Exercise 10.1

1. Identify the terms, their numerical as well as literal coefficients in each of the following expressions:
(i) $12x^2yz - 4xy^2$
(ii) 8 + mn + nl – lm
(iii) $x^2/3 + y/6 - xy^2$
(iv) -4p + 2.3q + 1.7r
Solution:

	Terms	Numerical coefficient	Literal coefficient
(i)	$12x^2yz$	12	x^2yz
	$-4xy^2$	−4	xy^2
(ii)	8	8	–
	mn	1	mn
	nl	1	nl
	$-lm$	−1	lm
(iii)	$\frac{x^2}{3}$	$\frac{1}{3}$	x^2
	$\frac{y}{6}$	$\frac{1}{6}$	y
	$-xy^2$	−1	xy^2
(iv)	$-4p$	−4	p
	$2.3q$	2.3	q
	$1.7r$	1.7	r

2. Identify monomials, binomials, and trinomials from the following algebraic expressions :
(i) $5p \times q \times r^2$
(ii) $3x^2 + y \div 2z$
(iii) $-3 + 7x^2$
(iv) $(5a^2 - 3b^2 + c)/2$
(v) $7x^5 - 3x/y$
(vi) $5p \div 3q - 3p^2 \times q^2$
Solution:

(i) $5p \times q \times r^2 = 5pqr^2$
As this algebraic expression has only one term, its therefore a monomial.

(ii) $3x^2 + y \div 2z = 3x^2/2z + y/2z$

As this algebraic expression has two terms, its therefore a binomial.

(iii) $-3 + 7x^2$
As this algebraic expression has two terms, its therefore a binomial.

(iv) $\frac{5a^2-3b^2+c}{2} = \frac{5a^2}{2} - \frac{3b^2}{2} + \frac{c}{2}$
As this algebraic expression has three terms, its therefore a trinomial.

(v) $7x^5 – 3x/y$
As this algebraic expression has two terms, its therefore a binomial.

(vi) $5p \div 3q – 3p^2 \times q^2 = 5p/3q - 3p^2q^2$
As this algebraic expression has two terms, its therefore a binomial.

3. Identify which of the following expressions are polynomials. If so, write their degrees.
(i) $2/5x^4 – \sqrt{3}x^2 + 5x – 1$
(ii) $7x^3 – 3/x^2 + \sqrt{5}$
(iii) $4a^3b^2 – 3ab^4 + 5ab + 2/3$
(iv) $2x^2y – 3/xy + 5y^3 + \sqrt{3}$
Solution:

(i) It is a polynomial and the degree of this expression is 4.
(ii) It is not a polynomial.
(iii) It is a polynomial and the degree of this expression is 5.
(iv) It is not a polynomial.

4. Add the following expressions:
(i) ab – bv, bv – ca, ca – ab
(ii) $5p^2q^2 + 4pq + 7, 3 + 9pq – 2p^2q$
(iii) $l^2 + m^2 + n^2$, lm + mn, mn + nl, nl + lm
(iv) $4x^3 – 7x^2 + 9, 3x^2 – 5x + 4, 7x^3 – 11x + 1, 6x^2 – 13x$
Solution:

(i) ab – bc, bc – ca, ca – ab
On adding the expressions, we have
⇒ ab – bc + bc – ca + ca – ab = 0

(ii) $5p^2q^2 + 4pq + 7,3 + 9pq – 2p^2q^2$
On adding the expressions, we have
$= 5p^2q^2 + 4pq + 7 + 3 + 9pq – 2p^2q^2$
$= 5p^2q^2 – 2p^2q^2 + 4pq + 9pq + 7 + 3$
$= 3p^2q^2 + 13pq + 10$

(iii) $l^2 + m^2 + n^2$, lm + mn, mn + nl, nl + lm
On adding the expressions, we have

$= l^2 + m^2 + n^2 + lm + mn + mn + nl + nl + lm$
$= l^2 + m^2 + n^2 + 2lm + 2mn + 2nl$

(iv) $4x^3 – 7x^2 + 9, 3x^2 – 5x + 4, 7x^3 – 11x + 1, 6x^2 – 13x$
On adding the expressions, we have
$= 4x^3 – 7x^2 + 9 + 3x^2 – 5x + 4 + 7x^3 – 11^2 + 1 + 6x^2 – 13x$
$= 4x^2 + 7x^3 – 7x^2 + 3x^2 + 6x^2 – 5x – 11x – 13x + 9 + 4 + 1$
$= 11x^3 – 2x^2 – 29x + 14$

5. Subtract:
(i) 8a + 3ab – 2b + 7 from 14a – 5ab + 7b – 5
(ii) 8xy + 4yz + 5zx from 12xy – 3yz – 4zx + 5xyz
(iii) $4p^2q – 3pq + 5pq^2 – 8p + 7q -10$ from $18 – 3p – 11q + 5pq – 2pq^2 + 5p^2q$
Solution:

(i) Subtracting 8a + 3ab – 2b + 7 from 14a – 5ab + 7b – 5, we have
= (14a – 5ab + 7b – 5) – (8a + 3ab – 2b + 7)
= 14a – 5ab + 7b – 5 – 8a – 3ab + 2b – 7
= 6a – 8ab + 9ab – 12

(ii) Subtracting 8xy + 4yz + 5zx from 12xy – 3yz – 4zx + 5xyz, we have
= (12xy – 3yz – 4zx + 5xyz) – (8xy + 4yz + 5zx)
= 12xy – 3yz – 4zx + 5xyz – 8xy – 4yz – 5zx
= 4xy – 7yz – 9zx + 5xyz

(iii) Subtracting $4p^2q – 3pq + 5pq^2 – 8p + 7q – 10$ from $18 – 3p – 11q + 5pq – 2pq^2 + 5p^2q$, we have
$= (18 – 3p – 11q + 5pq – 2pq^2 + 5p^2q) – (4p^2q – 3pq + 5pq^2 – 8p + 7q – 10)$
$= 18 – 3p – 11q + 5pq – 2pq^2 + 5p^2q – 7p^2q + 3pq – 5pq^2 + 8p – 7q + 10$
$= 28 + 5p – 78q + 8pq – 7pq^2 + p^2q$

6. Subtract the sum of $3x^2 + 5xy + 7y^2 + 3$ and $2x^2 – 4xy – 3y^2 + 7$ from $9x^2 – 8xy + 11y^2$
Solution:

First, adding $3x^2 + 5xy + 7y^2 + 3$ and $2x^2 – 4xy – 3y^2 + 7$, we have
$= 3x^2 + 5xy + 7y^2 + 3 + 2x^2 – 4xy – 3y^2 + 7$
$= 5x^2 + xy + 4y^2 + 10$
Now,
Subtracting $5x^2 + xy + 4y^2 + 10$ from $9x^2 – 8xy + 11y^2$
$= (9x^2 – 8xy + 11y^2) – (5x^2 + xy + 4y^2 + 10)$
$= 9x^2 – 8xy + 11y^2 – 5x^2 – xy – 4y^2 – 10$
$= 4x^2 – 9xy + 7y^2 – 10$

7. What must be subtracted from $3a^2 – 5ab – 2b^2 – 3$ to get $5a^2 – 7ab – 3b^2 + 3a$?
Solution:

From the question, its understood that we have to subtract $5a^2 – 7ab – 3b^2 + 3a$ from $3a^2 – 5ab – 2b^2 – 3$

$= 3a^2 – 5ab – 2b^2 – 3 – (5a^2 – 7ab – 3b^2 + 3a)$
$= 3a^2 – 5ab – 2b^2 – 3 – 5a^2 + 7ab + 3b^2 – 3a$
$= -2a^2 + 2ab + b^2 – 3a – 3$

8. The perimeter of a triangle is $7p^2 – 5p + 11$ and two of its sides are $p^2 + 2p – 1$ and $3p^2 – 6p + 3$. Find the third side of the triangle.
Solution:

Given,
Perimeter of a triangle = $7p^2 – 5p + 11$
And, two of its sides are $p^2 + 2p – 1$ and $3p^2 – 6p + 3$
We know that,
Perimeter of a triangle = Sum of three sides of triangle
$\Rightarrow 7p^2 – 5p + 11 = (p^2 + 2p – 1) + (3p^2 – 6p + 3)$ + (Third side of triangle)
$7p^2 – 5p + 11 = (4p^2 – 4p + 2)$ + (Third side of triangle)
$\Rightarrow$ Third side of triangle = $(7p^2 – 5p + 11) – (4p^2 – 4p + 2)$
$= (7p^2 – 4p^2) + (- 5p + 4p) + (11 – 2)$
$= 3p^2 – p + 9$
Thus, the third side of the triangle is $3p^2 – p + 9$.

