Exercise Solutions

Question 1: A thermally insulated closed copper vessel contains water at 15°C. When the vessel is shaken vigorously for 15 minutes, the temperature rises to 17°C. The mass of the vessel is 100 g and that of the water is 200 g. The specific heat capacities of copper and water are 420 J kg^{-1} K^{-1} and 4200 J kg^{-1} K^{-1} respectively.
Neglect any thermal expansion.
(a) How much heat is transferred to the liquid-vessel system?
(b) How much work has been done on this system?
(c) How much is the increase in internal energy of the system?

Solution:
Let T_1 and T_2 be Initial and Final temperature of water respectively.

m_c and m_w be Mass of copper vessel and water respectively.

c_c and c_w be Specific heat capacity of copper and water respectively.

Given:
T_1 = 15°C = 288K ; T_2 = 17°C = 290K

m_c = 100g = 100×10^{-3}kg ; mw = 200g = 200×10^{-3}kg

c_c = 420 J kg^{-1} K^{-1} ; cw = 4200 J kg^{-1} K^{-1}

(a) No heat from the surroundings can be transferred to the liquid-vessel system, as copper vessel is thermally insulated.

b) Work done on this system:

Using formula, $\Delta W = m_w c_w \Delta T + m_c c_c \Delta T$

Temp change = $\Delta T = T_2 - T_1$ = 290-288 = 2K

=> $\Delta W = 200 \times 10^{-3} \times 4200 \times 2 + 100 \times 10^{-3} \times 420 \times 2 = 1764$ J

c)
We know, $\Delta Q = \Delta U + \Delta W$ ...(1)
[first law of thermodynamics]

Where ΔQ = heat supplied to the system, ΔU = change in internal energy and ΔW = work done by the system

From part (a), we have ΔQ = 0

(1)=>ΔU = -ΔW

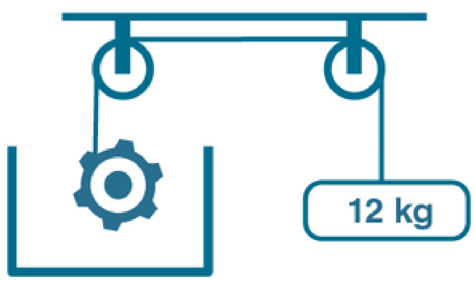
From part (2), ΔW = 1764 J

So, work done by the system ΔW = -1764J

=> ΔU = -(-1764) = 1764J

Increase in internal energy of the system is 1764J.

Question 2: Figure shows a paddle wheel coupled to a mass of 12 kg through fixed frictionless pulleys. The paddle is immersed in a liquid of heat capacity 4200 JK^{-1} kept in an adiabatic container. Consider a time interval in which the 12 kg block falls slowly through 70 cm.
(a) How much heat is given to the liquid?
(b) How much work is done on the liquid?
(c) Calculate the rise in the temperature of the liquid neglecting the heat capacity of the container and the paddle.



Solution:
(a) Heat given to liquid is zero.
As, Paddle immersed in liquid is kept in an adiabatic container. So, no heat can be either supplied or extracted to the liquid.

(b) Work done on the liquid = potential energy of mass
[As, no heat is supplied to liquid and pulley is frictionless]

We know, Potential energy of mass = mgh

Where, m = Mass attaches to the pulley = 12kg (Given)
and h = Height through which mass fall = 70cm = 0.7m (Given)
g = acceleration due to gravity = 10 m/s^2

Potential energy of mass = 12×10×0.7 = 84J

=> Work done on the liquid = 84 J

(c) We know, s = ΔQ/ΔT

Where ΔQ = heat supplied and ΔT = rise in temperature

ΔT = ΔQ/s

s = Heat capacity of liquids = 4200 JK^{-1}

Since work done is equal to heat supplied, therefore

ΔT = 84/4200 = 0.02 K

Thus, rise in temperature of the liquid will be 0.02K.

Question 3: A 100 kg block is started with a speed of 2.0 m s^{-1} on a long, rough belt kept fixed in a horizontal position. The coefficient of kinetic friction between the block and the belt is 0.20.
(a) Calculate the change in the internal energy of the block-belt system as the block comes to a stop on the belt.
(b) Consider the situation com a frame of reference moving at 2.0 m s^{-1} along the initial velocity of the block. As seen from this frame, the block is gently put on a moving belt and in due time the block starts moving with the belt at 2.0 ms^{-1}. Calculate the increase in the kinetic energy of the block as it stops slipping past the belt.
(c) Find the work done in this frame by the external force holding the belt.

