Exercise Solutions

Formula used:
By Gauss's law, flux of net electric field through a closed surface S equals the net charge enclosed (q_{in}) by the surface divided by ϵ_0.

$$\oint \vec{E}.d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

Question 1: The electric field in a region is given by

$$\vec{E} = \frac{3}{5}E_0\vec{i} + \frac{4}{5}E_0\vec{i} \text{ with } E_0 = 2.0 \times 10^3 NC^{-1}$$

Find the flux of this field through a rectangular surface of area 0.2 m^2 parallel to the y–z plane.

Solution:

The plane is parallel to y-z plane. The normal to this plane is parallel to x-axis. Therefore, area vector for this plane is given by $0.2m^2$ i

So, surface area of plane= 0.2 m^2 i

Flux of electric field through a surface of area = ϕ = E . dS

=> ϕ = [(3/5)E_0 i + (4/5)E_0 J] . (0.2 i) = (3/5) x 0.2 x E_0

=> ϕ = (3/5) x 0.2 x 2.0 × 10^3 = 240 Nm^2/C

Question 2: A charge Q is uniformly distributed over a rod of length l. Consider a hypothetical cube of edge l with the centre of the cube at one end of the rod. Find the minimum possible flux of the electric field through the entire surface of the cube.

Solution:
Linear charge density of rod = Q/l

Where Q is total charge on the rod

Length of rode = edge of cube = l

Portion of rod inside cube = l/2

Using Gauss's theorem, flux through the hypothetical cube is $\phi = (Q_{in}/\epsilon_o)$

Where Q_{in} is the charge enclosed inside the cube

Now, charge enclosed = Q_{in} = Q/l x l/2 = Q/2

So, $\phi = (Q/2/\epsilon_o) = Q/2\ \phi = (Q_{in}/\epsilon_o)$

Question 3: Show that there can be no net charge in a region in which the electric field is uniform at all points.

Solution:
The electric field is uniform (Given)
If we consider a plane perpendicular to electric field, we find this plane is an equipotential surface. If a test charge is introduced on the surface, then the work done will be zero in moving the test charge on this surface.

But if there is net charge is present in this region, the test charge will experience a force due to which non-zero work needs to be done to move our test charge from one point to another. This means, potential difference between these two points is finite. So this fact is contradiction to the fact that the surface is equipotential.

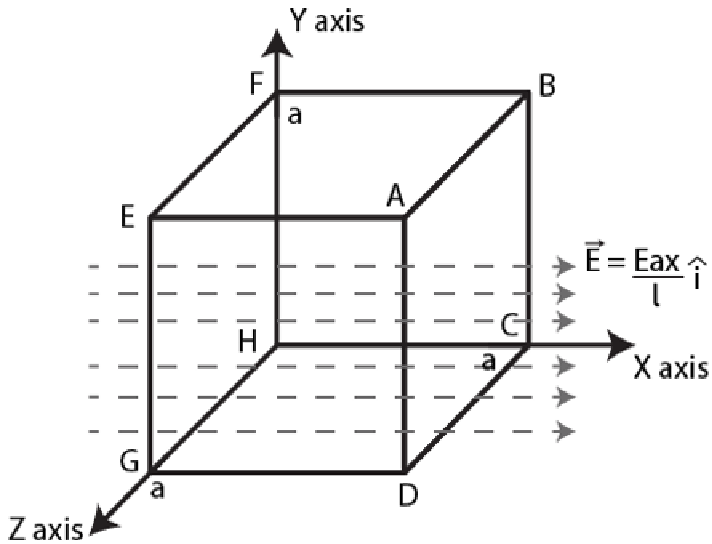
Therefore, net charge in the region of uniform electric field is zero.

Remember:
*Equipotential surfaces are perpendicular to the electric field lines

Question 4: The electric filed in a region is given by $E = [E_o x/l]$ i. Find the charge contained inside a cubical volume bounded by the surfaces x = 0, x = a, y = 0, y = a, z = 0 and z = a. Take $E_0 = 5 \times 10^3\ NC^{-1}$, l = 2 cm and α = 1 cm.

Solution:

$E = [E_o x/l]$ I (Given)
Edge of cube = a = 1 cm ; length = l = 2 cm



From figure,
we can see that electric field lines are parallel to the normal of the surfaces ABCD and EFGH therefore flux is nonzero through these surfaces, whereas for other faces of the cube electric field is perpendicular to normal of faces therefore flux through these surfaces is zero.

