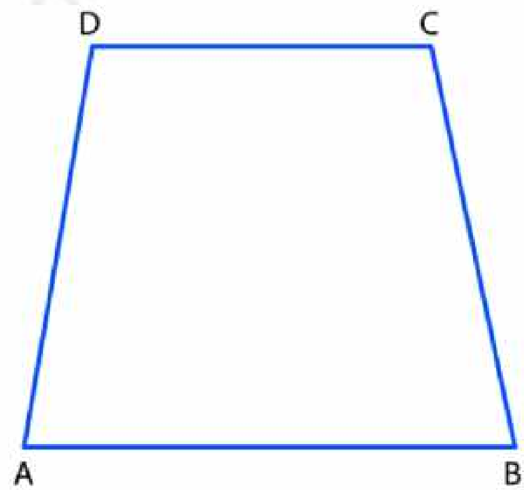
Exercise 13.1

1. If two angles of a quadrilateral are 40° and 110° and the other two are in the ratio 3 : 4, find these angles.
Solution:

We know that,
Sum of all four angles of a quadrilateral = 360^o
Sum of two given angles = $40^o + 110^o = 150^o$
So, the sum of remaining two angles = $360^o – 150^o = 210^o$
Also given,
Ratio in these angles = 3 : 4
Hence,
Third angle = $(210^o \times 3)/(3 + 4)$
= $(210^o \times 3)/7$
= 90^o
And,
Fourth angle = $(210^o \times 4)/(3 + 4)$
= $(210^o \times 4)/7$
= 120^o

2. If the angles of a quadrilateral, taken in order, are in the ratio 1 : 2 : 3 : 4, prove that it is a trapezium.
Solution:



Given,
In trapezium ABCD in which
∠A : ∠B : ∠C : ∠D = 1 : 2 : 3 : 4
We know,
The sum of angles of the quad. ABCD = 360^o
∠A = $(360^o \times 1)/10 = 36^o$
∠B = $(360^o \times 2)/10 = 72^o$
∠C = $(360^o \times 3)/10 = 108^o$
∠D = $(360^o \times 4)/10 = 144^o$
Now,
∠A + ∠D = $36^o + 114^o = 180^o$
Since, the sum of angles ∠A and ∠D is 180^o and these are co-interior angles
Thus, AB || DC
Therefore, ABCD is a trapezium.

3. If an angle of a parallelogram is two-thirds of its adjacent angle, find the angles of the parallelogram.
Solution:

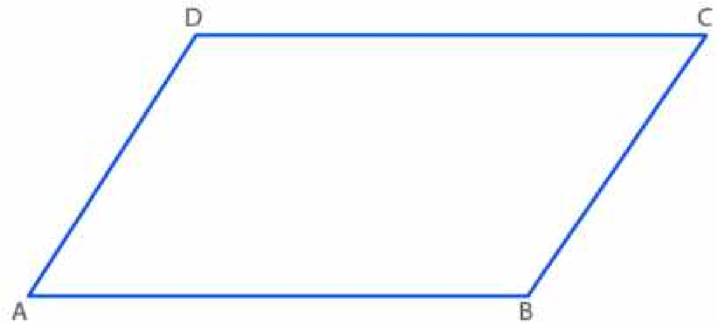
Here ABCD is a parallelogram.
Let ∠A = x^o

Then, ∠B = (2x/3)^o (Given condition)
So,
∠A + ∠B = 180^o (As the sum of adjacent angles in a parallelogram is 180^o)

$$x^o + \frac{2}{3}x^o = 180^o \Rightarrow \frac{3x+2x}{3} = 180$$

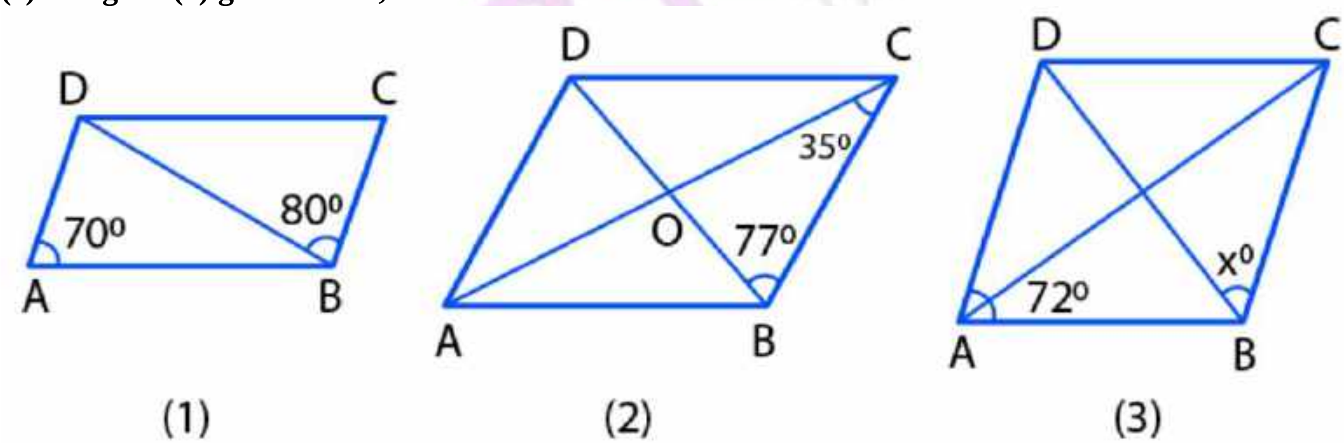
$$\frac{5x}{3} = 180 \Rightarrow 5x = 180 \times 3$$