Solution:
(a) Since the block comes to stop on the belt, final velocity will be zero, i.e. V = 0 m/s

we know that heat is produced due to friction between two surfaces. Now because of this heat, the internal energy of block will change.

Kinetic energy lost = initial kinetic energy- final kinetic energy

= (1/2) mu^2 - (1/2) mv^2

Here m = 100 kg and u (initial velocity) = 2 m/s and v (final velocity) = 0 m/s [given]

Kinetic energy lost = 200 J

In this frame of reference initial and final velocity of the block will change.

velocity of the frame of reference = u_o = 2m/s (Given)

New initial velocity = $u' = u - u_o$ = 2-2 = 0 m/s
New final velocity $v' = v - u_o$ = 0 - 2 = -2 m/s

Increase in kinetic energy = Final kinetic energy – Initial kinetic energy

$= (1/2)\ mv'^2 - (1/2)\ mu'^2$

$= (1/2) \times 100 \times (-2)^2 - 0$

= 200 J

(c) work done in this frame by the external force holding the belt.

We know, $f = \mu N$

Where, μ = coefficient of friction =0 .02 and
N = normal reaction = mg

=> f = 0.02 x 100 x 10 = 200N

Also, force= mass × acceleration [newton's second law of motion]

=> 200 = 100 x acceleration

=> acceleration = 2 m/s^2

Again, by third equation of motion, we have

$v'^2 - u'^2 = 2as$

=> $(-2)^2 - 0 = 2x2xs$

=> s = 1 m

Work done due to friction W_f = Force × Displacement

$W_f = 200 \times 1 = 200$ J

According to work-energy theorem, Work done = change in kinetic energy

$W' = (1/2)mv'^2 - (1/2)mu'^2$

$= (1/2) \times 100 \times (-2)^2 - 0$

= 200 J

Therefore, Total work done = work done in com frame of reference = $W = W' + W_f = 200 + 200 = 400J$.

Question 4: Calculate the change in internal energy of a gas kept in a rigid container when 100 J of heat is supplied to it.

Solution:
Given: $\Delta Q = 100J$

From first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$ ...(1)

Work done by the gas, $\Delta W = P \Delta V$

Since the gas is kept in a rigid container, so $\Delta V = 0$

=> $\Delta W = 0$

(1)=> $\Delta Q = \Delta U = 100$ J

change in internal energy will be 100 J.

Question 5: The pressure of gas changes linearly with volume from 10 kPa, 200 cc to 50 kPa, 50 cc.
(a) Calculate the work done by the gas.
(b) If no heat is supplied or extracted from the gas, what is the change in the internal energy of the gas?

Solution:
Given:
$P_1 = 10$ k Pa $= 10 \times 10^3$ Pa ; $P_2 = 50$ kPa $= 50 \times 10^3$ Pa
$V_1 = 200cc = 200 \times 10^{-6}$ m^3 ; $V_2 = 50cc = 50 \times 10^{-6}$ m^3

Where P_1 and P_2 be initial and final pressure
v_1 and v_2 be the Initial and final volume

(a) work done by the gas = $\Delta W = P \Delta V$
As, two values of pressure. So, we will take the average value of pressure

$P = [P_1 + P_2]/2 = (1/2)[10 + 50) \times 10^3 = 30 \times 10^3$ Pa

$\Delta V = V_2 - V_1 = (50 - 200) \times 10^{-6}$

work done on the gas = $30 \times 10^3 \times (50 - 200) \times 10^{-6} = -4.5$ J

(b) From first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$

Since $\Delta Q = 0$

Therefore $\Delta U = -\Delta W = -(-4.5)$ J = 4.5J

Question 6: An ideal gas is taken from an initial state i to a final state f in such a way that the ratio of the pressure to the absolute temperature remains constant. What will be the work done by gas?

Solution:

$P_1/T_1 = P_2/T_2$ (Given)

Where P_1 = initial pressure and P_2 = final pressure

T_1 and T_2 are absolute temp.,

Applying ideal gas equation for both processes,(PV = nRT) we get

$nR/V_1 = nR/V_2$

=> $V_1 = V_2$ ....(1)

We know, $\Delta W = P\Delta V$

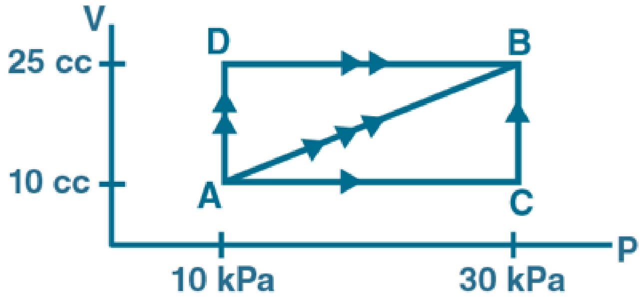
change in the volume of system = $\Delta V = V_2 - V_1 = 0$ [Using (1)]

=> $\Delta W = 0$

work done by the gas is zero.

