

# KBPE Class 10th Maths Question Paper With Solution 2019

QUESTION PAPER CODE S 1935

Answer any three questions from 1 to 4. Each question carries 2 scores. [3 \* 2 = 6]

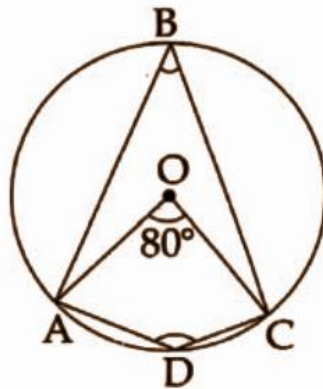
Question 1: In the figure, O is the centre of the circle.

$$\angle AOC = 80^\circ$$

[i] What is the measure of  $\angle ABC$ ?

[ii] What is the measure of  $\angle ADC$ ?

Solution:



Given  $\angle AOC = 80^\circ$

[i] The measurement  $\angle ABC = (1 / 2) * \angle AOC = 1 / 2 \times 80 = 40^\circ$ .

[ii]  $\angle ABC + \angle ADC = 180^\circ$

$$40^\circ + \angle ADC = 180^\circ$$

$$\angle ADC = 180^\circ - 40^\circ$$

$$\angle ADC = 140^\circ$$

**Question 2: [i] Write the first integer term of the arithmetic sequence  $(1/7)$ ,  $(2/7)$ ,  $(3/7)$  .....**

**[ii] What is the sum of the first 7 terms of the above sequence?**

**Solution:**

[i] Given arithmetic sequence =  $1/7 + 2/7 + 3/7$ , .....

Common difference  $d = 2/7 - 1/7 = 1/7$ .

Hence the first integer term =  $7/7 = 1$

[ii]  $a = (1/7)$

$d = 2/7 - 1/7$

$= 1/7$

$n = 7$

$S_n = (n/2) (2a + [n - 1]d)$

$S_7 = (7/2) (2 * [1/7] + [7 - 1] * [1/7])$

$= (7/2) ([2/7] + 6 * (1/7))$

$= (7/2) ([2/7] + [6/7])$

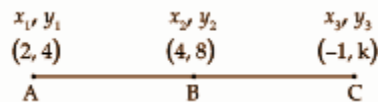
$= (7/2) (8/7)$

$= 4$

**Question 3: [i] If C  $(-1, k)$  is a point on the line passing through the points A  $(2, 4)$  and B  $(4, 8)$  which number is k?**

**[ii] What is the relation between the x coordinate and the y coordinate of any point on this line?**

**Solution:**



[i]

Points A, B and C are collinear.

Area of triangle ABC = 0

$(1/2) (x_1 [y_2 - y_3] + x_2 [y_3 - y_1] + x_3 [y_1 - y_2])$

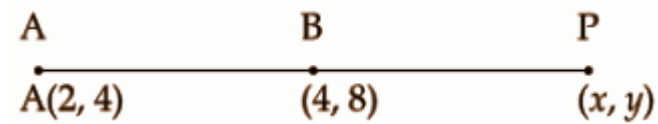
$|2(8 - k) + 4(k - 4) + (-1)(4 - 8)| = 0$

$16 - 2k + 4k - 16 - 4 + 8 = 0$

$2k = -4$

$$k = -2$$

[ii]



Area of triangle ABP = 0

$$(1/2) (x_1 [y_2 - y_3] + x_2 [y_3 - y_1] + x_3 [y_1 - y_2])$$

$$|2(8 - y) + 4(y - 4) + (x)(4 - 8)| = 0$$

$$16 - 2y + 4y - 16 - 4x = 0$$

$$2y - 4x = 0$$

$$2y = 4x$$

$$y = 2x$$

$$2x - y = 0$$

**Question 4: [i] Find P(1) if  $P(x) = x^2 + 2x + 5$**

**[ii] If  $(x - 1)$  is a factor of  $x^2 + 2x + k$ , what is the value of  $k$ ?**

**Solution:**

$$[i] P(x) = x^2 + 2x + 5$$

$$P(1) = 1^2 + 2 * 1 + 5$$

$$= 1 + 2 + 5$$

$$P(1) = 8$$

[ii] Since  $(x - 1)$  is the factor of  $x^2 + 2x + k$ , then

$$x - 1 = 0$$

$$x = 1$$

$$(1)^2 + 2(1) + k = 0$$

$$1 + 2 + k = 0$$

$$k = -3$$

**Answer any five questions from 5 to 11. Each question carries 3 scores. [5 \* 3 = 15]**

**Question 5:** [i] What is the remainder on dividing the terms of the arithmetic sequence 100, 107, 114 ..... by 7?

[ii] Write the sequence of all three-digit numbers. Which leaves the remainder 3 on division by 7? Which is the last term of this sequence?

**Solution:**

[i] Given sequence be 100, 107, 114, .....

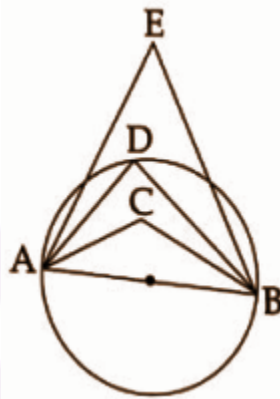
$$d = 7$$

$$\text{Remainder} = 100 / 7 = 2$$

[ii] 101, 108, 115 .....

Hence the last three-digit term = 997.

**Question 6:** AB is the diameter of the circle. D is the point on the circle.



$\angle ACB + \angle ADB + \angle AEB = 270^\circ$ . The measure of one among  $\angle ACB$ ,  $\angle ADB$  and  $\angle AEB$  is  $110^\circ$ . Write the measures of  $\angle ACB$ ,  $\angle ADB$ ,  $\angle AEB$ .

**Solution:**

$$\angle ADB = 90^\circ \text{ (Measurement of semi circle angle)}$$

$$\angle ACB + \angle ADB + \angle AEB = 270^\circ \text{ (given)}$$

$$\angle ACB + 90^\circ + \angle AEB = 270^\circ$$

$$\angle ACB + \angle AEB = 270^\circ - 90^\circ = 180^\circ$$

The given condition is that any one of the angles  $\angle ACB$ ,  $\angle AEB$  be  $110^\circ$ .

$$\text{Take } \angle ACB = 110^\circ$$

$$\text{Hence } \angle AEB = 180^\circ - 110^\circ = 70^\circ$$

So the angles ,  $\angle ADB = 90^0$  ,  $\angle ACB = 110^0$  ,  $\angle AEB = 70^0$ .

