

Exercise Solutions

Question 1: A vessel containing one mole of a monatomic ideal gas (molecular weight = 20 g mol⁻¹) is moving on a floor at a speed of 50 ms⁻¹. The vessel is stopped suddenly. Assuming that the mechanical energy lost has gone into the internal energy of the gas, find the rise in its temperature.

Solution:

$$\text{Change in internal energy} = \Delta U = nC_v \Delta T \dots(1)$$

Given: $n = 1$, $C_v = 3R/2 = 12.471 \text{ J/mol/K}$ and $\Delta T = \text{rise in temperature}$.

$$\text{and Mechanical energy lost} = E = -m [(v_i^2 - v_f^2)/2] \dots(2)$$

Equating, (1) and (2) and substituting the values, we have

$$\Rightarrow [0.02 \times (2500 - 0)]/2 = 12.471 \times \Delta T$$

$$\Rightarrow \Delta T = 2 \text{ K}$$

Question 2: 5g of a gas is contained in a rigid container and is heated from 15°C to 25°C. Specific heat capacity of the gas at constant volume is 0.172 cal g⁻¹°C⁻¹ and the mechanical equivalent of heat is 4.2 J cal⁻¹. Calculate the change in the internal energy of the gas.

Solution:

we know, Heat(Joule) = Mechanical equivalent of heat × Heat(cal)

$$dQ = du + dW$$

For rigid body, $V = 0$

$$\text{So } dW = 0$$

$$\Rightarrow dQ = du$$

$$\text{and } Q = ms dt = 5 \times 0.172 \times 10 = 8.6 \text{ cal} = 8.6 \times 4.2 = 36.12 \text{ Joule}$$

Question 3: The figure shows a cylindrical container containing oxygen ($\gamma = 1.4$) and closed by a 50 kg frictionless piston. The area of cross section is 100 cm², atmospheric pressure is 100 kPa and g is 10 ms⁻². The cylinder is slowly heated for some time. Find the amount of heat supplied to the gas if the piston moves out through a distance of 20 cm.



Solution:

Mass of piston (m) = 50 kg

Area (A) = $100 \text{ cm}^2 = (100 \times 10^{-4}) \text{ m}^2$

And $\gamma = 1.4$

Atmospheric pressure = $100 \text{ kPa} = 1,00,000 \text{ Pa}$

Pressure exerted by piston = force/area = mg/area

= $(50 \times 10)/(100 \times 10^{-4}) \text{ Pa} = 50,000 \text{ Pa}$

Total pressure (P) = $(50,000 + 1,00,000) \text{ Pa} = 1,50,000 \text{ Pa}$

Work done = Pressure \times change in volume = $P \, dV$

dV = distance moved by piston \times Area = $20 \times 100 = 2,000 \text{ cm}^3 = 2,000 \times 10^{-6} \text{ m}^3 = 2 \times 10^{-3} \text{ m}^3$

So, Work done = $(1,50,000 \times 2 \times 10^{-3}) \text{ J} = 300 \text{ J}$

We know, Work done, $W = P \Delta V = n R \, dT$

$\Rightarrow dT = 300/nR$

Now,

$dQ = n C_p \, dt = 300 \times C_p/R$

Also given, $\gamma = 1.4 = C_p/C_v$ and $C_p + C_v = R$

$\Rightarrow C_p = 7R/2, C_v = 5R/2.$

$\Rightarrow dQ = 300/R \times 7R/2 = 1050 \text{ J}$

Question 4: The specific heat capacities of hydrogen at constant volume and at constant pressure are $2.4 \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$ and $3.4 \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$ respectively. The molecular weight of hydrogen is 2 g mol^{-1} and the gas constant, $R = 8.3 \times 10^7 \text{ erg }^\circ\text{C}^{-1}\text{mol}^{-1}$. Calculate the value of J

Solution:

The gas constant = $R = (m \times C_p) - (m \times C_v) = 2 \times (C_p - C_v) = 2J$

Given: $C_v = 2.4 \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$ and $C_p = 3.4 \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$

Molecular mass of hydrogen = 2 g mol^{-1}

Gas constant = $R = 8.3 \times 10^7 \text{ erg mol}^{-1} \text{ }^\circ\text{C}^{-1}$

Now, $2J = R$

$\Rightarrow 2J = 8.3 \times 10^7 \text{ erg/mol }^\circ\text{C}$

or $J = 4.15 \times 10^7 \text{ erg/cal}$

Question 5: The ratio of the molar heat capacities of an ideal gas is $C_p/C_v = 7/6$. Calculate the change in internal energy of 1.0 mole of the gas when its temperature is raised by 50 K

- (a) keeping the pressure constant,
- (b) keeping the volume constant and
- (c) adiabatically.

