[c] How many boys are there in 10 B?
[d] What is the probability of both the selected students being boys?

Solution:

Class	XA	XB
Boys	20	30
Girls	30	20
Total	50	50

Given the probability of boys in XA = 2 / 5
Given the probability of boys in XB = 3 / 5

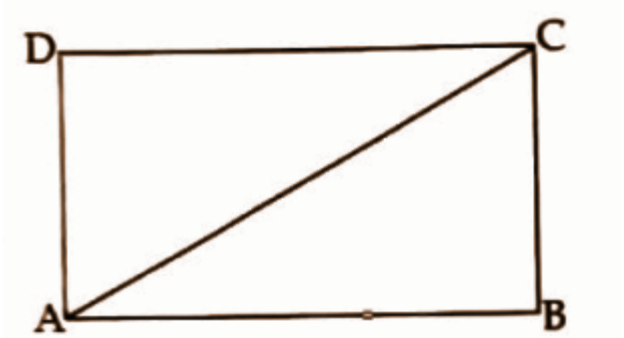
[a] Number of boys in XA
= 20 × [5 / 2]
= 50

[b] Probability of girl from XA
= 1 − [2 / 5]
= [5 − 2] / 5
= 3 / 5

[c] Number of boys in X B
= 50 × [3 / 5]
= 10 × 3
= 30

[d] Both being boys
= [2 / 5] × [3 / 5]
= 6 / 25

Question 15: Perimeter of the rectangle in the figure is 36cm. AC = √164 cm.



[a] What is AB + BC?

[b] Find the length of AB.

Solution:

Given the perimeter = 36cm

$AC = \sqrt{164}$ cm

[a] 2 (l + b) = 36

$\therefore$ AB + BC = 36 / 2

= 18cm

[b] Let AB = x , BC = 18 – x

Since ΔABC is right-angled, according to Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$164 = x^2 + (18 - x)^2$

$x^2 + 324 - 36x + x^2 = 164$

$2x^2 - 36x = 164 - 324 = -160$

Dividing by 2

$x^2 - 18x = -80$ [square completion method]

$x^2 - 18x + 81 = -80 + 81$

$(x - 9)^2 = 1$

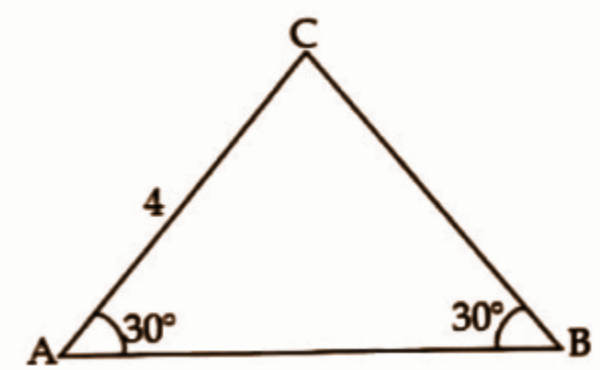
$x - 9 = \pm 1$

$x - 9 = 1$ or $x - 9 = -1$

$x = 10$ or $= 8$

So, AB = 10 cm.

Question 16: In triangle ABC, $\angle A = \angle B = 30^o$, AC = 4cm.



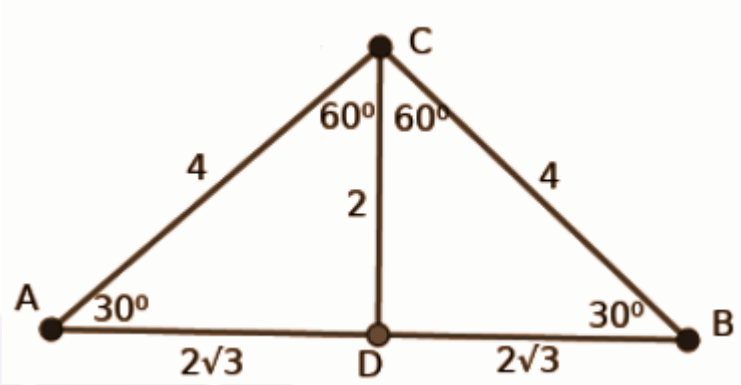
[a] What is the length of BC?