Exercise 10.2

1. Find the product of:
(i) $4x^3$ and $-3xy$
(ii) 2xyz and 0
(iii) $-(2/3)p^2q$, $(3/4)pq^2$ and 5pqr
(iv) -7ab, $-3a^3$ and $-(2/7)ab^2$
(v) $-½x^2 - (3/5)xy$, (2/3)yz and (5/7)xyz
Solution:

Product of:
(i) $4x^3$ and $-3xy = 4x^3 \times (-3xy) = -12x^{3+1} y = -12x^4y$
(ii) 2xyz and 0 = 2xyz × 0 = 0

(iii) $\left(-\frac{2}{3}p^2q\right) \times \left(\frac{3}{4}pq^2\right) \times (5pqr)$

$= -\frac{2}{3} \times \frac{3}{4} \times 5 \times p^2q \times pq^2 \times pqr$

$= -\frac{5}{2}p^4q^4r$

(iv) $(-7ab) \times (-3a^3) \times \left(-\frac{2}{7}ab^2\right)$

$= (-7) \times (-3) \times \left(-\frac{2}{7}\right) \times ab \times a^3 \times ab^2$

$= -6a^5b^3.$

(v) $\left(-\frac{1}{2}x^2\right) \times \left(-\frac{3}{5}xy\right) \times \left(\frac{2}{3}yz\right) \times \left(\frac{5}{7}xyz\right)$

$= \left(-\frac{1}{2}\right) \times \left(-\frac{3}{5}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{5}{7}\right) \times x^2 \times xy \times yz + xyz$

$= \frac{1}{7}x^4y^3z^2$

2. Multiply:
(i) (3x – 5y + 7z) by – 3xyz
(ii) $(2p^2 - 3pq + 5q^2 + 5)$ by – 2pq
(iii) $(2/3a^2b - 4/5ab^2 + 2/7ab + 3)$ by 35ab
(iv) $(4x^2 - 10xy + 7y^2 - 8x + 4y + 3)$ by 3xy
Solution:

(i) – 3xyz × (3x – 5y + 7z)
= (- 3xyz) × 3x + (- 3xyz) × (- 5y) + (- 3xyz) × (7z)
$= -9x^2yz + 15xyz^2 - 21xyz^2$

(ii) $-2pq \times (2p^2 - 3pq + 5q^2 + 5)$

$= (-2pq) \times 2p^2 + (-2pq) \times (-3pq) + (-2pq) \times (5q^2) + (-2pq) \times 5$
$= -4p^3q + 6p^2q^2 - 10pq^3 - 10pq$

(iii) $\left(\frac{2}{3}a^2b - \frac{4}{5}ab^2 + \frac{2}{7}ab + 3\right)$ by 35ab
$= (2/3)a^2b \times 35ab - (4/5)ab^2 \times 35ab + (2/7)ab \times 35ab + 3 \times 35ab$
$= (70/3)a^3b^2 - 28a^2b^3 + 10a^2b^2 + 105ab$

(iv) $(4x^2 - 10xy + 7y^2 - 8x + 4y + 3)$ by 3xy
$= 4x^2 \times 3xy - 10xy \times 3xy + 7y^2 \times 3xy - 8x \times 3xy + 4y \times 3xy + 3 \times 3xy$
$= 12x^3y - 30x^2y^2 + 21xy^3 - 24x^2y + 12xy^2 + 9xy$

3. Find the areas of rectangles with the following pairs of monomials as their lengths and breadths respectively:
(i) (p^2q, pq^2)
(ii) $(5xy, 7xy^2)$
Solution:

(i) Given, sides of a rectangle are p^2q and pq^2
Hence,
Area $= p^2q \times pq^2 = p^{2+1} \times q^{2+1} = p^3q^3$

(ii) Given, sides are 5xy and $7xy^2$
Hence,
Area $= 5xy \times 7xy^2 = 35x^{1+1} \times y^{1+2} = 35x^2y^3$

4. Find the volume of rectangular boxes with the following length, breadth and height respectively:
(i) $5ab, 3a^2b, 7a^4b^2$
(ii) $2pq, 4q^2, 8rp$
Solution:

Given are the length, breadth and height of a rectangular box:
(i) $5ab, 3a^2b, 7a^4b^2$
∴ Volume = Length × breadth × height
$= 5ab \times 3a^2b \times 7a^4b^2$
$= 5 \times 3 \times 7 \times a^{1+2+4} \times b^{1+1+2}$
$= 105a^7b^4$

(ii) $2pq, 4q^2, 8rp$
∴ Volume = Length × breadth × height
$= 2pq \times 4q^2 \times 8rp$
$= 2 \times 4 \times 8 \times p^{1+1} \times q^{1+2} \times r$
$= 64p^2q^3r$

5. Simplify the following expressions and evaluate them as directed:

(i) $x^2(3 – 2x + x^2)$ for x = 1; x = -1; x = 2/3 and x = –1/2
(ii) $5xy(3x + 4y – 7) – 3y(xy – x^2 + 9) – 8$ for x = 2, y = -1
Solution:

(i) $x^2(3 – 2x + x^2)$
For x = 1; x = -1; x = 2/3 and x = –1/2
$x^2(3 – 2x + x^2) = 3x^2 – 2x^3 + x^4$
(a) For x = 1
$3x^2 – 2x^3 + x^4 = 3(1)^2 – 2(1)^3 + (1)^4$
$= 3 \times 1 – 2 \times 1 + 1$
$= 3 – 2 + 1 = 2$
(b) For x = -1
$3x^2 – 2x^3 + x^4 = 3(-1)^2 – 2(-1)^3 + (-1)^4$
$= 3 \times 1 – 2 \times (-1) + 1$
$= 3 + 2 + 1 = 6$
(c) For x = 2/3
$3x^2 – 2x^3 + x^4 = 3(2/3)^2 – 2(2/3)^3 + (2/3)^4$
$= 3 \times (4/9) – 2 \times (8/27) + (16/81)$
$= (4/3) – (16/27) + (16/81)$
$= (108 – 48 + 16)/81$
$= (124 - 48)/81$
$= 76/81$

(d) For x = -1/2
$3x^2 – 2x^3 + x^4 = 3(-1/2)^2 – 2(-1/2)^3 + (-1/2)^4$
$= 3 \times (1/4) – 2 \times (-1/8) + (1/16)$
$= (3/4) + ¼ + (1/16)$
$= (12 + 4 + 1)/16$
$= 17/16$

(ii) $5xy(3x + 4y – 7) – 3y(xy – x^2 + 9) – 8$
$= 15x^2y + 20xy^2 – 35xy – 3xy^2 + 3x^2y – 21y – 8$
$= 18x^2y + 17xy^2 – 35xy – 27y – 8$
When x = 2, y = -1, we have
$= 18(2)^2 \times (-1) + 17(2)(-1)^2 – 35(2)(-1) – 27(-1) – 8$
$= 18 \times 4 \times (-1) + 17 \times 2 \times 1 – 35 \times 2 \times (-1) – 27 \times (-1) – 8$
$= -74 + 34 + 70 + 27 – 8$
$= 131 – 80 = 51$

6. Add the following:
(i) $4p(2 – p^2)$ and $8p^3 – 3p$
(ii) $7xy(8x + 2y – 3)$ and $4xy^2(3y – 7x + 8)$
Solution:

Adding,
(i) $4p(2 – p^2)$ and $8p^3 – 3p$

$= 8p – 4p^3 + 8p^3 – 3p$
$= 5p + 4p^3$
$= 4p^3 + 5p$

(ii) $7xy(8x + 2y – 3)$ and $4xy^2(3y – 7x + 8)$
$= 56x^2y + 14xy^2 – 21xy + 12xy^3 – 28x^2y^2 + 32xy^2$
$= 12xy^3 – 28x^2y^2 + 56x^2y +46xy^2 – 21xy$