$$x = \frac{180 \times 3}{5} \Rightarrow x = 36 \times 3 \Rightarrow x = 108$$



Hence, ∠A = 108^o
∠B = 2/3 x 108^o = 2 x 36^o = 72^o
∠B = ∠D = 72^o (opposite angles in a parallelogram is same)
Also,
∠A = ∠C = 108^o (opposite angles in a parallelogram is same)
Therefore, angles of parallelogram are 108^o, 72^o, 108^o and 72^o.

4. (a) In figure (1) given below, ABCD is a parallelogram in which ∠DAB = 70°, ∠DBC = 80°. Calculate angles CDB and ADB.
(b) In figure (2) given below, ABCD is a parallelogram. Find the angles of the AAOD.
(c) In figure (3) given below, ABCD is a rhombus. Find the value of x.



Solution:

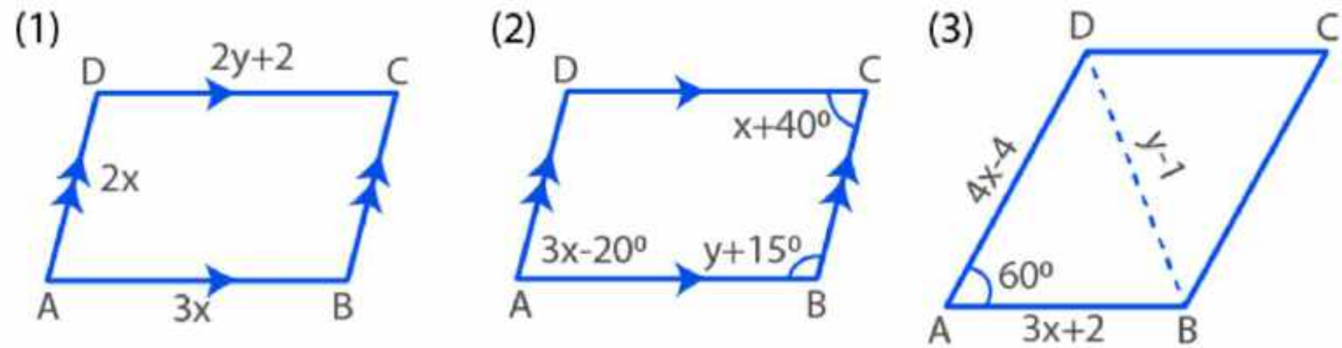
(a) Since, ABCD is a || gm
We have, AB || CD
∠ADB = ∠DBC (Alternate angles)
∠ADB = 80^o (Given, ∠DBC = 80^o)
Now,
In ΔADB, we have
∠A + ∠ADB + ∠ABD = 180^o (Angle sum property of a triangle)
70^o + 80^o + ∠ABD = 180^o
150^o + ∠ABD = 180^o
∠ABD = 180^o – 150^o = 30^o

Now, ∠CDB = ∠ABD (Since, AB || CD and alternate angles)
So,
∠CDB = 30°
Hence, ∠ADB = 80° and ∠CDB = 30°.

(b) Given, ∠BOC = 35° and ∠CBO = 77°
In ΔBOC, we have
∠BOC + ∠BCO + ∠CBO = 180° (Angle sum property of a triangle)
∠BOC = 180° – 112° = 68°
Now, in || gm ABCD
We have,
∠AOD = ∠BOC (Vertically opposite angles)
Hence, ∠AOD = 68°.

(c) ABCD is a rhombus
So, ∠A + ∠B = 180° (Sum of adjacent angles of a rhombus is 180°)
72° + ∠B = 180° (Given, ∠A = 72°)
∠B = 180° – 72° = 108°
Hence,
x = ½ B = ½ x 108° = 54°

5. (a) In figure (1) given below, ABCD is a parallelogram with perimeter 40. Find the values of x and y.
(b) In figure (2) given below. ABCD is a parallelogram. Find the values of x and y.
(c) In figure (3) given below. ABCD is a rhombus. Find x and y.



