Exercise Solutions

Important Formulas:

1. Formula for Electrostatic force,
By Coulomb's law:

$$F_e = \frac{1}{4\pi\epsilon_o}\left(\frac{q_1 q_2}{r^2}\right)$$ ....(A)

Where,
ϵ_o is the permittivity of vacuum
q_1, q_2 is the magnitude of the charges on the charge particles
r is the distance between the two charge particles.

2. By Newton's law of gravitation, the gravitational force:

$$F_g = G\frac{m_1 m_2}{r^2}$$ ....(B)

Where G is the Gravitational constant
m_1 and m_2 are the magnitude of charges
r is the distance of separation between the masses.

Question 1: Find the dimensional formula of ϵ_0.
Solution:
Dimensions of F = $[MLT^{-2}]$
Dimensions of q = [AT]
Dimensions of r = [L]

Now, using Equation (A), we have

Dimensional of ϵ_o = $[AT][AT]/[MLT^{-2}][L^2] = [M^{-1}L^{-3}T^4A^2]$

Question 2: A charge of 1.0 C is placed at the top of your college building and another equal charge at the top of your house. Take the separation between the two charges to be 2.0 km. Find the force exerted by the charges on each other. How many times of your weight is this force?

Solution:
$q_1 = q_2 = 1.0$ C (Given)

Let "r" be the distance between the charges.

=> r = 2 km = 2×10^3 m

By Coulomb's law,
using equation (A)

$F = 9 \times 10^9 \times (1 \times 1)/(2 \times 10^3)^2 = 2.25 \times 10^3$ N

Again, we know W = mg

Let the mass of my body, m = 70 kg.

W = 70 x 9.8 = 686 N

Now, on dividing the Electrostatic force and the body weight, we have

$F/W = [2.25 \times 10^3]/686 = 3.3$ (approx)

The electric force between the two charges is 3.3 times my weight.

Question 3: At what separation should two equal charges, 1.0 C each, be placed so that the force between them equals the weight of a 50 kg person?

Solution:

$q_1 = q_2 = 1.0$ C (Given)

Let "m" be the mass of the person and "w" be weight.

=> m = 50 kg and w = mg = 50 x 9.8 = 490 N

By Coulomb's law,
using equation (A)

Here $F_e = w = 490$ N

On rearranging and substituting the given values, we get

$r = 4.3 \times 10^3$ m (approx)

Question 4: Two equal charges are placed at a separation of 1.0 m. What should be the magnitude of the charges so that the force between them equals the weight of a 50 kg person?

Solution:

$q_1 = q_2 = q$ (Given)

Let "m" be the mass of the person and "w" be weight.

=> m = 50 kg and w = mg = 50 x 9.8 = 490 N

Two equal charges are placed at a separation of 1.0 m, so r = 1.0 m

By Coulomb's law,
using equation (A)

Here F_e = w = 490 N

On rearranging and substituting the given values, we get

q = 2.3 x 10^{-4} C (approx)

Question 5: Find the electric force between two protons separated by a distance of 1 fermi (1 fermi = 10^{-15} m). The protons are a nucleus remain at a separation of this order.

Solution:

$q_1 = q_2 = 1.6\times10^{-19}$ C (Given)

Two equal charges are placed at a separation, r = 1 fermi = 10^{-15} m

By Coulomb's law,
using equation (A)

Here $F_e = k [q_1q_2]/r^2$

$= [9x10^9x1.6x10^{-19}x1.6\times10^{-19}]/(10^{-15})^2$

F_e = 230 N (approx)

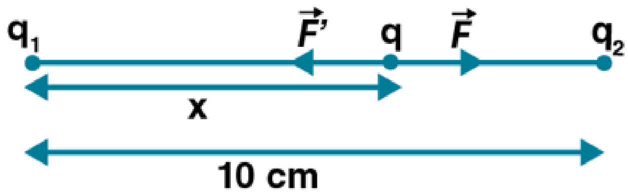
Question 6: Two charges 2.0×10^{-6} C and 1.0×10^{-6} C are placed at a separation of 10 cm. Where should a third charge be placed such that it experiences no net force due to these charges?