At EFGH, x = 0, electric field at EFGH = 0
At ABCD, x = a, electric field at ABCD is $E = E_o a/l$ i

The net flux through the whole cube is only through side ABCD is

$\phi = E . A = E_o a^2/l = [5x10^3 \text{ x } (1x10^{-2})^2]/[2x10^{-2}] = 25 \text{ NM}^2/C$

Now, net charge, $q = \epsilon_o \phi = 8.85 \text{ x } 10^{-12} \text{ x } 25 = 2.2125 \text{ x } 10^{-12}$ C

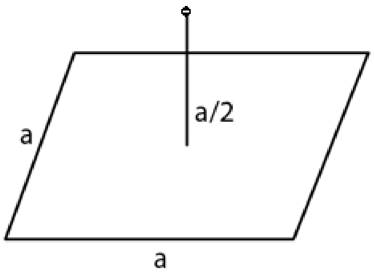
Question 5: A charge Q is placed at the centre of a cube. Find the flux of the electric field through the six surfaces of the cube.

Solution:
Total charge in the cube = Q at the center.
From Gauss's law we can conclude that total flux enclosed by the cube : $\phi = Q_{in}/\epsilon_o$
$= Q/\epsilon_o$

Since charge is placed at centre it is placed symmetrically with respect to all the faces of the cube. So, equal flux passes through all the six surfaces.
Flux passing through each surface = $\phi_0 = \phi/6$

=> $\phi_0 = Q/6\epsilon_o$

Question 6: A charge Q is placed at a distance a/2 above the centre of a horizontal, square of edge a as shown in figure. Find the flux of the electric field through the square surface.



Solution:
Length of square edge = a

Area of the plane = a^2

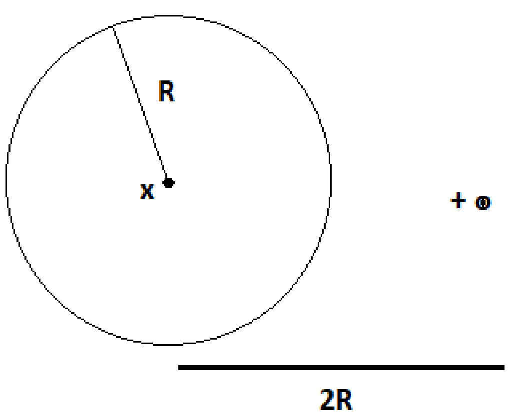
Distance of charge Q above the center = a/2

Since charge is placed at centre it is placed symmetrically with respect to all cube faces. So, equal flux passes through all the six surfaces.

Flux passing through each surface = $\phi_0 = Q/6\epsilon_o$

The flux through the given surface is $Q/6\epsilon_o$

Question 7: Find the flux of the electric field through a spherical surface of radius R due to a charge of 10^{-7} C at the centre and another equal charge at a point 2R way from the centre.



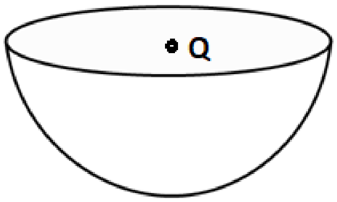
Solution:
Magnitude of charges = 10^{-7}C

By Gauss's law, flux of net electric field through a closed surface equals the net charge enclosed by the surface divided by ϵ_0

The only charge located inside the sphere will contribute to the flux passing through the sphere.

$\phi = \oint E \cdot ds = Q/\epsilon_0 = 10^{-7}/[8.85 \times 10^{-12}] = 1.1 \times 10^4\ Nm^2C^{-1}$

Question 8: A charge Q is placed at the centre of an imaginary hemispherical surface. Using symmetry arguments and the Gauss's law, find the flux of the electric field due to this charge through the surface of the hemisphere (figure).



Solution:
Consider charge Q is at the centre.
Net charge enclosed by the sphere $Q_{in} = Q$

From Gauss's law flux of electric field through this sphere is $\phi = Q/\epsilon_0$

For a hemisphere (open top), total flux through its curves surface, $\phi' = Q/2\epsilon_0$

Question 9: A spherical volume contains a uniformly distributed charge of density $2.0 \times 10^{-4}\ C\ m^{-3}$. Find the electric field at a point inside the volume at a distance 4.0 cm from the centre.

