Exercise Solutions

Question 1: An aluminum vessel of mass 0.5 kg contains 0.2 kg of water at 20°C. A block of iron of mass 0.2 kg at 100°C is gently put into the water. Find the equilibrium temperature of the mixture. Specific heat capacities of aluminum, iron and water are 910 J kg^{-1} K^{-1}; 470 J kg^{-1} and 4200 J kg^{-1} K^{-1} respectively.

Solution:
We know, ΔQ = mCΔT

where, ΔQ = heat exchange, ΔT = temperature change, M = mass and C = heat capacity

Form questions, we are given
Mass of aluminum = 0.5kg; mass of water=0.2kg

Mass of iron=0.2kg ; temperature of aluminum vessel and water=20^0c

Temp of iron=100^0c ; heat capacity of aluminum=910 J/kg-k and

heat capacity of iron=470J/kg-k ; heat capacity of water=4200J/kg-k

Now,
Heat gain by aluminum and water = 0.5 x 910 (T - 293) + 0.2 x 4200(T - 293)
Heat lost by iron = 0.2 x 470 x (373 - T)

As, heat gain by aluminum and water=heat loss by iron

=> 0.5 x 910 (T - 293) + 0.2 x 4200(T - 293) = 0.2 x 470 x (373 - T)

Solving above equation for T, we get

T = 298.41 K

Question 2: A piece of iron of mass 100 g is kept inside a furnace for a long time and then put in a calorimeter of water equivalent 10g containing 240 g of water at 20°C. The mixture attains an equilibrium temperature of 60°C. Find the temperature of the furnace. Specific heat capacity of iron = 470 J kg^{-1} $°C^{-1}$.

Solution:
We know, ΔQ = mCΔT

where, ΔQ=heat exchange, ΔT=temperature change, M=mass and C=heat capacity

Given
Mass of iron = 100g ; water equivalent of calorimeter = 10g

Mas of water = 240g ; temp of surface = 0^0c ; S_{iron} = 470J/kg-k

Now, Heat gained by water = heat lost by iron

=> (10/1000) x 470 x (T - 60) = (250/1000) x 4200 x (60-40)

Solving above equation for T, we get

T = 953.61 ^{0}C

Question 3: The temperatures of equal masses of three different liquids A, B and C are 12°C, 19°C and 28°C respectively. The temperature when A and B are mixed is 16°C, and when B and C are mixed, it is 23°C. What will be the temperature when A and C are mixed?

Solution:
From question,
Temp of A=12°C ; Temp of B=19° c and Temp of C=28° c

We know, ΔQ=mCΔT

where, ΔQ=heat exchange, ΔT=temperature change, M=mass and C=heat capacity

When A and B are mixed

M_A(16 - 12) = M_B(19 - 16)

M_B = ($4M_A$)/3

When B and C are mixed

$M_B(23 - 19) = M_C(28 - 23)$

$M_B = (5M_c)/4$

When A and C are mixed

$M_A(T - 12) = M_c(28 - T)$
Where "T" be the final temperature

Using vale of M_A and M_c

$(3/4)M_B(T - 12) = (4/5)M_B(28 - T)$

=> $T = 20.3^OC$

Question 4: Four 2 cm × 2 cm × 2 cm cubes of ice are taken out from a refrigerator and are put in 200 ml of a drink at 10°C.
(a) Find the temperature of the drink when thermal equilibrium is attained in it.
(b) If the ice cubes do not melt completely, find the amount melted. Assume that no heat is lost to the outside of the drink and that the container has negligible heat capacity. Density of ice = 900 kg m^{-3}, density of the drink = 4200 J kg^{-1} K^{-1}, latent heat of fusion of ice = 3.4×10^5 J kg^{-1}.