Solution:

$C_p/C_v = 7/6 \Rightarrow C_v = 6C_p/7$

$n = \text{number of moles} = 1$

$dT = \text{change in temperature} = 50\text{K}$

(a) change in internal energy = $dU = nC_v dT$

Also, $C_p - C_v = R$

$\Rightarrow C_p - 6C_p/7 = C_p/7 = R$

$\Rightarrow C_p = 7R$

Therefore, $C_v = C_p - R = 6R = (6 \times 8.314)\text{J/mol/K}$

$$\Rightarrow dU = 1 \times (6 \times 8.314) \times 50 = 2494.2 \text{ J}$$

(b) Volume constant, so $dV = 0$

By First law of thermodynamics: $dU = dQ - dW$

$$\text{Also, } dW = P dV = 0$$

$$\text{As, } dQ = nC_v dT$$

$$\Rightarrow dU = dQ = nC_v dT = 1 \times (6 \times 8.314) \times 50 = 2494.2 \text{ J}$$

(c) Adiabatic process: $dQ(\text{heat change}) = 0$

$$\text{We know, } dQ = dU + dW.$$

$$\Rightarrow dU = -dW$$

Where dU = change in internal energy.

$$\text{Also, For an adiabatic process, } dW = dT/(\gamma-1).$$

$$\text{and } \gamma = C_p/C_v = 7/6 \text{ (Given)}$$

$$\Rightarrow dU = -dW - (-nR dT)/(\gamma-1) = 2494.2 \text{ J}$$

Question 6: A sample of air weighing 1.18 g occupies $1.0 \times 10^3 \text{ cm}^3$ when kept at 300K and $1.0 \times 10^5 \text{ Pa}$. When 2.0 cal of heat is added to it at constant volume, its temperature increases by 1°C . Calculate the amount of heat needed to increase the temperature of air by 1°C at constant pressure if the mechanical equivalent of heat is $4.2 \times 10^7 \text{ erg cal}^{-1}$. Assume that air behaves as an ideal gas.

Solution:

$$\text{Ideal gas equation: } PV = nRT$$

$$\Rightarrow n = PV/RT = [100000 \times 0.001]/[8.314 \times 300] = 0.04 \text{ mol}$$

$$\text{By First law of thermodynamics: } dQ = dU + dW = dU + PdV$$

$$\Rightarrow dW = PdV$$

Since volume is constant, $dV = 0 \Rightarrow dW = 0$.

Hence, $dQ = dU$.

$dQ = \text{Heat} = 2 \text{ cal}$ (Given) and $dQ = n C_v dT$

$$\Rightarrow 2 = 0.04 \text{ mol} \times C_v \times 1\text{K}$$

$$\Rightarrow C_v = 50 \text{ cal/mol/K} = (50 \times 4.2 \times 10^7) \text{ erg/cal} \times \text{cal/mol/K} = 2.1 \times 10^9 \text{ erg/mol/K} = 210 \text{ J/mol/K}$$

We know, $C_p = (C_v + R)$

$$\Rightarrow C_p = (210 + 8.314) \text{ J/mol/K} = 218.314 \text{ J/mol/K}$$

Therefore, heat required to raise the temperature by 1°C at constant pressure = $nC_p dT$

$$\Rightarrow \text{Heat} = (0.04 \times 218.314 \times 1) \text{ J} = 8.737 \text{ J} = (8.737/4.2) \text{ cal} = 2.08 \text{ cal} \text{ (since } 1 \text{ J} = 4.2 \text{ cal)}$$

Question 7: An ideal gas expands from 100 cm^3 to 200 cm^3 at a constant pressure of $2.0 \times 10^5 \text{ Pa}$ when 50 J of heat is supplied to it. Calculate

- the change in internal energy of the gas.
- the number of moles in the gas if the initial temperature is 300 K .
- the molar heat capacity C_p at constant pressure and
- the molar heat capacity C_v at constant volume.

