Question 1: Circles $x^2 + y^2 - 2x - 4y = 0$ and $x^2 + y^2 - 8y - 4 = 0$

A) Touch each other internally

B) Touch each other externally

C) Cuts each other at two points

D) None of these

Solution:

$C_1(1, 2)$, $C_2(0, 4)$

$R_1 = \sqrt{5}$, $R_2 = 2\sqrt{5}$

$C_1C_2 = \sqrt{5}$ and $C_1C_2 = |R_2 - R_1|$

Hence, circles touch internally.

Question 2: The locus of the centre of a circle passing through (a, b) and cuts orthogonally to circle $x^2 + y^2 = p^2$ is

Solution:
Let equation of circle be $x^2 + y^2 + 2gx + 2fy + c = 0$ with $x^2 + y^2 = p^2$ cutting orthogonally, we get $0+0 = c - p^2$

$c = p^2$ and passes through (a, b), we get

$a^2 + b^2 + 2ga + 2fb + p^2 = 0$

$2ax + aby - (a^2 + b^2 + p^2) = 0$

Required locus as centre $(-g, -f)$ is changed to (x, y).

Question 3: The equation of the line passing through the points of intersection of the circles $3x^2 + 3y^2 - 2x + 12y - 9 = 0$ and $x^2 + y^2 + 6x + 2y - 15 = 0$, is

Solution:
Common chord = $S_1 - S_2$

$10x - 3y - 18 = 0$

Question 4: If two circles $(x - 1)^2 + (y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then

(a) $2 < r < 8$ (b) $r = 2$

(c) $r < 2$ (d) $r > 2$

Solution:

When two circles intersect each other, then

Difference between their radii < Distance between centers

$\Rightarrow r - 3 < 5 \Rightarrow r = 8$ …..(i)

Sum of their radii > Distance between centres …..(ii)

$\Rightarrow r + 3 > 5 \Rightarrow r > 2$

Hence by (i) and (ii), $2 < r < 8$

Question 5: The length of the chord joining the points in which the straight line $x/3 + y/4 = 1$ cuts the circle $x^2 + y^2 = 169/25$ is

(a) 1 (b) 2 (c) 4 (d) 8

Solution:
Length of the perpendicular to the line $4x + 3y = 12$ from $(0, 0) = 12/5$

Radius of the circle is $13/5$

Required length = $2\sqrt{\frac{169}{25} - \frac{144}{25}} = 2$

Question 6: The equation of the chord of the circle $x^2 + y^2 = a^2$ having (x_1, y_1) as its mid point is

$A) xy_1 + yx_1 = a^2$

$B) x_1 + y_1 = a$

$C) xx_1 + yy_1 = x_1^2 + y_1^2$

$D) xx_1 + yy_1 = a^2$

Solution:

T = S_1 is the equation of desired chord, hence

$$xx_1 + yy_1 - a^2 = x_1^2 + y_1^2 - a^2 \Rightarrow xx_1 + yy_1 = x_1^2 + y_1^2.$$

Question 7: From the origin chords are drawn to the circle $(x-1)^2 + y^2 = 1$. Find the equation of the locus of the middle points of these chords.

Solution:

The given circle is $x^2 + y^2 - 2x = 0$

Let (x_1, y_1) be the middle point of any chord of this circle, then its equation is $S_1 = T$.

$$x_1^2 + y_1^2 - 2x_1 = xx_1 + yy_1 - (x + x_1)$$

If it passes through (0, 0), then $x_1^2 + y_1^2 - 2x_1 = -x_1 \Rightarrow x_1^2 + y_1^2 - x_1 = 0$

Hence, the required locus of the given point (x_1, y_1) is $x^2 + y^2 - x = 0$.

Question 8: The radius of the circle, having a centre at (2,1) whose one of the chords is a diameter of the circle

$x^2 + y^2 - 2x - 6y + 6 = 0$ is

Solution:

The centre of the given circle is (1, 3) and radius is 2. The diameter of the given circle has its midpoint as (1, 3). The radius of the required circle is 3.

Question 9: If x/α + y/β = 1 touches the circle x2 + y2 = a2, then point (1/α, 1/β) lies on a/an

A) Straight line

B) Circle

C) Parabola

D) Ellipse

Solution:

$y = -\frac{\beta}{\alpha}x + \beta$ touches the circle,

$\beta^2 = a^2\left(1 + \frac{\beta^2}{\alpha^2}\right)$

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{1}{a^2}$

Locus of $\left(\frac{1}{\alpha}, \frac{1}{\beta}\right)$ $is\ x^2 + y^2 = \left(\frac{1}{a}\right)^2$.

