Exercise 11.1

Factorise the following (1 to 8) polynomials:

1. (i) $8xy^3 + 12x^2y^2$
(ii) $15ax^3 – 9ax^2$
Solution:

(i) $8xy^3 + 12x^2y^2 = 4xy^2 (2y + 3x)$
(ii) $15ax^3 – 9ax^2 = 3ax^2 (5x – 3)$

2. (i) $21 py^2 – 56py$
(ii) $4x^3 – 6x^2$
Solution:

(i) $21 py^2 – 56py = 7py (3y – 8)$
(ii) $4x^3 – 6x^2 = 2x^2 (2x – 3)$

3. (i) $25abc^2 – 15a^2b^2c$
(ii) $x^2yz + xy^2z + xyz^2$
Solution:

(i) $25abc^2 – 15a^2b^2c = 5abc (5c – 3ab)$
(ii) $x^2yz + xy^2z + xyz^2 = xyz(x + y + z)$

4. (i) $8x^3 – 6x^2 + 10x$
(ii) $14mn + 22m – 62p$
Solution:

(i) $8x^3 – 6x^2 + 10x = 2x (4x^2 – 3x + 5)$
(ii) $14mn + 22m – 62p = 2 (7mn + 11m – 31p)$

5. (i) $18p^2q^2 – 24pq^2 + 30p^2q$
(ii) $27a^3b^3 – 18a^2b^3 + 75a^3b^2$
Solution:

(i) $18p^2q^2 – 24 pq^2 + 30p^2q$
$= 6pq (3pq -4q + 5p)$

(ii) $27a^3b^3 – 18a^2b^3 + 75a^3b^2$
$= 3a^2b^2 (9ab – 6b + 25a)$

6. (i) $15a (2p – 3p) – 106 (2p – 3q)$
(ii) $3a (x^2 + y^2) + 6b (x^2 + y^2)$
Solution:

(i) 15a (2p – 3q) – 10b (2p – 3q)
= (2p – 3q)(15a – 10b)
= (2p – 3q) (5) (3a – 2b)
= 5 (2p- 3q) (3a – 2b)

(ii) 3a $(x^2 + y^2)$ + 66 $(x^2 + y^2)$
= $(x^2 + y^2)$ (3a + 6b)
= $(x^2 + y^2)$ (3) (a + 2b)
= 3 $(x^2 + y^2)$ (a + 2b)

7. (i) $6(x + 2y)^3 + 8(x + 2y)^2$
(ii) $14(a – 3b)^3 – 21p(a – 3b)$
Solution:

(i) $6(x + 2y)^3 + 8(x + 2y)^2$
$(x + 2y)^2$ [6 (x + 2y) + 8]
= $(x + 2y)^2$ [6x + 12y + 8]
= $(x + 2y)^2$ (2) (3x + 6y + 4)
= 2 $(x + 2y)^2$ (3x + 6y + 4)

(ii) $14(a – 3b)^3 – 21$ p(a – 3b)
= 7 [2 $(a – 3b)^3$ - 3p(a- 3b)]
= 7 [(a – 3b) {2 $(a – 3b)^2 – 3p$}]
= 7 (a – 3b) [2 $(a – 3b)^2 – 3p$]

8. $10a (2p + q)^3 – 15b (2p + q)^2 + 35(2p + q)$
Solution:

$10a (2p + q)^3 – 15b (2p + q)^2 + 35(2p + q)$
= 5 $[2a (2p + q)]^3 – 3b (2p + q)^2 + 7 (2p + q)$
= $5(2p + q) [2a (2p + q)^2 – 3b(2p + q) + 7]$

Exercise 11.2

Factorise the following (1 to 11) polynomials:

1. (i) $x^2 + xy – x – y$
(ii) $y^2 – yz – 5y + 5z$
Solution:

(i) $x^2 + xy – x – y$
$= x (x + y) -1 (x + y)$
$= (x + y) (x – 1)$

(ii) $y^2 – yz – 5y + 5z$
$= y (y – z) -5 (y – z)$
$= (y – z) (y – 5)$

2. (i) $5xy + 7y – 5y^2 – 7x$
(ii) $5p^2 – 8pq – 10p + 16q$
Solution:

(i) $5xy + 7y – 5y^2 – 7x$
$= 5xy – 5y^2 + 7y – 7x$
$= 5y (x – y) -7 (x – y)$
$= (x – y)(5y – 1)$

(ii) $5p^2 – 8pq – 10p + 16q$
$= 5p^2 – 10p – 8pq + 16q$
$= 5p (p – 2) – 8q (p – 2)$
$= (p – 2) (5p – 5q)$
$= (5p – 8q) (p – 2)$

3. (i) $a^2b – ab^2 + 3a – 3b$
(ii) $x^3 – 3x^2 + x – 3$
Solution:

(i) $a^2b – ab^2 + 3a – 3b$
$= ab (a – b) + 3 (a – b) = (a – b) (ab + 3)$

(ii) $x^3 – 3x^2 + x – 3$
$= x^2 (x – 3) + 1 (x – 3)$
$= (x – 3) (x^2 + 1)$

4. (i) $6xy^2 – 3xy – 10y + 5$
(ii) $3ax – 6ay – 8by + 4bx$
Solution:

(i) $6xy^2 – 3xy – 10y + 5$

3xy (2y – 1) -5(2y – 1)
= (2y – 1) (3xy – 5)

(ii) 3ax – 6ay – 8by + 4bx
= 3ax – 6ay + 4bx – 8by
= 3a (x – 2y) + 4b (x – 2y)
= (x – 2y) (3a + 4b)

5. (i) $x^2 + xy (1 + y) + y^3$
(ii) $y^2 – xy (1 – x) – x^3$
Solution:

(i) $x^2 + xy (1 + y) + y^3$
= $x^2 + xy + xy^2 + y^3$
= $x(x + y) + y^2(x + y)$
= $(x + y) (x + y^2)$

(ii) $y^2 – xy (1 – x) – x^3$
= $y^2 – xy + x^2y – x^3$
= $y (y – x) + x^2 (y – x)$
= $(y – x) (y + x^2)$

6. (i) $ab^2 + (a – 1) b – 1$
(ii) 2a – 4b – xa + 2bx
Solution:

(i) $ab^2 + (a – 1) b – 1$
= $ab^2 + ab – b – 1$
= ab (b + 1) -1 (b + 1)
= (b + 1) (ab – 1)

(ii) 2a – 4b – xa + 2bx
= 2 (a – 2b) -x (a – 2b)
= (a – 2b) (2 – x)

7. (i) 5ph – 10qk + 2rph – 4qrk
(ii) $x^2 – x(a + 2b) + 2a^2$
Solution:

(i) 5ph – 10qk + 2rph – 4qrk
= 5 (ph – 2qk) + 2r (ph – 2qk)
= (ph – 2qk) (5 + 2r)

(ii) $x^2 – x(a + 2b) + 2ab$
= $x^2 – xa – 2bx + 2ab$
= $x(x – a) – 2b(x – a)$

= (x – a) (x – 2b)

8. (i) ab $(x^2 + y^2)$ – xy $(a^2 + b^2)$
(ii) $(ax + by)^2 + (bx – ay)^2$
Solution:

(i) ab $(x^2 + y^2)$ – xy $(a^2 + b^2)$
= $abx^2 + aby^2 – a^2xy – b^2xy$
= $(abx^2 – b^2xy) + (aby^2 – a^2xy)$
= bx (ax – by) – ay (ax – by)
= (ax – by) (bx – ay)

(ii) $(ax + by)^2 + (bx – ay)^2$
= $(a^2x^2 + b^2y^2 + 2abxy) + (b^2x^2 + a^2y^2 – 2abxy)$
= $a^2x^2 + b^2y^2 + 2abxy + b^2x^2 + a^2y^2 – 2abxy$
= $a^2x^2 + b^2y^2 + b^2x^2 + a^2y^2$
= $a^2x^2 + a^2y^2 + b^2x^2 + a^2y$
= $a^2 (x^2 + y^2) + b^2 (x^2 + y^2)$
= $(a^2 + b^2) (x^2 + y^2)$

9. (i) $a^3 + ab(1 – 2a) – 2b^2$
(ii) $3x^2y – 3xy + 12x – 12$
Solution:

(i) $a^3 + ab – 2a^2b – 2b^2$
= $a^3 + ab – 2a^2b – 2b^2$
= $a (a^2 + b) – 2b (a^2 + b)$
= $(a^2 + b) (a – 2b)$

(ii) $3x^2y – 3xy + 12x- 12$
= $3 (x^2y – xy + 4x – 4)$
= 3 [xy (x – 1) +4 (x – 1)]
= 3 (x – 1) (xy + 4)

10. (i) $a^2b + ab^2 – abc – b^2c + axy + bxy$
(ii) $ax^2 – bx^2 + ay^2 – by^2 + az^2 – bz^2$
Solution:

(i) $a^2b + ab^2 – abc – b^2c + axy + bxy$
= ab (a + b) – bc (a + b) + xy (a + b)
= (a + b) (ab – bc + xy)

(ii) $ax^2 – bx^2 + ay^2 – by^2 + az^2 – bz^2$
= $x^2 (a – b) + y^2 (a – b) + z^2 (a – b)$
= $(a – b)(x^2 + y^2 + z^2)$

11. (i) $x – 1 – (x – 1)^2 + ax – a$
(ii) $ax + a^2x + aby + by – (ax + by)^2$
Solution:

(i) $x – 1 – (x – 1)^2 + ax – a$
$= (x – 1) – (x – 1)^2 + a (x – 1)$
$= (x – 1) [1 – (x – 1) + a]$
$= (x – 1) (1 – x + 1 + a)$
$= (x- 1) (2 – x + a)$