Solution:

$q_1 = 1.0 \times 10^{-6}$ C and $q_2 = 2.0\times10^{-6}$ C (Given)

Distance between the charge, r = 10 cm

Let "q" be the third charge and placed at a distance of x cm from the charge q_1.



By Coulomb's law,
using equation (A)

Force on charge q due to charge q_1

$F_1 = [9x10^9x2x10^{-6}xq]/x^2$

Again, Force on charge q due to charge q_2

$F_2 = [9x10^9x10^{-6}xq]/(10-x)^2$

According to question: $F_1 = F_2$

On equating above equations and solving, we get

=> $x^2 = 2(10 – x)^2$

=> $x^2 – 40x + 200 = 0$

=>$x = 20 + 10\sqrt{2}$ or $x = 20 - 10\sqrt{2}$

As x > 0. The correct option is $x = 20 + 10\sqrt{2}$

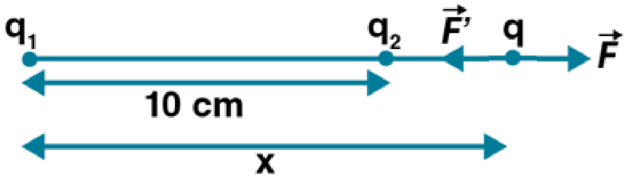
The charge q should be placed 5.9 cm from q_1.

Question 7: Suppose the second charge in the previous problem is $–1.0 \times 10^{–6}$ C. Locate the position where a third charge will not experience a net force.

Solution:

$q_1 = -1.0\times10^{-6}$ C, $q_2 = 2.0\times10^{-6}$ C (given)

3rd charge can not be placed between q_1 and q_2 as both the charges are opposite in nature.



By Coulomb's law,
using equation (A)

Force on charge q due to q_1:

$F_1 = [9x10^9x2x10^{-6}xq]/x^2$

Force on charge q due to q_2

$F_2 = [9x10^9x10^{-6}xq]/(x-10)^2$

According to question: $F_1 = F_2$

On equating above equations and solving, we get

=> $x^2 = 2(x-10)^2$

Or x = 5.86 and 34.1

Since x > 0, This implies, x = 34.14 cm

Thus, q should be placed 34.1 cm from the larger charge on the side of the smaller charge.

Question 8: Two charged particles are placed at a distance 1.0 cm apart. What is the minimum possible magnitude of the electric force acting on each charge?

Solution:
For the electric force to be minimum, the magnitude of charge on the particles must be minimum.

Magnitude of charges, $q_1 = q_2 = 1.6x10^{19}$ C

Distance between the charged particles, r = 1 cm = 0.01 m

By Coulomb's law,
using equation (A)
$F_e = [9x10^9x1.6x10^{-19}x1.6x10^{19}]/(0.01)^2$

$= 2.3 \times 10^{-24}$ N (approx)

Question 9: Estimate the number of electrons in 100 g of water. How much is the total negative charge on these electrons?

Solution:

mass of water = m = 100 g

Moles of water in 100g of water = (100/18) mol

[Molar mass of water = 18 g/mol]

Number of electrons in 1 molecules of water = (2x1)+8 = 10

Number of electrons in 6.023×10^{23}(Avogadro's number) molecules of water = $6.023 \times 10^{23} \times 10 = 6.023 \times 10^{24}$

=> Number of electrons in 18 g of water = 6.023×10^{24}

so, Number of electrons in 100 g of water = $[6.023 \times 10^{24}]/18 \times 100 = 3.34 \times 10^{25}$

We know, Negative charge on one electron = 1.6×10^{-19} C

Therefore, total charge = $3.34 \times 10^{25} \times 1.6 \times 10^{-19} = 5.35 \times 10^{6}$ C (approx)

Question 10: Suppose all the electrons of 100g water are lumped together to form a negatively charged particle and all the nuclei are lumped together to form a positively charged particle. If these two particles are placed 10.0 cm away from each other, find the force of attraction between them. Compare it with your weight.