7. Subtract:
(i) $6x(x – y + z)- 3y(x + y – z)$ from $2z(-x + y + z)$
(ii) $7xy(x^2 -2xy + 3y^2) – 8x(x^2y – 4xy + 7xy^2)$ from $3y(4x^2y – 5xy + 8xy^2)$
Solution:

Subtracting,
(i) $6x(x – y + z) – 3y(x + y – z)$ from $2z(-x + y + z)$
$\Rightarrow 6x^2 – 6xy + 6xz – 3xy – 3y^2 + 3yz$ from $– 2xz + 2yz + 2z^2$
$= (-2xz + 2yz + 2z^2) – (6x^2 – 6xy + 6xz – 3xy – 3y^2 + 3yz)$
$= – 2xz + 2yz + 2z^2 – 6x^2 + 6xy – 6xz + 3xy + 3y^2 – 3yz$
$= 9xy – yz – 8zx – 6x^2 + 3y^2 + 2z^2$

(ii) $7xy(x^2 – 2xy + 3y^2) – 8x(x^2y – 4xy + 7xy^2)$ from $3y(4x^2y – 5xy + 8xy^2)$
$\Rightarrow 7x^3y – 14x^2y^2 + 21xy^3 – 8x^3y + 32x^2y – 56x^2y^2$ from $12x^2y^2 – 15xy^2 + 24xy^3$
$= (12x^2y^2 – 15xy^2 + 24xy^3) – (7x^3y – 14x^2y^2 + 21xy^3 – 8x^3y + 32x^2y – 56x^2y^2$
$= 12x^2y^2 – 15xy^2 + 24xy^3 – 7x^3y + 14x^2y^2 – 12xy^3 + 8x^3y – 32x^2y + 56x^2y^2$
$= 82x^2y^2 + 3xy^3 + x^3y – 15xy^2 – 32x^2y$

Exercise 10.3

1. Multiply:
(i) (5x – 2) by (3x + 4)
(ii) (ax + b) by (cx + d)
(iii) (4p – 7) by (2 – 3p)
(iv) $(2x^2 + 3)$ by (3x – 5)
(v) (1.5a – 2.5b) by (1.5a + 2.56)
(vi) $\left(\frac{3}{7}p^2 + 4q^2\right)$ by $7\left(p^2 - \frac{3}{4}q^2\right)$
Solution:

(i) (5x – 2) by (3x + 4)
= (5x – 2) × (3x + 4)
= 5x (3x + 4) – 2 (3x + 4)
= $15x^2 + 20x - 6x - 8$
= $15x^2 + 14x - 8$

(ii) (ax + b) by (cx + d)
= (ax + b) × (cx + d)
= ax (cx + d) + b (cx + d)
= $acx^2 + adx + bcx + bd$

(iii) (4p – 7) by (2 – 3p)
= (4p – 7) × (2 – 3p)
= 4p(2 – 3p) -7(2 – 3p)
= $8p - 12p^2 - 14 + 21p$
= $29p - 12p^2 - 14$

(iv) $(2x^2 + 3)$ by (3x – 5)
= $(2x^2 + 3)(3x - 5)$
= $2x^2(3x - 5) + 3(3x - 5)$
= $6x^3 - 10x^2 + 9x - 15$

(v) (1.5a – 2.5b) by (1.5a + 2.5b)
= (1.5a – 2.5b) (1.5a + 2.5b)
= 1.5a(1.5 + 2.5b) – 2.5b(1.5a + 2.5b)
= $2.25a^2 + 3.75ab - 3.75a6 - 6.25b^2$
= $2.25a^2 - 6.25b^2$

(vi) $\left(\frac{3}{7}p^2+4q^2\right)$ by $7\left(p^2-\frac{3}{4}q^2\right)$

$$=\left(\frac{3}{7}p^2+4q^2\right)\times 7\left(p^2-\frac{3}{4}q^2\right)$$

$$=7\left(\frac{3}{7}p^2+4q^2\right)\left(p^2-\frac{3}{4}q^2\right)$$

$$=7\left[\frac{3}{7}p^2\left(p^2-\frac{3}{4}q^2\right)+4q^2\left(p^2-\frac{3}{4}q^2\right)\right]$$

$$=7\left[\frac{3}{7}p^4-\frac{9}{28}p^2q^2+4p^2q^2-3q^4\right]$$

$$=3p^4-\frac{9}{4}p^2q^2+28p^2q^2-21q^4$$

$$=3p^4-\frac{9p^2q^2+112p^2q^2}{4}-21q^4$$

$$=3p^4+\frac{103}{4}p^2q^2-21q^4$$

2. Multiply:
(i) $(x - 2y + 3)$ by $(x + 2y)$
(ii) $(3 - 5x + 2x^2)$ by $(4x - 5)$
Solution:

(i) $(x - 2y + 3)$ by $(x + 2y)$
$= (x - 2y + 3) \times (x + 2y)$
$= x (x + 2y) - 2y(x + 2y) + 3 (x + 2y)$
$= x^2 + 2xy - 2xy - 4y^2 + 3x + 6y$
$= x^2 - 4y^2 + 3x + 6y$

(ii) $(3 - 5x + 2x^2)$ by $(4x - 5)$
$= (4x - 5) (3 - 5x + 2x^2)$
$= 4x(3 - 5x + 2x^2) - 5(3 - 5x + 2x^2)$
$= 12x - 20x^2 + 8x^3 - 15 + 25x - 10x^2$
$= 8x^3 - 30x^2 + 37x - 15$

3. Multiply:
(i) $(3x^2 - 2x - 1)$ by $(2x^2 + x - 5)$
(ii) $(2 - 3y - 5y^2)$ by $(2y - 1 + 3y^2)$
Solution:

(i) $(3x^2 - 2x - 1)$ by $(2x^2 + x - 5)$
$= (3x^2 - 2x - 1) (2x^2 + x - 5)$

$= 3x^2(2x^2 + x – 5) – 2x(2x^2 + x – 5) -1(2x^2 + x – 5)$
$= 6x^4 + 3x^3 – 15x^2 – 4x^3 – 2x^2 + 10x – 2x^2 – x + 5$
$= 6x^4 – x^3 – 19x^2 + 9x + 5$

(ii) $(2 – 3y – 5y^2)$ by $(2y- 1 + 3y^2)$
$= (2 – 3y – 5y^2) \times (2y- 1 + 3y^2)$
$= 2(2y – 1 + 3y^2) - 3y (2y – 1 + 3y^2) -5y^2(2y – 1 + 3y^2)$
$= 4y – 2 + 6y^2 – 6y^2 + 3y – 9y^3 – 10y^3 + 5y^2 – 15y^4$
$= -15y^4 – 19y^3 + 5y^2 + 7y – 2$

4. Simplify:
(i) $(x^2 + 3) (x – 3) + 9$
(ii) $(x + 3) (x – 3) (x + 4) (x – 4)$
(iii) $(x + 5) (x + 6) (x + 7)$
(iv) $(p + q – 2r) (2p – q + r) – 4qr$
(v) $(p + q) (r + s) + (p – q)(r – s) – 2(pr + qs)$
(vi) $(x + y + z) (x – y + z) + (x + y – z) (-x + y + z) – 4zx$
Solution:

(i) $(x^2 + 3) (x – 3) + 9$
$= x^2 (x – 3) + 3(x – 3) + 9$
$= x^2 – 3x^2 + 3x – 9 + 9$
$= x^3 – 3x^2 + 3x$

(ii) $(x + 3) (x – 3) (x + 4) (x – 4)$
$= \{(x + 3) (x – 3)\} \times \{(x + 4) (x – 4)\}$
$= \{x (x – 3) + 3 (x – 3)\} \{x (x – 4) + 4 (x – 4)\}$
$= (x^2 – 3x + 3x – 9) \{x^2 – 4x + 4x – 16\}$
$= (x^2 – 9) (x^2 – 16)$
$= x^2 (x^2 – 16) – 9 (x^2 – 16)$
$= x^4 – 16x^2 – 9x^2 + 144$
$= x^4 – 25x^2 + 144$