Solution:

Volume charge density = $\rho = 2.0 \times 10^{-4}\ Cm^{-3}$ (given)

Let us assume that, "r" be spherical gaussian surface inside the sphere of radius, r = 4cm

And $q = \rho \times 4/3\ \pi r^3$
Where, "q" be the Charge enclose by this gaussian spherical surface

$= 2.0 \times 10^{-4} \times 4/3\ \pi r^3$

All the points on the surface are equivalent and by symmetry we can find that field at every point on this surface is equal in magnitude and radial in direction. Therefore, flux through this surface is

$$\phi = \oint \vec{E} . d\vec{S} = E \oint d\vec{S} = E . 4\pi r^2$$

By Gauss's law, $\oint E. dS = q_{in}/\epsilon_0 = \rho \times 4 \pi r^3/ \epsilon_0$

$\Rightarrow E . 4 \pi r^2 = \rho \times 4 \pi r^3/ \epsilon_0$

$\Rightarrow E = \rho r/\epsilon_0$

Putting the value of ρ, r and ϵ_0

$E = (2x10^{-4}) \times [4x10^{-2}]/[8.85x10^{12}] = 3 \times 10^5$ N/C

Question 10: The radius of a gold nucleus (Z = 79) is about 7.0×10^{-15} m. Assume that the positive charge is distributed uniformly throughout the nuclear volume. Find the strength of the electric field at
(a) the surface of the nucleus and
(b) at the middle point of a radius.
Remembering that gold is a conductor, is it justified to assume that the positive charge is uniformly distributed over the entire volume of the nucleus and does not come to the outer surface?

Solution:
Charge on the gold nucleus = Q = 79 x (1.6 x 10^{-19}) C
(a) The value of E is fixed for a particular radius:

$$E \oint ds = \frac{Q}{\epsilon_0}$$

$$E \times 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{\epsilon_0 \times 4\pi r^2}$$

$\Rightarrow E = [79x(1.6x10^{-19})]/[(8.85x10^{-12}) \times 4 \times 3.14x(7x10^{-10})^2] = 2.31513 \times 10^{21}\ NC^{-1}$

(b) Radius = r = (7/2) x 10^{-10} m and net charge = 79 x 1.6 x 10^{-19} C

$V = (4/3) \pi r^3$

$= (4/3) \times 22/7 \times 343/8 \times 10^{-30}$

Volume charge density = $[79x1.6x10^{-19}] /[(4/3) \pi \times 343 x10^{-30}]$

Now, Charge enclosed by imaginary sphere of radius 3.5 x 10^{-10} is

= Volume charge density x V

Putting values, we get

= $[79 \times 1.6 \times 10^{-19}]/8$

Now,

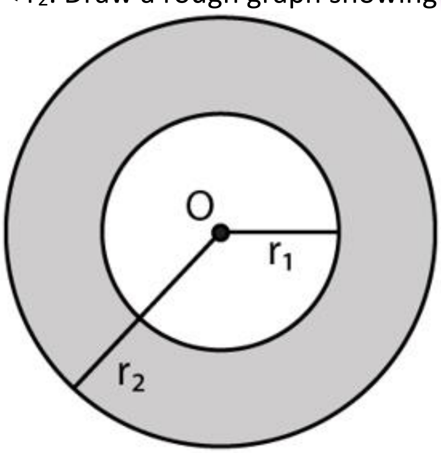
=> $E = [79 \times 1.6 \times 10^{-19}]/[8 \times 4\pi\epsilon_o r^2]$

Here $r = 3.5 \times 10^{-10}$

=> $E = 1.16 \times 10^{21}\ NC^{-1}$

when electric charge is given to a conductor it comes on its surface. But nucleons are bound by strong nuclear force inside nucleus which holds them and prevents them from coming out of conductor. So, it is justified to assume that electric charge is uniformly distributed in its entire volume.

Question 11: A charge Q is distributed uniformly within the material of a hollow sphere of inner and outer radii r_1 and r_2 figure. Find the electric field at a point P a distance x away from the centre for $r_1 < x < r_2$. Draw a rough graph showing the electric field as a function of x for $0 < x < 2r_2$ figure.



Solution:
Let r_1 and r_2 be the Inner and outer radius of hollow sphere.

Q be the amount of charge distributed within the hollow sphere.

Volume of charge = $(4/3)\,\pi\,(r_2^3 - r_1^3)$

Now, density = $Q/[(4/3)\,\pi\,(r_2^3 - r_1^3)]$

Charge enclosed by this surface:

q = Density x volume of the part of consisting charge

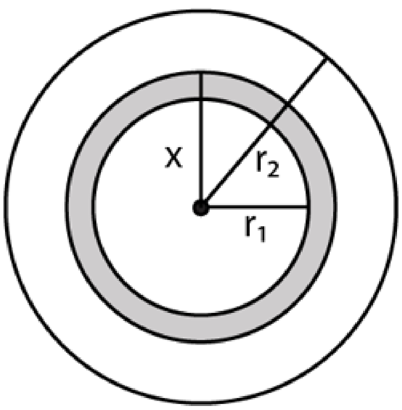
$$q = \frac{\frac{4}{3}\pi\left(x^3 - r_1^3\right)Q}{\frac{4}{3}\pi\left(r_2^3\right)}$$

$$q = \frac{\left(x^3 - r_1^3\right)}{\left(r_2^3 - r_1^3\right)}Q$$

By Gauss's law:

$\oint E.\, ds = q/\epsilon_o$

The surface integral is carried out on th esphere of radius x and q is the charged enclosed by this sphere.