(ii) $ax + a^2x + aby + by – (ax + by)^2$
$= (ax + by) + (a^2x + aby) – (ax + by)^2$
$= (ax + by) + a (ax + by) – (ax + by)^2$
$= (ax + by) [1 + a – (ax + by)]$
$= (ax + by) (1 + a – ax – by)$

Exercise 11.3

1. Factorise the following expressions using algebraic identities:
(i) $x^2 – 12x + 36$
(ii) $36p^2 – 60pq + 25q^2$
(iii) $9y^2 + 66xy + 121y^2$
(iv) $a^4 + 6a^2b^2 + 9b^4$
(v) $x^2 + 1/x^2 + 2$
(vi) $x^2 + x + 1/4$
Solution:

Using $(a + b)^2 = a^2 + 2ab + b^2$ and $(a – b)^2 = a^2 – 2ab + b^2$
(i) $y^2 – 12x + 36$
$= (x)^2 – 2 \times x \times 6 + (6)2^2$
$= (x – 6)^2$

(ii) $36p^2 – 60pq + 25q^2$
$= (6p)^2 – 2 \times 6p \times 5q + (5q)^2$
$= (6p – 5q)^2$

(iii) $9x^2 + 66xy + 121\ y^2$
$= (3x)^2 + 2 \times 3x \times 11y + (11y)^2$
$= (3x + 11\ y)^2$

(iv) $a^4 + 6a^2b^2 + 9b^4$
$= (a^2)^2 + 2 \times 2a^2 \times 3b^2 + (3b^2)^2$
$= (a^2 + 3b^2)^2$

(v) $x^2 + 1/x^2 + 2$
$= (x)^2 + 2 \times x \times 1/x + (1/x)^2$
$= (x + 1/x)^2$

(vi) $x^2 + x + ¼$
$= (x)^2 + 2 \times x \times 1/2 + (1/2)^2$
$= (x + 1/2)^2$

Factorise the following (2 to 13) expressions:
2. (i) $4p^2 – 9$
(ii) $4x^2 – 169y^2$
Solution:

(i) $4p^2 – 9$
$= (2p)^2 – (3)^2$
$= (2p + 3)\ (2p – 3)$

(ii) $4x^2 – 169y^2$

$= (2x)^2 – (13y)^2$
$= (2x + 13y) (2x – 13y)$

3. (i) $9x^2y^2 – 25$
(ii) $16x^2 – 1/144$
Solution:

(i) $9x^2y^2 – 25$
$= (3xy)^2 – (5)^2$
$= (3xy + 5) (3xy – 5)$

(ii) $16x^2 – 1/144$
$= (4x)^2 – (1/12)^2$
$= (4x + 1/12) (4x – 1/12)$

4. (i) $20x^2 – 45y^2$
(ii) $9/16 – 25a^2b^2$
Solution:

(i) $20x^2 – 45y^2$
$= 5 (4x^2 – 9y^2)$
$= 5[(2x)^2 – (3y)^2]$
$= 5 (2x + 3y) (2x – 3y)$

(ii) $9/16 – 25a^2b^2$
$= (¾)^2 – (5ab)^2$
$= (¾ + 5ab) (¾ - 5ab)$

5. (i) $(2a + 3b)^2 – 16c^2$
(ii) $1 – (b – c)^2$
Solution:

(i) $(2a + 3b)^2 – 16c^2$
$= (2a + 3b)^2 – (4c)^2$
$= (2a + 3b + 4c) (2a + 3b – 4c)$

(ii) $1 – (b – c)^2$
$= (1)^2 – (b – c)^2$
$= [1 + b – c)] [1 – (b – c)]$
$= (1 +b – c) (1 – b + c)$

6. (i) $9 (x + y)^2 – x^2$
(ii) $(2m + 3n)^2 – (3m + 2n)^2$
Solution:

(i) $9 (x + x)^2 – x^2$

$= [3 (x + y)]^2 – [x]^2$
$= [3 (x + y) + x] [3 (x + y) – x]$
$= (3x + 3y + x) (3x + 3y – x)$
$= (4x + 3y) (2x + 3x)$

(ii) $(2m + 3n)^2 – (3m + 2n)^2$
$= (4m^2 + 9n^2 + 12mn) – (9m^2 + 4n^2 + 12mn)$
$= 4m^2 + 9n^2 + 12mn – 9m^2 – 4m^2 – 12mn$
$= 4m^2 + 9n^2 – 9m^2 – 4n^2$
$= – 5m^2 + 5n^2$
$= 5 (n^2 – m^2)$
$= 5 (m + n) (n – m)$

7. (i) $25 (a + b)^2 – 16 (a – b)^2$
(ii) $9 (3x + 2)^2 – 4 (2x – 1)^2$
Solution:

(i) $25 (a + b)^2 – 16 (a – b)^2$
$= [5 (a + b)]^2 – [4 (a – b)]^2$
$= (5a + 5b)^2 – (4a – 4b)^2$
$= [(5a + 5b)^2 + (4a – 4b)] [(5a + 5b) – (4a – 4b)]$
$= (5a + 5b + 4a – 4b) (5a + 5b – 4a + 4b)$
$= (9a + b) (a + 9b)$