(iii) $(x + 5) (x + 6) (x + 7)$
$= \{(x + 5) \times (x + 6)\} (x + 7)$
$= (x^2 + 6x + 5x + 30) (x + 7)$
$= (x^2 + 11x + 30) (x + 7)$
$= x(x^2+ 11x + 30) + 7(x^2+ 11x + 30)$
$= x^3 + 11x^2 + 30x + 7x^2 + 77x + 210$
$= x^3 + 18x^2 + 107x + 210$

(iv) $(p + q – 2r)(2p – q + r) – 4qr$
$= p(2p – q + r) + q(2p – q + r) – 2r(2p – q + r) – 4qr$
$= 2p^2 – pq + pr + 2pq – q^2 + qr – 4pr + 2qr – 2r^2 – 4qr$
$= 2p^2 – q^2 – 2r^2 + pq – 3pr – 2qr$

(v) $(p + q)(r + s) + (p – q) (r – s) – 2(pr + qs)$
$= (pr + ps + qr + qs) + (pr – ps – qr + qs) – 2pr – 2qs$
$= 0$

(vi) $(x + y + z)(x – y + z) + (x + y – z)(-x + y + z) – 4zx$
$= x^2 – xy + xz + xy – y^2 + yz + xz – yz + z^2 – x^2 + xy + xz$
$– xy + x^2 + yx + xz – yz – z^2 – 4zx$
$= 0$

5. If two adjacent sides of a rectangle are $5x^2 + 25xy + 4y^2$ and $2x^2 – 2xy + 3y^2$, find its area.
Solution:

Given,
The adjacent sides of a rectangle are $5x^2 + 25xy + 4y^2$ and $2x^2 – 2xy + 3y^2$
So,
Area of rectangle = Product of two adjacent sides
$= (5x^2 + 25xy + 4y^2) (2x^2 – 2xy + 3y^2)$
$= 10x^4– 10x^3y+ 15x^2y^2 + 50x^3y – 50x^2y^2 + 75xy^3 + 8x^2y^2 – 8xy^3 + 12y^4$
$= 10x^4 + 40x^3y – 27x^2y^2 + 67xy^3 + 12y^4$
Thus,
The area of the rectangle is $10x^4 + 40x^3y – 27x^2y^2 + 67xy^3 + 12y^4$.

Exercise 10.4

1. Divide:

(i) $-39pq^2r^5$ by $-24p^3q^3r$

(ii) $-a^2b^3$ by a^3b^2

Solution:

(i) $-39pq^2r^5$ (÷) $-24p^3q^3r$

$= -39pq^2r^5 / -24p^3q^3r$

$$= \left(\frac{-39}{-24}\right) \times \left(\frac{pq^2r^5}{p^3q^3r}\right)$$

$$= \frac{13}{8} \times \frac{r^4}{p^2q} = \frac{13r^4}{8p^2q}$$

(ii) $\frac{-3}{4}a^2b^3 \div \frac{6}{7}a^3b^2$

$$= \frac{\frac{-3}{4}a^2b^3}{\frac{6}{7}a^3b^2}$$

$$= \left(\frac{\frac{-3}{4}}{\frac{6}{7}}\right) \times \left(\frac{a^2b^3}{a^3b^2}\right)$$

$$= \left(\frac{-3}{4} \times \frac{7}{6}\right) \times \left(\frac{b}{a}\right)$$

$$= \frac{-7}{8} \times \frac{b}{a} = \frac{-7b}{8a}$$

2. Divide:

(i) $9x^4 - 8x^3 - 12x + 3$ by $3x$

(ii) $14p^2q^3 - 32p^3q^2 + 15pq^2 - 22p + 18q$ by $-2p^2q$.

Solution:

(i) $\frac{9x^4 - 8x^3 - 12x + 3}{3x}$

$$= \frac{9x^4}{3x} - \frac{8x^3}{3x} - \frac{12x}{3x} + \frac{3}{3x}$$

$$= 3x^3 - \frac{8}{3}x^2 - 4 + \frac{1}{x}$$

(ii) $\frac{14p^2q^3 - 32p^3q^2 + 15pq^2 - 22p + 18q}{-2p^2q}$

$= \frac{14p^2q^3}{-2p^2q} - \frac{32p^3q^2}{-2p^2q} + \frac{15pq^2}{-2p^2q} - \frac{22p}{-2p^2q} + \frac{18q}{-2p^2q}$

$= -7q^2 + 16pq - \frac{15q}{2p} + \frac{11}{pq} - \frac{9}{p^2}$

3. Divide:
(i) $6x^2 + 13x + 5$ by $2x + 1$
(ii) $1 + y^3$ by $1 + y$
(iii) $5 + x - 2x^2$ by $x + 1$
(iv) $x^3 - 6x^2 + 12x - 8$ by $x - 2$
Solution:

(i) $6x^2 + 13x + 5 \div 2x + 1$

$$\begin{array}{l} 2x+1\,)\,6x^2 + 13x + 5\,(\,3x + 5 \\ \quad\quad\;\; 6x^2 + 3x \\ \quad\quad\;\; - \quad - \\ \hline \quad\quad\quad\quad 10x + 5 \\ \quad\quad\quad\quad 10x + 5 \\ \quad\quad\quad\quad - \quad - \\ \hline \quad\quad\quad\quad\quad 0 \end{array}$$

∴ Quotient = 3x + 5 and remainder = 0

(ii) $1 + y^3 \div 1 + y$

$$\begin{array}{l} y+1\,)\,y^3 + 1\,(\,y^2 - y + 1 \\ \quad\quad\; y^3 + y^2 \\ \quad\quad\; - \quad - \\ \hline \quad\quad\quad -y^2 + 1 \\ \quad\quad\quad -y^2 - y \\ \quad\quad\quad + \quad + \\ \hline \quad\quad\quad\quad y + 1 \\ \quad\quad\quad\quad y + 1 \\ \quad\quad\quad\quad - \quad - \\ \hline \quad\quad\quad\quad\quad 0 \end{array}$$

∴ Quotient = $y^2 - y + 1$ and remainder = 0

(iii) On arranging the terms of dividend in descending order of powers of x and then dividing, we get
$-2x^2 + x + 5 \div x + 1$

$$x + 1\ )\ -2x^2 + x + 5\ (\ -2x + 3$$
$$-2x^2 - 2x$$
$$+ \quad +$$
$$3x + 5$$
$$3x + 3$$
$$- \quad -$$
$$2$$

∴ Quotient = $-2x + 3$ and remainder = 2

(iv) $x^3 - 6x^2 + 12x - 8 \div x - 2$

$$x - 2\ )\ x^3 - 6x^2 + 12x - 8\ (\ x^2 - 4x + 4$$
$$x^3 - 2x^2$$
$$- \quad +$$
$$-4x^2 + 12x$$
$$-4x^2 + 8x$$
$$+ \quad -$$
$$4x - 8$$
$$4x - 8$$
$$- \quad +$$
$$0$$

∴ Quotient = $x^2 - 4x + 4$ and remainder = 0

4. Divide:
(i) $6x^3 + x^2 - 26x - 25$ by $3x - 7$
(ii) $m^3 - 6m^2 + 7$ by $m - 1$
Solution:

(i) $6x^3 + x^2 - 26x - 25 \div 3x - 7$

$$
\begin{array}{r|l}
3x-7 & 6x^3 + x^2 - 26x - 25 \;(\; 2x^2 + 5x + 3 \\
 & 6x^3 - 14x^2 \\
 & -\quad + \\ \hline
 & 15x^2 - 26x - 25 \\
 & 15x^2 - 35x \\
 & -\quad + \\ \hline
 & 9x - 25 \\
 & 9x - 21 \\
 & -\quad + \\ \hline
 & -4 \\ \hline
\end{array}
$$

∴ Quotient = $2x^2 + 5x + 3$ and remainder = – 4

(ii) $m^3 – 6m^2 + 7 ÷ m – 1$

$$
\begin{array}{r|l}
m-1 & m^3 - 6m^2 + 7 \;(\; m^2 - 5m - 5 \\
 & m^3 - m^2 \\
 & -\quad + \\ \hline
 & -5m^2 \quad + 7 \\
 & -5m^2 + 5m \\
 & +\quad - \\ \hline
 & -5m + 7 \\
 & -5m + 5 \\
 & +\quad - \\ \hline
 & 2 \\ \hline
\end{array}
$$

∴ Quotient = $m^2 – 5m – 5$ and remainder = 2.

