Question 1: Evaluate $\int \frac{(x-1)}{(x+1)^3}\, e^x\, dx$

Solution:

$$\int \frac{x-1}{(x+1)^3}\, e^x dx \;=\; \int e^x \left(\frac{(x+1)}{(x+1)^3} - \frac{2}{(x+1)^3} \right) dx$$

$$= \int e^x \left(\frac{1}{(x+1)^2} - \frac{2}{(x+1)^3} \right) dx$$

$$= \frac{e^x}{(x+1)^2} + c$$

Using formula $\frac{d}{dx}\left(\frac{1}{(x+1)^2}\right) = -\frac{2}{(x+1)^3}$

Question 2: Evaluate $\int \frac{e^x(1+\sin x)}{(1+\cos x)^3}\, dx$ = e^x f(x) + c, then find f(x).

Solution:

$$I = \int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx = \int e^x \left[\frac{1+2\sin(x/2)\,\cos(x/2)}{2\cos^2(x/2)}\right] dx$$

$$I = \int e^x \left[\tfrac{1}{2}\sec^2(x/2) + \tan(x/2)\right] dx = e^x.\tan(x/2) + c$$

Because, $\int e^x [f(x) + f'(x)]dx = e^x.\, f(x) + c\}$

Question 3: Evaluate $\int x \sin^2 x \; dx$

Solution:

$$\int x\sin^2 x\, dx = \int x \,.\, \frac{(1-\cos 2x)}{2}\, dx$$

$$= \frac{1}{2}\left[\int x\, dx - \int x \,.\, \cos 2x\, dx\right]$$

$$= \frac{x^2}{4} - \frac{x}{4}\sin 2x - \frac{1}{8}\cos 2x + c.$$

Question 4: Evaluate $\int x^2\, sin\, 2x \;\; dx$

Solution:

$$I = \int x^2 \sin 2x\, dx = \frac{-x^2 \cos 2x}{2} + \int \frac{2x\cos 2x}{2}\, dx + c$$

$$= -\frac{x^2\cos 2x}{2} + \frac{x\sin 2x}{2} + \frac{\cos 2x}{4} + c.$$

Question 5: Evaluate $\int e^x\, sin\, x \;\; dx$

Solution:

$$I = \int e^x \sin x\, dx = e^x \sin x - \int e^x \cos x\, dx + c$$

$$= e^x \sin x - e^x \cos x - \int e^x \sin x\, dx + c$$

$$2I = e^x(\sin x - \cos x) + c$$

$$I = \frac{1}{2}e^x(\sin x - \cos x) + c.$$

Question 6: Evaluate $\int \cos\sqrt{x} \;\; dx$

Solution:

Put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}}\, dx = dt \Rightarrow dx = 2t\, dt$

$$\int 2t \,.\cos t \, dt = 2\left[t \,.\, \sin t - \int \sin t \, dt\right]$$

$$= 2t\sin t + 2\cos t$$

$$= 2[\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}] + c.$$

$$Putting\ \sin^{-1}x = t => \frac{1}{\sqrt{1-x^2}}\,dx = dt$$

Question 7: Simplify $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

Solution:

$$\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \int t\sin t\,dt = -t\cos t + \sin t + c$$

$$= -\sin^{-1}x\cos(\sin^{-1}x) + \sin(\sin^{-1}x) + c$$

$$= x - \sin^{-1}x\sqrt{1-x^2} + c.$$

Question 8: Evaluate $\int \frac{dx}{\sin x+\cos x}$

Solution:

$$\int \frac{dx}{\sin x+\cos x} = \frac{1}{\sqrt{2}}\int \frac{dx}{\sin x\cos\frac{\pi}{4}+\cos x\sin\frac{\pi}{4}}$$

$$= \frac{1}{\sqrt{2}}\int \operatorname{cosec}\left(x+\frac{\pi}{4}\right)dx = \frac{1}{\sqrt{2}}\log\tan\left(\frac{\pi}{8}+\frac{x}{2}\right) + c.$$