Solution:

$$(a) dU = dQ - PdV = (50 - (2.0 \times 10^5 \times 10^{-4})) \text{ J} = 30 \text{ J}$$

(b) For constant pressure, from equation of state $PV/T = \text{constant}$

$$\Rightarrow V_1/T_1 = V_2/T_2$$

$$\text{Given: } V_1 = 100 \text{ cm}^3, V_2 = 200 \text{ cm}^3, T_1 = 300 \text{ K}$$

$$\Rightarrow T_2 = 600 \text{ K}$$

$$\text{So, } dT = T_2 - T_1 = 300 \text{ K}$$

Therefore, $PdV = nRdT$

$$\Rightarrow 2.0 \times 10^5 \times 10^{-4} = n \times 8.314 \times 300$$

$$\Rightarrow n = 0.008 \text{ mol}$$

(c) At constant pressure

$$dQ = 50 = nC_p dT$$

$$\Rightarrow 50 = 0.008 \times C_p \times 300$$

$$\Rightarrow C_p = 20.83 \text{ J/mol/K}$$

(d) At constant volume, $dU = dQ = nC_v dT$

$$dU = 30 \text{ J from part (a)}$$

$$\Rightarrow 30 = 0.008 \times C_v \times 300$$

$$\Rightarrow C_v = 12.5 \text{ J/mol/K.}$$

Question 8: An amount Q of heat is added to a monatomic ideal gas in a process in which the gas performs a work $Q/2$ on its surrounding. Find the molar heat capacity for the process.

Solution:

$$dQ = W + dU$$

$$\text{For monoatomic gas: } dU = Q - Q/2 = Q/2$$

$$\text{and } C_v = (3R/2) \text{ J/kg/mol}$$

$$\text{We know, } dU = nC_v dT \text{ and } dQ = nC dT$$

$$\text{When } dU = dQ$$

$$\Rightarrow nC_v dT = nC dT/2$$

$$\Rightarrow c = 2 C_v$$

$$\Rightarrow C = 2 \times (3R/2) = 3R \text{ J/kg/mol.}$$

Question 9: An ideal gas is taken through a process in which the pressure and the volume are changed according to the equation $p = kV$. Show that the molar heat capacity of the gas for the process is given by $C = C_v + R/2$.

Solution:

$$P = kV \dots (1) \text{ (Given)}$$

Multiplying by dV on both sides, we get

$$PdV = kVdV$$

on Integrating from $V = V_1$ to V_2 , we get

$$\int dV = k/2 [V_2^2 - V_1^2]$$

Equation of state of ideal gas: $PV = nRT = \text{constant} \dots (2)$

We can write, $V_1 = nRT_1/P_1$

P_1, V_1, T_1 = Pressure, volume, temperature of first gas

Since $P_1 = kV_1$

$kV_1^2 = nRT_1$. Similarly, $kV_2^2 = nRT_2$, where P_2, V_2, T_2 = Pressure, volume, temperature of second gas

on substituting, the above equation becomes,

$$\Rightarrow (k/2) \times nR/k [T_2 - T_1] = nR/2 dT \dots (3)$$

Now, $V = nRT/P$

$$\Rightarrow V^2 = nRT/k \dots (4)$$

[As, $P = kV$]

Using, first law of thermodynamics

$$Q = U + \int PdV$$

$$\Rightarrow nCdT = nC_v dT + (nR/2)dT$$

(As, $Q = nCdT$ and $U = nC_v dT$)

$$\Rightarrow C = C_v + nR/2 . \text{ Hence proved.}$$

Question 10: An ideal gas ($C_p/C_v = \gamma$) is taken through a process in which the pressure and the volume vary as $p = aV^b$. Find the value of b for which the specific heat capacity in the process is zero.

Solution:

$$\text{Given: } p = aV^b \dots (1)$$

specific heat capacity is zero ...(given)

Therefore, process is essentially an adiabatic process.

So, $PV^\gamma = \text{Constant}$

$$\Rightarrow P = c/V^{\gamma} = c v^{-\gamma} \dots(2)$$

Comparing (1) and (2), we get

$$a = c \text{ and } b = -\gamma$$

Question 11: Two ideal gases have the same value of $C_p/C_v = \gamma$. What will be the value of this ratio for a mixture of the two gases in the ratio 1: 2?

Solution:

$$C_p/C_v = \gamma \text{ (Given)}$$

$$\text{We know, } R = C_p - C_v$$

$$\text{Now, } C_v = R/(\gamma-1)$$

For the first ideal gas:

$$C_{v1} = R/(\gamma-1) \text{ and } C_{p1} = R\gamma/(\gamma-1)$$

Where C_{p1} and C_{v1} is the molar heat capacity at constant pressure and constant volume.

For the second ideal gas:

$$C_{v2} = R/(\gamma-1) \text{ and } C_{p2} = R\gamma/(\gamma-1)$$

Where C_{p2} and C_{v2} is the molar heat capacity at constant pressure and constant volume

Since we are given, $n_1 = n_2 = 1:2$

$$dU_1 = nC_{v1} dT \text{ and } dU_2 = 2n C_{v2} dT$$

$$\text{When gas is mixed: } nC_{v1} dT + 2n C_{v2} dT = 3 n C_v dT$$

$$\Rightarrow C_v = (1/3) [C_{v1} + 2C_{v2}]$$

on substituting C_{v1} and C_{v2} , we get

$$C_p/C_v = \gamma$$

Question 12: A mixture contains 1 mole of helium ($C_p = 2.5 R$, $C_v = 1.5R$) and 1 mole of hydrogen ($C_p = 3.5 R$, $C_v = 2.5 R$). Calculate the values of C_p , C_v and γ for the mixture.

