^2 = 931.5 MeV/u

Now,

=> $E = \Delta m c^2$

=> E = [238.0508 - (234.04363+4.00260)] x 931.5 = 4.2569 MeV

(b) When two protons and two neutrons are to be emitted one by one. The atomic masses of ^{238}U, ^{234}Th and ^4He are 238.0508 u, 234.04363 u and 4.00260 u respectively.

Reaction will be,

$U^{233} \rightarrow Th^{234} + 2n + 2p$

So, mass defect = $\Delta m = m(U^{233}) – m(Th^{234}) + 2(m_n) + 2(m_p)$

Where, m_n = mass of neutron = 1.008665 u and m_p = mass of proton = 1.007276 u

Δm = 238.0508 – [234.04363 + 2x1.008665 + 2x1.007276]

Δm = 0.024712 u

So, energy released = $E = \Delta m c^2$ = 0.024712 x 931.5 = 23.02 MeV

Question 7: Find the energy liberated in the reaction
$^{223}Ra \rightarrow {}^{209}Pb + {}^{14}C$.
The atomic masses needed are as follows.

^{223}Ra	^{209}Pb	^{14}C
223.018u	208.981u	14.003u

Solution:
Given reaction is: $^{223}Ra \rightarrow {}^{209}Pb + {}^{14}C$.

Energy = E = $[223.018u - (208.981u + 14.003u)]c^2$

= 0.034 x 931

= 31.65 MeV

Question 8: Show that the minimum energy needed to separate a proton from a nucleus with Z protons and N neutrons is
$\Delta E = (M_{z-1}, N + M_H - M_Z, N)\ c^2$
Where M_Z, N = mass of an atom with Z protons and N neutrons in the nucleus and M_H = mass of a hydrogen atom. This energy is known as proton-separation energy.

Solution:

As hydrogen contains only protons, so we can write energy equation as follows:

$E_{(Z,N)} \to E_{(Z-1,N)} + p_1$

=> $E_{(Z,N)} \to E_{(Z-1,N)} + {}^1H_1$

So, minimum energy needed to separate a proton from a nucleus with Z protons and N neutrons is
$\Delta E = (M_{z-1}, N + M_H - M_Z, N)\ c^2$

Question 9: Calculate the minimum energy needed to separate a neutron from a nucleus with Z protons and N neutrons in terms of the masses $M_{Z,\ N}$, $M_{Z,\ N-1}$ and the mass of the neutron.

Solution:
We know, energy released = Mass difference x c^2

The reaction is,
$E_{Z,N} = E_{Z,N-1} + {}^1n_0$

The minimum energy needed to separate the neutron will be,
$\Delta E = (M_{Z,N-1} + M_N - M_{Z,N}) \times c^2$

Question 10: ^{32}P beta-decays to ^{32}S. Find the sum of the energy of the antineutrino and the kinetic energy of the β-particle. Neglect the recoil of the daughter nucleus. Atomic mass of ^{32}P = 31.974 u and that of ^{32}S = 31.972 u.

Solution:
The reaction will be
$P^{32} \to S^{32} + {}_1V^0 + {}_{-1}\beta^0$

The sum of the energy of the antineutrino and the kinetic energy of the β-particle:

$E = m(P^{32}) – m(S^{32}) \times c^2$

= (31.974 – 31.972) x 931.5

= 1.863 MeV

Question 11: A free neutron beta-decays to a proton with a half-life of 14 minutes.
(a) What is the decay constant?
(b) Find the energy liberated in the process.

Solution:
(a) Decay constant:

Half-life of 14 minutes = 840 sec (Given)

We know, half life = ln(2)/λ

Where λ = decay constant

=> ln(2)/λ = 840
= $8.25 \times 10^{-4}\ s^{-1}$

(b) Find the energy liberated in the process:

$E = [m_n - (m_p + m_\beta)] \times c^2$

Where,
M_n = mass of Neutron = 1.008665 u
M_p = mass of proton = 1.007276 u
M_β = mass of β-particle = 0.0005486 u
c = Speed of light = 931.5 MeV/u

=> E = [1.008665 - (1.007276 + 0.0005486)] x 931.5

= 0.78283 MeV

= 782.83 KeV

Question 12: Complete the following decay schemes.