5. Divide:
(i) $a^3 + 2a^2 + 2a + 1$ by $a^2 + a + 1$
(ii) $12x^3 – 17x^2 + 26x – 18$ by $3x^2 – 2x + 5$
Solution:

(i) $a^3 + 2a^2 + 2a + 1 ÷ a^2 + a + 1$

$$
\begin{array}{r|l}
a^2+a+1 & a^3+2a^2+2a+1 \;(a+1 \\
 & a^3+a^2+a \\
 & -\quad -\quad - \\
\hline
 & \quad a^2+a+1 \\
 & \quad a^2+a+1 \\
\hline
 & \qquad 0
\end{array}
$$

∴ Quotient = a + 1 and remainder = 0.

(ii) $12x^3 – 17x^2 + 26x – 18 ÷ 3x^2 – 2x + 5$

$$
\begin{array}{r|l}
3x^2-2x+5 & 12x^3-17x^2+26x-18\;(4x-3 \\
 & 12x^3-8x^2+20x \\
 & -\quad +\quad - \\
\hline
 & \quad -9x^2+6x-18 \\
 & \quad -9x^2+6x-15 \\
 & \quad +\quad -\quad + \\
\hline
 & \qquad\qquad -3
\end{array}
$$

∴ Quotient = 4x – 3 and remainder = -3

6. If the area of a rectangle is $8x^2 – 45y^2 + 18xy$ and one of its sides is 4x + 15y, find the length of adjacent side.
Solution:

Given,
Area of rectangle = $8x^2 – 45y^2 + 18xy$
And, one side = 4x + 15y
∴ Second (adjacent) side = Area of rectangle/ One side
= $8x^2 – 45y^2 + 18xy ÷ 4x + 15y$

$$
\begin{array}{r|l}
 & 2x-3y \\
4x+15y & 8x^2+18xy-45y^2 \\
 & 8x^2+30xy \\
 & -\quad - \\
\hline
 & \quad -12xy-45y^2 \\
 & \quad -12xy-45y^2 \\
 & \quad +\quad + \\
\hline
 & \qquad 0
\end{array}
$$

Thus, length of the adjacent side is 2x – 3y.

Exercise 10.5

1. Using suitable identities, find the following products:

(i) (3x + 5) (3x + 5)

(ii) (9y – 5) (9y – 5)

(iii) (4x + 11y) (4x – 11y)

(iv) (3m/2 + 2n/3) (3m/2 – 2n/3)

(v) (2/a + 5/b) (2a + 5/b)

(vi) $(p^2/2 + 2/q^2)(p^2/2 - 2/q^2)$

Solution:

(i) (3x + 5) (3x + 5)

$= (3x + 5)^2$

$= (3x)^2 + 2 \times 3x \times 5 + (5)^2$ [Using, $(a + b)^2 = a^2 + 2ab + b^2$]

$= 9x^2 + 30x + 25$

(ii) (9y – 5) (9y – 5)

$= (9y - 5)^2$

$= (9y)^2 - 2 \times 9y \times 5 + (5)^2$ [Using, $(a - b)^2 = a^2 - 2ab + b^2$]

$= 81y^2 - 90y + 25$

(iii) (4x + 11y)(4x – 11y)

$= (4x)^2 - (11y)^2$

$= 16x^2 - 121y^2$ [Using, $(a + b)(a - b) = a^2 - b^2$]

(iv) (3m/2 + 2n/3) (3m/2 – 2n/3)

$= (3m/2)^2 - (2n/3)^2$

$= 9m^2/4 - 4n^2/9$ [Using, $(a + b)(a - b) = a^2 - b^2$]

(v) (2/a + 5/b) (2a + 5/b)

$= (2/a + 5/b)^2$

$= (2/a)^2 + 2(2/a)(5/b) + (5/b)^2$ [Using, $(a + b)^2 = a^2 + 2ab + b^2$]

$= 4/a^2 + 20a/b + 25/b^2$

(vi) $(p^2/2 + 2/q^2)(p^2/2 - 2/q^2)$

$= (p^2/2)^2 - (2/q^2)^2$ [Using, $(a + b)(a - b) = a^2 - b^2$]

$= p^4/4 - 4/q^4$

2. Using the identities, evaluate the following:

(i) 81^2

(ii) 97^2

(iii) 105^2

(iv) 997^2

(v) 6.1^2

(vi) 496 × 504

(vii) 20.5 × 19.5

(viii) 9.62
Solution:

(i) $(81)^2 = (80 + 1)^2$
$= (80)^2 + 2 \times 80 \times 1 + (1)^2$ [Using, $(a + b)^2 = a^2 + 2ab + b^2$]
$= 6400 + 160 + 1$
$= 6561$

(ii) $(97)^2 = (100 - 3)^2$
$= (100)^2 - 2 \times 100 \times 3 + (3)^2$ [Using, $(a - b)^2 = a^2 - 2ab + b^2$]
$= 10000 - 600 + 9$
$= 10009 - 600$
$= 9409$

(ii) $(105)^2 = (100 + 5)^2$
$= (100)^2 + 2 \times 100 \times 5 + (5)^2$ [Using, $(a + b)^2 = a^2 + 2ab + b^2$]
$= 10000 + 1000 + 25$
$= 11025$

(iv) $(997)^2 = (1000 - 3)^2$
$= (1000)^2 - 2 \times 1000 \times 3 + (3)^2$ [Using, $(a - b)^2 = a^2 - 2ab + b^2$]
$= 1000000 - 6000 + 9$
$= 1000009 - 6000$
$= 994009$

(v) $(6.1)^2 = (6 + 0.1)^2$
$= (6)^2 + 2 \times 6 \times 0.1 + (0.1)^2$ [Using, $(a + b)^2 = a^2 + 2ab + b^2$]
$= 36 + 1.2 + 0.01$
$= 37.21$

(vi) 496×504
$= (500 - 4)(500 + 4)$ [Using, $(a + b)(a - b) = a^2 - b^2$]
$= (500)^2 - (4)^2$
$= 250000 - 16$
$= 249984$

(vii) 20.5×19.5
$= (20 + 0.5)(20 - 0.5)$ [Using, $(a + b)(a - b) = a^2 - b^2$]
$= (20)^2 - (0.5)^2$
$= 400 - 0.25$
$= 399.75$

(viii) $(9.6)^2 = (10 - 0.4)^2$
$= (10)^2 - 2 \times 10 \times 0.4 + (0.4)^2$ [Using, $(a - b)^2 = a^2 - 2ab + b^2$]
$= 100 - 8.0 + 0.16$
$= 92.16$

3. Find the following squares, using the identities:
(i) $(pq + 5r)^2$ (ii) $(5a/2 – 3b/5)^2$
(iii) $(\sqrt{2}a + \sqrt{3}b)^2$ (iv) $(2x/3y – 3y/2x)^2$
Solution:

(i) $(pq + 5r)^2$
$= (pq)^2 + 2 \times pq \times 5r + (5r)^2$ [Using, $(a + b)^2 = a^2 + 2ab + b^2$]
$= p^2q^2 + 10pqr + 25r^2$

(ii) $(5a/2 – 3b/5)^2$
$= (5a/2)^2 – 2 \times (5a/2) \times (-3b/5) + (3b/5)^2$ [Using, $(a – b)^2 = a^2 – 2ab + b^2$]
$= 25a^2/4 – 3ab + 9b^2/25$

(iii) $(\sqrt{2}a + \sqrt{3}b)^2$
$= (\sqrt{2}a)^2 + 2 \times \sqrt{2}a \times \sqrt{3}b + (\sqrt{3}b)^2$ [Using, $(a + b)^2 = a^2 + 2ab + b^2$]
$= 2a^2 + 2\sqrt{6}ab + 3b^2$