Solution:

$C_{p1} = 2.5 R$, $C_{v1} = 1.5R$ for helium

$C_{p2} = 3.5 R$, $C_{v2} = 2.5 R$ for hydrogen

$n_1 = n_2 = 1$

We know, $dU = nC_v dT$

For the mixture,

$(n_1 + n_2)C_v dT = n_1C_{v1} dT + n_2C_{v2} dT$

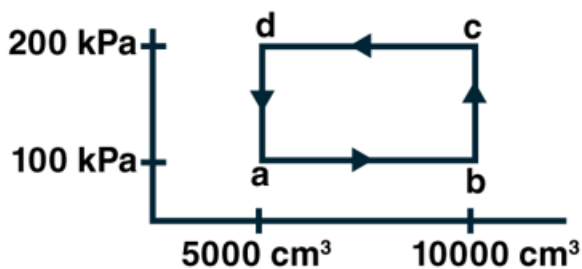
On substituting the values,

$C_v = 2R$

Also, $C_p = C_v + R = 3R$

$\Rightarrow \gamma = C_p/C_v = 1.5$

Question 13: Half mole of an ideal gas ($\gamma = 5/3$) is taken through the cycle abcda as shown in figure. Take $R = (25/3) \text{ J/Kmol}$.



- Find the temperature of the gas in the states a, b, c and d.
- Find the amount of heat supplied in the processes ab and bc.
- Find the amount of heat liberated in the processes cd and da.

Solution:

Given, $n = 1/2$, $\gamma = 5/3$, $R = 25/3 \text{ J/Kmol}$

a) By ideal gas equation, $PV = nRT$

Here, temperature at a, $T_a = PV/nR$

$$= [5000 \times 10^{-6} \times 100 \times 10^3] / [(1/2) \times (25/3)]$$

$$= 120 \text{ K}$$

Here, temperature at b

$$T_b = PV/nR = [10000 \times 10^{-6} \times 100 \times 10^3] / [(1/2) \times (25/3)]$$

$$= 240 \text{ K}$$

Here, temperature at c

$$T_c = PV/nR = [1000 \times 10^{-6} \times 200 \times 10^3] / [(1/2) \times (25/3)]$$

$$= 480 \text{ K}$$

Here, temperature at d

$$T_d = PV/nR = [5000 \times 10^{-6} \times 200 \times 10^3] / [(1/2) \times (25/3)]$$

$$= 240 \text{ K}$$

(b) For ab process

$$dQ = n c_p dT = (1/2) \times R / (\gamma - 1) [T_c - T_b] = 1500 \text{ J}$$

(c) Heat liberated in cd

$$dQ = -n c_p dT = (-1/2) \times R / (\gamma - 1) [T_d - T_c] = 2500 \text{ J}$$

Heat liberated in da:

$$dQ = -n c_v dT = (1/2) \times R / (\gamma - 1) [T_a - T_d] = 750 \text{ J}$$

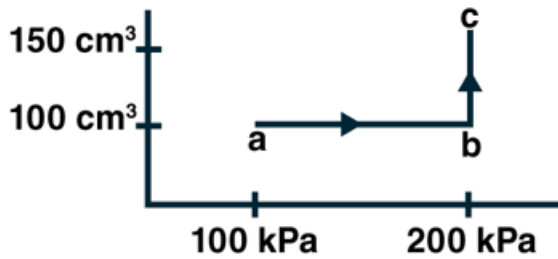
Question 14: An ideal gas ($\gamma = 1.67$) is taken through the process abc shown in figure. The temperature at the point a is 300K. Calculate

(a) the temperature at b and c,

(b) the work done in the process,

(c) the amount of heat supplied in the path ab and in the path bc and

(d) the change in the internal energy of the gas in the process.



Solution:

(a) For the process ab volume is constant

Ideal gas equation, $P_1/T_1 = P_2/T_2$

$$\Rightarrow 100/300 = 200/T_2$$

$$\Rightarrow T_2 = 600 \text{ k}$$

For the process bc, pressure is constant.