(a) $^{226}_{88}Ra \rightarrow \alpha$ +

(b) $^{19}_{8}O \rightarrow ^{19}_{9} F$ +

(c) $^{25}_{13}Al \rightarrow ^{25}_{12}Mg$ +

Solution:

(a) One α particle is produced so atomic number will decrease by 2 and the mass by 2,

Resultant reaction:

$$^{226}_{88}\text{Ra} \rightarrow ^{4}_{2}\alpha + ^{222}_{86}\text{Rn}$$

b) $^{19}_{8}O \rightarrow ^{19}_{9} F + \bar{e} + \bar{v}$

c) $^{25}_{13}\text{Ar} \rightarrow ^{25}_{12} \text{Mg} + e^{+} + v$

Question 13: In the decay 64Cu -> 64Ni + e^+ + v, the maximum kinetic energy carried by the positron is found to be 0.650 MeV.

(a) What is the energy of the neutrino which was emitted together with a positron of kinetic energy 0.150 MeV?

(b) What is the momentum of this neutrino in kg ms^{-1}? Use the formula applicable to a photon.

Solution:

(a) Energy the neutrino = 0.650 - K.E. of given positron

Maximum kinetic energy carried by the positron is found to be 0.650 MeV. (Given)

= 0.650 - 0.150

= 0.5 MeV

= 500 keV

(b) Momentum of this neutrino is

P = E/c = $[500 \times 1.6\times10^{-19}]/3\times10^{8} \times 10^{3}$

= 2.67×10^{22} Kg m/s

Question 14: Potassium -40 can decay in three modes. It can decay by β^--emission, β^+-emission or electron capture.
(a) Write the equations showing the end products.
(b) Find the Q-value in each of the three cases. Atomic masses of $^{40}Ar_{18}$, $^{40}K_{19}$ and $^{40}Ca_{20}$ are 39.9624 u, 39.9640 u and 39.9626 u respectively.

Solution:

(a) Decay of Potassium -40 by β^--emission, β^+-emission or electron capture:

$$_{19}K^{40} \rightarrow {}_{20}Ca^{40} + \beta^- + \bar{v}$$

$$_{19}K^{40} \rightarrow {}_{18}Ar^{40} + \beta^+ + v$$

And,

$$_{19}K^{40} + e^- \rightarrow {}_{18}Ar^{40} + v$$

(b) We know, Q- value = [mass of reactants - Mass of products]c^2
Where c = 931.5 MeV/u

Now,
Q-value in β^- decay = [39.9640 – 39.9626] x 931.5 = 1.30141 MeV
Q-value in β^+ decay = [39.9640 – (39.9626+2x0.005486)] x 931.5 = 0.4683 MeV
Q-value in electron capture = [39.9640 – 39.9624] x 931.5 = 1.490 MeV

Question 15: Lithium (Z = 3) has two stable isotopes ^6Li and ^7Li. When neutrons are bombarded on lithium sample, electrons and α-particles are ejected. Write down the nuclear processes taking place.

Solution:
Given: Lithium (Z = 3) has two stable isotopes ^6Li and ^7Li.

$$^{6}_{8}\mathrm{L\_i} + n \rightarrow {}^{7}_{3}L_i$$

$$^{8}_{3}L_i \rightarrow {}^{8}_{4}\mathrm{Be} + \bar{v} + e^-$$

$$^{8}_{4}\mathrm{Be} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} \rightarrow 2\ {}^{4}_{2}\mathrm{He}$$

Question 16: The masses of ^{11}C and ^{11}B are respectively 11.0114 u and 11.0093 u. Find the maximum energy a positron can have in the β^+-decay of ^{11}C to ^{11}B.