Solution:

(a) Since, ABCD is a parallelogram
So, AB = CD and BC = AD
⇒ 3x = 2y + 2
3x – 2y = 2 … (i)
Also, AB + BC + CD + DA = 40
⇒ 3x + 2x + 2y + 2 + 2x = 40
7x + 2y = 40 – 2
7x + 2y = 38 … (ii)
Now, adding (i) and (ii) we get
3x – 2y = 2

$7x + 2y = 38$

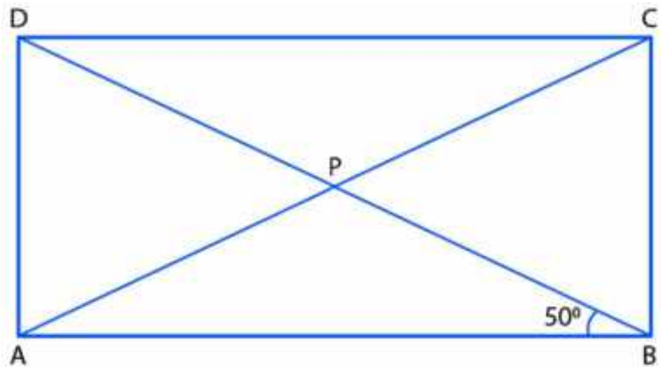
$10x = 40$
$\Rightarrow x = 40/10 = 4$
On substituting the value of x in (i), we get
$3(4) – 2y = 2$
$12 – 2y = 2$
$2y = 12 – 2$
$\Rightarrow y = 10/2 = 5$
Hence, $x = 4$ and $y = 5$

(b) In parallelogram ABCD, we have
$\angle A = \angle C$ (Opposite angles are same in || gm)
$3x – 20^o = x + 40^o$
$3x – x = 40^o + 20^o$
$2x = 60^o$
$x = 60^o/2 = 30^o$ … (i)
Also, $\angle A + \angle B = 180^o$ (Sum of adjacent angles in || gm is equal to 180^o)
$3x – 20^o + y + 15^o = 180^o$
$3x + y = 180^o + 20^o – 15^o$
$3x + y = 185^o$
$3(30^o) + y = 185^o$ [Using (i)]
$90^o + y = 185^o$
$y = 185^o – 90^o = 95^o$
Hence,
$x = 30^o$ and 95^o

(c) ABCD is a rhombus
So, AB = CD
$3x + 2 = 4x – 4$
$3x – 4x = -4 – 2$
$-x = -6$
$x = 6$
Now, in ΔABD we have
$\angle BAD = 60^o$ and AB = AD
$\angle ADB = \angle ABD$
So,
$\angle ADB = (180^o - \angle BAD)/ 2$
$= (180^o – 60^o)/ 2$
$= 120^o/2 = 60^o$
As ΔABD is an equilateral triangle, all the angles of the triangle are 60^o
Hence, AB = BD
$3x + 2 = y – 1$
$3(6) + 2 = y – 1$ (Substituting the value of x)
$18 + 2 = y – 1$
$20 = y – 1$

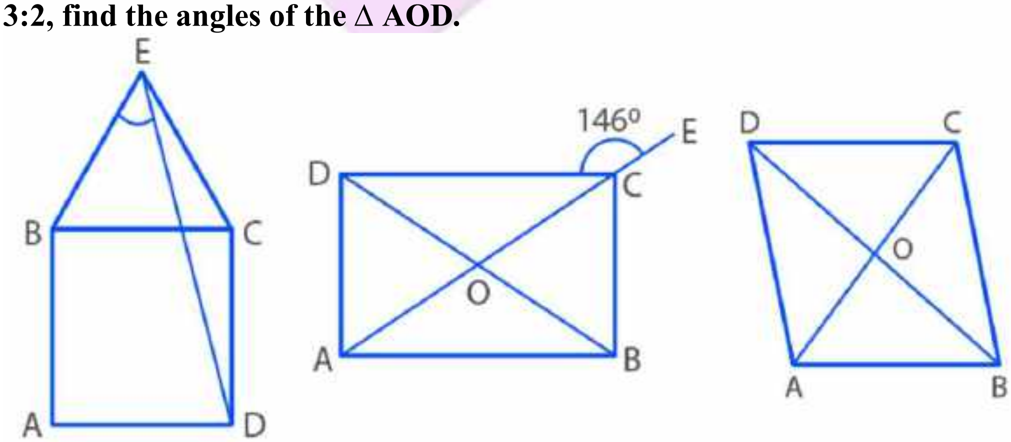
y = 20 + 1
y = 21
Thus, x = 6 and y = 21.