(ii) $9 (3x + 2)^2 – 4 (2x – 1)^2$
$= [3 (3x + 2)]^2 – [2 (2x – 1)]^2$
$= (9x + 6)^2 – (4x – 2)^2$
$= [(9x + 6) + (4x – 2)] [(9x + 6) – (4x – 2)]$
$= (9x + 6 + 4x – 2) (9x + 6 – 4x + 2)$
$= (13x + 4) (5x + 8)$

8. (i) $x^3 – 25x$
(ii) $63p^2q^2 – 7$
Solution:

(i) $x^3 – 25x$
$= x (x^2 – 25) = x [(x)^2 – (5)^2]$
$= x (x + 5) (x – 5)$

(ii) $63p^2q^2 – 7$
$= 7 (9p^2q^2 – 1)$
$= 7 [(3pq)^2 – (1)^2]$
$= 7 (3pq + 1) (3pq – 1)$

9. (i) $32a^2b – 72b^3$
(ii) $9 (a + b)^3 – 25 (a + b)$

Solution:

(i) $32\ a^2b - 72b^3$
$= 8b\ (4a^2 - 9b^2) \Rightarrow 8b\ [(2a)^2 - (3b)^2]$
$= 8b\ (2a + 3b)\ (2a - 3b)$

(ii) $9\ (a + b)^3 - 25\ (a + b)$
$= (a + b)\ [9\ (a + b)^2 - 25]$
$= (a + b)\ [\{3\ (a + b)\}^2 - (5)^2]$
$= (a + 6)\ [(3a + 3b)^2 - (5)^2]$
$= (a + b)\ [(3a + 3b + 5)\ (3a + 36 - 5)]$
$= (a + b)\ (3a + 3b + 5)\ (3a + 3b - 5)$

10. (i) $x^2 - y^2 - 2y - 1$
(ii) $p^2 - 4pq + 4q^2 - r^2$
Solution:

(i) $x^2 - y^2 - 2y - 1$
$= x^2 - (y^2 + 2y + 1)$
$= (x)^2 - (y + 1)^2$
$= [x + (y + 1)]\ [x - (y + 1)]$
$= (x + y + 1)\ (x - y - 1)$

(ii) $p^2 - 4pq + 4q^2 - r^2$
$= (p)^2 - 2 \times p \times 2q + (2q)^2 - r^2$ $[\because (a - b)^2 = a2 - 2ab + b^2]$
$= (p - 2q)^2 - (r)^2$
$= (p - 2q + r)(p - 2q - r)$ $[\because a^2 - b^2 = (a + b)(a - b)]$

11. (i) $9x^2 - y^2 + 4y - 4$
(ii) $4a^2 - 4b^2 + 4a + 1$
Solution:

(i) $9x^2 - y^2 + 4y - 4$
$= 9x^2 - (y^2 - 4y + 4)$
$= 9x^2 - (y - 2)^2$
$= (3x)^2\ (y - 2)^2$
$= [3x + (y - 2)]\ [3x - (y - 2)]$
$= (3x + y - 2)\ (3x - y + 2)$

(ii) $4a^2 - 4b^2 + 4a + 1$
$= (4a^2 + 4a + 1) - 4b^2$
$= (2a + 1)^2 - (2b)^2$
$= (2a + 2b + 1)\ (2a - 2b + 1)$

12. (i) $625 - p^4$
(ii) $5y^5 - 405y$

Solution:

(i) $625 - p^4$
$= (25)^2 - (p^2)^2$
$= (25 + p^2)(25 - p^2)$
$= (25 + p^2)[(5)^2 - (p)^2]$
$= (25 + p^2)(5 + p)(5 - p)$

(ii) $5y^5 - 405y$
$= 5y(y^4 - 81)$
$= 5y[(y^2)^2 - (9)^2]$
$= 5y(y^2 + 9)(y^2 - 9)$
$= 5y(y^2 + 9)[(y)^2 - (3)^2]$
$= 5y(y^2 + 9)(y + 3)(y - 3)$

13. (i) $x^4 - y^4 + x^2 - y^2$
(ii) $64a^2 - 9b^2 + 42bc - 49c^2$
Solution:

(i) $x^4 - y^4 + x^2 - y^2$
$= [(x^2)^2 - (y^2)^2] + (x^2 - y^2)$ [Using, $a^2 - b^2 = (a + b)(a - b)$]
$= (x^2 + y^2)(x^2 - y^2) + 1(x^2 - y^2)$
$= (x^2 - y^2)(x^2 + y^2 + 1)$
$= (x + y(x - y)(x^2 + y^2 + 1)$

(ii) $64a^2 - 9b^2 + 42bc - 49c^2$
$= 64a^2 - [9b^2 - 42bc + 49c^2]$
$= (8a)^2 - [(3b)^2 - 2 \times 3b \times 7c + (7c)^2]$ [$\because a^2 + b^2 - 2ab = (a - b)^2$ and $a^2 - b^2 = (a + b)(a - b)$]
$= (8a)^2 - (3b - 7c)^2$
$= (8a + 3b - 7c)(8a - 3b + 7c)$