Question 9: Evaluate $\int \sqrt{x^2+a^2}\;dx$

Solution:

$$I = \int \sqrt{x^2 + a^2}\, dx$$

$$= \int \sqrt{x^2 + a^2}\, .1\, dx$$

$$= \sqrt{x^2 + a^2} \int 1 dx - \int \left[\frac{d}{dx}\left(\sqrt{x^2 + a^2}\right) \int 1\, dx\right]\, dx$$

$$= x\sqrt{x^2 + a^2} - \int \left[\frac{2x}{2\sqrt{x^2+a^2}} x\right] dx$$

$$= x\sqrt{x^2 + a^2} - \int \left[\frac{x^2+a^2-a^2}{\sqrt{x^2+a^2}}\right] dx$$

$$= x\sqrt{x^2 + a^2} - \int \left[\sqrt{x^2 + a^2} - \frac{a^2}{\sqrt{x^2+a^2}}\right] dx$$

$$= x\sqrt{x^2 + a^2} - \int \sqrt{x^2 + a^2} dx + a^2 \int \frac{dx}{\sqrt{x^2+a^2}}$$

$$= \sqrt{x^2 + a^2} - I + a^2 \log\left[x + \sqrt{x^2 + a^2}\right] + C$$

$$2I = x\sqrt{x^2 + a^2} + a^2 \log\left[x + \sqrt{x^2 + a^2}\right] + C$$

$$I = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2} \log\left[x + \sqrt{x^2 + a^2}\right] + C.$$

Question 10: Evaluate $\int \frac{(x+1)^2}{x(x^2+1)}\, dx$

Solution:

$$\int \frac{(x+1)^2\, dx}{x(x^2+1)} = \int \frac{x^2+1+2x}{x(x^2+1)} dx = \int \frac{x^2+1}{x(x^2+1)} dx + 2\int \frac{x}{x(x^2+1)} dx$$

$$= \int \frac{dx}{x} + 2\int \frac{dx}{x^2+1} = \log_e x + 2\tan^{-1} x + c$$

Question 11: Find the value of A and B if $\int \frac{\sin x}{\sin (x-\alpha)}\, dx = Ax + B \log \sin(x - \alpha) + C$

Choose the right option:

(a) (sin α, cos α) (b) (cos α, sin α)

(c) (- sinα, cosα) (d) (-cosα, sinα)

Solution: Correct option is (b)

Explanation:

Let x – α = m

$$\int \frac{\sin(\alpha + m)}{\sin\ m}\ dm = \sin\alpha \int \cot m\ dm + \cos\alpha \int dm$$

= cos α(x – α) + sin α *ln*|sin m| + c

Therefore, on comparing we have

A = cos α and B = sin α

Question 12: Solve $\int\left(\frac{(\log x - 1)}{1 + (\log x)^2}\right)$^2 dx

Solution:

$$\int \left(\frac{(log\ x-1)}{1+(log\ x)^2}\right)^2 \text{ dx}$$

$$= \int \frac{(log\ x-1)^2}{(1+(log\ x)^2)^2} \text{ dx}$$

$$= \int \left[\frac{1}{1+(logx)^2} - \frac{2logx}{1+(logx)^2}\right]$$

Put log x = t so dx = e^t dt

$$\Rightarrow \int \left[\frac{e^t}{1+t^2} - \frac{2te^t}{(1+t^2)^2}\right] dt$$

$$\Rightarrow \int e^t \left[\frac{1}{1+t^2} - \frac{2t}{(1+t^2)^2}\right] dt$$

$$= \frac{e^t}{1+t^2} + c$$

$$= \frac{x}{1+(log\ x)^2} + c$$

Question 13: Find the value of

$$\sqrt{2}\int \frac{sin\ x}{sin(x-\pi/4)}\ dx$$

Solution:

$$\sqrt{2}\int \frac{sin\ x}{sin(x-\pi/4)}\ dx$$

$$= \sqrt{2}\int \frac{sin\ (x-\pi/4+\pi/4)}{sin(x-\pi/4)}\ dx$$

$$= \sqrt{2}\int [cos(\pi/4) + \ cot(x-\pi/4) sin\pi/4]\ dx$$

$$= \int dx + \ \int cot(x-\pi/4)\ dx$$

= x + ln|sin(x − π/4)| + c