Solution:
Mass of water = m = 100 g

Moles of water in 100g of water = (100/18) mol

[Molar mass of water = 18 g/mol]

Number of electrons in 1 molecules of electrons = 10 electrons x 6.023×10^{23} mol^{-1}

Number of electrons in 100 g of water = $10 \times 6.023 \times 10^{23} \times 100/18 = 3.35 \times 10^{25}$ electrons

We know, Negative charge on one electron = 1.6×10^{-19} C

Therefore, total negative charge = $3.35 \times 10^{25} \times 1.6 \times 10^{-19} = 5.35 \times 10^{6}$ C (approx)

Now, we have $q_1 = -5.35 \times 10^{6}$ C and $q_2 = 5.35 \times 10^{6}$ C
and r = 10 cm = 0.1 m

By Coulomb's law,
using equation (A)

The magnitude of electric force:

$F_e = [9\times10^{9} \times (5.35 \times 10^{6})^{2}]/(0.1)^{2}$

$= 2.57 \times 10^{25}$ N (approx)

Let my body mass be 70 kg, then

$F_w = mg = 70 \times 9.8 = 686$ N

Now, ratio is:

$F_e/F_w = 3.7 \times 10^{22}$

Question 11: Consider a gold nucleus to be a sphere of radius 6.9 fermi in which protons and neutrons are distributed. Find the force of repulsion between two protons situated at largest separation. Why do these protons not fly apart under this repulsion?

Solution:
Charge on each proton, $q_1 = q_2 = 1.6 \times 10^{-19}$ C
Let "d" be the diameter of the nucleus.

Radius of the sphere = R = 6.9 fermi

Largest Distance of Separation = Diameter of the nucleus = d = 2R = 13.8 fermi = 13.8×10^{-15} m

By Coulomb's law, using equation (A), we get

$F = 9 \times 10^{9} \times [1.6\times10^{-19})^{2}/(2R)^{2} = 1.2$ N

The protons do not fly apart due to the strong nuclear force which is attractive and balances the repulsive electric force.

Question 12: Two insulating small spheres are rubbed against each other and placed 1 cm apart. If they attract each other with a force of 0.1N, how many electrons were transferred from one sphere to the other during rubbing?

Solution:
force = F_e = 0.1N

r = 1 cm = 0.01 m and e = 1.6×10^{-19}

e = magnitude of charge on electron, r = Distance between the two spheres

According to question: $q_1 = q_2 = ne$

By Coulomb's law, the electric force:

$F_e = k(ne)^2/r^2$

rearranging and substituting the values, we get,

$n = 2 \times 10^{11}$

Thus, about 2×10^{11} electrons are transferred.

Question 13: NaCl molecule is bound due to the electric force between the sodium and the chlorine ions when one electron of sodium is transferred to chlorine. Taking the separation between the ions to be 2.75×10^{-8} cm, find the force of attraction between them. State the assumptions (if any) that you have made.

Solution:
Distance between the ions = r = 2.75×10^{-10} m

Charge on Na^+ ion = q = 1.6×10^{-19}

Charge on Cl^- ion = -q = -1.6×10^{-19}

By Coulomb's law, the electric force:

$F_e = kq^2/r^2$

$= [9\times10^9 \times (1.6 \times 10^{-19})^2]/[2.75 \times 10^{-10}]^2$

$= 3.05 \times 10^{-9}$ N (approx)

Question 14: Find the ratio of the electric and gravitational forces between two protons.