By ideal gas equation, $V_2/T_2 = V_3/T_3$

$$\Rightarrow 100/600 = 150/T_3$$

$$\Rightarrow T_3 = 900 \text{ k}$$

(b) Process ab is isochoric i.e. $W_{ab} = 0$

For process bc, $P=200 \text{ kPa}$, change in volume is 50 cm^3 from b to c

$$\text{Now, work done} = P \, dV = 200 \times 10^3 \times 50 \times 10^{-6} = 10 \text{ J}$$

(c) first law of thermodynamics: $dQ = dU + dW$

As ab is isochoric process $dW=0$

$$dQ_{ab} = dU = nC_v dT$$

$$= \frac{PV}{RT} \times \frac{R}{\gamma - 1} dT$$

$$= \frac{200 \times 10^3 \times 150 \times 10^{-6}}{600 \times .67} \times 300$$

$$= 14.925$$

Again,

$$dQ_{bc} = dU = nC_p dT$$

$$= \frac{PV}{RT} \times \frac{\gamma R}{\gamma - 1} dT$$

$$= \frac{200 \times 10^3 \times 150 \times 10^{-6}}{900 \times 8.3} \times \frac{1.67 \times 8.3}{.67} \times 300$$

$$= 24.925$$

(d) $dQ = dU + W$

or $dU = dQ - W$

$$= (24.925 + 14.925) - 10$$

$$= 29.85$$

Question 15: In Joly's differential steam calorimeter, 3g of an ideal gas is contained in a rigid closed sphere at 20°C. The sphere is heated by steam at 100°C and it is found that an extra 0.095 g of steam has condensed into water as the temperature of the gas becomes constant. Calculate the specific heat capacity of the gas in $J g^{-1} K^{-1}$. The latent heat of vaporization of water = 540 cal g^{-1} .

Solution:

In Joly's differential steam calorimeter,

$$C_v = 2mL/[m_1(\theta_2 - \theta_1)]$$

Given:

$m_1 =$ Mass of gas present = 3 g, $\theta_1 = 20^\circ\text{C}$, $\theta_2 = 100^\circ\text{C}$

$m_2 =$ Mass of steam condensed = 0.095 g, $L = 540 \text{ Cal/g} = 540 \times 4.2 \text{ J/g}$

$$\Rightarrow C_v = [0.095 \times 540 \times 4.2] / [3 \times 80]$$

$$= 0.89 \text{ J/gK}$$

Question 16: The volume of an ideal gas ($\gamma = 1.5$) is changed adiabatically from 4.00 litres to 3.00 litres. Find the ratio of

(a) the final pressure to the initial pressure and

(b) the final temperature to the initial temperature.

Solution:

(a) Since it is an adiabatic process, So $PV^\gamma = \text{Constant}$

$$P_2/P_1 = (V_1/V_2)^\gamma = 1.54$$

(b) In this case, $TV^{\gamma-1} = \text{Constant}$

$$T_1 V_1^{(\gamma-1)} = T_2 V_2^{(\gamma-1)}$$

$$\Rightarrow T_2/T_1 = (V_1/V_2)^{(\gamma-1)} = (4/3)^{(1.5-1)} = 1.154$$

Question 17: An ideal gas at pressure $2.5 \times 10^5 \text{ Pa}$ and temperature 300K occupies 100 cc. It is adiabatically compressed to half its original volume. Calculate

(a) the final pressure,

(b) the final temperature and

(c) the work done by the gas in the process. Take $\gamma = 1.5$.

Solution:

Given, $P_1 = 2.5 \times 10^5 \text{ Pa}$, $V_1 = 100 \text{ cc}$, $T_1 = 300 \text{ K}$, $V_2 = 50 \text{ cc}$ and $\gamma = 1.5$

(a) Since it is an adiabatic process, So $PV^\gamma = \text{Constant}$

$$P_2/P_1 = (V_1/V_2)^\gamma$$

Substituting values, we get

$$P_2 = 7.1 \times 10^5 \text{ Pa}$$

(b) In this case, $TV^{(\gamma-1)} = \text{Constant}$

$$T_1 V_1^{(\gamma-1)} = T_2 V_2^{(\gamma-1)}$$

$$\Rightarrow T_2/T_1 = (V_1/V_2)^{(\gamma-1)}$$

Substituting values, we get

$$T_2 = 300 \times 2^{0.5} = 425 \text{ K}$$

Question 18: Air ($\gamma = 1.4$) is pumped at 2 atm pressure in a motor tyre at 20°C . If the tyre suddenly bursts, what would be the temperature of the air coming out of the tyre. Neglect any mixing with the atmospheric air.