[b] Find the length of AB.

[c] In triangle PQR, $PQ = 4\sqrt{3}$ cm, $\angle P = \angle Q = 30^o$.

Draw the triangle.

Solution:



Given ,

$\angle A = \angle B = 30^0$

AC = 4cm

Draw $CD \perp AB$.

In right $\triangle$ ADC,

$30^0 : 60^0 : 90^0$

$1 : \sqrt{3} : 2$

DC : AD : AC

$x : x\sqrt{3} : 2x$

$2 : 2\sqrt{3} : 4$

DC = 2

$AD = 2\sqrt{3}$

AC = 4

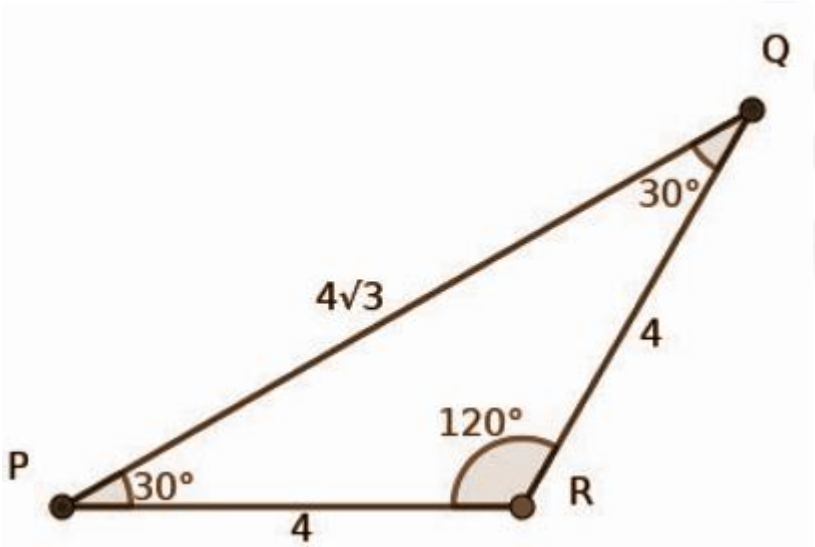
[a] Length of BC = AC = 4cm

[b] Length of AB = AD + DB

$= 2\sqrt{3} + 2\sqrt{3}$

$= 4\sqrt{3}cm$

[c] Draw PR = 4cm and make R be 120^0 and join PQ. △PRQ is the required triangle.



Question 17:

[a] If $p(x) = x^2 - 7x + 13$, what is p(3)?

[b] Write the polynomial p(x) - p(3) as the product of two first degree polynomials.

[c] Find the solutions of the equation p(x) - p(3) = 0.

Solution:

[a] Given polynomial

$p(x) = x^2 - 7x + 13$

$p(3) = 3^2 - 7 \times 3 + 13$

$= 9 - 21 + 13$

$= 1$

[b] $p(x) - p(3) = x^2 - 7x + 13 - 1$

$= x^2 - 7x + 12$

$= (x - 3)(x - 4)$

Hence the product two first degree polynomial = $(x - 3)(x - 4)$

[c] $p(x) - p(3) = 0$

$x^2 - 7x + 12 = 0$

$\Rightarrow (x - 3)(x - 4) = 0$

$\Rightarrow (x - 3) = 0$ or $(x - 4) = 0$

ie., $x = 3$ or $x = 4$.

Hence the solution is $x = 3$ and 4.

Question 18: In the figure, O is the centre of both circles. AB and AC touch the small circle at P and Q. A, B and C are points on the larger circle.

[a] If AP = 5cm, then what is the length of AQ?

[b] Prove that AB = AC.

[c] If AP = 5cm and $\angle A = 90^o$, then what is the radius of the small circle?