Solution:
The electric force: $F_e = kq^2/r^2$ ...(1)

The gravitational force, $F_g = G (m^2)/r^2$ ...(2)

Now,

$F_e/F_g = kq^2/Gm^2$

Here $k = 9\times 10^9\ NC^{-2}m^2$; $q = 1.6 \times 10^{-19}$ C ; $G = 6.67 \times 10^{-11}\ N\ Kg^{-2}m^2$ and $m = 1.67 \times 10^{-27}$ Kg

On putting all the values, we get

$F_e/F_g = 1.23 \times 10^{36}$ (approx)

Question 15: Suppose an attractive nuclear force acts between two protons which may be written as $F = CE^{-kx}/r^2$.
(a) Write down the dimensional formulae and appropriate SI units of C and κ.
(b) Suppose that $\kappa = 1\ fermi^{-1}$ and that the repulsive electric force between the protons is just balanced by the attractive nuclear force when the separation is 5 fermi. Find the value of C.

Solution:

force between two protons, $F = CE^{-kx}/r^2$ (Given)

(a) Dimensionless for k:

$[k] = 1/[r] = [L^{-1}]$

SI unit of k is m.

Nuclear force of attraction: $F = C\ e^{kr}/r^2$

Here e^{kr} is dimensionless quantity

$[C] = [F][r^2] = [MLT^{-2}][L^2]$

=> $[C] = [ML^3T^{-2}]$

And, SI unit of C is Nm^2

(b) Separation between the protons, $r = 5\text{fermi} = 5 \times 10^{-15}$ m (Given)
Charge on a proton, $q = 1.6 \times 10^{-19}$C.

The electric force between the two protons:
$F_e = kq^2/r^2$

Also, nuclear force between the protons:

$F_n = C\, e^{kr}/r^2$

These two forces balance each other, so $F_e = F_n$

=> $kq^2 = Ce^{-kr}$

Or $kq^2 /e^{-kr} = C$

=> $C = [9\text{x}10^9\text{x}(1.6 \times 10^{-19})^2]/ e^{-1\text{x}5} = 3.4 \text{ x } 10^{-26}$ Nm^2 (approx.)

Question 16: Three equal charges, 2.0×10^{-6} C each, are held fixed at the three corners of an equilateral triangle of side 5 cm. Find the Coulomb force experienced by one of the charges due to the rest two.

Solution:
Let ABC be an equilateral triangle of each side "a".

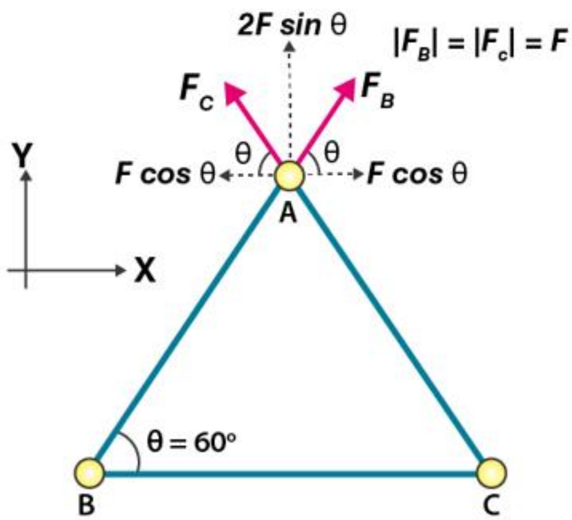
So, a = 5 cm = 0.05 m

Let the force due to charge at corner B, C be F_B, F_C.

And "q" be charge at each corner, $q = 2.0 \text{ x } 10^{-6}$.

By Coulomb's law,

$F_e = k(q_1q_2)/r^2 = 1/4\pi\epsilon_o\ [q_1q_2/r^2]$



$$F = |\overrightarrow{F_B}| = |\overrightarrow{F_C}| = k\frac{q^2}{a^2}$$

$$= \frac{9 \times 10^9 NC^{-2}m^2 \times (2.0 \times 10^{-6}C)^2}{(0.05m)^2}$$

$$= 14.4N \text{ (approx)}$$

From diagram, x and y components of F_C and F_B:

$F_{C,x} = F \cos 60^0$ and $F_{C,y} = F \sin 60^0$

$F_{b,x} = F \cos 60^0$ and $F_{b,y} = F \sin 60^0$

x's are in the opposite direction and same in magnitude, so they cancel each other.