**Question 7: If x is a natural number,**

**[a] What number is to be added to  $x^2 + 6x$  to get a perfect square?**

**[b] If  $x^2 + ax + 16$  is a perfect square number, then which number is a?**

**[c] If  $x^2 + ax + b$  is a perfect square, prove that  $a^2 = 4b$ .**

**Solution:**

Given  $x^2 + 6x$

[a]  $6x = 2ab$

$a = x$

$b = ?$

$b = 6x / 2x = 3$

Perfect square =  $b^2 = 3^2 = 9$ .

Hence 9 is to be added to them.

[b] Given,  $x^2 + ax + 16$  is perfect square

This is the form of  $a^2 + 2ab + b^2 = (a + b)^2$

$2ab = ax$

$a = x$

$b^2 = 16$

$b = \sqrt{16} = 4$

So,  $(x + 4)^2 = x^2 + ax + 16$

Hence  $a = 2ab = 2 \times 4 = 8$ .

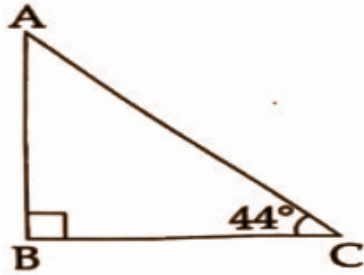
[c] Here  $b$  = the square of the half of  $a$

$b = (a / 2)^2$

$b = a^2 / 4$

$a^2 = 4b$

**Question 8: In the figure,  $\angle B = 90^\circ$ ,  $\angle C = 44^\circ$ .**



[a] What is the measure of A?

[b] Which among the following is  $\tan 44^\circ$ :

$(AB / BC)$  ,  $(AB / AC)$  ,  $(BC / AB)$  ,  $(BC / AC)$

[c] Prove that  $\tan 44^\circ * \tan 46^\circ = 1$

**Solution:**

$$[a] \angle A + \angle B + \angle C = 180^\circ$$

$$\angle A + 90^\circ + 44^\circ = 180^\circ$$

$$\angle A = 180^\circ - 90^\circ - 44^\circ$$

$$\angle A = 46^\circ$$

[b] In triangle ABC,

$$\tan 44^\circ = \text{opposite} / \text{adjacent}$$

$$= AB / BC \text{ \{from the figure\}}$$

$$[c] \text{ Take LHS} = \tan 44^\circ * \tan 46^\circ$$

$$= \tan 44^\circ * \cot (90^\circ - 46^\circ) [\tan \theta = \cot (90 - \theta)]$$

$$= \tan 44^\circ * \cot 44^\circ$$

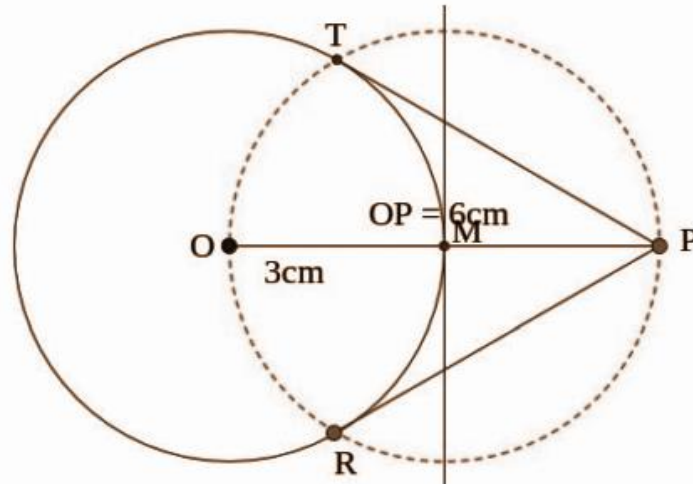
$$= \tan 44^\circ * [1 / \tan 44^\circ]$$

$$= 1$$

$$= \text{RHS}$$

**Question 9:** Draw a circle of radius 3 centimetres. Mark a point P at a distance of 6cm from the centre of the circle. Draw tangents from P to the circle.

**Solution:**



Steps of construction:

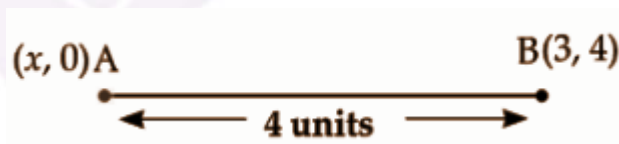
- Draw a circle of radius 3cm with O as the centre.
- From the centre O, draw  $OP = 6\text{cm}$  and perpendicular to OP marking it as M.
- Draw another circle with centre M cutting T and R respectively.
- Join PT and PR which are the required tangents.

**Question 10: [i] Find the coordinates of the point on the x-axis, which is at a distance of 4 units from (3, 4).**

**[ii] Find the coordinates of the point on the x-axis at a distance of 5 units from (3, 4).**

**Solution:**

[i]



$$4 = \sqrt{(x - 3)^2 + (0 - 4)^2}$$

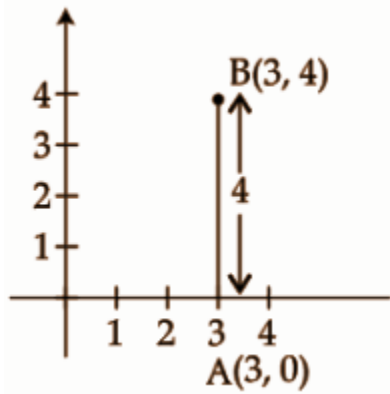
On squaring both sides,

$$4^2 = (x - 3)^2 + 16$$

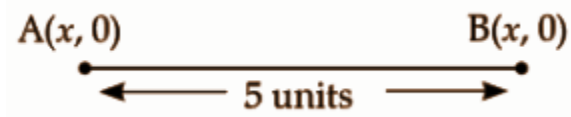
$$x - 3 = 0$$

$$x = 3$$

A (3, 0) is the required point.