6. The diagonals AC and BD of a rectangle ABCD intersect each other at P. If ∠ABD = 50°, find ∠DPC.
Solution:



Given, ABCD is a rectangle
We know that the diagonals of rectangle are same and bisect each other
So, we have
AP = BP
∠PAB = ∠PBA (Equal sides have equal opposite angles)
∠PAB = 50° (Since, given ∠PBA = 50°)
Now, in ΔAPB
∠APB + ∠ABP + ∠BAP = 180°
∠APB + 50° + 50° = 180°
∠APB = 180° – 100°
∠APB = 80°
Then,
∠DPB = ∠APB (Vertically opposite angles)
Hence,
∠DPB = 80°

7. (a) In figure (1) given below, equilateral triangle EBC surmounts square ABCD. Find angle BED represented by x.
(b) In figure (2) given below, ABCD is a rectangle and diagonals intersect at O. AC is produced to E. If ∠ECD = 146°, find the angles of the Δ AOB.
(c) In figure (3) given below, ABCD is rhombus and diagonals intersect at O. If ∠OAB : ∠OBA = 3:2, find the angles of the Δ AOD.



(1) (2) (3)

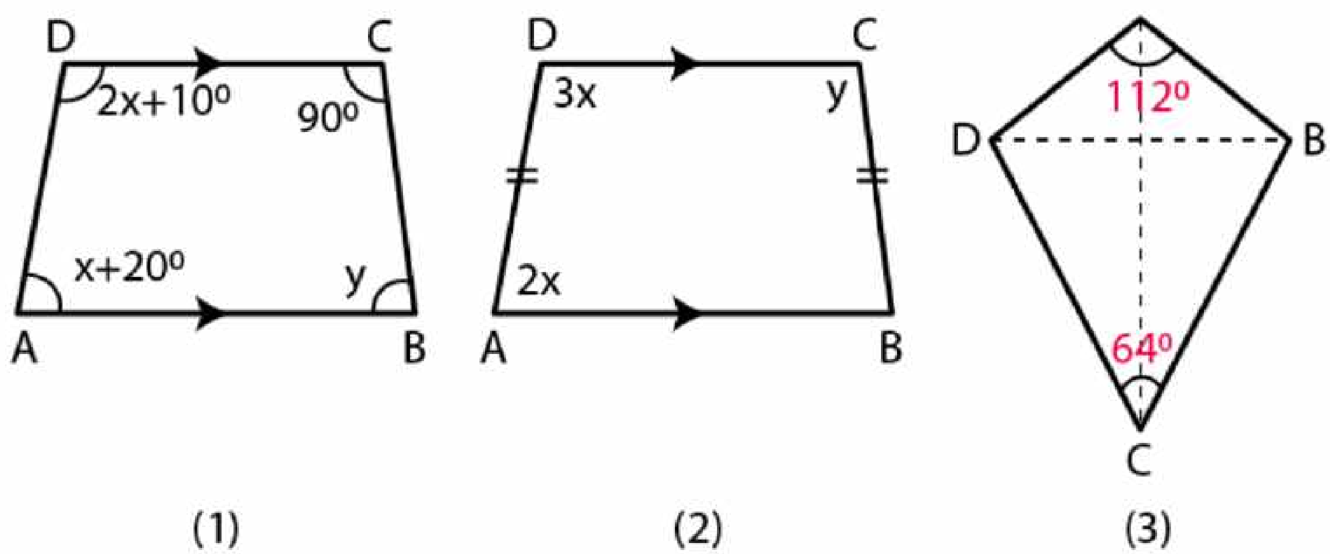
Solution:

Since, EBC is an equilateral triangle, we have
EB = BC = EC … (i)
Also, ABCD is a square
So, AB = BC = CD = AD … (ii)
From (i) and (ii), we get
EB = EC = AB = BC = CD = AD … (iii)
Now, in ΔECD
∠ECD = ∠BCD + ∠ECB
= $90^o + 60^o$
= 150^o … (iv)
Also, EC = CD [From (iii)]
So, ∠DEC = ∠CDE … (v)
∠ECD + ∠DEC + ∠CDE = 180^o [Angles sum property of a triangle]
150^o + ∠DEC + ∠DEC = 180^o [Using (iv) and (v)]
2 ∠DEC = $180^o - 150^o = 30^o$
∠DEC = $30^o/2$
∠DEC = 15^o … (vi)
Now, ∠BEC = 60^o [BEC is an equilateral triangle]
∠BED + ∠DEC = 60^o
$x^o + 15^o = 60^o$ [From (vi)]
$x = 60^o - 15^o$
$x = 45^o$
Hence, the value of x is 45^o.

(b) Given, ABCD is a rectangle
∠ECD = 146^o
As ACE is a straight line, we have
146^o + ∠ACD = 180^o [Linear pair]
∠ACD = $180^o - 146^o = 34^o$ … (i)
And, ∠CAB = ∠ACD [Alternate angles] … (ii)
From (i) and (ii), we have
∠CAB = 34^o ⇒ ∠OAB = 34^o … (iii)
In ΔAOB
AO = OB [Diagonals of a rectangle are equal and bisect each other]
∠OAB = ∠OBA … (iv) [Equal sides have equal angles opposite to them]
From (iii) and (iv),
∠OBA = 34^o … (v)
Now,
∠AOB + ∠OBA + ∠OAB = 180^o
∠AOB + $34^o + 34^o = 180^o$ [Using (3) and (5)]
∠AOB + $68^o = 180^o$
∠AOB = $180^o - 68^o = 112^o$
Hence, ∠AOB = 112^o, ∠OAB = 34^o and ∠OBA = 34^o