Exercise 11.4

1. (i) $x^2 + 3x + 2$,
(ii) $z^2 + 10z + 24$
Solution:

(i) $x^2 + 3x + 2$
$= x^2 + 2x + x + 2$
$= x(x + 2) + 1 (x + 2)$
$= (x + 2) (x + 1)$

(ii) $z^2 + 10z + 24$
$= z^2 + 6z + 4z + 24$
$= z(z + 6) + 4 (z + 6)$
$= (z + 6) (z + 4)$

2. (i) $y^2 – 7y + 12$
(ii) $m^2 – 23m + 42$
Solution:

(i) $y^2 – 7y + 12$
$= y^2 – 3y – 4y + 12$ [Since, $12 = -3 \times (-4)1$ and $– 7 = -3 – 4$}
$= y(y – 3) -4(y – 3)$
$= (y – 3) (y – 4)$

(ii) $m^2 – 23m + 42$
$= m^2 – 2m – 21m + 42$ [Since, $42 = -2 \times (-21)$ and $– 23 = -21 – 2$]
$= m(m – 2) – 21(m – 2)$
$= (m – 2) (m – 21)$

3. (i) $y^2 – 5y – 24$,
(ii) $t^2 + 23t – 108$
Solution:

(i) $y^2– 5y – 24$
$= y^2 – 8y + 3y – 24$
$= y(y – 8) + 3(y – 8)$
$= (y – 8) (y + 3)$

(ii) $t^2 + 23t - 108$
$= t^2 + 27t – 4t – 108$
$= t (t + 27) – 4 (t + 27)$
$= (t + 27) (t – 4)$

4. (i) $3x^2 + 14x + 8$,
(ii) $3y^2 + 10y + 8$

Solution:

(i) $3x^2 + 14x + 8$
$= 3x^2 + 12x + 2x + 8$
$= 3x (x + 4) + 2 (x + 4)$
$= (x + 4) (3x + 2)$

(ii) $3y^2 + 10y + 8$
$= 3y^2 + 6y + 4y + 8$
$= 3y (y + 2) + 4 (y + 2)$
$= (y + 2) (3y + 4)$

5. (i) $14x^2 – 23x + 8$,
(ii) $12x^2 – x – 35$
Solution:

(i) $14x^2 – 23x + 8$
$= 14x^2 – 16x – 7x + 8$
$= 2x (7x – 8) – 1 (7x – 8)$
$= (7x – 8) (2x – 1)$

(ii) $12x^2 – x – 35$
$= 12x^2 – 21x + 20x – 35$
$= 3x (4x – 7) + 5 (4x – 7)$
$= (4x – 7) (3x + 5)$

6. (i) $6x^2 + 11x – 10$
(ii) $5 – 4x – 12x^2$
Solution:

(i) $6x^2 + 11x – 10$
$= 6x^2 + 15x – 4x – 10$
$= 3x (2x + 5) – 2 (2x + 5)$
$= (2x + 5) (3x – 2)$

(ii) $5 – 4x- 12x^2$
$= 5 – 10x + 6x – 12x^2$
$= 5 (1 – 2x) + 6x (1 – 2x)$
$= (1 – 2x) (5 + 6x)$

7. (i) $1 – 18y – 63y^2$,
(ii) $3x^2 – 5xy – 12y^2$
Solution:

(i) $1 – 18y – 63y^2$
$= 1 – 21y + 3y – 63y^2$

= 1(1- 21y) + 3y (1 – 21y)
= (1 – 21y) (1 + 3y)

(ii) $3x^2 – 5xy – 12y^2$
$= 3x^2 – 9xy + 4xy- 12y^2$
= 3x (x – 3y) + 4y (x – 3y)
= (x – 3y) (3x + 4y)

8. (i) $x^2 – 3xy – 40y^2$
(ii) $10p^2q^2 – 21pq + 9$
Solution:

(i) $x^2 – 3xy – 40y^2$
$= x^2 – 8xy + 5xy – 40y^2$
= x (x – 8y) + 5y (x – 8y)
= (x – 8y) (x + 5y)

(ii) $10p^2q^2 – 21pq + 9$
$= 10p^2q^2 – 15pq – 6pq + 9$
= 5pq (2pq – 3) – 3 (2pq – 3)
= (2pq – 3) (5pq – 3)

9. (i) $2a^2b^2 + ab – 45$
(ii) x (12x + 7) – 10
Solution:

(i) $2a^2b^2 + ab – 45$
$= 2a^2b^2 + 10ab – 9ab – 45$
= 2ab (ab + 5) – 9 (ab + 5)
= (ab + 5) (2ab – 9)

(ii) x (12x + 7) – 10
$= 12x^2 + 7x – 10$
$= 12x^2 + 15x – 8x – 10$
= 3x (4x + 5) – 2 (4x + 5)
= (4x + 5) (3x – 2)