[ii]



$$AB = 5$$

$$\sqrt{(x - 3)^2 + (0 - 4)^2} = 5$$

On squaring both sides,

$$(x - 3)^2 + 16 = 5^2$$

$$(x - 3)^2 + 16 = 25$$

$$(x - 3)^2 = 25 - 16$$

$$(x - 3)^2 = 9$$

$$(x - 3) = \pm 3$$

$$x - 3 = 6$$

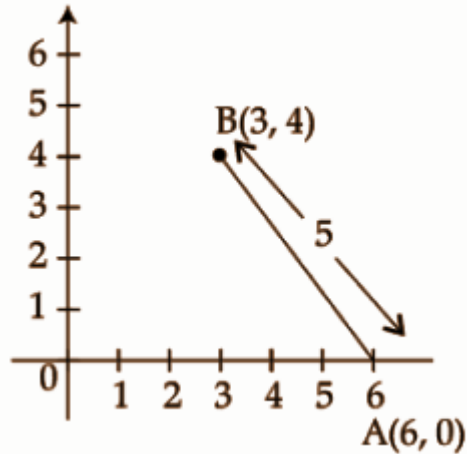
$$x = 6$$

$$x - 3 = -3$$

$$x = 0$$

Hence, (6, 0) and (0, 0) is the required point.

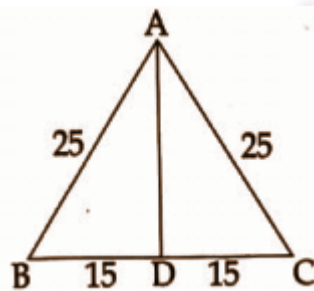




**Question 11:** The given figure is the lateral face of a square pyramid.  $AB = AC = 25$  centimeters and  $BD = DC = 15$  centimeters.

[i] What is the length of its base edge?

[ii] Find the lateral surface area of the pyramid.



**Solution:**

Side of the base = diagonal /  $\sqrt{2}$

$$= (30 / \sqrt{2}) * (\sqrt{2} / \sqrt{2})$$

$$= 15 \sqrt{2} \text{ cm}$$

$$= 15 * 1.414$$

$$= 17.210$$

Side of the base = 17.210 cm

Lateral surface area =  $(1 / 2) * \text{perimeter of the base} * \text{slant height}$

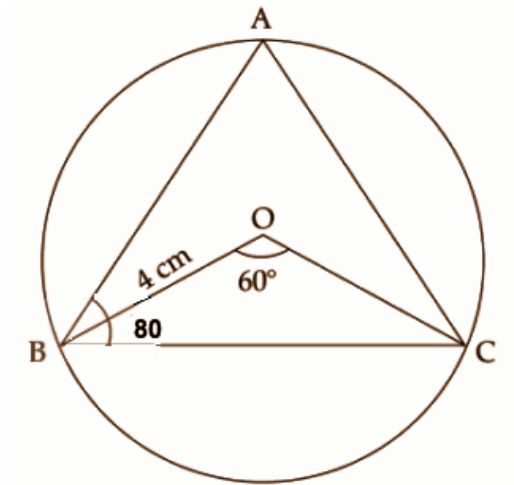
$$= (1 / 2) * (17.21) * 4 * 25$$

$$= 860.50 \text{ cm}^2$$

**Answer any 7 questions from 12 to 21. Each question carries 4 scores. [7 \* 4 = 28]**

**Question 12:** In triangle ABC,  $\angle A = 30^\circ$ ,  $\angle B = 80^\circ$ , the circumradius of the triangle is 4 centimetres. Draw the triangle. Measure the length of its smallest side.

**Solution:**



Steps of construction:

- Draw a circle of radius 4cm having a centre at O.
- Make an angle  $\angle BOC = 60^\circ$ .
- Construct an angle  $\angle CBA = 80^\circ$ .
- Join AC.
- $\triangle ABC$  is the required triangle.

$$\angle A + \angle B + \angle C = 180^\circ$$

$$30^\circ + 80^\circ + \angle C = 180^\circ$$

$$\angle C = 180^\circ - 110^\circ$$

$$\angle C = 70^\circ$$

$$30^\circ < 70^\circ < 80^\circ$$

$$\angle A < \angle C < \angle B$$

The smallest angle is  $\angle A$ .

BC is the smallest side of  $\triangle ABC$ .

**Question 13:** Find the following sums:

[i]  $1 + 2 + 3 + \dots + 100$

[ii]  $1 + 3 + 5 + \dots 99$

[iii]  $2 + 4 + 6 + \dots 100$

[iv]  $3 + 7 + 11 + \dots 199$

**Solution:**

[i]  $1 + 2 + 3 + \dots 100$

$$a = 1$$

$$d = 2 - 1 = 1$$

$$\text{Last term} = 100 = l$$

$$l = a + (n - 1) d$$

$$100 = 1 + (n - 1) 1$$

$$100 = 1 + n - 1$$

$$n = 100$$

$$S_n = (n / 2) (n + 1)$$

$$S_{100} = (100 / 2) (100 + 1)$$

$$= (50) * (101)$$

$$= 5050$$

[ii]  $1 + 3 + 5 + \dots 99$

$$a = 1$$

$$d = 3 - 1 = 2$$

$$\text{Last term} = 99 = l$$

$$l = a + (n - 1) d$$

$$99 = 1 + (n - 1) 2$$

$$99 = 1 + 2n - 2$$

$$99 = 2n - 1$$

$$100 = 2n$$

$$100 / 2 = n$$

$$50 = n$$

$$S_n = (n / 2) (a + a_n)$$

$$S_{50} = (50 / 2) (1 + 99)$$

$$= (25) * (100)$$

$$= 2500$$

[iii]  $2 + 4 + 6 + \dots 100$

$$a = 2$$

$$d = 4 - 2 = 2$$

$$\text{Last term} = 100 = l$$

$$l = a + (n - 1) d$$

$$100 = 2 + (n - 1) 2$$

$$100 = 2 + 2n - 2$$

$$100 = 2n$$

$$n = 100 / 2$$

$$n = 50$$

$$S_n = (n / 2) (a + a_n)$$

$$S_{50} = (50 / 2) (2 + 100)$$

$$= (25) * (102)$$

$$= 2550$$

$$[\text{iv}] 3 + 7 + 11 + \dots\dots 199$$

$$a = 3$$

$$d = 7 - 3 = 4$$

$$\text{Last term} = 199 = l$$

$$l = a + (n - 1) d$$

$$199 = 3 + (n - 1) 4$$

$$199 = 3 + 4n - 4$$

$$199 = 4n - 1$$

$$200 / 4 = n$$

$$n = 50$$

$$S_n = (n / 2) (a + a_n)$$

$$S_{50} = (50 / 2) (3 + 199)$$

$$= (25) * (202)$$

$$= 5050$$

**Question 14: A box contains some green and blue balls. 7 red balls are put into it. Now the probability of getting a red ball from the box is  $7 / 24$  and that of the blue ball is  $1 / 6$ .**

**[i] How many balls are there in the box?**

**[ii] How many of them are blue?**

**[iii] What is the probability of getting a green ball from the box?**

**Solution:**

Let the number of green balls be  $x$ .