(c) Here, ABCD is a rhombus and diagonals intersect at O and ∠OAB : ∠OBA = 3 : 2
Let ∠OAB = $2x^o$
Then, ∠OBA = $2x^o$
We know that diagonals of rhombus intersect at right angle,
So, ∠OAB = 90^o
Now, in ΔAOB
∠OAB + ∠OBA = 180^o
$90^o + 3x^o + 2x^o = 180^o$
$90^o + 5x^o = 180^o$
$5x^o = 180^o – 90^o = 90^o$
$x^o = 90^o/5 = 18^o$
Hence,
∠OAB = $3x^o = 3 \times 18^o = 54^o$
OBA = $2x^o = 2 \times 18^o = 36^o$ and
∠AOB = 90^o

8. (a) In figure (1) given below, ABCD is a trapezium. Find the values of x and y.
(b) In figure (2) given below, ABCD is an isosceles trapezium. Find the values of x and y.
(c) In figure (3) given below, ABCD is a kite and diagonals intersect at O. If ∠DAB = 112° and ∠DCB = 64°, find ∠ODC and ∠OBA.



Solution:

(a) Given: ABCD is a trapezium
∠A = x + 20^o, ∠B = y, ∠C = 92^o, ∠D = 2x + 10^o
We have,
∠B + ∠C = 180^o [Since AB || DC]
y + 92^o = 180^o
y = $180^o – 92^o = 88^o$
Also, ∠A + ∠D = 180^o
x + 20^o + 2x + 10^o = 180^o

$3x + 30^o = 180^o$
$3x = 180^o – 30^o = 150^o$
$x = 150^o/3 = 50^o$
Hence, the value of $x = 50^o$ and $y = 88^o$.

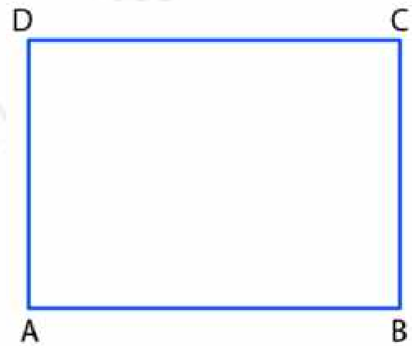
(b) Given: ABCD is an isosceles trapezium BC = AD
$\angle A = 2x$, $\angle C = y$ and $\angle D = 3x$
Since, ABCD is a trapezium and AB || DC
$\Rightarrow \angle A + \angle D = 180^o$
$2x + 3x = 180^o$
$5x = 180^o$
$x = 180^o/5 = 36^o$ … (i)
Also, AB = BC and AB || DC
So, $\angle A + \angle C = 180^o$
$2x + y = 180^o$
$2 \times 36^o + y = 180^o$
$72^o + y = 180^o$
$y = 180^o – 72^o = 108^o$
Hence, value of $x = 72^o$ and $y = 108^o$.

(c) Given: ABCD is a kite and diagonal intersect at O.
$\angle DAB = 112^o$ and $\angle DCB = 64^o$
As AC is the diagonal of kite ABCD, we have
$\angle DCO = 64^o/2 = 32^o$
And, $\angle DOC = 90^o$ [Diagonal of kites bisect at right angles]
In ΔOCD, we have
$\angle ODC = 180^o – (\angle DCO + \angle DOC)$
$= 180^o – (32^o + 90^o)$
$= 180^o – 122^o$
$= 58^o$
In ΔDAB, we have
$\angle OAB = 112^o/2 = 56^o$
$\angle AOB = 90^o$ [Diagonal of kites bisect at right angles]
In ΔOAB, we have
$\angle OBA = 180^o – (\angle OAB + \angle AOB)$
$= 180^o – (56^o + 90^o)$
$= 180^o – 146^o$
$= 34^o$
Hence, $\angle ODC = 58^o$ and $\angle OBA = 34^o$.

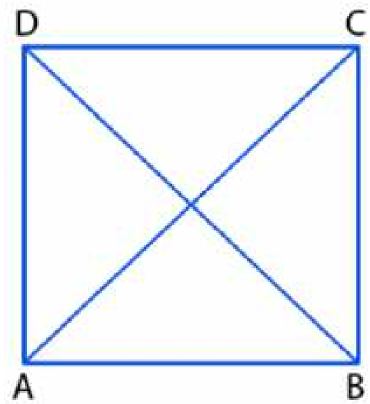
9. (i) Prove that each angle of a rectangle is 90°.
(ii) If the angle of a quadrilateral are equal, prove that it is a rectangle.
(iii) If the diagonals of a rhombus are equal, prove that it is a square.
(iv) Prove that every diagonal of a rhombus bisects the angles at the vertices.
Solution:

(i) Given: ABCD is a rectangle
To prove: Each angle of rectangle = 90°
Proof:
In a rectangle opposite angles of a rectangle are equal
So, ∠A = ∠C and ∠B = ∠C
But, ∠A + ∠B + ∠C + ∠D = 360° [Sum of angles of a quadrilateral]
∠A + ∠B + ∠A + ∠B = 360°
2(∠A + ∠B) = 360°
(∠A + ∠B) = 360°/2
∠A + ∠B = 180°
But, ∠A = ∠B [Angles of a rectangle]
So, ∠A = ∠B = 90°
Thus,
∠A = ∠B = ∠C = ∠D = 90°
Hence, each angle of a rectangle is 90°.