10. (i) $(a + b)^2 – 11(a + b) – 42$
(ii) 8 + 6(p + q) – 5(p + q)
Solution:

(i) $(a + b)^2 – 11(a + b) – 42$
Let (a + b) = x, then we have
$= x^2 – 11x – 42$
$= x^2 – 14x + 3x – 42$ [∵ -42 = -14 × 3 and -11 = -14 + 3]

= x(x – 14) + 3 (x – 14)
= (x – 14) (x + 3)
Substituting the value of x we get,
= (a + b – 14) (a + b + 3)

(ii) $8 + 6(p + q) – 5(p + q)^2$
Let p + q = x, then we have
$= 8 + 6x – 5x^2$
$= -5x^2 + 6x + 8$
$= -(5x^2 – 6x – 8)$
$= 5x^2 – 10x + 4x – 8$ [$\because 5 \times (-8) = 40 \Rightarrow -40 = -10 \times 4$ and $-6 = -10 + 4$]
= (x – 2) (5x + 4)
Substituting the value of x, then
= -(p + q – 2) (5p + 5q +4)
= (4 + 5p + 5q) (-p – q + 2)
= (4 + 5p + 5 q) (2 – p – q)

11. (i) $(x – 2y)^2 – 6(x – 2y) + 5$
(ii) $7 + 10(2x – 3y) – 8(2x – 3y)^2$
Solution:

(i) Let x – 2y = z
Then, $(x – 2y)^2 – 6 (x – 2y) + 5$ becomes
$= z^2 – 6z + 5$
$= z^2 – 5z – z + 5$
= z(z – 5) – 1 (z – 5)
= (z – 5)(z – 1)
Now, on substituting z = x – 2y, we get
= [(x - 2y) – 5] [(x – 2y) – 1]
= (x – 2y – 5) (x – 2y – 1)

(ii) $7 + 10 (2x – 3y) – 8 (2x – 3y)^2$
Let 2x – 3y = z
Then, $7 + 10 (2x – 3y) – 8 (2x – 3y)^2$ becomes
$= 7 + 10z – 8z^2$
$= 7 + 14z – 4z – 8z^2$
= 7 (1 + 2z) – 4z (1 + 2z)
= (1 + 2z) (7 – 4z)
Now, on substituting z = 2x – 3y, we get
= [(1 + 2 (2x – 3y)] [7 – 4 (2x – 3y)]
= (1 + 4x – 6y) (7 – 8x + 12y)

Exercise 11.5

Work out the following divisions:
(i) (35x + 28) ÷ (5x + 4)
(ii) $7p^2q^2(9r – 27) ÷ 63pq(r – 3)$
Solution:

(i) (35x + 28) ÷ (5x + 4)

$$\frac{7(5x+4)}{(5x+4)} = 7$$

(ii) $7p^2q^2(9r – 27) ÷ 63pq(r – 3)$

$$= \frac{7p^2q^2 \times 9(r-3)}{63pq(r-3)}$$

$= p^{2-1}\ q^{2-1} \times 9 = 9pq$

2. Divide as directed:
(i) 6(2x + 7) (5x – 3) ÷ 3(5x – 3)
(ii) 33pq (p + 3) (2q – 5) ÷ 11p (2q – 5)
Solution:

(i) 6(2x + 7) (5x – 3) ÷ 3(5x – 3)

$$= \frac{6(2x+7)(5x-3)}{3(5x-3)}$$

= 2(2x + 7)

(ii) 33pq (p + 3) (2q – 5) ÷ 11p (2q – 5)

$$= \frac{33pq(p+3)(2q-5)}{11p(2q-5)}$$

= 3q(p + 3)

3. Factorise the expression and divide them as directed:
(i) $(7x^2 – 63x) ÷ 7(x – 3)$
(ii) $(3p^2 + 17p + 10) ÷ (p + 5)$
(iii) $10xy(14y^2 + 43y – 21) ÷ 5x(7y – 3)$
(iv) $12pqr(6p^2 – 13pq + 6q^2) ÷ 6pq(2p – 3q)$
Solution:

(i) $(7x^2 – 63x) ÷ 7(x – 3)$

$$= \frac{7x(x^2-9)}{7(x-3)}$$
$$= \frac{7x[(x)^2-(3)^2]}{7(x-3)}$$
$$= \frac{7x(x+3)(x-3)}{7(x-3)}$$
$$= x(x+3)$$

(ii) $(3p^2 + 17p + 10) \div (p + 5)$
$= (3p^2 + 17p + 10)/ (p + 5)$

$$= \frac{3p^2+2p+15p+10}{p+5} \qquad \left\{\begin{array}{l}\because 3\times10=30\\ \therefore 30=2\times15\\ \quad 17=2+15\end{array}\right\}$$
$$= \frac{p(3p+2)+5(3p+2)}{p+5}$$
$$= \frac{(3p+2)(p+5)}{(p+5)}$$
$$= 3p+2$$