Solution:

Given $\gamma = 1.4$, $P_1 = 2 \text{ atm}$, $P_2 = 1 \text{ atm}$, $T_1 = 20^\circ\text{C} = 293 \text{ K}$

Here bursting of tire is an adiabatic process,

$$T_1^\gamma P_1^{(1-\gamma)} = T_2^\gamma P_2^{(1-\gamma)}$$

$$\Rightarrow T_2^\gamma = [T_1^\gamma P_1^{(1-\gamma)}] / P_2^{(1-\gamma)}$$

$$\Rightarrow T_2^{1.4} = 2153.78$$

$$\Rightarrow T_2 = (2153.78)^{1/4} = 240.3 \text{ K}$$

Question 19: A gas is enclosed in a cylindrical can fitted with a piston. The walls of the can and the piston are adiabatic. The initial pressure, volume and temperature of the gas are 100 kPa, 400 cm^3 and 300 K respectively. The ratio of the specific heat capacities of the gas is $C_p/C_v = 1.5$. Find the pressure and the temperature of the gas if it is

- suddenly compressed
- slowly compressed to 100 cm^3 .

Solution:

Given, $V_1 = 400 \text{ cm}^3$; $P_1 = 100 \text{ KPa} = 10^5 \text{ Pa}$; $T_1 = 300 \text{ K}$,

and $C_p/C_v = 1.5$

(a) Suddenly compressed to $V_2 = 100 \text{ cm}^3$ [adiabatic process]

$$\Rightarrow PV^\gamma = \text{const.}$$

$$P_2/P_1 = (V_1/V_2)^\gamma = (400/100)^{1.5}$$

$$\Rightarrow P_2 = 10^5 \times 4^{1.5} = 800 \text{ KPa}$$

$$\text{Also, } T_1 P_1^{1-\gamma} = T_2 P_2^{1-\gamma}$$

$$\Rightarrow T_2 = [T_1 P_1^{1-\gamma}] / P_2^{1-\gamma} = 300 \times (100/400)^{-0.5} = 600 \text{ K}$$

(b) Even though the container is slowly compressed the walls are adiabatic so heat transferred is zero. Thus the values remain unchanged.

$$\Rightarrow P_2 = 800 \text{ KPa and } T_2 = 600 \text{ K}$$

Question 20: The initial pressure and volume of a given mass of a gas ($C_p/C_v = \gamma$) are P_0 and V_0 . The gas can exchange heat with the surrounding.

(a) It is slowly compressed to a volume $V_0/2$ and then suddenly compressed to $V_0/4$. Find the final pressure.

(b) If the gas is suddenly compressed from the volume V_0 to $V_0/2$ and then slowly compressed to $V_0/4$, what will be the final pressure?

Solution:

Here, $C_p/C_v = \gamma$; $P_0 =$ Initial Pressure and $V_0 =$ Initial Volume

(a)

(i) Isothermal compression, $P_1 V_1 = P_2 V_2$

$$\Rightarrow P_0 V_0 = P_2 V_0/2$$

$$\Rightarrow P_2 = 2 P_0$$

(ii) Adiabatic compression, $P V^\gamma = \text{Constant}$

$$\text{Now, } P_3/P_2 = (V_2/V_3)^\gamma$$

$$\text{Put } V_2 = V_0/2 \text{ and } V_3 = V_0/4 \text{ and } P_2 = 2P_0$$

$$\Rightarrow P_3 = P_0 (2)^{(\gamma+1)}$$

(b) Since the volume is suddenly compressed.

Let P_2 and $V_2 = V_0/2$ be the pressure and volume after compression

$$P_2/P_1 = (V_1/V_2)^\gamma = [V_0/(V_0/2)]^\gamma = (2)^\gamma$$

Since $P_1 = P_0$

$$\Rightarrow P_2 = P_0 (2)^\gamma$$

As, volume is slowly compressed, temperature remains constant [Isothermal compression]

$$P_2 V_2 = P_3 V_3$$

$$\Rightarrow P_0 \times 2^\gamma (V_0/2) = P_3 \times (V_0/4)$$

$$\Rightarrow P_3 = P_0 2^{\gamma+1}$$

Question 21: Consider a given sample of an ideal gas ($C_p/C_v = \gamma$) having initial pressure P_0 and volume V_0 .

(a) The gas is isothermally taken to a pressure $P_0/2$ and from there adiabatically to a pressure $P_0/4$. Find the final volume.

(b) The gas is brought back to its initial state. It is adiabatically taken to a pressure $P_0/2$ and from there isothermally to a pressure $P_0/4$. Find the final volume.