(iv) $(2x/3y – 3y/2x)^2$

$$= \left(\frac{2x}{3y}\right)^2 - 2 \times \frac{2x}{2y} \times \frac{3y}{2x} + \left(\frac{3y}{2x}\right)^2 \quad \{(a-b)^2 = a^2 - 2ab + b^2\}$$

$$= \frac{4x^2}{9y^2} - 2 + \frac{9y^2}{4x^2}$$

4. Using the identity, $(x + a) (x + b) = x^2 + (a + b)x + ab$, find the following products:
(i) $(x + 7) (x + 3)$
(ii) $(3x + 4) (3x – 5)$
(iii) $(p^2 + 2q) (p^2 – 3q)$
(iv) $(abc + 3) (abc – 5)$
Solution:

(i) $(x + 7) (x + 3)$
$= (x)^2 + (7 + 3)x + 7 \times 3$
$= x^2 + 10x + 21$

(ii) $(3x + 4) (3x – 5)$
$= (3x)^2 + (4 – 5) (3x) + 4 \times (-5)$
$= 9x^2 – 3x – 20$

(iii) $(P^2 + 2q)(p^2 – 3q)$
$= (p^2)^2 + (2q – 3q)p^2 + 2q \times (-3q)$
$= p^4 – p^2q – 6pq$

(iv) $(abc + 3) (abc – 5)$

$= (abc)^2 + (3 – 5)abc + 3 \times (-5)$
$= a^2b^2c^2 – 2abc – 15$

5. Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$, evaluate the following:
(i) 203 × 204
(ii) 8.2 × 8.7
(iii) 107 × 93
Solution:

(i) 203 × 204
$= (200 + 3)(200 + 4)$
$= (200)^2 + (3 + 4) \times 200 + 3 \times 4$
$= 40000 + 1400 + 12$
$= 41412$

(ii) 8.2 × 8.7
$= (8 + 0.2)(8 + 0.7)$
$= (8)^2 + (0.2 + 0.7) \times 8 + 0.2 \times 0.7$
$= 64 + 8 \times (0.9) + 0.14$
$= 64 + 7.2 + 0.14$
$= 71.34$

(iii) 107 × 93
$= (100 + 7)(100 – 7)$
$= (100)^2 + (7 – 7) \times 100 + 7 \times (-7)$
$= 10000 + 0 – 49$
$= 9951$

6. Using the identity $a^2 – b^2 = (a + b)(a – b)$, find
(i) $53^2 – 47^2$
(ii) $(2.05)^2 – (0.95)^2$
(iii) $(14.3)^2 – (5.7)^2$
Solution:

(i) $53^2 – 47^2$
$= (50 + 3)(50 – 3)$
$= (50)^2 – (3)^2$
$= 2500 – 9$
$= 2491$

(ii) $(2.05)^2 – (0.95)^2$
$= (2.05 + 0.95)(2.05 – 0.95)$
$= 3 \times 1.10$
$= 3.3$

(iii) $(14.3)^2 – (5.7)^2$

$= (14.3 + 5.7)(14.3 – 5.7)$
$= 20 \times 8.6$
$= 172$

7. Simplify the following:
(i) $(2x + 5y)^2 + (2x – 5y)^2$
(ii) $(7a/2 – 5b/2)^2 – (5a/2 – 7b/2)^2$
(iii) $(p^2 – q^2r)^2 + 2p^2q^2r$
Solution:

(i) $(2x + 5y)^2 + (2x – 5y)^2$ [Using, $(a \pm b)^2 = a^2 \pm 2ab + b^2$]
$= (2x)^2 + 2 \times 2x \times 5y + (5y)^2 + (2x)^2 – 2 \times 2x \times 5y + (5y)^2$
$= 4x^2 + 20xy + 25y^2 + 4x^2 – 20xy + 25y^2$
$= 8x^2 + 50y^2$

(ii) $(7a/2 – 5b/2)^2 – (5a/2 – 7b/2)^2$

$$= \left[\left(\frac{7}{2}a\right)^2 - 2\times\frac{7}{2}a\times\frac{5}{2}b - \left(\frac{5}{2}b\right)^2\right] - \left[\left(\frac{5}{2}a\right)^2 - 2\times\frac{5}{2}a\times\frac{7}{2}b + \left(\frac{7}{2}b\right)^2\right]$$

$$= \left[\frac{49}{4}a^2 - \frac{35}{2}ab + \frac{25}{4}b^2\right] - \left[\frac{25}{4}a^2 - \frac{35}{2}ab + \frac{49}{4}b^2\right]$$

$$= \frac{49}{4}a^2 - \frac{35}{2}ab + \frac{25}{4}b^2 - \frac{25}{4}a^2 + \frac{35}{2}ab - \frac{49}{4}b^2$$

$$= \frac{49}{4}a^2 - \frac{25}{4}a^2 + \frac{25}{4}b^2 - \frac{49}{4}a^2$$

$$= \frac{24}{4}a^2 + \frac{-24}{4}b^2$$

$$= 6a^2 - 6b^2$$

(iii) $(p^2 – q^2r)^2 + 2p^2q^2r$ [Using, $(a – b)^2 = a^2 – 2ab + b^2$]
$= (p^2)^2 – 2 \times p^2 \times q^2r + (q^2r)^2 + 2p^2q^2r$
$= p^4 – 2p^2q + q^4r^2 + 2p^2q^2r$
$= p^4 + q^4r^2$

8. Show that:
(i) $(4x + 7y)^2 – (4x – 7y)^2 = 112xy$
(ii) $(3p/7 – 7q/6)^2 + pq = 9p^2/49 + 49q^2/36$
(iii) $(p – q)(p + q) + (q – r)(q + r) + (r – p)(r + p) = 0$
Solution:

(i) Taking LHS, we have
LHS $= (4x + 7y)^2 – (4x – 7y)^2$ [Using, $(a \pm b)^2 = a^2 \pm 2ab + b^2$]
$= [(4x)^2 + 2 \times 4x \times 7y + (7y)^2] – [(4x)^2 – 2 \times 4x + 7y + (7y)^2]$
$= (16x^2 + 56xy + 49y^2) – (16x^2 – 56xy + 49y^2)$
$= l6x^2 + 56xy + 49y^2 – 16x^2 + 56xy – 49y^2$

= 112xy = RHS

(ii) Taking LHS, we have

$$\text{LHS} = \left(\frac{3}{7}p - \frac{7}{6}q\right)^2 + pq$$

$$= \left(\frac{3}{7}p\right)^2 - 2 \times \frac{3}{7}p \times \frac{7}{6}q + \left(\frac{7}{6}q\right)^2 + pq \quad \{(a-b)^2 = a^2 - 2ab + b^2\}$$

$$= \frac{9}{49}p^2 - pq + \frac{49}{36}q^2 + pq$$

$$= \frac{9}{49}p^2 + \frac{49}{36}q^2 = \text{RHS}$$

(iii) Taking LHS, we have

LHS = (p – q) (p + q) + (q – r) (q + r) + (r – p)(r + p)

= $p^2 - q^2 + q^2 - r^2 + r^2 - p^2$ [Using, $(a + b)(a - b) = a^2 - b^2$]

= 0 = RHS

9. If x + 1/x = 2, evaluate:
(i) $x^2 + 1/x^2$ (ii) $x^4 + 1/x^4$
Solution:

(i) We have, x + 1/x = 2

On squaring on both sides, we get

$(x + 1/x)^2 = 2^2$

$x^2 + 2 \times x \times 1/x + 1/x^2 = 4$

$x^2 + 2 + 1/x^2 = 4$

$x^2 + 1/x^2 = 4 - 2$

Thus,

$x^2 + 1/x^2 = 2$

(ii) Again squaring, we get

$(x^2 + 1/x^2)^2 = 2^2$

$x^4 + 2 \times x^2 \times 1/x^2 + 1/x^4 = 4$

$x^4 + 2 + 1/x^4 = 4$

$x^4 + 1/x^4 = 4 - 2$

Thus,

$x^4 + 1/x^4 = 2$

10. If x – 1/x = 7, evaluate:
(i) $x^2 + 1/x^2$ (ii) $x^4 + 1/x^4$
Solution:

We have, x – 1/x = 7

On squaring on both sides, we get

$(x - 1/x)^2 = 7^2$
$x^2 - 2 \times x^2 \times 1/x^2 + 1/x^2 = 49$
$x^2 - 2 + 1/x^2 = 49$
$x^2 + 1/x^2 = 49 + 2$
Thus,
$x^2 + 1/x^2 = 51$