The number of blue balls is  $y$ .

Number of red balls = 7

Total number of balls =  $x + y + 7$

$P(\text{red ball}) = 7 / 24$

$P(\text{blue ball}) = 1 / 3$

[i] Since  $P(\text{red ball}) = 7 / 24$ ,

$$7 / [x + y + 7] = 7 / 24$$

$$24 = x + y + 7$$

$$24 - 7 = x + y$$

$$17 = x + y \text{ ---- (1)}$$

$P(\text{blue ball}) = 1 / 3$

$$y / [x + y + 7] = 1 / 3$$

$$3y = x + y + 7$$

$$2y = x + 7$$

$$-x + 2y = 7 \text{ ---- (2)}$$

On adding equation (1) and (2),

$$17 = x + y$$

$$-x + 2y = 7$$

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$$3y = 24$$

$$y = 24 / 3$$

$$y = 8$$

Put  $y = 8$  in equation (1),

$$17 = x + 8$$

$$17 - 8 = x$$

$$x = 9$$

Total number of balls =  $8 + 9 + 7 = 24$

[ii] Number of blue balls

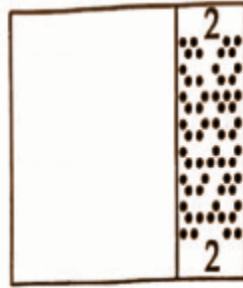
$$y / 24 = 1 / 3$$

$$3y = 24$$

$$y = 8$$

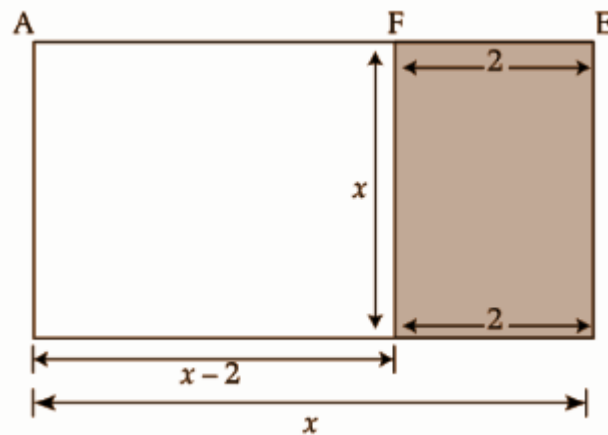
$$[\text{iii}] P (\text{green ball}) = x / 24 = 9 / 24 = 3 / 8$$

**Question 15:** Land is acquired for road widening from a square ground, as shown in the figure. The width of the acquired land is 2 meters. Area of the remaining ground is 440 square meters.



- [i] What is the shape of the remaining ground?
- [ii] What is the length of the remaining ground?

**Solution:**



- [i] The shape of the remaining ground is rectangular.
- [ii] Let the length be  $x$  and breadth be  $x - 2$ .

Given,

$$\text{Area} = 440 \text{ m}^2$$

$$L * B = 440$$

$$x * (x - 2) = 440$$

$$x^2 - 2x = 440$$

$$x^2 - 2x - 440 = 0$$

$$(x - 22)(x + 20) = 0$$

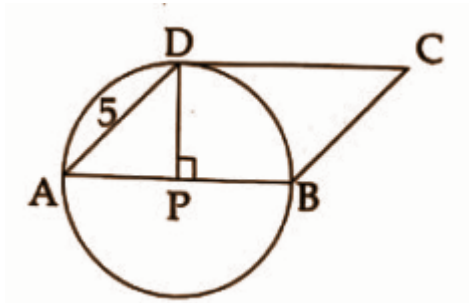
$$x = 22, -20$$

Since the values must be positive,  $x = 22$  is taken.

$$\text{Length} = 22\text{m}$$

$$\text{Breadth} = 22 - 2 = 20\text{m}$$

**Question 16:** In the figure, P is the centre of the circle. A, B and D are points on the circle.  $\angle P = 90^\circ$ ,  $AD = 5\text{cm}$ .



- What is the measure of  $\angle A$ ?
- What is the area of the triangle APD?
- Find the area of the parallelogram ABCD.

**Solution:**

[a] In triangle APD,  $\angle P = 90^\circ$

$\angle A = \angle D$  [angle opposite to equal side are equal]

$\angle A + \angle ADP + \angle APD = 180^\circ$  [angle sum property of a triangle]

$$\angle A + \angle A + 90^\circ = 180^\circ$$

$$2\angle A = 90^\circ$$

$$\angle A = 45^\circ$$

[b] In triangle APD,

$$\sin 45^\circ = PD / AD$$

$$1 / \sqrt{2} = PD / 5$$

$$5 / \sqrt{2} = PD = AP$$

$$\text{Area of } \triangle ADP = (1 / 2) * AP * PD$$

$$= (1 / 2) * (5 / \sqrt{2}) * (5 / \sqrt{2})$$

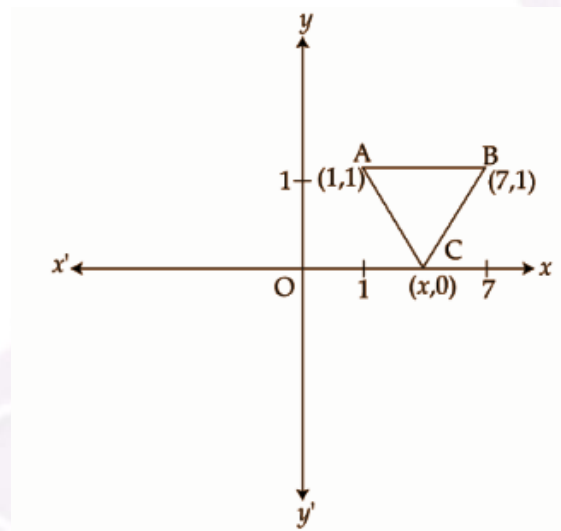
$$= 25 / 4 \text{ cm}^2$$

$$\begin{aligned}
 \text{[c] Area of a parallelogram} &= \text{base} * \text{height} \\
 &= AB * PD \\
 &= 2AP * PD \\
 &= 2 * (5 / \sqrt{2}) * (5 / \sqrt{2}) \\
 &= 25 \text{ cm}^2
 \end{aligned}$$