$$|\overrightarrow{F_{net}}| = F_{c,x} + F_{c,y} = 2F \sin 60°$$

$$= 2 \times 14.4 \sin(60°)N$$

$$= 24.9N \text{ (approx)}$$

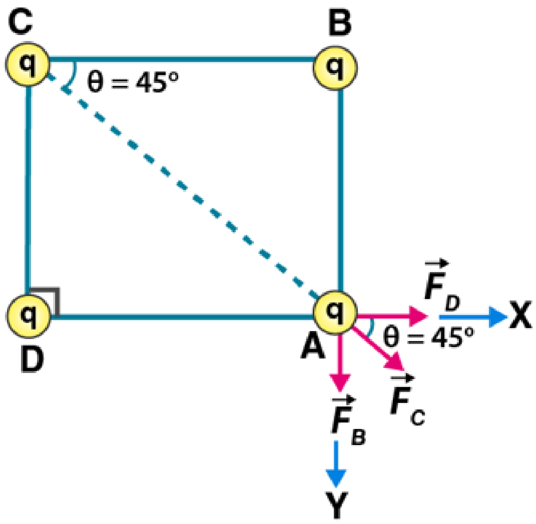
Question 17: Four equal charges 2.0 × 10^{-6} C each are fixed at the four corners of a square of side 5 cm. Find the Coulomb force experienced by one of the charges due to the rest three.

Solution:
Let ABCD be a square of each side "a".

So, a = 5 cm = 0.05 m

Let the force due to charge at corner B, C and D be F_B, F_C, F_D.

And "q" be charge at each corner, q = 2.0×10^{-6}.



From figure,

The length of diagonal = a√2 = 0.05√2 m

$$\overrightarrow{F_D} = k\frac{q^2}{a^2}\hat{\imath} = \frac{9\times10^9 NC^{-2}m^2 \times (2\times10^{-6}C)^2}{(0.05m)^2}\hat{\imath} = 14.4N\hat{\imath}$$

$$\overrightarrow{F_B} = k\frac{q^2}{a^2}\hat{\jmath} = \frac{9\times10^9 NC^{-2}m^2 \times (2\times10^{-6}C)^2}{(0.05m)^2}\hat{\jmath} = 14.4N\hat{\jmath}$$

$$|\overrightarrow{F_C}| = k\frac{q^2}{a^2} = \frac{9\times10^9 NC^{-2}m^2 \times (2\times10^{-6}C)^2}{(0.05\sqrt{2}m)^2} = 7.2N$$

Now,
$F_x = |F_C| \cos\theta = 7.2N \cos(45^o) = (7.2/\sqrt{2})$ N

$F_y = |F_C| \sin\theta = 7.2N \sin(45^o) = (7.2/\sqrt{2})$ N

$$\overrightarrow{F_C} = F_x\hat{\imath} + F_y\hat{\jmath} = \frac{14.4}{2\sqrt{2}}(\hat{\imath} + \hat{\jmath})N$$

$$\overrightarrow{F_{net}} = \overrightarrow{F_b} + \overrightarrow{F_C} + \overrightarrow{F_d} = 19.49\ (\hat{\imath} + \hat{\jmath})N$$

$$|\overrightarrow{F_{net}}| = 27.5\ N\ (approx)$$

Question 18: A hydrogen atom contains one proton and one electron. It may be assumed that the electron revolves in a circle of radius 0.53 angstrom(1 angstrom = 10^{-19} m and is abbreviated as A^o) with the proton at the center. The hydrogen atom is said to be in the ground sate in this case. Find the magnitude of the electric force between the proton and the electron of a hydrogen atom in its ground state.

Solution:
The magnitude of electric force:
[Using Coulomb's law]

$F_e = 1/4\pi\epsilon_o\ [e^2/r^2]$

$F_e = 9 \times 10^9 \times [1.6\times10^{-19}]/[(0.53\times10^{-10})^2]$

=> $F_e = 8.2 \times 10^{-8}$ N

Question 19: Find the speed of the electron in the ground state of a hydrogen atom. The description of ground state is given in the previous problem.