Solution:

Here, $C_p/C_v = \gamma$; $P_0 =$ Initial Pressure and $V_0 =$ Initial Volume

(a)

(i) Isothermal compression, $P_1 V_1 = P_2 V_2$

Here $P_2 = P_0/2$

$$\Rightarrow P_0 V_0 = (P_0/2) V_2$$

$$\Rightarrow V_2 = 2 V_0$$

(ii) Adiabatic compression, $PV^\gamma = \text{Constant}$

Here $P_3 = P_0/4$

$$\text{Now, } P_3/P_2 = (V_2/V_3)^\gamma$$

Put $V_2 = P_0/4$ and $V_3 = P_0/2$ and $V_2 = 2V_0$

$$\Rightarrow V_3 = V_0 (2)^{(1/\gamma + 1)}$$

(b) For adiabatic Process

Here $P_1 = P_0$, $P_2 = P_0/2$

Let $V_1 = V_0$ be the initial volume and V_2 be the volume after process.

and $PV^\gamma = \text{Constant}$

Now,

$$P_2/P_1 = (V_1/V_2)^\gamma = [(P_0/2)/P_0]^\gamma = (V_0/V_2)^\gamma$$

$$\Rightarrow V_2 = V_0 (2)^{1/\gamma}$$

Again,

After isothermal process,

Let $P_3 = P_0/4$ and V_3 be the pressure and volume

$$\Rightarrow P_2 V_2 = P_3 V_3$$

$$\Rightarrow (P_0/2) \times V_0 \times 2^{1/\gamma} = V_3 \times (P_0/4)$$

$$\Rightarrow V_3 = V_0 2^{(1+1/\gamma)}$$

Question 22: A sample of an ideal gas ($\gamma = 1.5$) is compressed adiabatically from a volume of 150 cm^3 to 50 cm^3 . The initial pressure and the initial temperature are 150 kPa and 300 K . Find

- the number of moles of the gas in the sample,
- the molar heat capacity at constant volume,
- the final pressure and temperature,
- the work done by the gas in the process and
- the change in internal energy of the gas.

Solution:

Given : $V_1 = 150 \text{ cm}^3$, $V_2 = 50 \text{ cm}^3$, $P_1 = 150 \text{ kPa} = 150 \times 10^3 \text{ Pa}$ and $T_1 = 300 \text{ K}$

(a) By ideal gas equation, $PV = nRT$

$$\Rightarrow n = PV/RT = [150 \times 10^3 \times 150 \times 10^{-6}] / [8.3 \times 300] = 0.009 \text{ moles}$$

(b) We know $C_p/C_v = \gamma$ and $R = C_p - C_v$

$$\Rightarrow C_v = R/(\gamma - 1) = 8.3/0.5 = 16.6 \text{ J}$$

(c) For adiabatic process, $PV^\gamma = \text{constant}$.

$$\Rightarrow P_2/P_1 = (V_1/V_2)^\gamma$$

$$\Rightarrow P_2 = 150 \times 10^3 \times (150/50)^{1.5} = 780 \text{ KPa}$$

For adiabatic process:

$$T_1^\gamma P_1^{1-\gamma} = T_2^\gamma P_2^{1-\gamma}$$

$$\Rightarrow T_2^\gamma = [T_1^\gamma P_1^{1-\gamma}] / P_2^{1-\gamma} = 300^{1.5} \times (150/780)^{-0.5}$$

$$\Rightarrow T_2 = 519.74 \text{ K}$$

(d) first law of thermodynamics, $dQ = dU + dW$

In an adiabatic process, amount of heat supplied which is zero i.e. $dQ = 0$

$$\Rightarrow -dU = dW$$

Also, $dU = nC_v dT$

$$\Rightarrow dW = -nC_v dT = -0.009 \times 16.6 \times (520 - 300) = -33 \text{ J}$$

(e) We know, $dU = nC_v dT = 33 \text{ J}$

Question 23: Three samples A, B and C of the same gas ($\gamma = 1.5$) have equal volumes and temperatures. The volume of each sample is doubled, the process being isothermal for A, adiabatic for B and isobaric for C. If the final pressure is equal for the three samples, find the ratio of the initial pressures.

Solution:

Given, $T_A = T_B = T_C$ and $V_A = V_B = V_C$

Where, V_A, V_B, V_C be the volume of three gases and T_A, T_B, T_C be the temperature of A, B, C gas

A is undergoing an isothermal process, where $V_1 = V_A, V_2 = 2V_A$

Let P_{1A} and P_{2A} be the initial and final pressures

$$\Rightarrow P_{1A} V_A = P_{2A} (2V_A)$$

$$\Rightarrow P_{2A} = P_{1A}/2$$

B is undergoing an adiabatic process, $PV^\gamma = \text{constant}$, where $V_1 = V_B$, $V_2 = 2V_B$.