Solution:
The maximum energy for the positron in the β^+-decay = energy due to the mass defect (ΔM)

=> ΔM = [11.0114 – 11.0093] x 931 = 1.955 MeV

Question 17: ^{228}Th emits an alpha particle to reduce to ^{224}Ra. Calculate the kinetic energy of the alpha particle emitted in the following decay:

$^{228}Th \rightarrow ^{224}Ra^* + \alpha$
$^{224}Ra^* \rightarrow ^{224}Ra + \gamma$ (217 keV).
Atomic mass of ^{228}Th is 228.028726u, that of ^{224}Ra is 224.020196 u and that of $^4H_2$ is 4.00260 u.

Solution:
Mass of ^{224}Ra = 224.020196 x 931 + 0.217 = 208563.0195 MeV

K.E. of α particle = (mass of Th - mass of Ra - mass of alpha particle) x (speed of light)^2

= (228.028726 x 931) – (208563.0195 + 4.00260 x 931)

= 5.304 MeV

Question 18: Calculate the maximum kinetic energy of the beta particle emitted in the following decay scheme:

$^{12}N \rightarrow ^{12}C^* + e+ + v$
$^{12}C^* \rightarrow ^{12}C + \gamma$ (4.43 MeV).
The atomic mass of ^{12}N is 12.018613 u

Solution:

Given reaction are:
$^{12}N \rightarrow ^{12}C^* + e^+ + v$
$^{12}C^* \rightarrow ^{12}C + \gamma$ (4.43 MeV).

Adding both the reactions, we get

$^{12}N \rightarrow ^{12}C + e^+ + v + \gamma$ (4.43 MeV).

Now, find $K.E._{max}$

Max K.E of β-particle = [(mass of ^{12}N – mass of ^{12}C) × c^2] – 4.43Mev

= [12.018613 – 12] x 931 – 4.43

= 12.89 MeV

Question 19: The decay constant of $^{197}Hg_{80}$ (electron capture to $^{197}Au_{79}$) is $1.8 \times 10^{-4}\ s^{-1}$.
(a) What is the half-life? (b) What is the average-life? (c) How much time will it take to convert 25% of this isotope of mercury into gold?

Solution:
(a) Half life = ln(2)/[decay constant]

= ln(2)/[1.8 x 10^{-4}]

= 3850.81 sec

= 64 minutes (approx)

(b) Average life = 1/[decay constant]

=1/[1.8 x 10^{-4}]

= 5555.56 sec

= 92 minutes (approx.)

(c) Using relation,
$A/A_0 = (1/2)^N$

where N = Number of half lives
A= Activity of the substance
A_0 = Initial activity

As per question,

Present activity = (1 - 0.25) A_0 = 0.75 A_0

=> $(0.75A_0)/A_0 = (1/2)^N$

Taking "*ln*" both the sides,

ln 0.75 = N x *ln*(1/2)

=> N = 1598.23 sec

Question 20: The half-life of ^{198}Au is 2.7 days.
(a) Find the activity of a sample containing 1.00 µg of ^{198}Au.
(b) What will be the activity after 7 days? Take the atomic weight of ^{198}Au to 198 g mol^{-1}.

Solution:
The half-life of ^{198}Au is 2.7 days.

(a) Disintegration constant = $\lambda = 0.693/T_{1/2}$

= 0.693/[2.7x24x60x60]

= $2.97 \times 10^{-6}\ s^{-1}$

Let N be the number of atoms left undecayed. then

N = $[1\times10^{-6}\times6.023\times10^{23}]/198$

Activity, A = λN

= $2.97 \times 10^{-6} \times [1\times10^{-6}\times6.023\times10^{23}]/198$ Ci

= 0.244 Ci

(b) Using relation,
$A/A_0 = (1/2)^N$

where N = Number of half lives
A= Activity of the substance
A_0 = Initial activity

Here, N = 2.592

A = $(1/2)^{2.592}$ x 0.244 Ci

=>A = 0.0404 Ci