Question 7: Figure shows three paths through which a gas can be taken from the state A to the state B. Calculate the work done by the gas in each of the three paths.

Solution:



We know that work done by the gas: $\Delta W = P\Delta V$

Work done in path ADB:
$W_{ADB} = W_{AD} + W_{DB}$

$W_{ADB} = P_A (V_D - V_A) + 0$
[Because V_D-V_B, so W_{DB} =0)

From graph,

$= 10 \times 10^3 \times (25 - 10) \times 10^{-6}$

= 0.15J

Work done in path AB:

$W_{AB} = P_{avg}(V_B - V_A)$

Here, $P_{avg} = [P_B + P_A]/2 = [10x10^3 + 30x10^3]/2 = 20 \times 10^3$ Pa

=> $W_{AB} = 20 \times 10^3 (25 - 10)10^{-6} = 0.30$ J

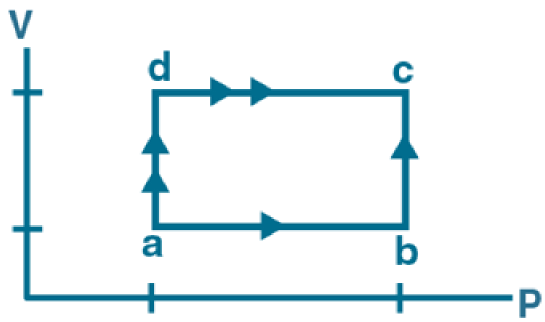
Work done in path ACB:

$W_{ACB} = W_{AC} + W_{BC}$
Here $W_{AC} = 0$ because $V_A=V_C$

$W_{ACB} = 0 + P_B(V_B - V_C)$

$= 30\times10^3\times(25-10)\times10^{-6}$
=0.45J

Question 8: When a system is taken through the process abc shown in figure shown below of heat is absorbed by the system and 30 J of work is done by it. If the system does 10 J of work during the process adc, how much heat flows into it during the process?



Solution:
Given: $\Delta Q_1 = 80J$; $\Delta W_1 = 30J$; $\Delta W_2 = 10J$

Let heat absorbed into the system during process adc = ΔQ_2

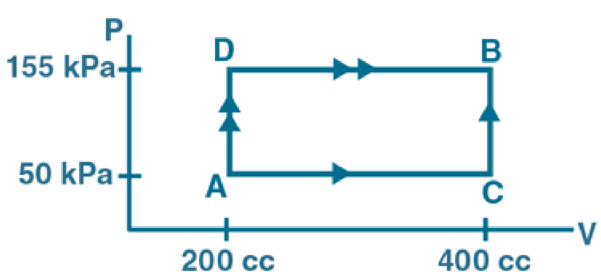
Using first law of thermodynamics for process abc

$\Delta Q_1 = \Delta U_1 + \Delta W_1$ and $\Delta Q_2 = \Delta U_2 + \Delta W_2$

$\Delta U_1 = \Delta Q_1 - \Delta W_1 = 80 - 30 = 50J$

and
$\Delta Q_2 = \Delta U_2 + \Delta W_2 = \Delta U_1 + \Delta W_2 = 50 + 10 = 60\ J$

Question 9: 50 cal of heat should be supplied to take a system from the state A to the state B through the path ACB as shown in the figure. Find the quantity of heat to be supplied to take it from A to B via ADB.



Solution:
Heat absorbed in process ABC = $\Delta Q_1 = 50cal = 50\times 4.2\ J = 210J$

Let heat absorbed into the system during process ADC = ΔQ_2

We know that work done by the gas: $\Delta W = P\Delta V$

Work done in path ACB:

$W_{ACB} = \Delta W_1 = W_{AC} + W_{BC}$

$=> \Delta W_1 = P_A(V_C - V_A) + 0$
(In figure, $V_B = V_C => W_{BC} = 0$)

$= 50 \times 10^3 \times (400-200) \times 10^{-6}$

= 10J

Work done in path ADB:

$W_{ADB} = \Delta W_2 = W_{AD} + W_{DB}$

$\Delta W_2 = P_B$ (VD-VB) + 0
(In figure, $V_A = V_D => W_{AD}=0$)

