Exercise Solutions

Question 1: The guru of a yogi lives in a Himalayan cave, 1000 km away from the house of the yogi. The yogi claims that whenever he thinks about his guru, the guru immediately knows about it. Calculate the minimum possible time interval between the yogi thinking about the guru and the guru knowing about it.

Solution:
The maximum velocity that can be attained is speed of light.
Velocity = 3×10^8 m/sec

Distance = 1000 km = 10^6 m

We know, Time = Distance/velocity

=> Time = $10^6/3x10^8$ = 1/300 sec

Question 2: A suitcase kept on a shop's rack is measured 50 cm × 25 cm × 10 cm by the shop's owner. A traveler takes this suitcase in a train moving with velocity 0.6c. If the suitcase is placed with its length along the train's velocity, find the dimensions measured by
(a) the traveler and (b) a ground observer

Solution:
(a) Dimensions of the suitcase for the traveler will remain same because suitcase is in rest with respect to traveler.

(b) Here, the frame of reference of observer and the suitcase is not same. The suitcase is moving with the speed of 0.6c. Since, the suitcase is placed with its length along the train's velocity, so only change can be observed in the length, other dimensions remain same.

Let L' be the length observed by the observer on the ground.

=> $L' = L\sqrt{[1 - v^2/c^2]}$

=> $L' = 50\sqrt{[1-(0.6/c^2]} = 50 \times 0.8 = 40$ cm

Length observed is 40 cm, which is less than the original length of the suitcase.

Question 3: The length of a rod is exactly 1m when measured at rest. What will be its length when it moves at a speed of
(a) 3×10^5 m s^{-1}
(b) 3×10^6 m s^{-1} and
(c) 3×10^7 m s^{-1}?

Solution:
(a) length of the rod = 1m
Speed = 3×10^5 m s–1

New length L' is

$$L' = L\sqrt{1-\frac{v^2}{c^2}} = 1\sqrt{1-\frac{9 \times 10^{10}}{9 \times 10^{16}}} = \sqrt{1-10^{-6}}$$

The length observed = 0.99999995 m

(b) length = 1m and Speed = 3×106 m s^{-1}

=> L' = L√[1 - v^2/c^2]

= 1 √[1-(9x10^{12})/(9x10^{16}] = 0.99995 m

(c) length = 1m and Speed = 3×10^7 m s^{-1}

=> L' = L √[1 - v^2/c^2]

= 1 √[1-(9x10^{14})/(9x10^{16}] = 0.9949 m

Question 4: A person standing on a platform finds that a train moving with velocity 0.6c takes one second to pass by him. Find
(a) the length of the train as seen by the person and
(b) the rest length of the train.

Solution:

(a) Let l' be the length of the train as seen by the person.

L’ = vt = 0.6 x 3×10^8 = 1.8 x 10^8 m

(b) Rest length of the train be L, so using below relation

=> L' = L √[1 - v^2/c^2]

1.8 x 10^8 = L √[1-(0.6^2)/(9x10^{16}]

=>L = 1.8x10^8/0.8 = 2.25 x 10^8 m

Question 5: An Airplane travels over a rectangular field 100 m × 50m, parallel to its length. What should be the speed of the plane so that the field becomes square in the plane frame?

Solution:
An Airplane travels over a rectangular field 100 m × 50m, parallel to its length.
So its original length = L = 100 and contracted length = L' = 50

Let v be the speed of the airplane where field becomes square in the plane frame.
Find v:

we know, $L' = L\sqrt{[1 - v^2/c^2]}$

$\Rightarrow 50 = 100\sqrt{[1 - v^2/c^2]}$

$\Rightarrow (1/2)^2 = [1 - v^2/c^2]$

$\Rightarrow [1 - v^2/c^2] = ¼$

Or $v^2 = 3c^2/4$

Or $v = [\sqrt{3}\, c]/2 = 0.866\, c$

Question 6: The rest distance between Patna and Delhi is 1000 km. A nonstop train travels at 360 km h^{-1}.
(a) What is the distance between Patna and Delhi in the train frame?
(b) How much time elapses in the train frame between Patna and Delhi?

Solution:

The distance between Patna and Delhi = 1000km = 10^6 m

Speed of the train = 360 km h^{-1} = 100 m/s

(a)Distance between the two cities in the train frame L

we know, $L' = L\sqrt{[1 - v^2/c^2]}$

$\Rightarrow L' = 10^6\sqrt{[1 - 10^4/9x10^{16}]} = 10^9$

(b) let t be the actual time taken by the train

$t = L/v = 10^6/100 = 10^4$ s = 500/3 min

Change in time = $\Delta t = \Delta L/v = L'/v = [56x10^{-9}]/100$

$= 0.56 \times 10^{-9}$ s

= 0.56 ns

So, the time lapse in the train between Patna and Delhi will be 0.56 ns less than 10^4 s.

Question 7: A person travels by a car at a speed of 180 km h^{-1}. It takes exactly 10 hours by his wristwatch to go from the station A to the station B.
(a) What is the rest distance between the two stations?
(b) How much time is taken in the road frame by the car to go from the station A to the station B?

Solution:
Speed of the car = 180 km h^{-1}= 50 m/sec

Time taken by car to reach from station A to station B = 10 hours

(a) Let L' be the distance travelled by person in car.

This implies, L' = speed x time = 180x10 =1800 km

we know, $L' = L\sqrt{[1 - v^2/c^2]}$

Where, L is the rest distance between the two stations.