Solution:
Number of ice cubes = 4
Volume of each ice cube = $(2 \times 2 \times 2) = 8$ cm^3
Density of ice = 900 kg m^{-3}
Total mass of ice, $m_i = (4\times8 \times10^{-6} \times 900) = 288\times10^{-4}$ kg
Latent heat of fusion of ice, $L_i = 3.4 \times 10^5$ J kg^{-1}
Density of the drink = 1000 kg m^{-3}
Volume of the drink = 200 ml and
Mass of the drink = $(200\times10^{-6})\times1000$ kg

We know, $\Delta Q = mC\Delta T$

where, ΔQ=heat exchange, ΔT=temperature change, M=mass and C=heat capacity

(a)
Heat required to convert four 8 cm^3 ice cubes into water (H_i) = m_iL_i = $(288\times10^{-4}) \times (3.4\times10^5)$ = 9792 J

Heat released when temperature of 200 ml changes from 10°C to 0°C.
$H_w = (200\times10^{-6})\times1000\times4200\times(10-0) = 8400$ J

Since $H_i > H_w$, some ice will remain solid and there will be equilibrium between ice and water. Thus, the thermal equilibrium will be attained at 0°C.

(b) Let M be the mass of melted ice.

$M \times (3.4 \times 10^5) = 8400$

=> M = 0.0247 kg = 25 g (approx)

Question 5: Indian style of cooling drinking water is to keep it in a pitcher having porous walls. Water comes to the outer surface very slowly and evaporates. Most of the energy needed for evaporation is taken from the water itself and the water is cooled down. Assume that a pitcher contains 10 kg of water and 0.2 g of water comes out per second. Assuming no backward heat transfer from the atmosphere to the water, calculate the time in which the temperature decreases by 5°C. Specific heat capacity of water = 4200 J kg^{-1} $°C^{-1}$ and latent heat of vaporization of water = 2.27×10^6 J kg^{-1}.

Solution:
Heat lost per second from the atmosphere = Heat gained by the water

$(0.2\times 10^{-3})\times 2.27 \times 10^6 = 4200\times(10-0.2\times 10^{-3})\times \Delta T$

=> ΔT= 0.01080974°C

Therefore time required for 5° change = 5/(0.01080974) = 462.5 sec = 7.70 minutes

Question 6: A cube of iron (density = 8000 kg m^{-3}, specific heat capacity = 470 J kg^{-1} K^{-1}) is heated to a high temperature and is placed on a large block of ice at 0°C. The cube melts the ice below it, displaces the water and sinks. In the final equilibrium position, its upper surface just goes inside the ice. Calculate the initial temperature of the cube. Neglect any loss of heat outside the ice and the cube. The density of ice = 900 kg m^{-3} and the latent heat of fusion of ice = 3.36×10^5 J kg^{-1}.

Solution:

Heat lost by cube of iron = Heat gained by ice

$8000 \times V \times 470 \times T = 900 \times V \times 3.36 \times 10^5$

=> T = 80.42° C

Question 7: 1 kg of ice at 0°C is mixed with 1 kg of steam at 100°C. What will be the composition of the system when thermal equilibrium is reached? Latent heat of fusion of ice = 3.36×10^5 J kg^{-1} and latent heat of vaporization of water = 2.26×10^6 J kg^{-1}.

Solution:

Heat released when ice changed = $H_1 = 1 \times 3.36 \times 10^5$ J

Heat when temp of water changes from 0^oC to 100^oC = $H_2 = 1 \times 4200 \times 100 = 420000$ J

Total heat absorbed by the ice to raise the temp to 100^oC = $H = H_1 + H_2 = 756000$ J

let "m" be the mass changed into water

Heat = m x latent heat = 756000 J

=> $m = 756000 \times 2.26 \times 10^6 = 0.3345$

Total mass of water = 1 + 0.3345 = 1.3345 kg

So, mass of steam = 1 - 0.3345 = 0.66548 kg

Question 8: Calculate the time required to heat 20 kg of water from 10°C to 35°C using an immersion heater rated 1000 W. Assume that 80% of the power input is used to heat the water. Specific heat capacity of water = 4200 J kg^{-1} K^{-1}.

Solution:

Heat given by heater = 0.8 × 1000 = 800

Heat taken by water = 20 × 4200 × (35-10) = 21 x 10^5

Time = Heat/Power = [21 x 10^5]/800 = 2625 sec = 43.75 minutes

Question 9: On a winter day the temperature of the tap water is 20°C whereas the room temperature is 5°C. Water is stored in a tank of capacity 0.5 m^3 for household use. If it were possible to use the heat liberated by the water to lift a 10 kg mass vertically, how high can it be lifted as the water comes to the room temperature? Take g = 10 m s^{-2}.

