**Question 1:** ABC is an isosceles triangle. If the coordinates of the base are B (1,3) and C (- 2, 7), the coordinates of vertex A can be
**Solution:**

The vertex A (x, y) is equidistant from B and C because ABC is an isosceles triangle.

Therefore,

$(y - 3)^2 = (x + 2)^2 + (y - 7)^2$

⇨ $6x - 8y + 43 = 0$

Thus, any point lying on this line can be the vertex A except the midpoint (-1/2, 5) of BC.

**Question 2:** If the vertices P, Q, R of a triangle PQR are rational points, which of the following points of the triangle PQR is (are) always rational point(s)?

A) Centroid

B) Incentre

C) Circumcentre

D) Orthocentre (A rational point is a point both of whose coordinates are rational numbers)

**Solution:**

If A = $(x_1, y_1)$ and B $(x_2, y_2)$ and C = $(x_3, y_3)$, where $x_1$, $y_1$ are rational numbers then $\sum x_1$, $\sum y_1$ are also rational. So, the coordinates of the centroid $\left(\frac{\sum x_1}{3}, \frac{\sum y_1}{3}\right)$ will be rational. As

$$c = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

Here, c may or may not be rational and it may be an irrational number of the form √p. Hence, the coordinates of the incentre

$\left(\frac{\Sigma a x_1}{\Sigma a}, \frac{\Sigma a y_1}{\Sigma a}\right)$

may or may not be rational. If (α, β) be the circumcentre or orthocentre, a and b are found by solving two linear equations in α, β with rational coefficients. So α, β must be rational numbers.

**Question 3:** The new coordinates of a point (4, 5), when the origin is shifted to the point (1, -2) are

A) (5, 3)

B) (3, 5)

C) (3, 7)

D) None of these

**Solution:**

We know that if the origin is shifted to (h, k), then new coordinates (x, y) becomes (x - h, y - k).

Therefore, the new coordinate of (4, 5) with respect to new origin (1, - 2) is (3, 7).

**Question 4:** If P = (1, 0), Q = (-1, 0) and R = (2, 0) are three given points, then the locus of a point S satisfying the relation

$SQ^2 + SR^2 = 2SP^2$ is

A) A straight line parallel to the x-axis

B) A circle through the origin

C) A circle with centre at the origin

D) A straight line parallel to the y-axis

**Solution:**

Let S (x, y), then $(x+1)^2 + y^2 + (x-2)^2 + y^2 = 2[(x-1)^2 + y^2]$

$\Rightarrow 2x + 1 + 4 - 4x = -4x + 2$

=> x = -3/2

Hence it is a straight line parallel to the y-axis.

**Question 5:** The coordinates of the points O, A and B are (0, 0), (0, 4) and (6, 0), respectively. If a points P moves such that the area of ΔPOA is always twice the area of ΔPOB, then find the equation to both parts of the locus of P.

**Solution:**

The three given points are O(0,0), A(0,4) and B(6, 0) and let P (x, y) be the moving point.

Area of ΔPOA = 2 * Area of ΔPOB

$$\Rightarrow \frac{1}{2} \times 4 \times x = \pm 2 \times \frac{1}{2} \times 6 \times y$$

$$x = \pm 3y$$

Hence, the equation to both parts of the locus of P is (x – 3y)(x + 3y) = 0

**Question 6:** Find the Orthocentre of the triangle formed by the lines x + y = 1 and xy = 0.

**Solution:**

Given lines are x + y = 1 and xy = 0 when x = 0, then y = 1 when x = 1, then y = 0

Therefore, (0, 1) and (1, 0) are the vertices of triangle.

Clearly, the triangle is right-angled isosceles.

Orthocentre of a right-angled triangle is the same as the vertex of the right angle.

Therefore, the point of intersection of x + y = 1 and xy = 0 is (0, 0).

**Question 7:** Find the circumcentre of a triangle formed by the line xy + 2x + 2y + 4 = 0 and x + y + 2 = 0 .

**Solution:**

xy + 2x + 2y + 4 = 0 .....(i) and

x + y + 2 = 0 ......(ii)

From (i) and (ii), xy = 0 ⇒ x = y = 0

Vertices of triangle are (−2, 0) (0, 0) (0, −2) (In a right angled triangle circumcentre is mid point of hypotenuse)

(−1, −1) is the circumcircle.