$=155 \times 10^3 \times (400-200) \times 10^{-6}$

=31 J

From first law of thermodynamics,
For process ABC

$\Delta Q_1 = \Delta U_1 + \Delta W_1$

Or ΔU_1 = 2 10 – 10 = 200J

For process ADC

$\Delta Q_2 = \Delta U_2 + \Delta W_2$

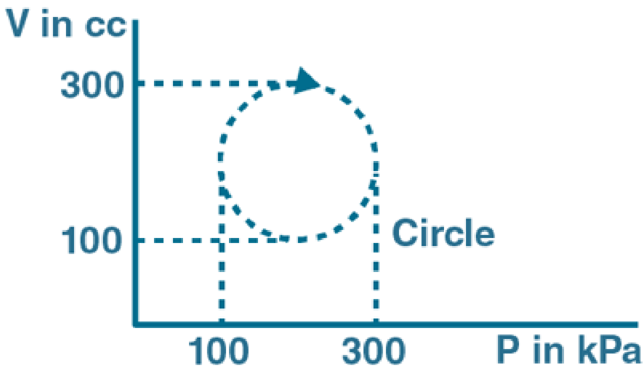
$= \Delta U_1 + \Delta W_2$

[$\Delta U_1 = \Delta U_2 = \Delta U$ as, initial point and final point for both the processes is the same. So, change in internal energy will be the same.]

= 200 + 31 = 231 J.

Heat supplied to the system during process ADC is 231 J.

Question 10: Calculate the heat absorbed by a system in going through the cyclic process shown in the figure below.



Solution:
In the given case heat absorbed = Area of the circle

and, ΔQ = ΔW = area of circle

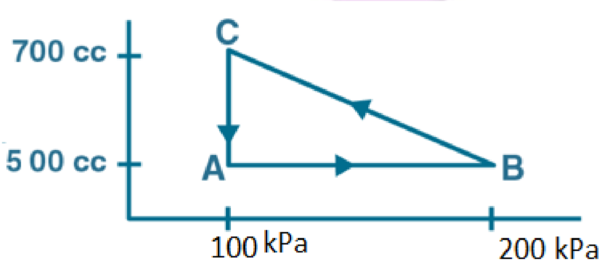
Diameter of the circle =300-100=200, so radius = 100

Area of circle = $\pi \times 100 \times 100 \times 10^{-6} \times 10^{3} = 31.4$

[$10^{-6} \times 10^{3}$ added because volume and pressure are given in cc and kPa respectively)

Thus, Heat absorbed by a system = 31.4 J.

Question 11: A gas is taken through a cyclic process ABCA as shown in the figure. If 2.4 cal of heat is given in the process, what is the value of J?



Solution:
work done by the gas: $\Delta W = P\Delta V$

Work done in path AB = 0 as $V_A = V_B$.
Work done in path CA = $\Delta W_1 = P_A(V_A - V_C) = 100 \times 10^{3} \times (500-700) \times 10^{-6} = -20$J

Work done in path BC = $\Delta W_2 = P_{avg}(V_C - V_B)$ ...(1)

$P_{avg} = [P_B + P_C]/2 = [100x10^3+200x10^3]/2 = 150 \times 10^3$ Pa

(1)=> $\Delta W_2 = 150 \times 10^3 (700-500)10^{-6} = 30$ J

Total work done in process ABCA = $\Delta W = \Delta W_1 + \Delta W_2 = 30 - 20 = 10$ J

From first law of thermodynamics,
$\Delta Q = \Delta U + \Delta W$

In cyclic process, $\Delta U = 0$

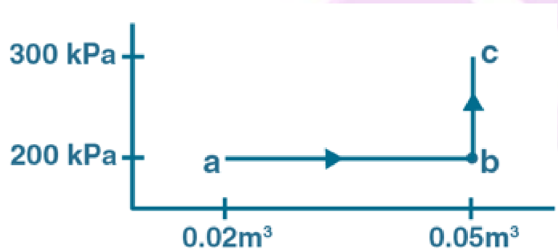
=> $\Delta Q = \Delta W$

$\Delta Q = 2.4$ cal (Given)

2.4 x J = 10 Joule

J = 10/2.4 = 4.17 Joule/cal

Question 12: A substance is taken through the process abc as shown in the figure. If the internal energy of the substance increases by 5000 J and heat of 2625 cal is given to the system, calculate the value of J.