(ii) Given: In quadrilateral ABCD, we have
∠A = ∠B = ∠C = ∠D
To prove: ABCD is a rectangle
Proof:
∠A = ∠B = ∠C = ∠D
⇒ ∠A = ∠C and ∠B = ∠D
But these are opposite angles of the quadrilateral.
So, ABCD is a parallelogram
And, as ∠A = ∠B = ∠C = ∠D
Therefore, ABCD is a rectangle.



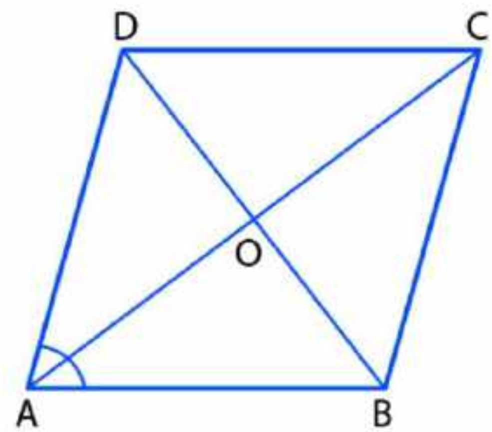
(iii) Given: ABCD is a rhombus in which AC = BD
To prove: ABCD is a square
Proof:
Join AC and BD.
Now, in ΔABC and ΔDCB we have
∠AB = ∠DC [Sides of a rhombus]
∠BC = ∠BC [Common]
∠AC = ∠BD [Given]
So, ΔABC ≅ ΔDCB by S.S.S axiom of congruency
Thus,
∠ABC = ∠DBC [By C.P.C.T]
But these are made by transversal BC on the same side of parallel lines AB and CD.
So, ∠ABC + ∠DBC = 180°
∠ABC = 90°
Hence, ABCD is a square.



(iv) Given: ABCD is rhombus.
To prove: Diagonals AC and BD bisects ∠A, ∠C, ∠B and ∠D respectively
Proof:
In ΔAOD and ΔCOD, we have
AD = CD [sides of a rhombus are all equal]
OD = OD [Common]
AO = OC [Diagonal of rhombus bisect each other]
So, ΔAOD ≅ ΔCOD by S.S.S axiom of congruency
Thus,
∠AOD = ∠COD [By C.P.C.T]
So, ∠AOD + ∠COD = 180^o [Linear pair]
∠AOD = 180^o
∠AOD = 90^o
And, ∠COD = 90^o
Thus,
OD ⊥ AC ⇒ BD ⊥ AC
Also, ∠ADO = ∠CDO [By C.P.C.T]
So,
OD bisect ∠D BD bisect ∠D
Similarly, we can prove that BD bisect ∠B and AC bisect the ∠A and ∠C.

10. ABCD is a parallelogram. If the diagonal AC bisects ∠A, then prove that:
(i) AC bisects ∠C
(ii) ABCD is a rhombus
(iii) AC ⊥ BD.
Solution:

Given: In parallelogram ABCD in which diagonal AC bisects ∠A
To prove: (i) AC bisects ∠C
(ii) ABCD is a rhombus
(iii) AC ⊥ BD



Proof:
(i) As AB || CD, we have [Opposite sides of a || gm]
∠DCA = ∠CAB
Similarly, ∠DAC = ∠DCB
But, ∠CAB = ∠DAC [Since, AC bisects ∠A]
Hence,
∠DCA = ∠ACB and AC bisects ∠C.

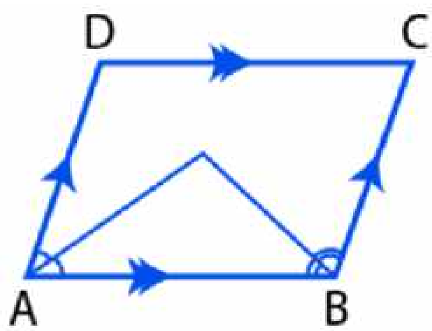
(ii) As AC bisects ∠A and ∠C
And, ∠A = ∠C
Hence, ABCD is a rhombus.