$1800 = L\sqrt{[1 – (180)^2/(9x10^{16})]}$

$\Rightarrow L = 1800 + 25x10^{-9}$
Rest distance is 25nm more than 1800 km.

(b) Time taken by car to cover the distance in the road frame = distance/ speed

$t = [1.8 \times 10^5 + 25x10^{-9}]/50$

$t = 0.36 \times 10^5 + 5 \times 10^{-8}$

t = 10 h + 0.5 nm

Question 8: A person travels on a spaceship moving at a speed of 5c/13.
(a) Find the time interval calculated by him between the consecutive birthday celebrations of his friend on the earth.

(b) Find the time interval calculated by the friend on the earth between the consecutive birthday celebrations of the traveller.

Solution:
Let t' be the time interval observed by the person in spacecraft.

$t' = t/\sqrt{(1-u^2/c^2)}$ .....(1)

Here, t = 1 year and

$(1-u^2/c^2) = 1 - 25/169 = 12/13$

(1)=>t = 13/12 yr

Thus, the time interval observed by the person between the consecutive birthday celebrations of his friend on earth will be more than one year i.e. 1.08 years (approx.)

(b) The person on Earth will also consider the same speed, so the time interval calculated by him is same.

Question 9: According to the station clocks, two babies are born at the same instant, one in Howrah and other in Delhi.
(a) Who is elder in the frame of 2301 UP Rajdhani Express going from Howrah to Delhi?
(b) Who is elder in the frame of 2302 Dn Rajdhani Express going from Delhi to Howrah.

Solution:

Both stations are in the floor frame. The station clocks therefore record the correct time interval. However, the clocks on the train record the wrong time because trains in different places and run at different speeds.

The proper time interval ΔT is less than improper i.e. $\Delta T' = v\Delta T$.

(a) In the frame of 2301 Up rajdhani express going from Howrah to Delhi, the baby born in Delhi will be elder.
(b) In the frame of 2302 Dn rajdhani express going from Delhi to Howrah, Howrah baby will be elder.

Question 10: Two babies are born in a moving train, one in the compartment adjacent to the engine and other in the compartment adjacent to the guard. According to the train frame, the babies are born at the same instant of time. Who is elder according to the ground frame?

Solution:
As the frame moves, the clocks do not record the synchronized time. The clock at the far end moves

the clock at the other end by Lv/c^2, where L is the rest separation between the clocks and v is the speed of the moving frame,(train). This means that the baby born next to the guard cell is older to the baby adjacent to the engine.

Question 11: Suppose Swarglok (heaven) is in constant motion at a speed of 0.9999c with respect to the earth. According to the earth's frame, how much time passes on the earth before one day passes on Swarglok?

Solution:
Given speed of Swarglok (heaven) = v = 0.9999c

Let ΔT' days pass on earth before one day passes on Swarglok(heaven)

$$\Delta T' = \frac{\Delta T}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{1}{\sqrt{1-\frac{(0.9999c)^2}{c^2}}} = \frac{1}{0.01414} = 70.712$$

Therefore, ΔT' = 70.71 days

Question 12: If a person lives on the average 100 years in his rest frame, how long does he live in the earth frame if he spends all his life on a spaceship going at 60% of the speed of light.

Solution:
Time in rest frame = T_0 = 100 years

Speed of Spaceship = v = 60/100 = 0.60 c

We know relation between time dilation for a moving object w.r.t. rest frame (Lorentz transformation] is v' T_0 …(1)

Where v' = velocity of moving object

$v' = 1/\sqrt{(1-v^2/c^2)}$

Given : v = 60% of c, V = (3/5)c

$(1-v^2/c^2) = 1 - (3/5)^2 = 16/25$

=> v' = 4/5

(1)=> v'T_o = 100 x 5/4 = 125

So, the person will live 125 years in earth frame.

Question 13: An electric bulb, connected to a make and break power supply, switches off and on every second in its rest frame. What is the frequency of its switching off and on as seen from a spaceship travelling at a speed 0.8c?

Solution:
Speed of spaceship = v = 0.8 c (given)

Proper frequency of bulb = f = 1 Hz

We know, f' = $f(1-v^2/c^2)$

= 1 x $[(1-0.64c^2)/c^2]$

= 0.6 Hz

Question 14: A person travelling by a car moving at 100 km h^{-1} finds that his wristwatch agrees with the clock on a tower A. By what amount will his wristwatch lag or lead the clock on another tower B, 1000 km (in the earth's frame) from the tower A when the car reaches there?

Solution:
Velocity of car = v = 100 km h^{-1}
Distance between tower A and B = 1000 km
And c = 3 x 10^8 m/s

Here, time passed in the tower clock = ΔT = velocity/distance = 1000/100 = 10 hr = 36000

So, time elapsed in the wristwatch = ΔT' = $\Delta T / \sqrt{[1-v^2/c^2]}$

= $[36000]/[\sqrt{(1-(1000/(36x3x10^8))^2)}]$

= 36000 + 0.154

= ΔT + 0.154

Or ΔT' - ΔT = 0.154

Therefore, time will lag by 0.154 ns.

Question 15: At what speed the volume of an object shrinks to half its rest value?

Solution: As per question, V' = V/2

We know, $V' = V\sqrt{[1-v^2/c^2]}$

$\Rightarrow V/2 = V\sqrt{[1-v^2/c^2]}$

$\Rightarrow c/2 = \sqrt{[c^2 - v^2]}$

$\Rightarrow c^2/4 = c^2 - v^2$

$\Rightarrow v = (\sqrt{3}/2)\, c$