Solution:
Work done in process abc = $\Delta W = W_{ab} + W_{bc}$

$\Delta W = P_a(V_b - V_a) + 0$

(As, $V_b = V_c$ => $W_{bc} = 0$)

$\Delta W = 200 \times 10_3 \times (0.05-0.02) = 6000$J

From first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$

Change in internal energy= $\Delta U = 5000$J (Given)

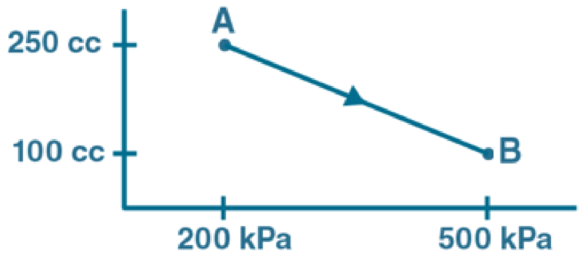
=> ΔQ = 5000J + 6000J = 11000 J

But Heat given to system = 2625cal = 2625×J J (Given)

=> 2625×J = 11000

or J = 4.19 Joule/cal

Question 13: A gas is taken along the path AB as shown in the figure. If 70 cal of heat is extracted from the gas in the process, calculate the change in the internal energy of the system.



Solution:

ΔQ = -70 cal = -70×4.2 = -294J (Given)

and, $\Delta W = P\Delta V$

Where $P = P_{avg} = [P_A+P_B]/2 = [200\text{x}10^3+500\text{x}10^3]/2 = 350\text{x}10^3$ Pa

$\Delta W = 350 \times 10^3 \times (250 - 100) \times 10^{-6}$

= -52.5 J

From first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$

=> ΔU = -294 - (-52.5) = -241.5 J

Question 14: The internal energy of a gas is given by U = 1.5 pV. It expands from 100 cm3 to 200 cm^3 against a constant pressure of 1.0×10^5 Pa. Calculate the heat absorbed by the gas in the process.

Solution:
Change in volume $\Delta V = (200-100)\times10^{-6}\ m^3 = 10^{-4}\ m^3$

change in internal energy $\Delta U = 1.5\ p\Delta V = 1.5\times1.0\times10^5\times10^{-4}$ = 15J

work done by the gas: $\Delta W = p\Delta V = 1.0\times10^5\times10^{-4}$ = 10J

From first law of thermodynamics, $\Delta Q = \Delta U + \Delta W = 15 + 10 = 25$ J

Question 15: A gas is enclosed in a cylindrical vessel fitted with a frictionless piston. The gas is slowly heated for some time. During the process, 10 J of heat is supplied, and the piston is found to move out 10 cm. Find the increase in the internal energy of the gas. The area of cross section of the cylinder = 4 cm2 and the atmospheric pressure = 100 kPa.

Solution:
Heat supplied to system $= \Delta Q = 10$J

Atmospheric pressure $= P = 100$ kPa $= 100 \times 10^3$ Pa

Displacement of the piston d = 10 cm

Area of cross section of cylinder = A = 4 cm^2

Now, $\Delta V = A \times 10\ cm^3 = 4 \times 10\ cm^3 = 40 \times 10^{-6}\ m^3$

$\Delta W = P\ \Delta v = 100 \times 10^3 \times 40 \times 10^{-6} = 4\ cm^3$

Now, $10 = \Delta U + \Delta W$

$\Rightarrow \Delta U = 6$J

Question 16: A gas is initially at a pressure of 100 kPa and its volume is 2.0 m^3. Its pressure is kept constant and the volume is changed from 2.0 m^3 to 2.5 m^3. Its volume is now kept constant and the pressure is increased from 100 kPa to 200 kPa. The gas is brought back to its initial state, the pressure varying linearly with its volume.
(a) Whether the heat is supplied to or extracted from the gas in the complete cycle?
(b) How much heat was supplied or extracted?

Solution:
From graph
$V_a = 2m^3$; $V_b = V_c = 2.5m^3$; $P_a = P_b = 100kPa = 100 \times 10^3$ Pa

$P_c = 200$ kPa $= 200 \times 10^3$ Pa

Work done in process ABCA = area enclosed by the triangle ABC

$\Rightarrow \Delta W = 0.5 \times BC \times AB$

$= 0.5 \times (200-100) \times 10^3 \times (2.5-2)$

= 25000 J

(a) From first law of thermodynamics: ΔQ = ΔU + ΔW

Here ΔU = 0, as Process ABCA is a cyclic process => change in internal energy will be zero.

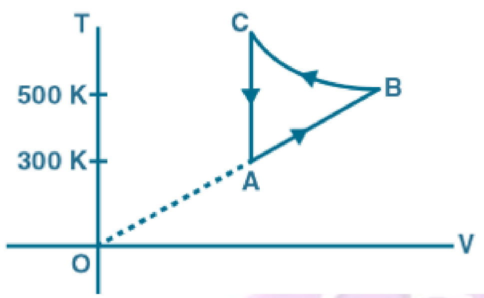
=> ΔQ = ΔW =25000 J

the heat is extracted from the system.