$E(4\pi x^2) = q/\ \epsilon_o$

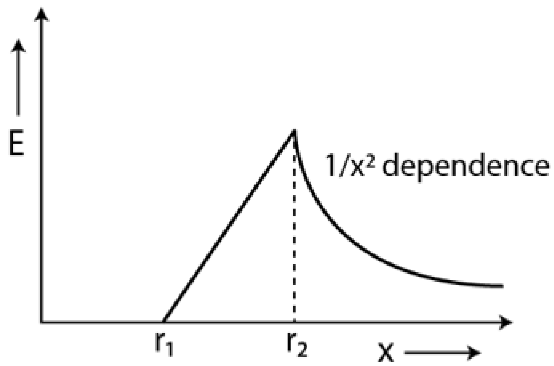
This electric field is directly proportional to x for $r_1 < x < r_2$

For $r_2 < x < 2r_2$

Total electric flux through this surface = $(4\pi x^2)$ E

or $E = Q/4\pi x^2\epsilon_o$

The graph showing electric field as a function of x



Question 12: A charge Q is placed at the centre of an uncharged, hollow metallic sphere of radius a.

(a) Find the surface charge density on the inner surface and on the outer surface.
(b) If a charge q is put on the sphere, what would be the surface charge densities on the inner and the outer surfaces?
(c) Final the electric field inside the sphere at a distance x from the centre in the situations (a) and (b).

Solution:

Surface area of sphere = $4\pi a^2$

where "a" is radius of sphere.

Also, electric fields at all points inside the conductor is zero. And Q be the amount of charge present at the centre of the hollow sphere.

Due to induction, the charge induced at the inner surface be −Q and the charge induced on the outer surface is +Q

(a) σ = charge/(total surface area)

Surface charge density of the inner surface and outer surface:

$\sigma_{in} = -Q/(4\pi a^2)$ and $\sigma_{out} = Q/(4\pi a^2)$

(b) If another charge "q" is added to the outer surface, all the charge on the metal surface will move to the outer surface. Thus, inner charge density remain same but outer charge density on the surface become Q + q.

$\sigma_{in} = -q/(4\pi a^2)$ and $\sigma_{out} = Q+q/(4\pi a^2)$

(c) Applying Gauss law on this sphere;

$\oint$ E. dS = Q/ϵ_o

Where $\oint$ dS = $4\pi\ x^2$

=> $E = Q/4\pi\epsilon_o x^2$

Where, Q be the charge enclosed by the sphere.

For situation(b) the charge enclosed by the Gaussian surface remains the same.
[since point is inside the sphere, there is no effect of the charge q given to the shell.]
Therefore, for situations (a) and (b) the electric field inside the sphere at a distance x from the centre is same.

Question 13: Consider the following very rough model of a beryllium atom. The nucleus has four protons and four neutrons confined to a small volume of radius 10^{-15} m. The two 1s electrons make a spherical charge cloud at an average distance of 1.3×10^{11} m from the nucleus, whereas the two 2s electrons make another spherical cloud at an average distance of 5.2×10^{-11} m from the nucleus. Find the electric field at

(a) a point just inside the 1s cloud and (b) a point just inside the 2s cloud.

Solution:
(a) Let r_1 is the gaussian spherical surface of radius just equal to the radius of 1s cloud.
Here, flux = $\phi = \oint E.dS = E\oint ds = E.\ 4\pi r_1^2$ ...(1)

Also, Charge enclosed by this sphere = total charge on the nucleus = q

$q = 4 \times 1.6 \times 10^{-19}$ C

Now, Using gauss's law and equation (1), we get

$E.\ 4\pi\ r^2 = q/\epsilon_o$

Here, E = electric field at a point just inside the 1s cloud.

=> $E = q/4\pi r_1^2\epsilon_o$

We are given:
Radius of 1-s charge cloud = $r_1 = 1.3\times 10^{-11}$m

=> $E = [2x1.6x10^{-19}]/[4x3.14x(1.3\times 10^{-11})^2x8.85x10^{-12} = 3.4 \times 10^{13}$ N/C

(b) Charge enclosed by this sphere = Total charge on the nucleus + charge of 2 1s electrons = q