(iii) Since, AC and BD are the diagonals of a rhombus and
AC and BD bisect each other at right angles

Hence, AC ⊥ BD

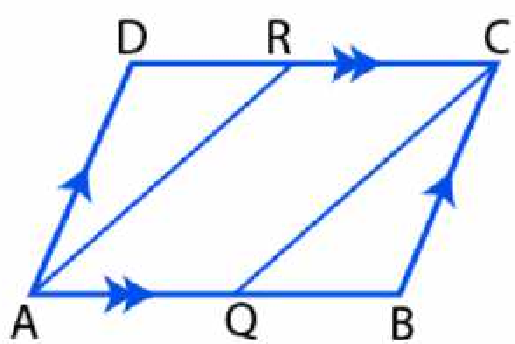
11. (i) Prove that bisectors of any two adjacent angles of a parallelogram are at right angles.
(ii) Prove that bisectors of any two opposite angles of a parallelogram are parallel.
(iii) If the diagonals of a quadrilateral are equal and bisect each other at right angles, then prove that it is a square.
Solution:

(i) Given AM bisect angle A and BM bisects angle of || gm ABCD.



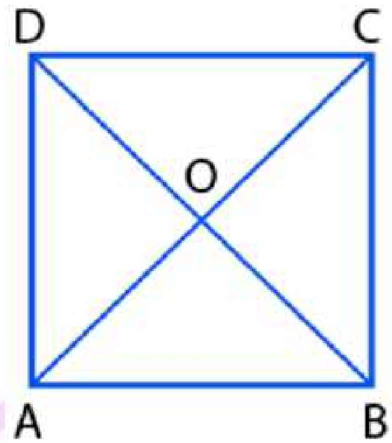
To prove: ∠AMB = 90°
Proof:
We have,
∠A + ∠B = 180° [AD || BC and AB is the transversal]
⇒ ½ (∠A + ∠B) = 180°/2
½ ∠A + ½ ∠B = 90°
∠MAB + ∠MBA = 90° [Since, AM bisects ∠A and BM bisects ∠B]
Now, in ΔAMB
∠AMB + ∠MAB + ∠MBA = 180° [Angles sum property of a triangle]
∠AMB + 90° = 180°
∠AMB = 180° – 90° = 90°
Hence, bisectors of any two adjacent angles of a parallelogram are at right angles.

(ii) Given: A || gm ABCD in which bisector AR of ∠A meets DC in R and bisector CQ of ∠C meets AB in Q



To prove: AR || CQ
Proof:
In || gm ABCD, we have
∠A = ∠C [Opposite angles of || gm are equal]
½ ∠A = ½ ∠C
∠DAR = ∠BCQ [Since, AR is bisector of ½ ∠A and CQ is the bisector of ½ ∠C]
Now, in ΔADR and ΔCBQ
∠DAR = ∠BCQ [Proved above]
AD = BC [Opposite sides of || gm ABCD are equal]
So, ΔADR ≅ ΔCBQ, by A.S.A axiom of congruency
Then by C.P.C.T, we have
∠DRA = ∠BCQ
And,
∠DRA = ∠RAQ [Alternate angles since, DC || AB]
Thus, ∠RAQ = ∠BCQ
But these are corresponding angles,
Hence, AR || CQ.

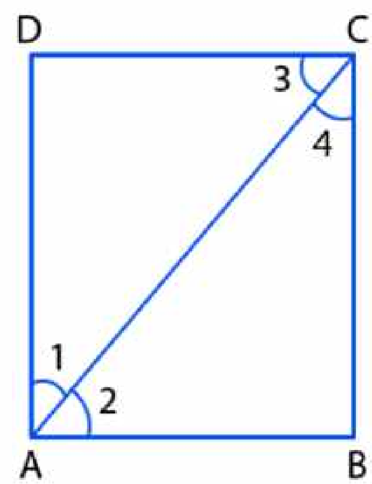
(iii) Given: In quadrilateral ABCD, diagonals AC and BD are equal and bisect each other at right angles
To prove: ABCD is a square



Proof:
In ΔAOB and ΔCOD, we have
AO = OC [Given]
BO = OD [Given]
∠AOB = ∠COD [Vertically opposite angles]
So, ΔAOB ≅ ΔCOD, by S.A.S axiom of congruency
By C.P.C.T, we have
AB = CD
and ∠OAB = ∠OCD
But these are alternate angles
AB || CD
Thus, ABCD is a parallelogram
In a parallelogram, the diagonal bisect each other and are equal
Hence, ABCD is a square.