Solution:

Heat liberated by water = Energy required to lift

=> mass x specific heat x change in temp = mgh

0.5 × 1000× 4200× (20-5) = 10 × 10 × h

=> h = 315000m = 315km

Question 10: A bullet of mass 20g enters into a fixed wooden block with a speed of 40 m s^{-1} and stops in it. Find the change in internal energy during the process.

Solution:
Mass of bullet=20g = 0.02 kg and Initial speed = 40 m/s

Total energy of bullet = (1/2)mv^2 = (1/2)(0.02) 40^2 = 16 J

Question 11: A 50 kg man is running at a speed of 18 km h^{-1}. If all the kinetic energy of the man can be used to increase the temperature of water from 20°C to 30°C, how much water can be heated with this energy?

Solution:
Mass of man = 50kg
Speed of man = 5m/s
Change in temp = $\Delta T = 10^oC$

Total energy = $(1/2)\ mv^2$

and we know, $\Delta Q = MC\Delta T$

Kinetic energy of person = heat absorbed by the water

=> $(1/2)\ mv^2 = MC\Delta T$

=> $(1/2) \times 50 \times 5^2 = M \times 4200 \times 10$

=> M = 0.01488 Kg = 14.88 gm

Question 12: A brick weighing 4.0 kg is dropped into a 1.0 m deep river from a height of 2.0 m. Assuming that 80% of the gravitational potential energy is finally converted into thermal energy, find this thermal energy in calorie.

Solution:
we know, potential energy = mgh = 4 × 10 × 3 = 120J
Given: m = 4 kg, h = 3m and g = 9.8 m/s

Now, energy converted to thermal energy = 0.8 × 120=96J
and, thermal energy in calories = 96/4.2 = 22.8 cal

Question 13: A van of mass 1500 kg travelling at a speed of 54 km h^{-1} is stopped in 10 s. Assuming that all the mechanical energy lost appears as thermal energy in the brake mechanism, find the average rate of production of thermal energy in cal s^{-1}.

Solution:
we know, Mechanical energy = kinetic energy = $(1/2)\ mv^2$
Given: mass of van = 1500 kg, Speed = 15m/s and Time taken to stop = 10 sec
Now,
Kinetic energy = $1/2 \times 1500 \times 15^2 = 168750$ J

And,

Average rate of production of thermal energy = Energy produced/time

= (168750/10) J/s

= (4017.8) cal/s

Question 14: A block of mass 100g slides on a rough horizontal surface. If the speed of the block decreases from 10 m s^{-1} to 5 m s^{-1}, find the thermal energy developed in the process.

Solution:
mass of block = 100g = 0.1kg (Given)

Change in kinetic energy = thermal energy developed ..(1)

Change in kinetic energy = $(1/2)mv_1^2 - (1/2)mv_2^2$

Given, v_1 = 10 m s^{-1} and v_2 = 5 m s^{-1}

=> Change in kinetic energy = $(1/2)\times 0.1\,[10^2 - 5^2]$ = 3.75 J

(1)=> thermal energy developed = 3.75 J

Question 15: Two blocks of masses 10 kg and 20 kg moving at speeds of 10 m s^{-1} and 20 m s^{-1} respectively in opposite directions, approach each other and collide. If the collision is completely inelastic, find the thermal energy developed in the process.

Solution:
Given: Mass of blocks = 10kg and 20kg and Speeds of blocks = 10m/s and 20m/s

We know, Momentum = mv

Applying conservation of momentum:

$m_2u_2 - m_1u_1 = (m_1 + m_2)v$

Where v = velocity of the blocks after collision.

Substituting the values, we get

=> 20x20 - 10x10 = (10+20)v

=> v = 10 m/s

Initial K.E. say K_i

$K_i = (1/2)\ m_1v_1^2 + (1/2)\ m_2v_2^2$

$= (1/2)\text{x } 10\text{x } 10^2 + (1/2)\text{x}20\text{x}20^2$

= 4500

Final K.E., K_f

$K_f = (1/2)(m_1 + m_2)v^2$

$= (1/2)(10+20)10^2 = 1500$

Total change in K.E. = 4500 - 1500 = 3000 J

therefore, thermal energy developed in the process is 3000 J

Question 16: A ball is dropped on a floor from a height of 2.0 m. After the collision it rises up to a height of 1.5 m. Assume that 40% of the mechanical energy lost goes as thermal energy into the ball. Calculate the rise in the temperature of the ball in the collision. Heat capacity of the ball is 800 J K^{-1}.