Solution:

[a] Given AP = 5 cm

Hence the length of AQ = 5cm. [$\because$ Same tangents from A]

[b] AB and AC are tangents

$OP \perp AB$ and $OQ \perp AC$ [$\because$ Chord bisector theorem]

AP = BP and AQ = QC

AB = AC

[c] Given $\angle A = 90^0$

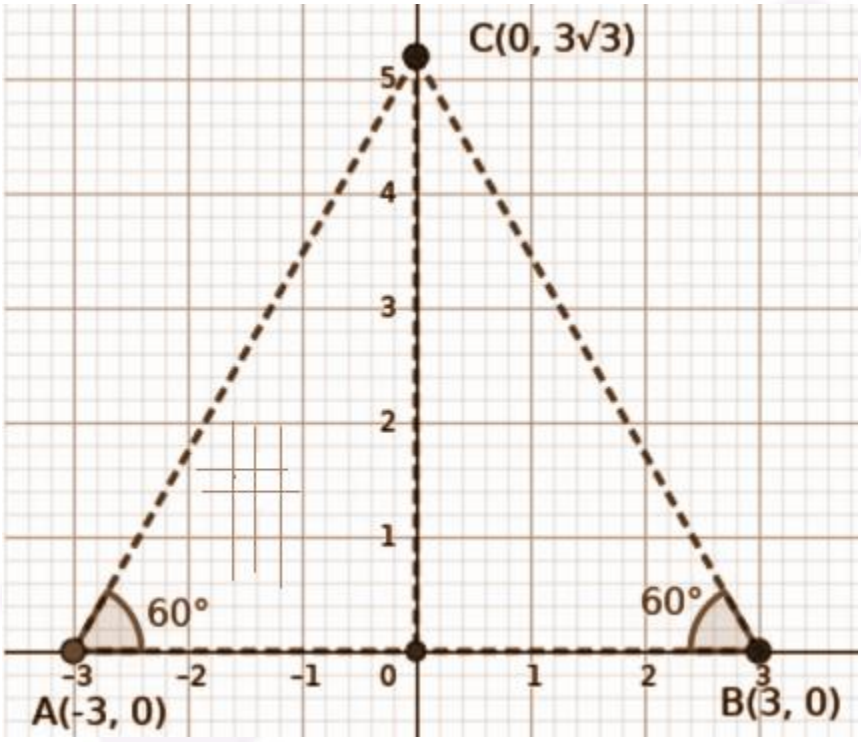
So it can be seen that APOQ is a square.

[$\because$ OP and OQ be radii $\therefore$ $\angle APO = \angle AQO = 90^0$. ie., $\angle POQ = 90^0$]

Hence the radius of the small circle = 5cm.

Question 19: Draw the coordinate axes and mark the points A (−3, 0), B (3, 0), C (0, 3√3).

Solution:



Question 20: A sector of radius 12cm and the central angle 120°is rolled up to a cone.

[a] What is the slant height of the cone?

[b] Find the radius and height of the cone.

[c] What is the central angle of the sector to be used to make a cone of base radius √2 cm and height 4cm?

Solution:

[a] The radius of the sector = 12cm [The radius of the sector to be the slant height of the cone]

[b] $r / l = x^0 / 360^o$

$\Rightarrow r / 12 = 120 / 360$

$\Rightarrow 360r = 12 \times 120$

$\Rightarrow r = [12 \times 120] / 360$

$\therefore r = 4cm$

Radius = 4cm

$h = \sqrt{l^2 - r^2}$

$= \sqrt{12^2 - 4^2}$

$= \sqrt{144 - 16}$

$= \sqrt{128}$

$= 8\sqrt{2}$ cm

[c] $r / l = x^0 / 360^o$

Central angle $(x^0) = [360 \times r] / l$

To find 'l'

$l = \sqrt{h^2 + r^2}$

$r = \sqrt{2}$, $h = 4cm$

$\therefore l = \sqrt{4^2 + \sqrt{2}^2}$

$= \sqrt{16 + 2}$

$= \sqrt{18}$

$= 3\sqrt{2}$

$\therefore$ Center angle $(x^0) = 360 \times rl$

$= [360 \times \sqrt{2}] / [3\sqrt{2}]$

$= 120^0$

Question 21:

[a] What is the slope of the line passing through the points (5, 0) and (3, 2)? Write the equation of the line.

[b] The x coordinate of a point on the line x - y = 5 is 5. What is the y coordinate of that point?

[c] Write the coordinates of the point of intersection of the lines x + y = 5 and x - y = 5.