$q = 2 \times 1.6 \times 10^{-19}$ C

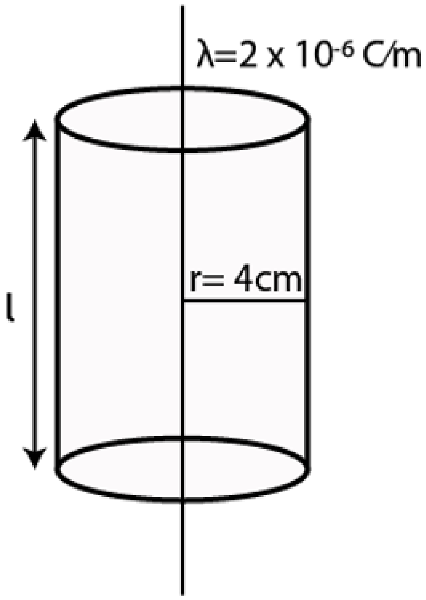
We have, $E = q/4\pi r_2^2 \epsilon_o$

Here, E = electric field at a point just inside the 2s cloud

$E = [2x1.6x10^{-19}]/[4x3.14x(5.2\times 10^{-11})^2x8.85x10^{-12} = 1.1 \times 10^{12}$ N

Question 14: Find the magnitude of the electric field at a point 4 cm away from a line charge of density 2×10^{-6} C m^{-1}.

Solution: Charge density of line charge =r = 2×10^{-6} Cm^{-1}
Distance of point from line charge = λ = 4cm = 4×10^{-2} m



Given: Radius of 2-s charge cloud = $r_2 = 5.2\times 10^{-11}$m

The charge enclosed by the gaussian surface = $\lambda \times l$

Using gauss's law: $\phi = \oint E.dS = E\oint ds = E. 2\pi rl$

=> $E. 2\pi rl = \lambda l/ \epsilon_o$

=>$E = 2 \times [10^{-6}]/[2x3.14x4x10^{-2}x8.85x10^{-12}]$ $NC^{-1} = 8.99 \times 10^5$ N/C

Which is the magnitude of electric field at a distance 4cm away from the line charge.

Question 15: A long cylindrical wire carries a positive charge of linear density 2.0×10^{-6} C m^{-1}. An electron revolves around it in a circular path under the influence of the attractive electrostatic force. Find the kinetic energy of the electron. Note that it is independent of the radius.

Solution:
Electric field E due to a linear charge distribution of linear charge density λ at a distance r from the line
$E = \lambda/2\pi\epsilon_o r$ ...(1)

And Magnitude of Force experienced by q in an electric field of intensity = F = qE

Here, charge particle is electron so the charge = q = e

As per given, $qE = mv^2/r$

We know, K.E. = $(1/2)\ mv^2$

=> $(1/2)\ mv^2 = (1/2)\ qE\ r$

or K = qEr/2

Using (1), we have

$K = q\lambda/4\pi\epsilon_o$

Thus, kinetic energy is independent of radius.

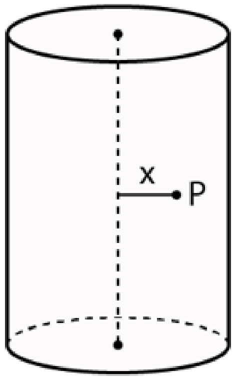
We are given:
q = charge of electron = 1.6×10^{-19}C

$\lambda = 2.0\times 10^{-6}$C and $\epsilon_o = 8.85 \times 10^{-12}$

On substituting values and solving, we get

$K = 2.88 \times 10^{-17}$ J

Question 16: A long cylindrical volume contains a uniformly distributed charge of density ρ. Find the electric field at a point P inside the cylindrical volume at a distance x from its axis.



Solution:

Volume charge density inside cylinder = ρ

Let h be the height of cylinder.

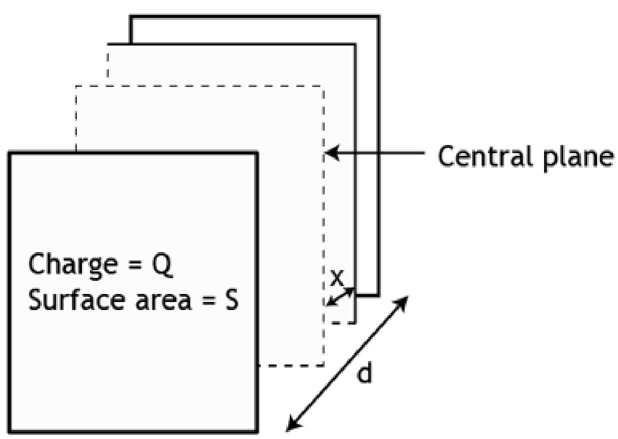
Therefore, charge Q at point P = $\rho\pi x^2 h$

For electric field $\oint E.ds = Q/\epsilon_o$

=> $E = [\rho\pi x^2 h]/[2\pi x h\epsilon_o] = \rho x/2\epsilon_o$

Question 17: A nonconducting sheet of large surface area and thickness d contains uniform charge distribution of density ρ. Find the electric field at a point P inside the plate, at a distance x from the central plane. Draw a qualitative graph of E against x for 0 < x < d.