Solution:
The magnitude of electric force:
[Using Coulomb's law]

$F_e = 1/4\pi\epsilon_o\ [e^2/r^2]$

$F_e = 9 \times 10^9 \times [1.6\times10^{-19}]/[(0.53\times10^{-10})^2]$

=> $F_e = 8.2 \times 10^{-8}$ N
(using previous problem given)

Now,
Let F_C be the centripetal force on the electron.

$F_C = mv^2/r$

As, electric force acts as centripetal force.

So , $F_e = F_c$

=>$v = \sqrt{(F_e r)/m}$

=>$v = \sqrt{[8.2\text{x}10^{-8}\text{x}0.53\text{x}10^{-10}]/[9.11\text{x}10^{-31}]}$

=>$v = 2.18 \text{ x } 10^6 \text{ ms}^{-1}$

Question 20: Ten positively charged particles are kept fixed on the x-axis at points x = 10 cm, 20 cm 30 cm, ..., 100 cm. The first particle has a charge 1.0×10^{-8} C, the second 8×10^{-8} C, the third 27×10^{-8} C and so on. The tenth particle has a charge 1000×10^{-8} C. Find the magnitude of the electric force acting on a 1C charge placed at the origin.

Solution:

Charge on the i^{th} particle = $q_i = i^3 \text{ x } 10^{-8}$C

Distance of i^{th} particle from origin = r_1 = 10i cm = 0.1i m

Magnitude of force on the 1C charge due to i^{th} particle:

$F_i = 1/4\pi\epsilon_o\ [q_1q_2/r^2] = 1/4\pi\epsilon_o\ [1\text{x } 10^{-8}\text{x}q_i]/r_i^2]$

=>F_i = 9000i N

Now, magnitude of net force:

$$F_{x,net} = \sum_{i=1}^{10} 9000 \times i\ N$$

$$= 9000N \sum_{i=1}^{10} i$$

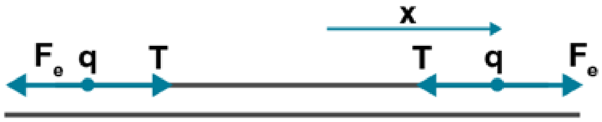
$$= 9000N \times 55N$$

$$= 4.95 \times 10^5 N$$

Question 21: Two charged particles having charge 2.0×10^{-8} C each are joined by an insulating string of length 1m and the system is kept on a smooth horizontal table. Find the tension in the string.

Solution:
Let "q" be the charge on each particle that is, 2.0×10^{-8} C

And, Distance between the two charges = r = 1m



The tension will adjust such that the net force is zero.

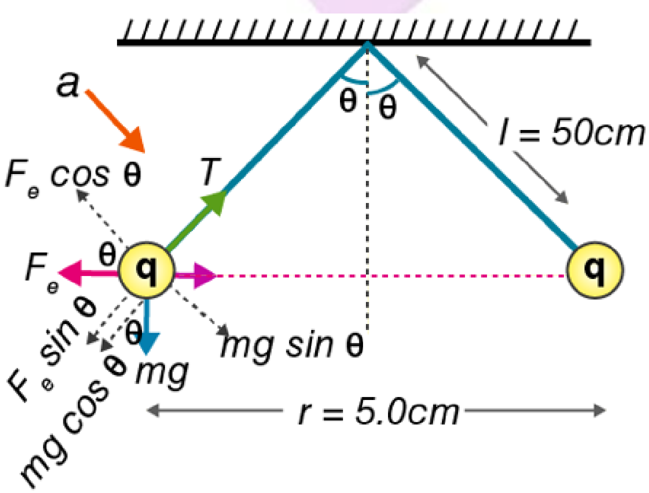
Using Coulomb's law,

$T = F_e = 1/4\pi\epsilon_o\ [q^2/r^2]$

$\Rightarrow T = [9\times10^9 x(2\times10^{-8})]/[1] = 3.6 \times 10^{-6}$ N

Question 22: Two identical balls, having a charge of 2.00×10^{-7} C and a mass of 100g, are suspended from a common point by two insulating strings each 50 cm long. The balls are held at a separation 5.0 cm apart and then released. Find
(a) the electric force on one of the charged balls
(b) the components of the resultant force on it along and perpendicular to the string
(c) the tension in the string
(d) the acceleration of one of the balls. Answers are to be obtained only for the instant just after the release.