Solution:

[a] Given points are (5, 0) and (3, 2).

Slope = $[y_2 - y_1] / [x_2 - x_1]$

$= [2 - 0] / [3 - 5]$

$= 2 / -2$

$= - 1$

Equation of the line is

$y - y_1 = m(x - x_1)$

$= y - 0$

$= -1 (x - 5)$

$y = -x + 5$

$x + y - 5 = 0$ is the equation.

[b] If x = 5, then

$5 - y = 5$

$-y = 5 - 5$

$= 0$

The y coordinate of that point is 0.

[c] Given $x + y = 5$ and $x - y = 0$.

$x + y = 5 \rightarrow (1)$

$x - y = 0 \rightarrow (2)$

Solve (1) and (2),

x and y = 5, and 0

So the coordinates = (5, 0).

Answer any five questions from 22 to 28. Each question carries 5 scores. [5 * 5 = 25]

Question 22: Sum of the first four terms of an arithmetic sequence is 72. Sum of first n terms is also 72.

[a] What is the 5^{th} term of the sequence?
[b] Find the sum of the first five terms.
[c] Write the sequence.

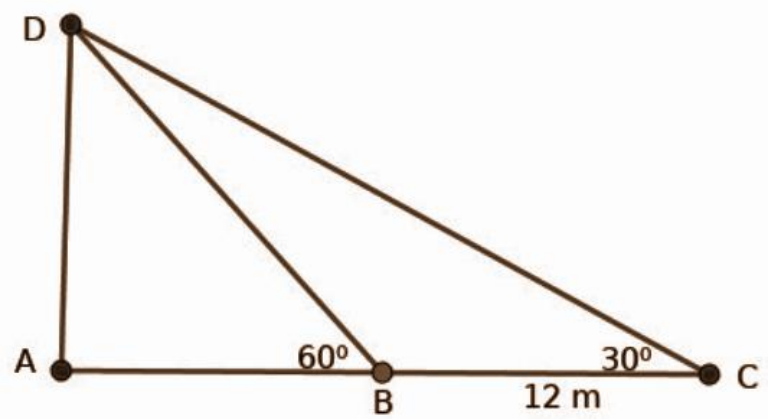
Solution:
Given sum of the first 4 terms = 72
Sum of the first 9 terms = 72
[a] 5^{th} term $(x_5) = (S_9) / 9$
$= 72 / 9$
$= 8$

[b] Sum of the first 5 terms (S_5)
$= S_4 + x_5$
$= 72 + 8$
$= 80$

[c] $X_3 = (S_5) / 5$
$= 80 / 5$
$= 16$
$X_3 + 2d = X_5$
$16 + 2d = 8$
$2d = – 8$
$d = – 4$
$X_1 = X_3 – 2d$
$= 16 – [2 \times – 4]$
$= 24$
The sequence = 24, 20, 16, 12, 8,.....

Question 23: A boy standing at the edge of a canal sees the top of a tree on the other edge at an elevation of 60º. Stepping 12m back, he sees it at an elevation of 30º. Find the height of the tree.

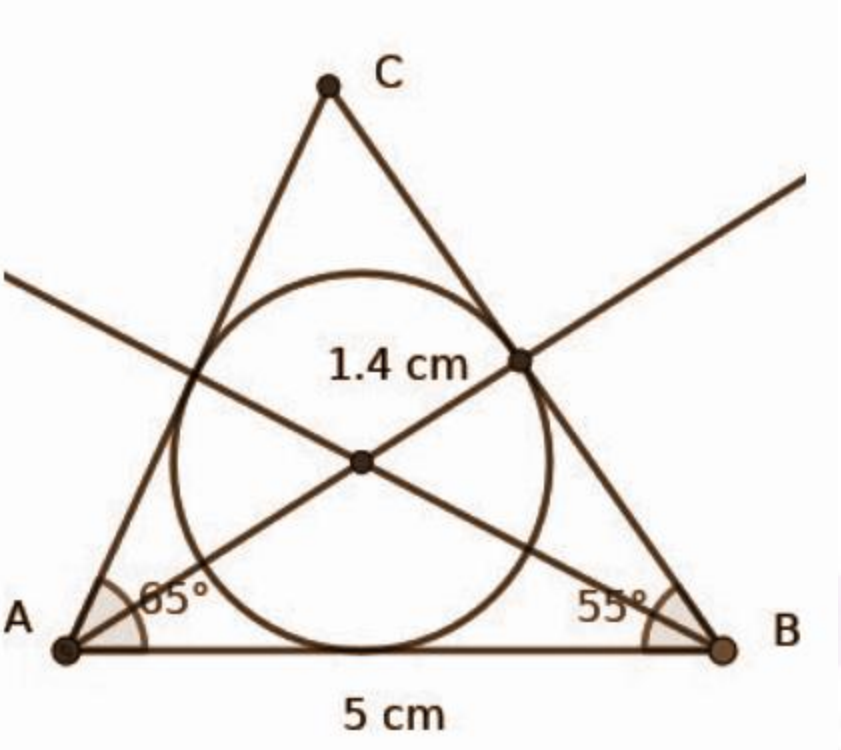
Solution:



Let AB be the height of the tree.
B be the first position of the boy.
C be the second position of the boy.
BC = 12
$\angle C = 30^0$
$\angle BPA = 60^0$
$\angle PBA = 30^0$
$\angle A = 90^0$
$\triangle CBD$ is an isosceles triangle.
$\therefore BC = BD = 12$
From right $\triangle BAP$,
$30^0 ; 60^0 ; 90^0$
$1 : \sqrt{3} : 2$
$\Rightarrow AD : AB : BD$
$\Rightarrow x : x\sqrt{3} : 2x$
$\Rightarrow 6 : 6\sqrt{3} : 12$
$\therefore AB = 6\sqrt{3}$
Hence the height of the tree = $6\sqrt{3}$m.

Question 24: In $\triangle ABC$, AB = 5cm, $\angle A = 65^0$, $\angle B = 55^0$. Draw the triangle ABC and the incircle. Measure the radius of the incircle.

Solution:



Question 25: A circle is drawn with (5, 3) as a centre. (5, 6) is a point on the circle.
[a] What is the radius of the circle?
[b] Write the equation of the circle.
[c] What is the distance from the centre of the circle to the x-axis?
[d] What is the length of the tangents from the origin to the circle?

Solution:
[a] Radius of the circle = 6 – 3 = 3 units

[b] Equation of the circle
$= (x - a)^2 + (y - b)^2 = r^2$
$\Rightarrow (x - 5)^2 + (y - 3)^2 = 3^2$
$\Rightarrow x^2 - 10x + 25 + y^2 - 6y + 9 = 9$
$\Rightarrow x^2 + y^2 - 10x - 6y + 25 = 0$

[c] The distance from the centre of the circle to the x-axis is the radius of the circle = 3 units.

[d] The length of the tangents from the origin to the circle = 5 units. [The x-axis being itself a tangent.]

Question 26:
[a] The radius of a solid sphere is 6cm. Find its volume and surface area.
[b] It is cut into two equal halves. What is the total surface area of each hemisphere? What is the volume of a hemisphere?

Solution:
[a] Given radius = 6cm;
Volume = $[4 / 3] \pi r^3$
$= [4 / 3] \pi 6^3$
$= 288\pi \text{ cm}^3$

Total Surface Area = $4\pi r^2$
$= 4\pi \times 6^2$
$= 144\pi \text{cm}^2$

[b] Total Surface Area of hemisphere = $3\pi r^2$
$= 3\pi \times 6^2$
$= 108\pi \text{cm}^2$

The volume of hemisphere = $[2 / 3] \pi r^3$
$= [2 / 3] \pi 6^3$
$= 144\pi \text{cm}^3$

Question 27: The table below shows, children of a class sorted according to their marks in the examination.

Marks	Number of Children
0-10	4
10-20	7
20-30	10
30-40	12
40-50	8
	41

[a] If we arrange the children from the one with the least mark to the one with the greatest, then what will be the assumed mark of the 12th student?
[b] Compute the median mark.