Solution:



Volume of this cuboidal surface = V = Ax
Total flux passing through the gaussian surface = $\oint E.ds = EA$

Volume charge density of the sheet = Q/sd

Where Q = charge on the sheet and s = SA of the sheet and d= sheet thickness

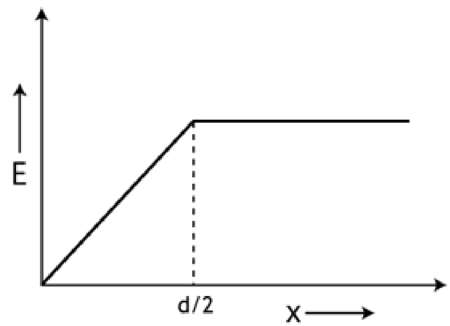
Now, $EA = \rho Ax/\epsilon_o$

=> $E = \rho x/\epsilon_o$; valid only inside the sheet i.e. when $x < d/2$

For outside the sheet (when $x > d/2$)

$EA = \rho A(d/2)/\epsilon_o = \rho d/2\epsilon_o$

Graph of electric field for $0<x<d$ is as shown below:



Question 18: A charged particle having a charge of -2.0×10^{-6} C is placed close to a nonconducting plate having a surface charge density 4.0×10^{-6} C m^{-2}. Find the force of attraction between the particle and the plate.

Solution:
The electric field due to a conducting thin sheet,

$E = \sigma/2\epsilon_o$

Force on a charge particle of charge q in presence of electric field is: $F = qE$

=> $F = q\sigma/2\epsilon_o$

The magnitude of attractive force between the particle and the plate,

$F = qE$

Putting values of σ and q,

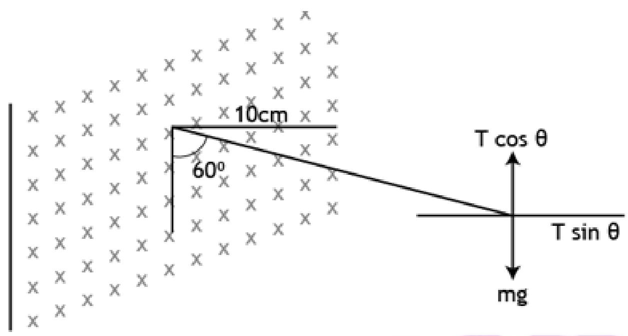
$F = [(-2.0 \times 10^{-6}) \times (4.0 \times 10^{-6})]/[2 \times (8.55 \times 10^{-12})]$

$F = 0.45$ N

Therefore, force of attraction between the particle and the plate is 0.45N.

Question 19: One end of a 10 cm long silk thread is fixed to a large vertical surface of a charged nonconducting plate and the other end is fastened to a small ball having a mass of 10 g and a charge of 4.0×10^{-6} C. In equilibrium, the thread makes an angle of 60° with the vertical. Find the surface charge density on the plate.

Solution:
Mass of ball = 10g
Length of silk thread = 10cm
Charge of ball = 4.0×10^{-6} C
Equilibrium angle of thread with vertical is 60°
Let σ be the surface charge density on the plate.



There are 2 forces acting on the ball:
W = mg (weight of the ball downwards)

And
Electric force due to non-conducting plate producing electric field E = F = qE

Due to these forces, tension develops in the thread

$E = \sigma/2\, \epsilon_o$

The tension force T due to string is divided into horizontal and vertical components i.e. T sin60° and T cos60°.

=> T sin60° = qE and T cos60° = mg

=> tan 60° = qE/mg

or E = mg tan60°/q

Also, elecrtic field due to plate,

$E = \sigma/2\,\epsilon_o$

=> $mg\tan60^o/q = \sigma/2\,\epsilon_o$

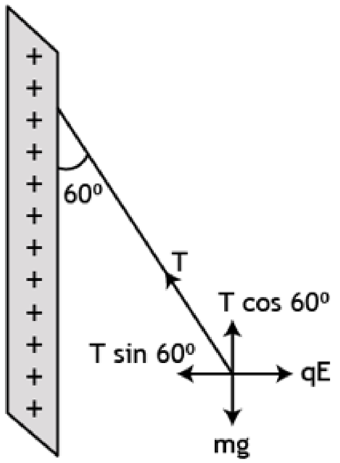
or $\sigma = 2\epsilon_o\, mg\tan60^o/q$

$$\sigma = \frac{2 \times 8.85 \times 10^{-12} \times 10 \times 10^{-3} \times 9.8 \times \sqrt{3}}{4 \times 10^{-6}}$$

$$\sigma = 7.5 \times 10^{-7}\ cm^{-2}$$

Question 20: Consider the situation of the previous problem.
(a) Find the tension in the string in equilibrium.
(b) Suppose the ball is slightly pushed aside and released. Find the time period of the small oscillations.