- Question 17:** [a] Draw the coordinates and mark the points A (1, 1), B (7, 1).  
 [b] Draw an isosceles triangle ABC with AB as the hypotenuse.  
 [c] Write the coordinates of C.

**Solution:**

[a]



$$AC = BC$$

$$\sqrt{(x - 1)^2 + 1} = \sqrt{(x - 7)^2 + 1}$$

On squaring both sides,

$$(x - 1)^2 + 1 = (x - 7)^2 + 1$$

$$x^2 + 1 - 2x + 1 = x^2 + 49 - 14x + 1$$

$$-2x + 14x = 49 - 1$$

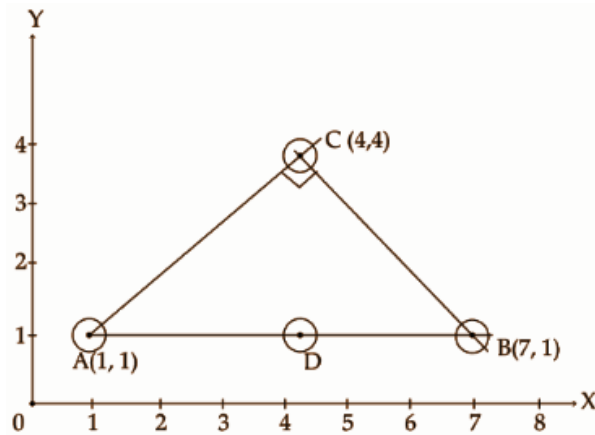
$$12x = 48$$

$$x = 48 / 12$$

$$x = 4$$



[b]

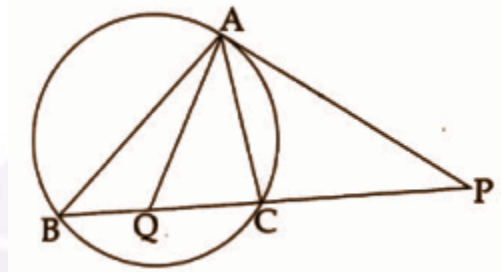


$C(4, 0)$  is the required point.

Coordinate of  $C(4, 4)$  [ $AD = BC = CD$ ]

The midpoint of the hypotenuse is equal distance from the vertex of the triangle.

**Question 18:** In the figure, chord  $BC$  is extended to  $P$ . Tangent from  $P$  to the circle is  $PA$ .  $AQ$  is the bisector of  $\triangle BAC$ .

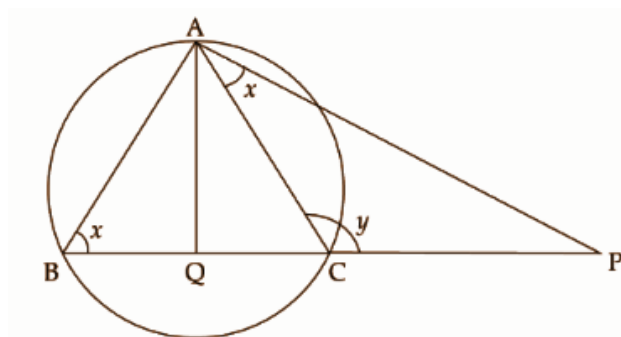


[a] Write one pair of equal angles from the figure.

[b] If  $\angle PAC = x$  and  $\angle PCA = y$ , then prove that  $\angle BAC = y - x$ .

[c] Prove that  $\angle PAQ = \frac{y + x}{2}$

**Solution:**



$$[a] \triangle BAC = \triangle PAC$$

$$[b] \triangle PAC = \triangle ABC$$

$$\triangle ACP = \triangle BAC + \triangle ABC \text{ [exterior angle property]}$$

$$y = \triangle BAC + x$$

$$\triangle BAC = y - x$$

$$[c] \triangle PAQ = \triangle PAC + \triangle CAQ$$

$$= x + \left(\frac{1}{2}\right) * \triangle BAC$$

$$= x + \left(\frac{1}{2}\right) * (y - x)$$

$$= x + \left(\frac{1}{2}\right) y - \left(\frac{1}{2}\right) x$$

$$\triangle PAQ = \left(\frac{1}{2}\right) (x + y)$$

**Question 19: If  $(x - 1)$  is a factor of the second-degree polynomial  $P(x) = ax^2 + bx + c$  and  $P(0) = -5$ .**

**[a] What is the value of  $c$ ?**

**[b] Prove that  $a + b = 5$ .**

**[c] Write a second-degree polynomial whose one factor is  $x - 1$ .**

**Solution:**

[a] Given that  $x - 1$  is a factor of the polynomial  $ax^2 + bx + c$

$$x - 1 = 0$$

$$x = 1$$

$$P(1) = 0$$

$$a(1)^2 + b * 1 + c = 0$$

$$a + b + c = 0 \text{ ---- (1)}$$

Now, at  $x = 0$ ,  $P(0) = -5$

$$a * 0 + b * 0 + c = -5$$

$$c = -5$$

$$[b] a + b + c = 0$$

$$a + b - 5 = 0$$

$$a + b = 5$$

[c] Second-degree polynomial =  $3x^2 + 2x - 5$  or  $2x^2 - 3x + 5$  or  $4x^2 + x - 5$  [any of them]

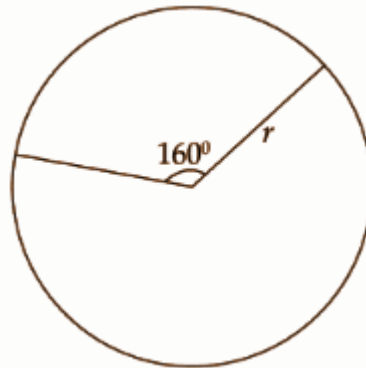
**Question 20:** A circular sheet of paper is divided into two sectors. The central angle of one of them is  $160^\circ$ .