Solution:



(a)The magnitude of the electric force is:

Using Coulomb's law,

$F_e = 1/4\pi\epsilon_o\ [q^2/r^2] = [(9x10^9)(2x10^{-7})^2]/[0.05] = 0.144$ N

Where, Distance between spheres = r = 5 cm = 0.05 m (given)
And, Magnitude of charge on each ball = $q = 2x10^{-7}$ (given)

(b) From figure,

From figure, along the string, there is component of acceleration. So, y-component of force along y-axis is zero.

Now, consider the direction perpendicular to the string

$\Sigma F_x = F_e \cos\theta - mg \sin\theta$

= 0.144 x 0.99875 - 0.1 x 9.8 x 0.05

= 0.095 N (approx)

Therefore, component of net force perpendicular to the string is 0.095N and away from the other charge.

(c) We have, $\Sigma F_y = T - F_e \sin\theta - mg \cos\theta = 0$ from figure]

Or $T = F_e \sin\theta + mg \cos\theta =$ 0.144 x 0.05 + 0.1 x 9.8 x 0.99875

= 0.986N (approx.)

(d) Apply Newton's second law along the y-direction,

$\Sigma F_y = m\, a_y$

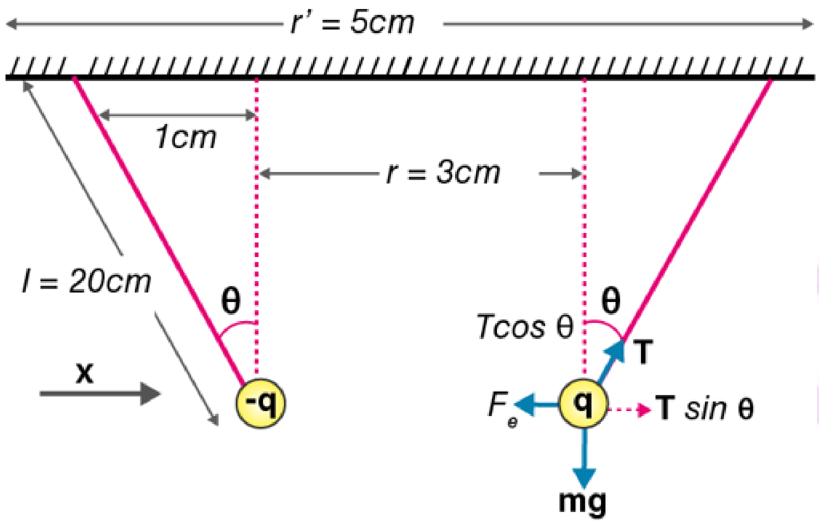
=> $a_y = 0.095/0.1 = 0.95\ m/s^2$

And we have, $a_x = 0$

So, net acceleration = a = 0.95 m/s^2

Question 23: Two identical pith balls are charged by rubbing against each other. They are suspended from a horizontal rod through two strings of length 20 cm each, the separation between the suspension points being 5 cm. In equilibrium, the separation between the balls is 3 cm. Find the mass of each ball and the tension in the strings. The charge on each ball has a magnitude 2.0×10^{-8} C.

Solution:



Since r' > r, the balls must be attracted to each other as both are oppositely charged.