(iii) $10xy(14y^2 + 43y – 21) \div 5x(7y – 3)$

$$= \frac{10xy[14y^2+49y-6y-21]}{5x(7y-3)} \qquad \left\{\begin{array}{l}\because -21\times14=-294\\ \therefore -294=49\times(-6)\\ \quad 43=49-6\end{array}\right\}$$
$$= \frac{10xy[7y(2y+7)-3(2y+7)]}{5x(7y-3)}$$
$$= \frac{10xy(2y+7)(7y-3)}{5x(7y-3)}$$
$$= 2xy(2y+7)$$

(iv) $12pqr(6p^2 – 13pq + 6q^2) \div 6pq(2p – 3q)$

$$= \frac{12pqr[6p^2-9pq-4pq+6q^2]}{6pq(2p-3q)} \qquad \left\{\begin{array}{l}\because 6\times6=36\\ \therefore 36=-9\times(-4)\\ \quad -13=-9-4\end{array}\right\}$$
$$= \frac{12pqr[3p(2p-3q)-2q(2p-3q)]}{6pq(2p-3q)}$$
$$= \frac{12pqr(2p-3q)(3p-2q)}{6pq(2p-3q)}$$
$$= 2r(3p-2q)$$

Check Your Progress

1. Find the HCF of the given polynomials:
(i) $14pq, 28p^2q^2$
(ii) $8abc, 24ab^2, 12a^2b$
Solution:

(i) $14pq, 28p^2q^2$
HCF of 14, 28 = 14
HCF of $14pq, 28p^2q^2 = 14pq$

(ii) $8abc, 24ab^2, 12a^2b$
HCF of 8, 24, 12 = 4
HCF of $8abc, 24ab^2, 12a^2b = 4ab$

2. Factorise the following:
(i) $10x^2 – 18x^3 + 14x^4$
(ii) $5x^2y + 10xyz + 15xy^2$
(iii) $p^2x^2 + c^2x^2 – ac^2 – ap^2$
(iv) $15(x + y)^2 – 5x – 5y$
(v) $(ax + by)^2 + (ay – bx)^2$
(vi) $ax + by + cx + bx + cy + ay$
(vii) $49x^2 – 70xy + 25y^2$
(viii) $4a^2 + 12ab + 9b^2$
(ix) $49p^2 – 36q^2$
(x) $100x^3 – 25xy^2$
(xi) $x^2 – 2xy + y^2 – z^2$
(xii) $x^8 – y^8$
(xiii) $12x^3 – 14x^2 – 10x$
(xiv) $p^2 – 10p + 21$
(xv) $2x^2 – x – 6$
(xvi) $6x^2 – 5xy – 6y^2$
(xvii) $x^2 + 2xy – 99y^2$
Solution:

(i) $10x^2 – 18x^3 + 14x^4$
HCF of 10, 18, 14 = 2
So, $10x^2 – 18x^3 + 14x^4$
$= 2x^2 (5 – 9x + 7x^2)$

(ii) $5x^2y + 10xyz + 15xy^2$
HCF of 5, 10, 15 = 5
So, $5x^2y + 10xyz + 15xy^2$
$= 5xy (x + 2z + 3y)$

(iii) $p^2x^2 + c^2x^2 – ac^2 – ap$

$= p^2x^2 – ap^2 + c^2x^2 – ac^2$
$= p^2(x^2 – a) + c^2(x^2 – a)$
$= (x^2 – a) (p^2 + c^2)$

(iv) $15(x + y)^2 – 5x – 5y$
$= 15(x + y)^2 – 5(x + y)$
$= 5(x + y) [3(x + y) – 1]$
$= 5(x + y) (3x + 3y – 1)$

(v) $(ax + by)^2 + (ay – bx)^2$
On expanding, we have
$= a^2x^2 + b^2y^2 + 2abxy + a^2y^2 + b^2x^2 – 2abxy$
$= a^2x^2 + a^2y^2 + b^2x^2 + b^2y^2$
$= a^2(x^2 + y^2) + b^2(x^2 + y^2)$
$= (x^2 + y^2) (a^2 + b^2)$

(vi) $ax + by + cx + bx + cy + ay$
$= ax + bx + cx + ay + by + cy$ [On grouping the like variables]
$= x(a + b + c) + y(a + b + c)$
$= (a + b + c) (x + y)$

(vii) $49x^2 – 70xy + 25y^2$
$= (7x)^2 – 2 \times 7x \times 5y + (5y)^2$ [$\because (a – b)^2 = a^2 – 2ab + b^2$]
$= (7x – 5y)^2$

(viii) $4a^2 + 12ab + 9b^2$
$= (2a)2^2 + 2 \times 2a \times 3b + (3b)^2$ [$\because (a + b)^2 = a^2 + 2ab + b^2$]
$= (2a + 3b)^2$

(ix) $49p^2 – 36q^2$
$= (7p)^2 – (6q)^2$
$= (7p + 6q) (7p – 6q)$ [$\because a^2 – b^2 = (a + b) (a – b)$]