[a] What is the central angle of the remaining sector?

[b] These sectors are bent into cones of maximum volume. If the radius of the small cone is 8 centimetres, what is the radius of the other?

[c] What is the slant height of the cone?

**Solution:**



[a] Central angle of the remaining sector =  $360^\circ - 160^\circ = 200^\circ$

[b]  $R_1$  is the radius of the small cone = 8cm

$$2\pi R_1 = 2\pi r (\theta_1) / 360^\circ$$

$$8 = r * (160^\circ / 360^\circ)$$

$$r = (360^\circ * 8) / 160^\circ$$

$$r = 18\text{cm}$$

$$2\pi R_2 = 2\pi r (\theta_2) / 360^\circ$$

$$R_2 = (18 * 200^\circ) / 360^\circ$$

$$R_2 = 10\text{cm}$$

[c] Slant height ( $l_1$ ) = 18cm

Slant height ( $l_2$ ) = 18cm

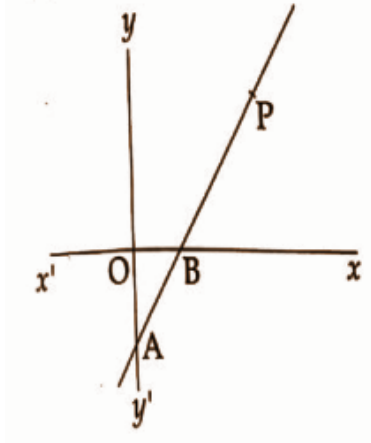
**Question 21:** Equation of the line AB is  $3x - 2y = 6$ . P is a point on the line. The line intersects the y-axis at A and the x-axis at B.

[a] What is the x coordinate of A?

[b] What is the length of OA?

[c] What is the length of OB?

[d] The x coordinate and the y coordinate of P are the same. Find the coordinates of P.



**Solution:**

Given, the equation of line AB is  $3x - 2y = 6$

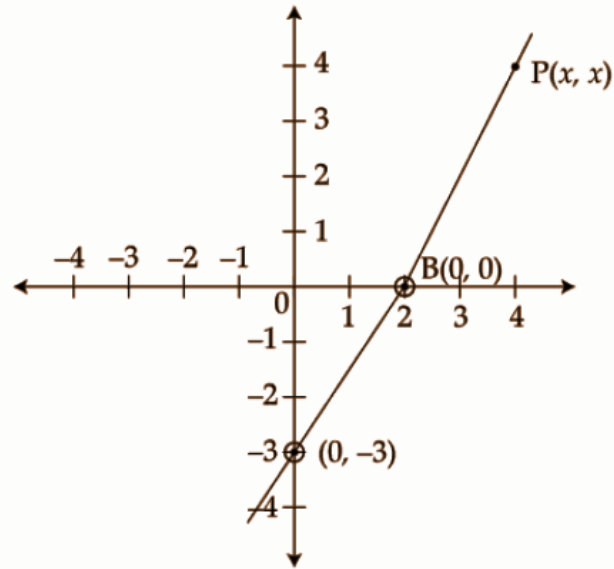
x	0	2
y	-3	0

[a] x coordinate of A = 0

[b] OA = 3 units

[c] OB = 2 units

[d]



A, B, P are collinear.

Area of  $\triangle ABP = 0$

$$(1/2) [0(0 - x) + 2(x + 3) + x(-3 - 0)] = 0$$

$$2x + 6 - 3x = 0$$

$$x = 6$$

Hence, the coordinates of P are (6, 6).

**Answer any 5 questions from 22 to 28. Each question carries 5 scores. [5 \* 5 = 25]**

**Question 22: If the terms of the arithmetic sequence  $(2/9), (3/9), (4/9), (5/9), \dots$  are represented as  $x_1, x_2, \dots$  then**

**[a]  $x_1 + x_2 + x_3 =$**

**[b]  $x_4 + x_5 + x_6 =$**

**[c] Find the sum of the first 9 terms.**

**[d] What is the sum of the first 300 terms?**

**Solution:**

**[a]  $x_1 + x_2 + x_3$**

$$= (2/9) + (3/9) + (4/9)$$

$$= 9/9$$

$$= 1$$

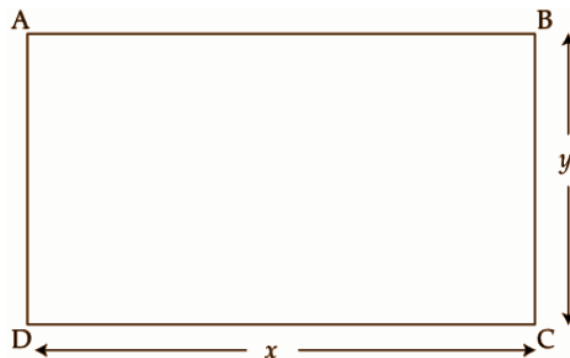
$$\begin{aligned}
 & \text{[b] } x_4 + x_5 + x_6 \\
 & = (5/9) + (6/9) + (7/9) \\
 & = (18/9) \\
 & = 2
 \end{aligned}$$

$$\begin{aligned}
 & \text{[c] } n = 9 \\
 & a = 2/9 \\
 & d = (3/9) - (2/9) = (1/9) \\
 & S_n = (n/2) (2a + [n - 1]d) \\
 & S_9 = (9/2) (2 * [2/9] + [9 - 1] * (1/9)) \\
 & = (9/2) [(4/9) + (8/9)] \\
 & = (9/2) (12/9) \\
 & = 6
 \end{aligned}$$

$$\begin{aligned}
 & \text{[d] } n = 300 \\
 & a = 2/9 \\
 & d = (3/9) - (2/9) = (1/9) \\
 & S_n = (n/2) (2a + [n - 1]d) \\
 & S_{300} = (300/2) (2 * [2/9] + [300 - 1] * (1/9)) \\
 & = (300/2) [(4/9) + (299/9)] \\
 & = (150) (303/9) \\
 & = 5050
 \end{aligned}$$