Solution:

Class	Frequency	Marks	Cumulative Frequency
0 - 10	4	<10	4
10 - 20	7	<20	11
20 - 30	10	<30	21
30 - 40	12	<40	33
40 - 50	8	<50	41
Total	**41**		

[a] Assumed mark of the 12th student

$= 20 + [30 - 20] / [10 \times 2]$

$= 20 + 1 / 2$

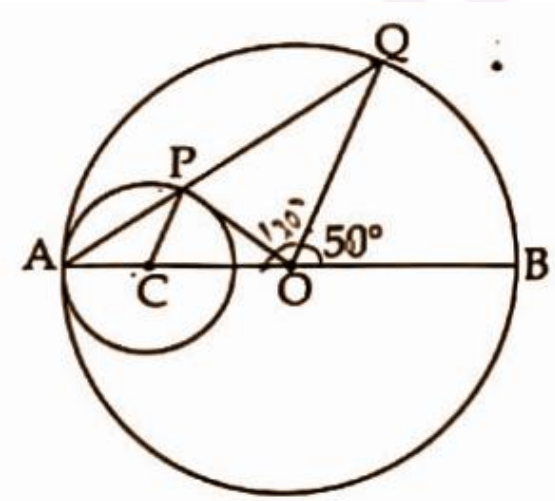
$= 20.5$

[b] $N / 2 = 41 / 2 = 20.5$

Median class = 20 - 30

l = 20
F = 11
f = 10
Median = l + [(N / 2 − F] / f) / c
= 20 + ([20.5 − 11] / 10) × 10
= 20 + [9.5 / 10] × 10
= 20 + 9.5
= 29.5
∴ Median mark = 29.5

Question 28: In the figure, O is the centre of the large circle. The centre of the small circle is C. OP is a tangent to the small circle. ∠BOQ = 50°.



[a] ∠OAQ = ?
[b] ∠OCP = ?
[c] ∠APO = ?
[d] ∠POQ = ?

Solution:
Given BOQ = 50^0

[a] Since △AOQ is isosceles, their base angles are equal.

AOQ = 180 – 50 = 130

$\angle A = \angle Q = [180 - 130] / 2$

$= 50 / 2$

$= 25^0$

$\therefore \angle OAQ = 25^0$

[b] $\angle OCP = 25^0 \times 2 = 50^0$

[c] $\angle APO = 25^0 + 90^o = 115^0$

[d] $\angle POQ = 180^0 - (50 + \angle AOP)$

$= 180^o – 50^o – 40^o$

$= 90^0$

Question 29: Read the following passage. Understand the mathematical concept in it and answer the questions that follow. Each question carries 1 score. [6 * 1 = 6]

The common difference of the arithmetic sequence 15, 14, 13, 12, 11, 10 ….. is 14 - 15 = -1. The first term of the sequence is 15 and the 15^{th} term is 15 + 14 * -1 = 15 - 14 = 1. Similarly, the 4^{th} term is 12 and the 12^{th} term is 4. Its 16^{th} term is x_{16} = 15 + 15 * -1 = 15 - 15 = 0. So, the sum of the first 31 terms is also 0. That is if the n^{th} term of an arithmetic sequence with common difference -1 is m, then the m^{th} term is n and the $(m + n)^{th}$ term is 0.

[a] 7^{th} term of an arithmetic sequence is 10 and the 10^{th} term is 7. What is the common difference?

[b] What is the 21^{st} term of the arithmetic sequence 21, 20, 19….

[c] 5^{th} term of an arithmetic sequence is 17 and the 17^{th} term is 5. Which term of the sequence is 0?

[d] 5^{th} term of an arithmetic sequence is 17 and the 17^{th} term is 5. What is the 44^{th} term?

[e] 1^{st} term of an arithmetic sequence is n and the n^{th} term is 1. What is the $(n + 1)^{th}$ term?

[f] The 1^{st} term of an arithmetic sequence is n and the n^{th} term is 1. Sum of how many terms, starting from the first term, of this sequence is 0?

Solution:

[a] The common difference d = - 1

[b] 21^{st} term becomes 1.

[c] 22^{nd} term becomes 0.

[d] 44^{th} term = 0 – 22 = -22

[e] The $(n + 1)^{th}$ term is 0.

[f] 0 is the sum of $(2n + 1)^{th}$ term.