Solution:



There are 2 forces acting on the ball:
W = mg (weight of the ball downwards)

And
Electric force due to non-conducting plate producing electric field E = F = qE

Due to these forces, tension develops in the thread

$E = \sigma/2\epsilon_o$ ...(1)

The tension force T due to string is divided into horizontal and vertical components i.e. T sin60° and T cos60°.

Since the ball is in equilibrium, the net horizontal and vertical force on the ball is zero

By equilibrium along horizontal direction,

=> $T \sin60° = qE$ and $T \cos60° = mg$

=> $\tan 60^o = qE/mg$

or $E = mg \tan60^o/q$

Also, electric field due to plate,

(1)=> $\sigma/2\epsilon_o = mg \tan60/q$

=> $\sigma = [2x8.85x10^{-12}x10x10^{-3}x9.8x\sqrt{3}]/[4x10^{-6}]$

$= 7.5 \times 10^{-7}\ Cm^{-2}$

(a) The tension in the string in equilibrium condition

$T \cos60° = mg$

$T = 2mg = 2 \times 10 \times 10^{-3} \times 9.8 = 0.196$ N

(b) When the ball will come in mean position, tension, weight and electric force will balance.

$F = \sqrt{[(mg)^2+(q\sigma/2\epsilon o)^2]}$

[F = ma]

Time period of small oscillations = $T = 2\pi \sqrt{(l/g)}$

$= 2\pi \sqrt{((10x10^{-2}/9.8)}$

= 0.45 s

Question 21: Two large conducting plates are placed parallel to each other with a separation of 2.00 cm between them. An electron starting from rest near one of the plates reaches the other plate in 2.00 microseconds. Find the surface charge density on the inner surfaces.

Solution:
Let "a" acceleration of electron and σ is surface charge density of the plate

Using 2nd law of motion: $s = ut + (1/2) at^2$

Since electron starts from rest, so u = 0

=> $s = (1/2) at^2$

=> $a = 2s/t^2$ is the acceleration is provided by the force due to electric field between plates.

Force applied to the particle = $F = ma = 2ms/t^2$

Also, force is provided by the electric field (E) between the plates = F = qE

=> $2ms/t^2 = qE$

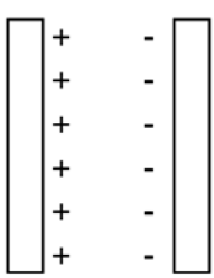
we know, Electric field due to a conducting plate of surface charge density σ is $E = \sigma/\epsilon_o$

=> $q\sigma/\epsilon_o = 2ms/t^2$

=> $\sigma = 2ms\epsilon_o/qt^2 = 2 \times 9.1 \times 10^{-31} \times 8.85 \times 10^{-12}/[1.6\times10^{-19} \times (2 \times10^{-6})^2]$ C m^{-2}

=> $\sigma = 0.503 \times 10^{-12}$ Cm^{-2}

Question 22: Two large conducting plates are placed parallel to each other and the carry equal and opposite charges with surface density σ as shown in figure. Find the electric field
(a) at the left of the plates.
(b) in between the plates and
(c) at the right of the plates.



Solution:

Let σ be the surface charge density on the plates.

The electric field due to plate 1 = $E_1 = \sigma/2\epsilon_o$

The electric field due to plate 2 = $E_2 = \sigma/2\epsilon_o$

(a) Directions of the plates will be opposite to each other on any point.

Net electric field at a point on the plates = $\sigma/2\epsilon_o - \sigma/2\epsilon_o = 0$

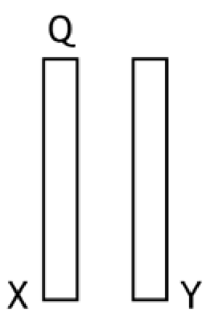
(b) Direction of the fields will be same. So, total electric field:

$\sigma/2\epsilon_o + \sigma/2\epsilon_o = \sigma/\epsilon_o$

(c) The strength of the electric field due to both the plates will be same but their directions will be opposite.