**Question 23: Draw a rectangle of area 12 square centimetres. Draw a square having the same area.**

**Solution:**



For the given rectangle,

$$\text{Area} = 12\text{cm}^2$$

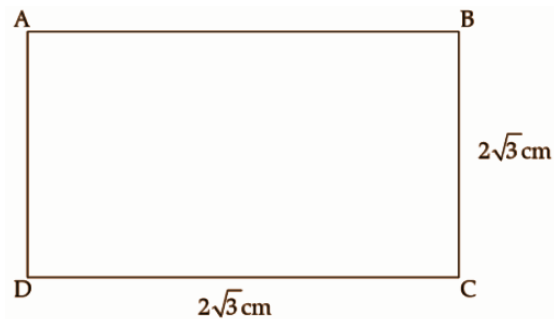
$$x * y = 12\text{cm}^2$$

For a square,  $x = y$ .

$$x * x = 12\text{cm}^2$$

$$x^2 = 12\text{cm}^2$$

$$x = 2\sqrt{3} \text{ cm}$$

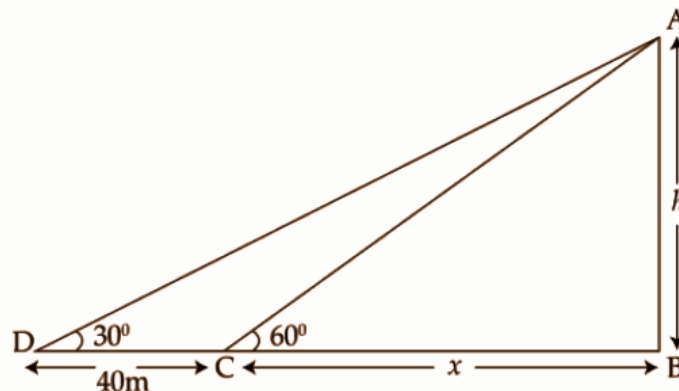


**Question 24:** A boy standing at one bank of a river sees the top of a tree on the other bank directly opposite to the boy at an elevation of  $60^\circ$ . Stepping 40 meters back, he sees the top of the elevation at  $30^\circ$ .

[a] Draw a rough sketch and find the height of the tree.

[b] What is the width of the river?

**Solution:**



Let AB be  $h$  and CB be  $x$ .

In  $\triangle ABC$ ,

$$\tan 60^\circ = AB / BC$$

$$\sqrt{3} = h / x$$

$$h = \sqrt{3}x \text{ ---- (1)}$$

In  $\triangle ABD$ ,

$$\tan 30^\circ = AB / BD$$

$$1 / \sqrt{3} = h / x + 40$$

$$x + 40 = \sqrt{3} (\sqrt{3}x) \text{ ---- (2)}$$

$$x + 40 = 3x$$

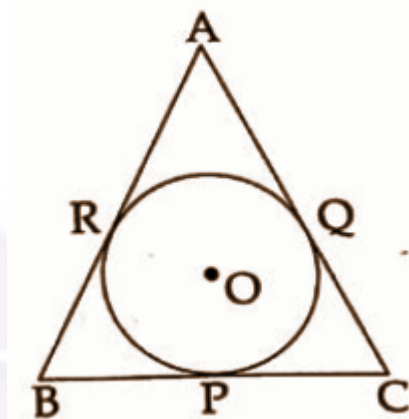
$$40 = 2x$$

$$x = 20$$

$$h = 20\sqrt{3} \text{ m}$$

Width of the river is 20m and the height of the tree is  $20\sqrt{3}$ m.

**Question 25:** Circle with centre O touches the sides of a triangle at P, Q and R,  $AB = AC$ ,  $AQ = 4$ cm and  $CQ = 6$ cm.



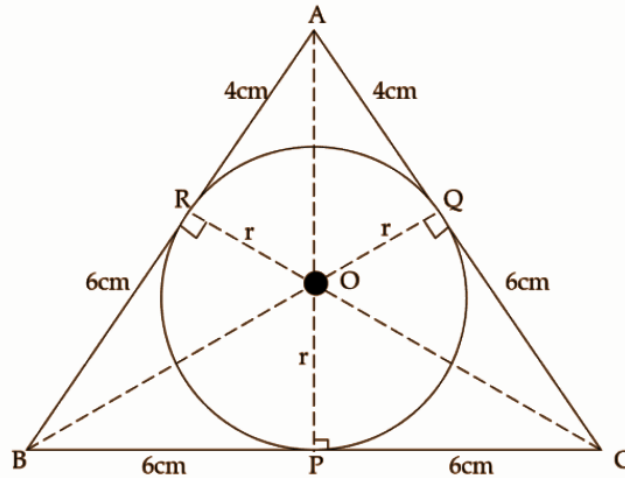
[a] What is the length of CP?

[b] Find the perimeter and the area of the triangle.

[c] What is the radius of the circle?

**Solution:**





[a]  $CP = CQ$  [Length of external tangents are equal]

$CP = 6\text{cm}$

[b] Perimeter of triangle =  $4 + 6 + 6 + 6 + 4 + 6 = 32\text{ cm}$

For the area of  $\triangle ABC$ ,

$$s = [AB + BC + CA] / 2$$

$$= [10 + 12 + 10] / 2$$

$$= 16\text{cm}$$

$$\text{Area of } \triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{(16)(16-10)(16-12)(16-10)}$$

$$= \sqrt{16 * 6 * 4 * 6}$$

$$= 48\text{cm}^2$$

[c] Area of  $\triangle ABC = \text{area of } \triangle AOB + \text{area of } \triangle BOC + \text{area of } \triangle COA$

$$48 = (1/2) * 10 * r + (1/2) * 12 * r + (1/2) * 10 * r$$

$$48 * 2 = r(10 + 12 + 10)$$

$$48 * 2 = 32 * r$$

$$r = 3\text{cm}$$

**Question 26: Radius of a cylinder is equal to its height. If the radius is taken as 'r', the volume of the cylinder is  $\pi r^2 * r = \pi r^3$ . Like this find the volumes of the solids, with the following measures.**

Solids	Measures	Volume
Cone	radius = height = r	
Hemisphere	radius = r	
Sphere	radius = r	

[a] What is the ratio of the volumes of the cone, hemisphere, cylinder and the sphere?