At equilibrium, all forces must cancel out in accordance with newton's first law. Hence net force = 0

The magnitude of the electric force:

$F_e = 1/4\pi\epsilon_o\ [q_1q_2/r^2]$

[By Coulomb's law]

$F_e = [9\text{x}10^9\text{x}(2\text{x}10^{-8})^2]/[0.03^2] = 0.004\text{N}$

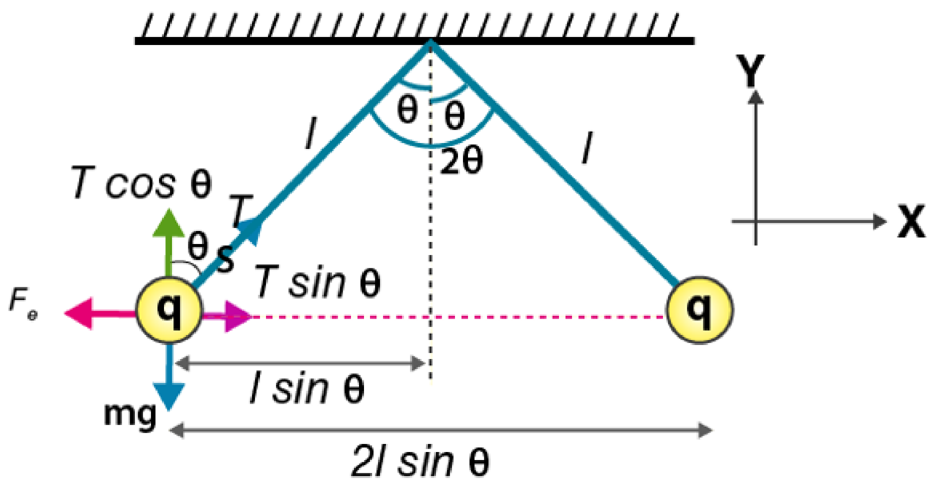
Since $T \sin\theta = F_e$ and $T \cos\theta = mg$

=> T = 0.004/0.05 = 0.08 N and m = [0.08x0.99875]/9.8 = 0.008153 kg = 8.2 g

Therefore, Tension in each of the ropes is 0.08N and Mass of each of the balls is 8.2 grams.

Question 24: Two small spheres, each having a mass of 20g, are suspended from a common point by two insulating strings of length 40 cm each. The spheres are identically charged and the separation between the balls at equilibrium is found to be 4 cm. find the charge on each sphere.

Solution:



Length of strings = l = 40cm = 0.4 m
Let "T" = magnitude of Tension, F_e = electric force between the spheres and q = magnitude of charge on each pitch ball.

Form figure, sinθ = 0.05 and cosθ = 0.99875

At equilibrium, all forces must cancel out in accordance with newton's first law. [x-coordinate and y-coordinate of the force is zero.]

=> F_e = T sinθ and mg = T cosθ

Now,

T = mg/cosθ = [0.02 x 9.8]/[0.99875] = 0.1962 N (approx)

Also, T sinθ = $1/4\pi\epsilon_o\ [q^2/r^2]$

=> $q^2 = [0.1962 \times 0.05 \times (0.04)^2]/[9 \times 10^9]$

=>q = 4.17×10^{-8} C

Question 25: Two identical pith balls, each carrying a charge q, are suspended from a common point by two strings of equal length ℓ. Find the mass of each ball if the angle between the strings is 20 in equilibrium.

Solution:
At equilibrium,
F_e = T sinθ and mg = T cosθ

Let "r" be the distance between the two charges.

$\sin\theta = (r/2)/l$ or $r = 2l \sin\theta$

Using Coulomb's law,

$F_e = 1/4\pi\epsilon_o \ [q^2/(4l^2\sin^2\theta)]$

Or $T \sin\theta = F_e = 1/4\pi\epsilon_o \ [q^2/(4l^2\sin^2\theta)]$

Or $T = 1/4\pi\epsilon_o \ [q^2/(4l^2\sin^3\theta)]$

And $m = (T \cos\theta)/g = 1/4\pi\epsilon_o \ [q^2/(4l^2\sin^3\theta)] \times \cos\theta/g$

Or $T = [q^2\cot\theta]/[16\pi\epsilon_o g\ l^2\sin\theta]$