(b) Amount of heat supplied = work done by the gas in the cyclic process

i.e. ΔQ = ΔW = 25000J.

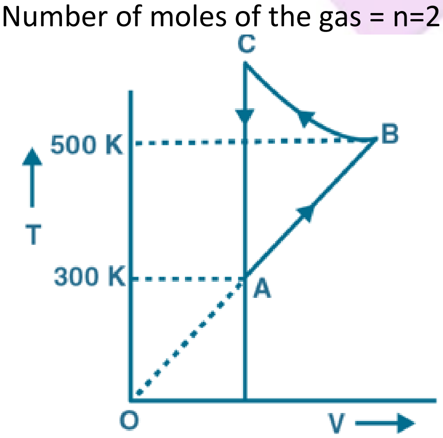
Question 17: Consider the cyclic process ABCA, shown in the figure, performed on a sample 2.0 mol of an ideal gas. A total of 1200 J of heat is withdrawn from the sample in the process. Find the work done by the gas during the part BC.



Solution:

Heat extracted from the system = ΔQ = -1200J (given)

Number of moles of the gas = n=2.0



We know, ΔW = P ΔV

Since $V_A = V_C$ => work done along line CA will be zero.

Total work done = $\Delta W = W_{AB} + W_{BC} = P(V_B - V_A) + W_{BC}$

But we know that ideal gas equation, PV = nRT

Therefore, $P\Delta V = nR\Delta T$

=> $\Delta W = nR(T_B - T_A) + W_{BC}$ ...(1)

From first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$

Since Process ABCA is a cyclic process, $\Delta U = 0$

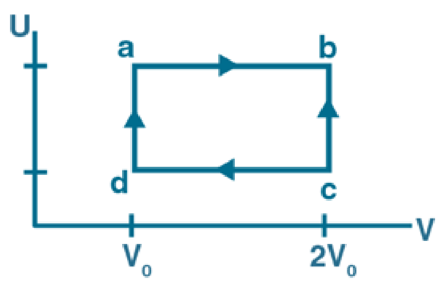
=> $\Delta Q = \Delta W = nR(T_B - T_A) + W_{BC}$

[Using (1)]

$-1200 = 2 \times 8.31(500 - 300) + W_{BC}$

=> $W_{BC} = -4524$ J

Question 18: Figure shows the variation in the internal energy U with the volume V of 2.0 mol of an ideal gas in a cyclic process abcda. The temperatures of the gas at b and c are 500 K and 300 K respectively. Calculate the heat absorbed by the gas during the process.



Solution:
Here n = 2 moles

From figure, $\Delta V = 0$ for path bc and da.

Hence $\Delta Q = \Delta W$

We know that, work done in an isothermal process: $W = nRT \ln[V_f/V_i]$

=> $\Delta W = \Delta W_{ab} + \Delta W_{cd}$

$= nRT_a \ln [V_b/V_a] + nRT_c \ln [V_d/V_c]$

$= 2 \times 8.31 \times 500 \times \ln(2V_0/V_0) + 2 \times 8.31 \times 300 \times \ln(V_0/2V_0)$

$= 8310 \times \ln(2) - 4986 \times \ln(2)$

= 2304.02 J

Question 19: Find the change in the internal energy of 2 kg of water as it is heated from 0°C to 4°C. The specific heat capacity of water is 4200 J kg^{-1} K^{-1} and its densities at 0°C and 4°C are 999.9 kg m^{-3} and 1000 kg m^{-3} respectively. Atmospheric pressure = 10^5 Pa.

Solution:

m = 2kg, ΔT = 4°C -0°C =4°C

c = 4200 J $kg^{-1}K^{-1}$

Density of water at 0°C = 999.9 kg m^{-3}

Density of water at 4°C=1000 kg m^{-3}

We know that specific heat capacity: $c = \Delta Q/m\Delta T$

=> $\Delta Q = cm \Delta T$

= 4200 x 2 x 4 = 33600J

Volume at 0°C:

V_1 = m/[density at 0°C] = 2/999.9 m^3

Volume at 4°C:

V_2 = m/[density at 4°C] = 2/1000 m^3

Work done by the gas: $\Delta W = P\Delta V = P(V_2 - V_1)$

$= 10^5 \times (2/1000 - 2/999.9)$

= -0.02 J

From first law of thermodynamics: $\Delta U = \Delta Q - \Delta W$

= 33600 - (-0.02)

= 33599.98J

change in internal energy is 33599.98J.