Question 10: Find the number of common tangents to circles $x^2 + y^2 + 2x + 8y - 23 = 0$ and $x^2 + y^2 - 4x - 10y + 9 = 0$

Solution:

$x^2 + y^2 + 2x + 8y - 23 = 0$

$therefore\ C_1(-1, -4), r_1 = 2\sqrt{10}$

Again $x^2 + y^2 - 4x - 10y + 9 = 0$

$C_2(2, 5), r_2 = 2\sqrt{5}$

Now C_1C_2 = distance between centres.

$C_1C_2 = \sqrt{9 + 81} = 3\sqrt{10} = 9.486$ and

$r_1 + r_2 = 2(\sqrt{10} + \sqrt{5}) = 10.6$

$r_1 - r_2 = 2\sqrt{5}(\sqrt{2} - 1)$

$= 2 \times 2.2 \times 0.4$

$= 4.4 \times 0.4 = 1.76$

$C_1C_2 = 2\sqrt{10} > r_1 - r_2$

$r_1 - r_2 < C_1C_2 < r_1 + r_2$

Two tangents can be drawn.

Question 11: The area of a triangle formed by the tangent, normally drawn at (1, √3) to the circle $x^2 + y^2 = 4$ and positive x-axis, is

Solution:

T = x + √3y – 4 = 0

The required area = ½ x 4 x √3 = 2√3

Question 12: A tangent to the circle $x^2 + y^2 = 5$ at the point (1,2) and the circle $x^2 + y^2 - 8x + 6y + 20 = 0$

A) Touches

B) Cuts at real points

C) Cuts at imaginary points

D) None of these

Solution:

Tangent is x − 2y − 5 = 0 and points of intersection with circle $x^2 + y^2 - 8x + 6y + 20 = 0$ are given by $4y^2 + 25 + 20y + y^2 - 16y - 40 + 6y - 20 = 0$

=> $5y^2 + 10y + 5 = 0$

=>y = -1, x = -3

i.e., touches. Answer: (a)

Question 13: If a circle passes through the points of intersection of the coordinate axis with the lines λx – y + 1 = 0 and x – 2y + 3 = 0 then find the value of λ.

Solution:

Points of the intersection with coordinate axes are, (0,1) and (-3, 0), (0, 3/2) (-1/λ , 0)

Equation of circle through (0, 1), (3, 0) and $x^2 + y^2 + 7x/2 - 5y/2 + 3/2 = 0$

It passes through (-1/λ , 0)

$$\Rightarrow \frac{1}{\lambda^2} - \frac{7}{2\lambda} + \frac{3}{2} = 0 \Rightarrow 3\lambda^2 - 7\lambda + 2 = 0$$

$$\Rightarrow \lambda = \frac{7 \pm \sqrt{49-24}}{6} = \frac{7 \pm 5}{6} = 2, \frac{1}{3}.$$

Question 14: If a > 2b > 0 then the positive value of m for which y = mx – b$\sqrt{1+m^2}$

is a common tangent to $x^2 + y^2 = b^2$ and $(x – a)^2 + y^2 = b^2$ is

Solution:

Any tangent to $x^2 + y^2 = b^2$ is y = mx - b$\sqrt{1+m^2}$

It touches $(x – a)^2 + y^2 = b^2$ if

$$\frac{ma - b\sqrt{1+m^2}}{\sqrt{m^2+1}} = b$$

$$ma = 2b\sqrt{1+m^2}$$

$$m^2a^2 = 4b^2 + 4b^2m^2,$$

$$m = \pm \frac{2b}{\sqrt{a^2-4b^2}}.$$

Question 15: The lines 2x −3y = 5 and 3x − 4y = 7 are the diameters of a circle of area 154 square units.

The equation of the circle is

Solution:

Centre of circle = Point of intersection of diameters = (1, -1)

Now, area = 154

=>πr^2 = 154 => r = 7

Hence, the equation of required circle is

$$(x-1)^2 + (y+1)^2 = 7^2 \Rightarrow x^2 + y^2 - 2x + 2y = 47.$$