Solution:

Heat capacity of ball=800 J/K (Given)

Let "m" kg be the mass of ball.
Let v_1 and v_2 be the speed of the ball when it falls from the height h_1 and h_2

$v_1^2 = 2gh_1 = 40$ m/s

$v_2^2 = 2gh_2 = 30$ m/s

Change in K.E. = ΔK = (1/2)m x 40 - (1/2)m x 30 = (10/2)m = 5 m

The change in K.E. is utilised in increasing the temp of the ball as loss in P.E. is zero as PE of the ball is same at the position just before hitting the ground and after its first collision.

The change in K.E. is utilised in increasing the temp of the ball.

=> (40/100) ΔK = m x 800 x ΔT
Where ΔT is the change in temp.

Substituting the value of ΔK, we get

ΔT = 0.0025 = 2.5×10^{-3} °C

Question 17: A copper cube of mass 200 g slides down on a rough inclined plane of inclination 37° at a constant speed. Assume that any loss in mechanical energy goes into the copper block as thermal energy. Find the increase in the temperature of the block as it slides down through 60 cm. Specific heat capacity of copper = 420 J kg^{-1} K^{-1}.

Solution:
Heat capacity of copper=420J/Kg/K

copper cube of mass = m = 200 g = 0.2 kg
Length through which the block has slided = l = 60 cm = 0.6 m

We know, force of friction, f = mg

Loss in mechanical energy of the copper block = work done by the frictional force on the copper block to a distance of 60 cm

W = mgl sin θ = 0.2 x 10 x 0.6 sin 37° = 0.72

Let ΔT be the temperature change of the block,

Thermal energy = ms ΔT = 0.2 x 420 x ΔT = 84 ΔT

[But 84 ΔT = 0.72]

=> ΔT = 0.72/84 = 0.00857 = 8.57 x 10^{-3} ^{o}C

Question 18: A metal block of density 6000 kg m^{-3} and mass 1.2 kg is suspended through a spring of spring constant 200 N m^{-1}. The spring-block system is dipped in water kept in a vessel. The water has a mass of 260 g and the bock is at a height 40 cm above the bottom of the vessel. If the support to the spring is broken, what will be the rise in the temperature of the water. Specific heat capacity of the block is 250 J kg^{-1} K^{-1} and that of water is 4200 J kg^{-1} K^{-1}. Heat capacities of the vessel and the spring are negligible.

Solution:
Density of metal block = d = 6000 kg m^{-3}
Mass of metal block = m = 1.2 kg
and Spring constant of the spring = k = 200 N m^{-1}

Volume of the block, say V = 1.2/6000 = 2 x 10^{-4} m^3

When the mass is dipped in water, it experiences a buoyant force and in the spring there is potential energy stored in it.

Net force on the block is zero before breaking of the support of the spring.

So balancing forces:

kx + Vρg = mg

=> (200x) + (2 × 10−4)× (1000) × (10) = 12

=> x = (12-2)/200 = 10/200 = 0.05 m

The mechanical energy of the block is transferred to both block and water. ΔT be the rise in temperature of the block and the water.

Applying conservation of energy, we get

$(1/2)kx^2 + mgh - V\rho gh = m_1 s_1 \Delta T + m_2 s_2 \Delta T$

=> $1/2 \times 200 \times 0.0025 + 1.2 \times 10 \times 40 \times 10\text{-}2 - 2 \times 10^{-4} \times 1000 \times 10 \times 40 \times 10^{-2} = (260/1000) \times 4200 \times \Delta T + 1.2 \times 250 \times \Delta T$

=> $0.25 + 4.8 - 0.8 = 1092\ \Delta T + 300\Delta T$

=> $\Delta T = 4.25/1392 = 0.0030531 = 3 \times 10^{-3}$ °C