[b] A solid metal sphere of radius 6cm is melted and recast into solid cones of radius 6cm and height 6cm. Find the number of cones.

**Solution:**

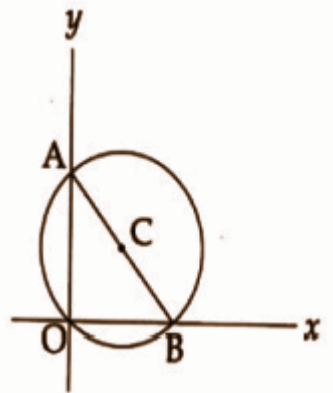
[a]

Solids	Measures	Volume
Cone	radius = height = r	$\frac{1}{3}\pi r^2 h \Rightarrow \frac{1}{3}\pi r^2 \times r \Rightarrow \frac{1}{3}\pi r^3$
Hemisphere	radius = r	$\frac{2}{3}\pi r^3$
Sphere	radius = r	$\frac{4}{3}\pi r^3$

$$\begin{aligned}
 [b] V_c : V_h : V_{cy} : V_s &= [\pi r^3 / 3] : [2 / 3] \pi r^3 : \pi r^3 : [4 / 3] \pi r^3 \\
 &= (1 / 3) : (2 / 3) : 1 : (4 / 3) \\
 &= 1 : 2 : 3 : 4
 \end{aligned}$$

$$\begin{aligned}
 \text{Number of cones} &= \text{Volume of the sphere} / \text{Volume of the cone} \\
 &= [(4 / 3) \pi r^3] / [\pi R^2 h / 3] \\
 &= \{[4 * \pi * 6^3] / (3)\} / \{[\pi * 6^2 * 6] / (3)\} \\
 &= 4
 \end{aligned}$$

**Question 27:** C is at the centre of the circle passing through the origin. Circle cuts the y-axis at A (0, 4) and the x-axis at B(4, 0).



[a] Write the coordinates of C.

[b] Write the equation of the circle.

[c] (0, 0) is a point on the circle. There is one more point on the circle with x and y coordinates equal. Which is that?

**Solution:**

[a] C is the midpoint of AB.

$$x = [4 + 0] / 2$$

$$= 4 / 2$$

$$x = 2$$

$$y = [4 + 0] / 2$$

$$= 4 / 2$$

$$y = 2$$

The coordinates of C are (2, 2).

[b] The equation of the circle is given by  $(x - a)^2 + (y - b)^2 = r^2$

$$(x - 2)^2 + (y - 2)^2 = [\sqrt{(4 - 2)^2 + (0 - 2)^2}]^2$$

$$x^2 + 4 - 4x + y^2 + 4 - 4y = 8$$

$$x^2 + y^2 - 4x - 4y = 0$$

[c] Let P(x, x) be a point on the circle.

$$x^2 + x^2 - 4x - 4x = 0$$

$$2x^2 - 8x = 0$$

$$x = 0, 4$$

The required point is (4, 4).

**Question 28:** The table below shows the number of children in a class, sorted according to their heights.

Height (Centimetres)	Number of Children
130 -140	7
140 - 150	9
150 -160	10
160 -170	10
170 -180	9

If the students are directed to stand in a line according to the order of their heights starting from the smallest, then

- [a] The height of the child at what position is taken as the median?
- [b] What is the assumed height of the child in the 17th position?
- [c] Find the median height.

**Solution:**

Class interval	frequency	Cumulative frequency
130 - 140	7	7
140 - 150	9	16
150 - 160	10	26
160 - 170	10	36
170 - 180	9	45

[a]  $N = 45$

Median is taken as  $[N + 1] / 2$

$$= [45 + 1] / 2$$

$$= 46 / 2$$

$$= 23$$

The height of the child at the 23<sup>rd</sup> position is taken as the median.

[b] Height of the child in the 17<sup>th</sup> position between 150 - 160. Assumed height is 152cm.

$$\begin{aligned} \text{[c] Median} &= [l_1] + \left\{ \frac{(N/2) - C}{cf} \right\} * h \\ &= 150 + \frac{[22.5 - 16]}{10} * (10) \\ &= 150 + 6.5 \\ &= 156.5 \end{aligned}$$

**Question 29: Read the following. Understand mathematical concepts in it and answer the questions that follow.**

The remainders obtained on dividing the powers of two by 7 have an interesting property.

We can understand it from the table given below.

Number	2 <sup>1</sup>	2 <sup>2</sup>	2 <sup>3</sup>	2 <sup>4</sup>	2 <sup>5</sup>	2 <sup>6</sup>	2 <sup>7</sup>	.....
Remainder	2	4	1	2	4	1	2	.....

If the powers are 1, 4, 7 ..... the remainder is 2.

If the powers are 3, 6, 9 ..... the remainder is 1.

[a] What is the remainder on dividing 2<sup>8</sup> by 7?

[b] Write the sequence of powers of 2 leaving remainder 1 on division by 7.

[c] Check whether 2019 is a term of arithmetic sequence 3, 6, 9 .....

[d] What is the remainder on dividing 2<sup>2019</sup> by 7?

[e] Write the algebraic form of the arithmetic sequence 1, 4, 7 .....

[f] Write the algebraic form of the sequence 2<sup>1</sup>, 2<sup>4</sup>, 2<sup>7</sup> ..... [powers of two leaving remainder 2 on division by 7].

**Solution:**

[a] If 2<sup>8</sup> is divided by 7, then the remainder is 7.

[b] 2<sup>3</sup>, 2<sup>6</sup>, 2<sup>9</sup> ..... when divided by 7 leaves a remainder 1.

[c] Yes

$$2019 = 3(n - 1) + 3$$

$$2019 = 3n - 9$$

$$2019 + 9 = 3n$$

$$2018 = 3n$$

$$2018 / 3 = n$$

$$n = 673 \text{ terms}$$

[d] 1 is the remainder on dividing  $2^{2019}$  by 7.

$$[e] a_n = a + (n - 1)d$$

$$a_n = 1 + (n - 1)3$$

$$= 1 + 3n - 3$$

$$a_n = 3n - 2$$

[f] 1, 4, 7 .....  $n^{\text{th}}$  term is  $3n - 2$ .

So, the algebraic form is  $2^{3n-2}$ .