Let P_{1B} and P_{2B} be the initial and final pressures

Here $\gamma = 1.5$

$$\Rightarrow P_{2B}/P_{1B} = (V_B/(2V_B))^\gamma$$

$$\Rightarrow P_{2B} = P_{1B} (1/2)^{1.5}$$

C is undergoing an isobaric process, where pressure remains constant and equal to P_{1C}

$$\Rightarrow P_{1A}/2 = P_{1B} (1/2)^{1.5} = P_{1C}$$

$$\Rightarrow P_{1A} : P_{1B} : P_{1C} = 2 : 2^{1.5} : 1 = 2 : 2\sqrt{2} : 1$$

Question 24: Two samples A and B of the same gas have equal volumes and pressures. The gas in sample A is expanded isothermally to double its volume and the gas in B is expanded adiabatically to double its volume. If the work done by the gas is the same for the two cases, show that γ satisfies the equation $1 - 2^{(1-\gamma)} = (\gamma - 1) \ln 2$.

Solution:

Given $v_2 = 2V_1$

Isothermal work done = $W_A = nRT_1 \ln (V_2/V_1)$

Also, B is expanded adiabatically:

$$W_B = [P_1V_1 - P_2V_2]/[\gamma-1]$$

Given $W_A = W_B$

$$\Rightarrow nRT_1 \ln (V_2/V_1) = [P_1V_1 - P_2V_2]/[\gamma-1] \dots(1)$$

For adiabatic process, $PV^\gamma = \text{constant}$

$$\Rightarrow (P_2/P_1) = (V_1/(2V_1))^\gamma$$

$$\Rightarrow P_2 = P_1 (1/2)^\gamma$$

(1) \Rightarrow

$$nRT_1 \ln\left(\frac{V_2}{V_1}\right) = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

$$nRT_1 \ln(2) = \frac{P_1V_1\left(1 - \frac{1}{2^\gamma} \times 2\right)}{\gamma - 1}$$

For ideal gas equation, $PV = nRT$

$$\ln(2) = [1 - (1/2^\gamma) \times 2]/(\gamma - 1)$$

$$\Rightarrow \ln 2(\gamma - 1) = 1 - 2^{1-\gamma}$$

Question 25: 1 litre of an ideal gas ($\gamma = 1.5$) at 300 K is suddenly compressed to half its original volume.

- Find the ratio of the final pressure to the initial pressure.
- If the original pressure is 100 kPa, find the work done by the gas in the process.
- What is the change in internal energy?
- What is the final temperature?
- The gas is now cooled to 300 K keeping its pressure constant. Calculate the work done during the process.
- The gas is now expanded isothermally to achieve its original volume of 1 litre. Calculate the work done by the gas.
- Calculate the total work done in the cycle.

Solution:

Given : $\gamma = 1.5$, $T = 300$ K and $V_2 = L/2$

(a) In adiabatic process since volume is changed suddenly,

$$P_2/P_1 = (V_1/V_2)^\gamma$$

$$\Rightarrow P_2/P_1 = (2)^{1.5}$$

(b)

Given: $P_1 = 100$ KPa = 10^5 Pa, $P_2 = 2^{1.5}(10^5)$ KPa

In adiabatic process:

$$\begin{aligned}
 W &= \frac{P_1V_1 - P_2V_2}{\gamma - 1} \\
 &= \frac{10^5 \times 10^{-3} - 2^{1.5}(10^5)500 \times 10^{-3}}{1.5 - 1} \\
 &= -82 \text{ J}
 \end{aligned}$$

(c) Since it is an adiabatic process, $dQ = 0$

By first law of thermodynamics, $dQ = dU + dW$

$$\Rightarrow dU = -dW = 82 \text{ J}$$

(d) In this case, for an adiabatic process, $TV^{(\gamma-1)} = \text{constant}$

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\Rightarrow T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{1.5-1}$$

$$= 300 \times 2^{0.5}$$

$$\Rightarrow T_2 = 424 \text{ K}$$

(e) Work done in an isobaric process, here the pressure is kept constant.

$$W = P dV = nRdT$$

$$\text{Here, } n = PV/RT = [10^5 \times 10^{-3}]/[R \times 300] = 1/3R$$

$$\Rightarrow W = nRdT = (1/3R) \times R \times (300-424)$$

$$= -41.4 \text{ J}$$

(f) In this case, process is isothermal.

$$\text{Work done} = nR dT \ln(V_2/V_1) = (1/3R) \times R \times \ln(2) = 103 \text{ J}$$

$$(g) \text{ Total Work done in the cycle} = -82 - 41.4 + 103 = -20.4 \text{ J}$$