(ii) Again squaring, we get
$(x^2 + 1/x^2)^2 = 51^2$
$x^4 + 1/x^4 + 2 \times x^2 \times 1/x^2 = 2601$
$x^4 + 1/x^4 + 2 = 2601$
$x^4 + 1/x^4 = 2601 - 2$
Thus,
$x^4 + 1/x^4 = 2599$

11. If $x^2 + 1/x^2 = 23$, evaluate:
(i) $x + 1/x$ (ii) $x - 1/x$
Solution:

We have, $x^2 + 1/x^2 = 23$
(i) $(x + 1/x)^2 = x^2 + 1/x^2 + 2$
$= 23 + 2$
$= 25$
Taking square root on both sides, we get
$(x + 1/x) = \pm 5$
Thus, $x + 1/x = 5$ or -5

(ii) $(x - 1/x)^2 = x^2 + 1/x^2 - 2$
$= 23 - 2$
$= 21$
Taking square root on both sides, we get
$(x + 1/x) = \pm\sqrt{21}$
Thus, $x + 1/x = \sqrt{21}$ or $-\sqrt{21}$

12. If $a + b = 9$ and $ab = 10$, find the value of $a^2 + b^2$.
Solution:

Given,
$a + b = 9$ and $ab = 10$
Now, squaring $a + b = 9$ on both sides, we have
$(a + b)^2 = (9)$
$a^2 + b^2 + 2ab = 81$
$a^2 + b^2 + 2 \times 10 = 81$
$a^2 + b^2 + 20 = 81$
$a^2 + b^2 = 81 - 20 = 61$
$\therefore a^2 + b^2 = 61$

13. If $a - b = 6$ and $a^2 + b^2 = 42$, find the value of
Solution:

Given
$a - b = 6$ and $a^2 + b^2 = 42$
$a - b = 6$
Now, squaring $a - b = 6$ on both sides, we have
$(a - b)^2 = (6)^2$
$a^2 + b^2 - 2ab = 36$
$42 - 2ab = 36$
$2ab = 42 - 36 = 6$
$ab = 6/2 = 3$
$\therefore ab = 3$

14. If $a^2 + b^2 = 41$ and $ab = 4$, find the values of
(i) $a + b$
(ii) $a - b$
Solution:

Given, $a^2 + b^2 = 41$ and $ab = 4$
(i) $(a + b)^2 = a^2 + b^2 + 2ab$
$= 41 + 2 \times 4$
$= 41 + 8$
$= 49$
$\therefore a + b = \pm 7$

(ii) $(a - b)^2 = a^2 + b^2 - 2ab$
$= 41 - 2 \times 4$
$= 41 - 8$
$= 33$
$\therefore a - b = \pm\sqrt{33}$

Check Your Progress

1. Add the following expressions:
(i) $-5x^2y + 3xy^2 – 7xy + 8$, $12x^2y – 5xy^2 + 3xy – 2$
(ii) $9xy + 3yz – 5zx$, $4yz + 9zx – 5y$, $-5xz + 2x – 5xy$
Solution:

(i) $(-5x^2y + 3xy^2 – 7xy + 8) + (12x^2y – 5xy^2 + 3xy – 2)$
$= 7x^2y – 2xy^2 – 4xy + 6$

(ii) $(9xy + 3yz – 5zx) + (4yz + 9zx – 5y, -5xz + 2x – 5xy)$
$= 4xy + 7yz – zx + 2x – 5y$

2. Subtract:
(i) $5a + 3b + 11c – 2$ from $3a + 5b – 9c + 3$
(ii) $10x^2 – 8y^2 + 5y – 3$ from $8x^2 – 5xy + 2y^2 + 5x – 3y$
Solution:

(i) $5a – 3b + 11c – 2$ from $3a + 5b – 9c + 3$
$= (3a + 5b – 9c + 3) – (5a – 3b + 11c – 2)$
$= 3a + 5b – 9c + 3 – 5a + 3b – 11c + 2$
$= -2a + 8b – 20c + 5$

(ii) $10x^2 – 8y^2 + 5y – 3$ from $8x^2 – 5xy + 2y^2 + 5x – 3y$
$= (8x^2 – 5xy + 2y^2 + 5x – 3y) – (10x^2 – 8y^2 + 5y – 3)$
$= 8x^2 – 5xy + 2y^2 + 5x – 3y – 10x^2 + 8y^2 – 5y + 3$

3. What must be added to $5x^2 – 3x + 1$ to get $3x^3 – 7x^2 + 8$?
Solution:

From the question, the required expression is
$= (3x^3 – 7x^2 + 8) – (5x^2 – 3x + 1)$
$= 3x^3 – 7x^2 + 8 – 5x^2 + 3x – 1$
$= 3x^3 – 12x^2 + 3x + 7$

4. Find the product of
(i) $3x^2y$ and $-4xy^2$
(ii) $–(4/5)xy$, $(5/7)yz$ and $–(14/9)zx$
Solution:

Product of:
(i) $3x^2y$ and $-4xy^2$
$= 3x^2 \times (-4xy^2)$
$= -12x^{2+1} y^{1+2}$
$= 12x^3y^3$

(ii) $-(4/5)xy$, $(5/7)yz$ and $-(14/9)zx$
$= -(4/5)xy \times (5/7)yz \times -(14/9)zx$
$= -(4/5) \times (5/7) \times -(14/9)\ x^2y^2z^2$
$= (8/9)x^2y^2z^2$

5. Multiply:
(i) $(3pq - 4p^2 + 5q^2 + 7)$ by $-7pq$
(ii) $(3/4x^2y - 4/5xy + 5/6xy^2)$ by $-15xyz$
Solution:

(i) $(3pq - 4p^2 + 5q^2 + 7) \times (-7pq)$
$= -7pq \times 3pq - 7pq \times (-4p^2) + (-7pq)(5q^2) - 7pq \times 7$
$= -21p^2q^2 + 28p^3q - 35pq^3 - 49pq$

(ii) $(3/4x^2y - 4/5xy + 5/6xy^2) \times (-15xyz)$

$$= -15xyz\left(\frac{3}{4}x^2y - \frac{4}{5}xy + \frac{5}{6}xy^2\right)$$

$$= -15xyz \times \frac{3}{4}x^2y - 15xyz \times \left(\frac{-4}{5}xy\right) - 15xyz\left(\frac{5}{6}xy^2\right)$$

$$= \frac{-45}{4}x^3y^2z + 12x^2y^2z - \frac{25}{2}x^2y^3z$$

6. Multiply:
(i) $(5x^2 + 4x - 2)$ by $(3 - x - 4x^2)$
(ii) $(7x^2 + 12xy - 9y^2)$ by $(3x^2 - 5xy + 3y^2)$
Solution:

(i) $(5x^2 + 4x - 2) \times (3 - x - 4x^2)$
$= 5x^2(3 - x - 4x^2) + 4x(3 - x - 4x^2) - 2(3x - x - 4x^2)$
$= 15x^2 - 5x^3 - 20x^4 + 12x - 4x^2 - 16x^3 - 6x + 2x + 8x^2$
$= -20x^4 - 21x^3 + 19x^2 + 14x - 6$

(ii) $(7x^2 + 12xy - 9y^2)$ x $(3x^2 - 5xy + 3y^2)$
$= 7x^2(3x^2 - 5xy + 3y^2) + 12xy(3x^2 - 5xy + 3y^2) - 9y^2(3x^2 - 5xy + 3y^2)$
$= 21x^4 - 35x^3y + 21x^2y^2 + 36x^3y - 60x^2y^2 + 36xy^3 - 27x^2y^2 + 45xy^3 - 27y^4$
$= 21x^4 + x^3y + 81xy^3 - 66x^2y^2 - 27y^4$

7. Simplify the following expressions and evaluate them as directed:
(i) $(3ab - 2a^2 + 5b^2)$ x $(2b^2 - 5ab + 3a^2) + 8a^3b - 7b^4$ for $a = 1$, $b = -1$
(ii) $(1.7x - 2.5y)(2y + 3x + 4) - 7.8x^2 - 10y$ for $x = 0$, $y = 1$.
Solution:

(i) $(3ab - 2a^2 + 5b^2) \times (2b^2 - 5ab + 3a^2) + 8a^3b - 7b^4$
$= 3ab(2b^2 - 5ab + 3a^2) - 2a^2(2b^2 - 5ab + 3a^2) + 5b^2(2b^2 - 5ab + 3a^2) + 8a^3b - 7b^4$

$= 6ab^{32} – 15a^2b^2 + 9a^3b – 4a^2b^2 + 10a^3b – 6a^4 + 10b^4 – 25ab^3 + 15a^2b^2 + 8a^3b – 7b^4$
$= 27a^3b – 4a^2b^2 – 19ab^3 – 6a^4 + 3b^4$
Putting, a = 1 and b = (-1)
$= 27(1)^3 (-1) – 4(1)^2 (-1)^2 – 19 (1) (-1)^3 – 6(1)^4 + 3(-1)^4$
= -27 – 4 + 19 – 6 + 3
= -37 + 22
= -15

(ii) $(1.7x – 2.5y) (2y + 3x + 4) – 7.8x^2 – 10y$
$1.7x(2y + 3x + 4) – 2.5y(2y + 3x + 4) – 7.8x^2 – 10y$
$= 3.4xy + 5.1x^2 + 6.8x – 5y^2 – 7.5xy – 10y – 7.8x^2 – 10y$
$= -2.7x^2 – 4.1xy – 5y^2 + 6.8x – 20y$
Putting, x = 0 and y = 1
$= -2.7 \times 0 – 4.1 \times 0 \times 1 – 5(1)^2 + 6.8 \times 0 – 20 \times 1$
= 0 + 0 – 5 + 0 – 20
= -25

8. Carry out the following divisions:
(i) $66pq^2r^3 \div 11qr^2$
(ii) $(x^3 + 2x^2 + 3x) \div 2x$
Solution:

(i) $66pq^2r^3/ 11qr^2$
$= 6pq^{2-1}r^{3-2}$
= 6pqr

(ii) $(x^3 + 2x^2 + 3x)/ 2x$
$= x^3/2x + 2x^2/2x + 3x/2x$
$= ½ x^2 + x + 3/2$

9. Divide $10x^4 – 19x^3 + 17x^2 + 15x – 42$ by $2x^2 – 3x + 5$.
Solution:

$(10x^4 – 19x^3 + 17x^2 + 15x – 42) \div (2x^2 – 3x + 5)$
Performing long division, we have

$$\begin{array}{r|l} & 5x^2 - 2x - 7 \\ 2x^2 - 3x + 5 & 10x^4 - 19x^3 + 17x^2 + 15x - 42 \\ & 10x^4 - 15x^3 + 25x^2 \\ & - \quad + \quad - \\ & -4x^3 - 8x^2 + 15x \\ & -4x^3 + 6x^2 - 10x \\ & + \quad - \quad + \\ & -14x^2 + 25x - 42 \\ & -14x^2 + 21x - 35 \\ & + \quad - \quad + \\ & 4x - 7 \end{array}$$

Thus, Quotient = $5x^2 – 2x – 7$ and Remainder = $4x – 7$

10. Using identities, find the following products:
(i) (3x + 4y) (3x + 4y)
(ii) (5a/2 - b) (5a/2 – b)
(iii) (3.5m – 1.5n) (3.5m + 1.5n)
(iv) (7xy – 2) (7xy + 7)
Solution:

(i) $(3x + 4y) (3x + 4y)$
$= (3x + 4y)^2$
$= (3x)^2 + 2 \times 3x \times 4y + (4y)^2$ [Using, $(a + b)^2 = a^2 + 2ab + b^2$]
$= 9x^2 + 24xy + 16y^2$

(ii) $(5a/2 - b) (5a/2 – b)$
$= (5a/2 - b)^2$
$= (5a/2)^2 + 2 \times 5a/2 \times (-b) + (b)^2$ [Using, $(a – b)^2 = a^2 – 2ab + b^2$]
$= 25a^2/4 – 5ab + b^2$

(iii) $(3.5m – 1.5n) (3.5m + 1.5n)$
$= (3.5m)^2 – (1.5n)^2$ [Using, $(a – b)(a + b) = a^2 – b^2$]
$= 12.25m^2 – 2.25n^2$

(iv) $(7xy – 2)(7xy + 7)$
$= (7xy)^2 + (-2 + 7) \times (7xy) + (-2) \times 7$ [Using, $(x + a)(x + b) = x^2 + (a + b)x + ab$]
$= 49x^2y^2 + 35xy – 14$

11. Using suitable identities, evaluate the following:
(i) 105^2
(ii) 97^2
(iii) 201 × 199
(iv) $87^2 – 13^2$
(v) 105 × 107
Solution:

(i) $(105)^2 = (100 + 5)^2$
$= (100)^2 + 2 \times 100 \times 5 + (5)^2$ [Using, $(a + b)^2 = a^2 + 2ab + b^2$]
$= 10000 + 1000 + 25$
$= 11025$

(ii) $(97)^2 = (100 – 3)^2$
$= (100)^2 – 2 \times 100 \times 3 + (3)^2$ [Using, $(a – b)^2 = a^2 – 2ab + b^2$]
$= 10000 – 600 + 9$
$= 10009 – 600$
$= 9409$

(iii) $201 \times 199 = (200 + 1)(200 - 1)$
$= (200)^2 - (1)^2$ [Using, $(a + b)(a - b) = a^2 - b^2$]
$= 40000 - 1$
$= 39999$

(iv) $87^2 - 13^2$
$= (87 + 13)(87 - 13)$ [Using, $a^2 - b^2 = (a + b)(a - b)$]
$= 100 \times 74$
$= 7400$

(v) 105×107
$= (100 + 5)(100 + 7)$
$= (100)^2 + (5 + 7) \times 100 + 5 \times 7$ [Using, $(x + a)(x - b) = x^2 + (a + b)x + ab$]
$= 10000 + 1200 + 35$
$= 11235$

12. Prove that following:
(i) $(a + b)^2 - (a - b)^2 + 4ab$
(ii) $(2a + 3b)^2 + (2a - 3b)^2 = 8a^2 + 18b^2$
Solution:

(i) Taking the RHS, we have
RHS $= (a - b)^2 + 4ab$
$= a^2 - 2ab + b^2 + 4ab$
$= a^2 + 2ab + b^2$
$= (a + b)^2 =$ L.H.S.

(ii) Taking the LHS, we have
LHS $= (2a + 3b)^2 + (1a - 3b)^2$
$= (2a)^2 + 2 \times 2a \times 3b + (3b)^2 + (2a)^2 - 2 \times 2a \times 3b + (3b)^2$
$= 4a^2 + 12ab + 9b^2 + 4a^2 - 12ab + 9b^2$
$= 8a^2 + 18b^2 =$ RHS

13. If $x + 1/x = 5$, evaluate
(i) $x^2 + 1/x^2$ (ii) $x^4 + 1/x^4$
Solution:

(i) We have, $x + 1/x = 5$
On squaring on both sides, we get
$(x + 1/x)^2 = 5^2$
$x^2 + 1/x^2 + 2 \times x \times 1/x = 25$
$x^2 + 2 + 1/x^2 = 25$
$x^2 + 1/x^2 = 25 - 2$
Hence, $x^2 + 1/x^2 = 23$

(ii) Again, squaring $x^2 + 1/x^2 = 23$ on both sides, we get

$(x^2 + 1/x^2)^2 = 23^2$
$x^4 + 1/x^4 + 2 \times x^4 \times 1/x^4 = 529$
$x^4 + 1/x^4 + 2 = 529$
$x^4 + 1/x^4 = 529 - 2$
Hence,
$x^4 + 1/x^4 = 527$

14. If $a + b = 5$ and $a^2 + b^2 = 13$, find ab.
Solution:

Given,
$a + b = 5$ and $a^2 + b^2 = 13$
On squaring $a + b = 5$ both sides, we get
$(a + b)^2 = (5)^2$
$a^2 + b^2 + 2ab = 25$
$13 + 2ab = 25 \Rightarrow 2ab = 25 - 13 = 12$
$\Rightarrow ab = 12/2 = 6$
$\therefore ab = 6$