(x) $100x^3 – 25xy^2$
$= 25x(x^2 – y^2) = 25x\{(x)^2 – (y)^2\}$
$= 25x(x + y) (x – y)$

(xi) $x^2 – 2xy + y^2 – z^2$
$= (x – y)^2 – (z)^2$ [$\because a^2$ -2ab + $b^2 = (a – b)^2$ and $a^2 – b^2 = (a + b) (a – b)$]
$= (x – y + z)(x – y – z)$

(xii) $x^8 – y^8$
$= (x^4)^2 – (y^4)^2$ [$\because a^2 – b^2 = (a + b)(a- b)$]
$= (x^4 + y^4) (x^4 – y^4)$
$= (x^4 + y^4) [(x^2)^2 – (y^2)^2]$
$= (x^4 + y^4) (x^2 + y^2) (x^2 – y^2)$

$= (x^4 + y^4 (x^2 + y^2) (x + y) (x – y)$

(xiii) $12x^3 – 14x^2 – 10x$
$= 2x(6x^2 – 7x – 5)$ [Now, as $6 \times (-5) = -30 \Rightarrow -30 = -10 \times 3$ and $-7 = -10 + 3$]
$= 2x(6x^2 + 3x – 10x – 5)$
$= 2x\{3x(2x + 1) – 5(2x + 1)\}$
$= 2x(2x + 1) (3x – 5)$

(xiv) $p^2 – 10p + 21$
$= p^2 – 3p – 7p + 21$ [Now, as $21 = -3 \times (-7)$ and $-10 = -3 – 7$]
$= p(p – 3) – 7(p – 3)$
$= (p – 3)(p – 7)$

(xv) $2x^2 – x – 6$
$= 2x^2 – 4x + 3x – 6$ [Now, as $-6 \times 2 = -12 \Rightarrow -12 = -4 \times 3$ and $-1 = -4 + 3$]
$= 2x (x – 2) + 3(x – 2)$
$= (x – 2) (2x + 3)$

(xvi) $6x^2 – 5xy – 6y^2$
$= 6x^2 – 9xy + 4xy – 6y^2$ [Now, as $6 \times (-6) = -36 \Rightarrow – 36 = -9 \times 4$ and $– 5 = -9 + 4$]
$= 3x(2x – 3y) + 2y(2x – 3y)$
$= (2x – 3y) (3x + 2y)$

(xvii) $x^2 + 2xy – 99y^2$
$= x^2 + 11xy – 9xy – 99y^2$ [Now, as $-99 = -11 \times 9$ and $-2 = -11 + 9$ }
$= x(x + 11y) – 9y(x + 11y)$
$= (x + 11y) (x – 9y)$

3. Divide as directed:
(i) $15(y + 3)(y^2 – 16) \div 5(y^2 – y – 12)$
(ii) $(3x^3 – 6x^2 – 24x) \div (x – 4) (x + 2)$
(iii) $(x^4 – 81) \div (x^3 + 3x^2 + 9x + 27)$
Solution:

(i) $15(y + 3)(y^2 – 16) \div 5(y^2 – y – 12)$
$y^2 – 16 = (y)^2 – (4)^2$
$= (y + 4)(y – 4)$
$y^2 – y – 12 = y^2 – 4y + 3y – 12$
$= y(y – 4) + 3(y – 4)$
$= (y – 4)(y + 3)$
Now,
$$\frac{15(y+3)\left(y^2-16\right)}{5(y^2-y-12)}$$
$$= \frac{15\times(y+3)(y+4)(y-4)}{5(y-4)(y+3)}$$

= 3(y + 4)

(ii) $(3x^3 – 6x^2 – 24x) ÷ (x – 4) (x + 2)$

$3x^3 – 6x^2 – 24x = 3x(x^2 – 2x – 8)$

$= 3x\{x^2 – 4x + 2x – 8\}$

$= 3x\{x(x – 4) + 2(x – 4)\}$

$= 3x(x – 4) (x + 2)$

Now,

$$\frac{3x^3-6x^2-24x}{(x-4)(x+2)}$$

$$= \frac{3x(x-4)(x+2)}{(x-4)(x+2)}$$

$$= 3x$$

(iii) $(x^4 – 81) ÷ (x^3 + 3x^2 + 9x + 27)$

$x^4 – 81 = (x^2)^2 – (9)^2 = (x^2 + 9) (x^2 – 9)$

$= (x^2 + 9) [(x)^2 – (3)^2]$

$= (x^2 + 9) (x + 3) (x – 3)$

And,

$x^3 + 3x^2 + 9x + 27 = (x)^2 + (x + 3) + 9 (x + 3)$

$= (x^2 + 9) (x + 3)$

Now,

$$\frac{x^4-81}{x^3+3x^2+9x+27}$$

$$=\frac{\left(x^2+9\right)(x+3)(x-3)}{(x^2+9)(x+3)}$$

= (x – 3)