**Question 8:** Find the area of the triangle formed by the points (a,b + c), (b, c + a), (c, a + b).

**Solution:**

$$\text{Area} = \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} a & a+b+c & 1 \\ b & a+b+c & 1 \\ c & a+b+c & 1 \end{vmatrix}$$

$$= \frac{(a+b+c)}{2}\begin{vmatrix} a & 1 & 1 \\ b & 1 & 1 \\ c & 1 & 1 \end{vmatrix}$$

$$= 0.$$

**Question 9:** Three points are A (6, 3), B (−3, 5), C (4, −2) and P (x, y) is a point, then the ratio of the area of ΔPBC and ΔABC is

**Solution:**

$$\frac{\Delta PBC}{\Delta ABC} = \left| \frac{[-3(-2-y)+4(y-5)+x(5+2)]}{[6\,(5+2)-3(-2-3)+4(3-5)]} \right|$$

$$= \left| \frac{7x+7y-14}{49} \right| = \left| \frac{x+y-2}{7} \right|.$$

**Question 10:** If P (1, 2), Q (4, 6) R (5, 7) and S (a, b) are the vertices of a parallelogram PQRS, then find the values of a and b.

**Solution:**

Diagonals cut each other at middle points. Hence,

$$\frac{a+4}{2} = \frac{1+5}{2} \Rightarrow a = 2$$

$$\frac{b+6}{2} = \frac{2+7}{2} \Rightarrow b = 3.$$

**Question 11:** The locus of the point P (x, y) satisfying the relation

$\sqrt{(x-3)^2+(y-1)^2} + \sqrt{(x+3)^2+(y-1)^2} = 6$ is

**Solution:**

$$\sqrt{(x-3)^2+(y-1)^2} + \sqrt{(x+3)^2+(y-1)^2} = 6$$

$$\sqrt{(x-3)^2+(y-1)^2} = 6 - \sqrt{(x+3)^2+(y-1)^2}$$

Squaring both sides.

$$12x + 36 = 12\sqrt{(x+3)^2+(y-1)^2}$$

Again, squaring we get the given equation is a pair of straight lines.

**Question 12:** The equations to a pair of opposite sides of a parallelogram are $x^2 - 5x + 6 = 0$ and $y^2 - 6y + 5 = 0$. Find the equations of its diagonals.

**Solution:**

Equation of diagonal $d_1$ is $y - 1 = \frac{5-1}{3-2}(x - 2)$

$$\Rightarrow y - 1 = \frac{4}{1}(x - 2) \Rightarrow y = 4x - 7$$

Equation of diagonal $d_2$ is $y - 1 = \frac{5-1}{2-3}(x - 3)$

$$\Rightarrow y - 1 = -4(x - 3) \Rightarrow 4x + y = 13$$

So equations are, $4x + y = 13$ and $y = 4x - 7$.

**Question 13:** The area bounded by the angle bisectors of the lines $x^2 - y^2 + 2y = 1$ and the line $x + y = 3$, is

**Solution:**

The angle bisectors of the lines given by $x^2 - y^2 + 2y = 1$ are $x = 0$ and $y = 1$.

Therefore, required area = ½ x 2 x 2 = 2 sq. unit.

**Question 14:** The lines represented by the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ will be equidistant from the origin, if

$A) f^2 + g^2 = c(b - a)$

$B) f^4 + g^4 = c(bf^2 + ag^2)$

$C) f^4 - g^4 = c(bf^2 - ag^2)$

$D) f^2 + g^2 = af^2 + bg^2$

**Solution:**

Let the equations represented by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ be $lx + my + n = 0$ and $l'x + m'y + n' = 0$. Then, the combined equation represented by these lines is given by $(lx + my + n)(l'x + m'y + n') = 0$. So, it must be similar with the given equation.

On comparing, we get

$$ll' = a, \ mm' = b \ nn' = c, \ lm' + ml' = 2h,$$

$$ln' + l'n = 2g, \ \ mn' + nm' = 2f$$

According to the condition, the length of perpendiculars drawn from the origin to the lines are the same.

So,

$$\frac{n}{\sqrt{l^2+m^2}} = \frac{n'}{\sqrt{l'^2+m'^2}} = \frac{(nn')^2}{(l^2+m^2)(l'^2+m'^2)}$$

Now on eliminating l, m, l', m' and n, n', we get the required condition

$f^4 - g^4 = c(bf^2 - ag^2)$.