Net electric field at a point = $\sigma/2\epsilon_o - \sigma/2\epsilon_o = 0$

Question 23: Two conducting plates X and Y, each having large surface area A (on one side), are placed parallel to each other as shown in figure. The plate X is given a charge Q whereas the other is neutral. Find:
(a) the surface charge density at the inner surface of the plate X,
(b) the electric field at a point to the left of the plates
(c) the electric field at a point in between the plates and
(d) the electric field at a point to the right of the plates



Solution:
(a) Charge present on the plate is Q ...(Given)

So due to convection, other plate will get the same charge i.e. Q.

Let σ_1 and σ_2 the surface charge densities on each side of the plate.

Electric field due to a plate = $E = \sigma/2\epsilon_o$

So, magnitudes of the electric fields become: $\sigma_1/2\epsilon_o$ and $\sigma_2/2\epsilon_o$

The charge on each side = $q_1 = q_2 = Q/2$

=> Net surface charge density on each side = Q/2A
Where Area of plate = A.

(b) Electric field to the left of the plates

Charge density on the left side of the plate surface = σ = Q/2A

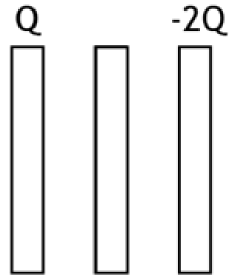
So, electric field = $Q/2A\epsilon_o$

As "x" is positively-charged plate, so above result must be directed towards the left

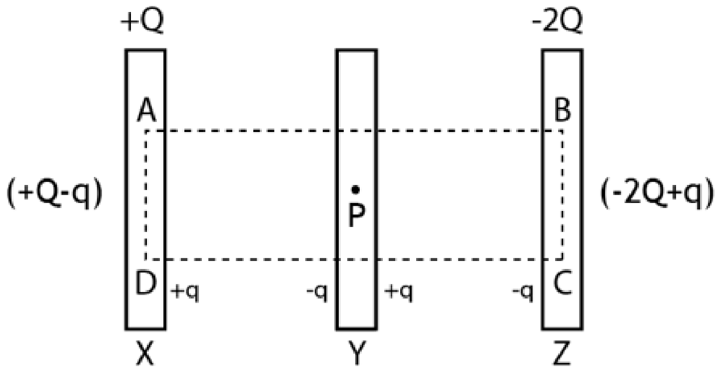
(c) Charged plate "x" acts as the only source of electric field, with positive in the inner side. A negative charge will be induced on inner side of plate Y, as it is neutral and plate Y attracts the charged particle towards itself. The middle portion E is towards the right and is equal to $Q/2A\epsilon_o$

d) The outer side of plate 'Y' acts as positive and hence it repels to the right with E = $Q/2A\epsilon_o$

Question 24: Three identical metal plates with large surface areas are kept parallel to each other as shown in figure. The leftmost plate is given a charge Q, the rightmost a charge –2Q and the middle one remains neutral. Find the charge appearing on the outer surface of the rightmost plate.



Solution:
Let "Q" be the charge on the left plate and "-2Q" be the charge on the rightmost plate.
Two faces of this closed surface lie completely in the conductor, where the electric field is zero. The other parts of the closed surface that are outside are parallel to the electric field and therefore the flow on these parts is zero. The total flow of the electric field through this closed surface is zero. According to Gauss's law, the total charge within the closed surface is zero.



The electric field due to a thin plate of charge Q : $E = Q/2A\epsilon_o$

Electric field at point P:
Due to outer surface of plate "X" towards right= $(Q-q)/2A\epsilon_o$

Due to inner surface of plate "X" towards right= $q/2A\epsilon_o$

Due to right surface of plate "Y" towards left = $q/2A\epsilon_o$

Due to Left surface of plate "Y" toward left towards left = $q/2A\epsilon_o$

Due to outer surface of plate "Z" towards right = $(2Q-q)/2A\epsilon_o$

Due to inner surface of plate "Z" towards right = $q/2A\epsilon_o$

Now, Net field at point P:

$$\frac{Q-q}{2A\epsilon_0} + \frac{q}{2A\epsilon_0} - \frac{q}{2A\epsilon_0} - \frac{q}{2A\epsilon_0} + \frac{q}{2A\epsilon_0} + \frac{2Q-q}{2A\epsilon_0} = \frac{3Q-2q}{2A\epsilon_0}$$

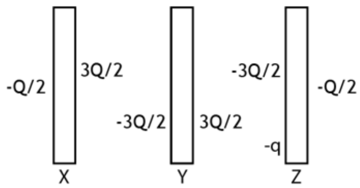
[By considering field towards right as positive and towards left as negative]

We know, field inside a point inside conductor = 0.

$(3Q-2q)/2A\epsilon_o = 0$

=> $q = 3Q/2$

The final charge distribution as:



Charge on the outer surface of rightmost plate = -2Q + q = -2Q +3Q/2 = -Q/2