Question 20: Calculate the increase in the internal energy of 10g of water when it is heated from 0°C to 100°C and converted into steam at 100 kPa. The density of steam = 0.6 kg m^{-3}. The specific heat capacity of water = 4200 J kg^{-1} $°C^{-1}$ and the latent heat of vaporization of water = 2.25×10^6 J kg^{-1}.

Solution:

Pressure = P = 100kPa = 100×10^5 Pa
Change in temperature = ΔT = (100-0) °C = 100°C
Density of water ρ = 1000 kg m^{-3}

specific heat capacity: $c = \Delta Q/m\Delta T$

=> $\Delta Q = cm\,\Delta T$

In this question,

=> $\Delta Q = mL + cm\Delta T$ ...(1)

Where,

Mass of water m=10g =0.010kg
latent heat of vaporization of water= L = 2.25×10^6 J kg^{-1}.

c = 4200 J kg^{-1} $°C^{-1}$

Putting all the values in (1), we get

ΔQ = 26700J

Now, work done by the gas: $\Delta W = P\Delta V$

here $\Delta V = (m/\rho' - m/\rho)$

=> $\Delta W = 10^5$ x [0.01/0.06 - 0.01/1000]

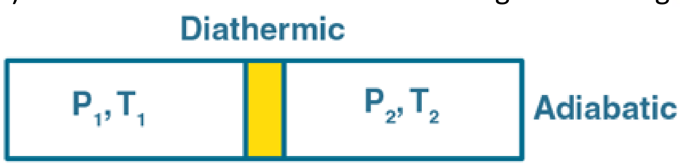
= 1699J

By first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$

= 26700 - 1699

=> $\Delta Q = 25001J$

Question 21: Figure shows a cylindrical tube of volume V with adiabatic walls containing an ideal gas. The internal energy of this ideal gas is given by 1.5 nRT. The tube is divided into two equal parts by a fixed diathermic wall. Initially, the pressure and the temperature are p_1, T_1 on the left and p_2, T_2 on the right. The system is left for sufficient time so that the temperature becomes equal on the two sides.
(a) How much work has been done by the gas on the left part?
(b) Find the final pressures on the two sides.
(c) Find the final equilibrium temperature.
(d) How much heat has flown from the gas on the right to the gas on the left?



Solution:

(a) Since all wall can not be moved thus $\Delta U = 0$ and $\Delta Q = 0$

=> $\Delta W = 0$

(b) Applying ideal gas equation in the left chamber before and after equilibrium
$P_1V/2 = nRT_1$ and $(P_1' V)/2 = nRT$

=> $P_1/T_1 = P_1' V/T$

=> $P_1' = P_1T/T_1$

using value of T (Solved in part (c))

$$P'_1 = \frac{P_1}{T_1} \times \frac{(T_1T_2) \times (P_1 + P_2)}{T_2P_1 + P_2T_1}$$

Where,

$$T = \frac{(T_1T_2) \times (P_1 + P_2)}{T_2P_1 + P_2T_1}$$

Similarly, P'_2 is

$$P'_2 = \frac{P_2}{T_2} \times \frac{(T_1T_2) \times (P_1 + P_2)}{T_2P_1 + P_2T_1}$$

(c) Let final pressure in LHS and RHS be P_1 and P_2

=> $n_1 = P_1V/2RT_1$ and $n_2 = P_2V/2RT_2$

As, Total number of moles $n=n_1 +n_2$

$n = P_1V/2RT_1 + n_2 = P_2V/2RT_2$

$n = V/2R [(T_2P_1 + P_2T_1)/(T_1T_2)]$ ...(1)

The internal energy of ideal gas : $U = nC_v T$

Where C_v = molar specific heat at constant volume

According to question, U = 1.5nRT

=> $nC_v T = 1.5 nRT$
=> $C_v = 1.5R$

Internal energy of the left chamber = $U_1 = n_1C_vT_1$

Internal energy of right chamber = $U_2 = n_2C_vT_2$

Total internal energy $U = U_1 + U_2$

$(1.5)nRT= n_1C_vT_2 + n_2C_vT_2$

$(1.5)nRT = 1.5R(n_1T_1 + n_2T_2)$

$nT = n_1T_1 + n_2T_2$

substituting the value of n, n_1 and n_2, we get

$$nT = \frac{P_1V}{2RT_1} \times T_1 + \frac{P_2V}{2RT_2} \times T$$

$$nT = \frac{V(P_1 + P_2)}{2R}$$

$$T = \frac{V(P_1 + P_2)}{2R} \times \frac{T_1T_2}{T_2P_1 + P_2T_1} \times \frac{2R}{V}$$

$$T = \frac{(T_1T_2) \times (P_1 + P_2)}{T_2P_1 + P_2T_1}$$

which is the final equilibrium temperature.

(d) The internal energy of ideal gas: $U = nC_vT$

From part (a), $\Delta W = 0$ for the right chamber of the tube.

=> $\Delta Q = \Delta U$ …(a)

Change in internal energy of the right chamber after equilibrium will be

$\Delta U = n_2c_vT_2 - n_2c_vT$

$$\Delta U = \frac{P_2V}{2RT_2} \times 1.5 \times R \times T_2 - \frac{P_2V}{2RT_2} \times 1.5 \times R \times \frac{(T_1T_2) \times (P_1 + P_2)}{T_2P_1 + P_2T_1}$$

From (a),

$$\Delta Q = \Delta U = \frac{P_2V}{2} \times 1.5\left(1 - \frac{(T_1) \times (P_1 + P_2)}{T_2P_1 + P_2T_1}\right)$$

$$\Delta Q = \frac{3P_2V}{4} \times \left(\frac{T_2P_1 - T_1P_1}{T_2P_1 + P_2T_1}\right)$$

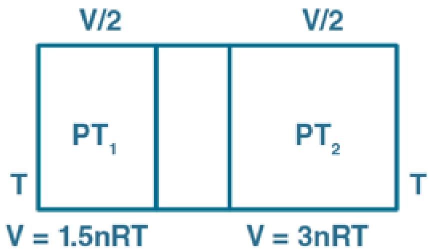
$$\Delta Q = \frac{3P_1P_2V}{4} \times \left(\frac{T_2 - T_1}{T_2P_1 + P_2T_1}\right)$$

Which is the heat flown from left to right chamber.

Question 22: An adiabatic vessel of total volume V is divided into two equal parts by a conducting separator. The separator is fixed in this position. The part on the left contains one mole of an ideal gas (U = 1.5 nRT) and the part on the right contains two moles of the same gas. Initially, the pressure on each side is p. The system is left for sufficient time so that a steady state is reached. Find
(a) the work done by the gas in the left part during the process.

(b) the temperature on the two sides in the beginning,
(c) the final common temperature reached by the gases,
(d) the heat given to the gas in the right part and
(e) the increase in the internal energy of the gas in the left part.

Solution:



(a) As the conducting wall is fixed the work done by the gas on the left part during the process is zero. Because change in volume will be zero due to fixed position of the wall.

(b)For left side:
Let p = initial pressure on both sides of the wall.
T_1 = initial temperature
number of moles = 1

Also, volume = V/2

We know, PV = nRT
In this situation, above equation can be written as

$PV/2 = nRT_1$

$\Rightarrow T_2 = PV/2R$

Similarly, for the right part
number of moles = 2

$\Rightarrow T_2 = PV/4R$

(c) The internal energy of the gas: U = 1.5 nRT

and internal energy of ideal gas: $U = n C_v T$

Total moles = $n = n_1 + n_2 = 1 + 2 = 3$

$\Rightarrow U = 3 C_v T$

Let U_1 and U_2 be the internal energy of the left and right part respectively.

So, $U_1 = n_1 C_v T_1 = C_v T_1$ and $U_2 = n_2 C_v T_2 = 2 C_v T_2$

Here, C_v will be same since gas is same on both the parts

=> $U = U_1 + U_2$

=> $3 C_v T = C_v T_1 + 2 C_v T_2$

=> $3T = T_1 + 2T_2$

Using values of T_1 and T_2

=> $T = pV/3R$

(d) As per part (a), ΔW=0 for the right part of the vessel.

We know, ΔQ=ΔU+ΔW

=> ΔQ = ΔU

U = 1.5 nRT (Given)

Heat given to the right part = ΔU = 1.5 n R ΔT

For the right part, change in internal energy after equilibrium has reached will be due to the change in temperature from T_2 to T.

$\Delta U = 1.5 n_2 R (T - T_2)$

Substituting Values of n_2, T_1 and T_2, we get

$$\Delta U = 1.5 \times 2 \times R \times \left(\frac{pV}{3R} - \frac{pV}{4R}\right)$$

$$\Delta U = \frac{pV}{4}$$

(e) From part (a), Since ΔW is zero as the volume is fixed.

Here, ΔQ = -ΔU (left part)

=> ΔU = -PV/4