12. (i) If ABCD is a rectangle in which the diagonal BD bisect ∠B, then show that ABCD is a square.
(ii) Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.
Solution:

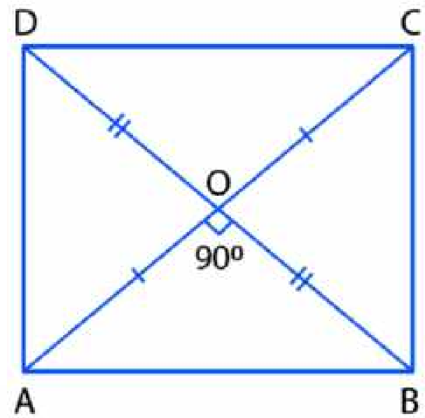
(i) ABCD is a rectangle and its diagonals AC bisects ∠A and ∠C
To prove: ABCD is a square



Proof:
We know that the opposite sides of a rectangle are equal and each angle is 90º
As AC bisects ∠A and ∠C
So, ∠1 = ∠2 and ∠3 = ∠4
But, ∠A = ∠C = 90º
∠2 = 45º and ∠4 = 45º
And, AB = BC [Opposite sides of equal angles]
But, AB = CD and BC = AD
So, AB = BC = CD = DA
Therefore, ABCD is a square.

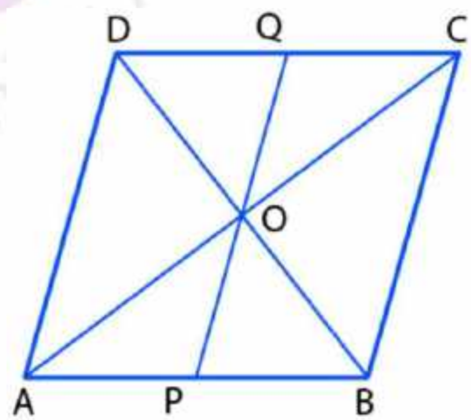
(ii) In quadrilateral ABCD diagonals AC and BD are equal and bisect each other at right angle
To prove: ABCD is a square
Proof:
In ΔAOB and ΔBOC, we have
AO = CO [Diagonals bisect each other at right angles]
OB = OB [Common]
∠AOB = ∠COB [Each 90º]
So, ΔAOB ≅ ΔBOC, by S.A.S axiom
By C.P.C.T, we have

AB = BC … (i)
Similarly, in ΔBOC and ΔCOD
OB = OD [Diagonals bisect each other at right angles]
OC = OC [Common]
∠BOC = ∠COD [Each 90^o]
So, ΔBOC ≅ ΔCOD, by S.A.S axiom
By C.P.C.T, we have
BC = CD … (ii)
From (i) and (ii), we have
AB = BC = CD = DA
Hence, ABCD is a square.

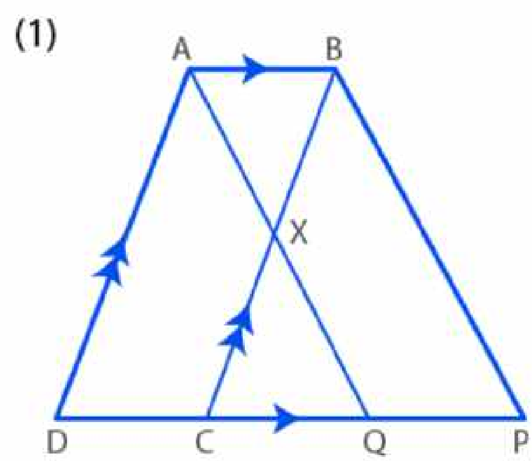


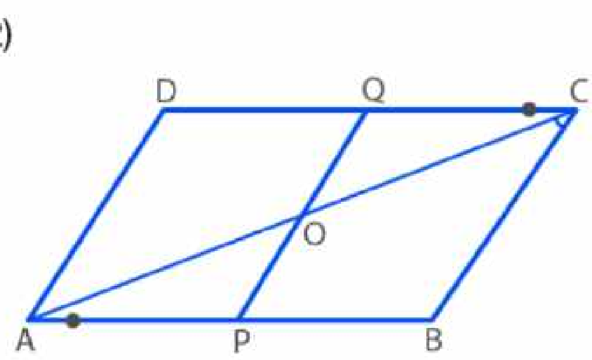
13. P and Q are points on opposite sides AD and BC of a parallelogram ABCD such that PQ passes through the point of intersection O of its diagonals AC and BD. Show that PQ is bisected at O.
Solution:

Given: ABCD is a parallelogram, P and Q are the points on AB and DC. Diagonals AC and BD intersect each other at O.
To prove:
Diagonals of || gm ABCD bisect each other at O
So, AO = OC and BO = OD
Now, in ΔAOP and ΔCOQ we have
AO = OC and BO = OD
Now, in ΔAOP and ΔCOQ
AO = OC [Proved]
∠OAP = ∠OCQ [Alternate angles]
∠AOP = ∠COQ [Vertically opposite angles]
So, ΔAOP ≅ ΔCOQ by S.A.S axiom
Thus, by C.P.C.T
OP = OQ
Hence, O bisects PQ.



14. (a) In figure (1) given below, ABCD is a parallelogram and X is mid-point of BC. The line AX produced meets DC produced at Q. The parallelogram ABPQ is completed.
Prove that:
(i) the triangles ABX and QCX are congruent;
(ii)DC = CQ = QP
(b) In figure (2) given below, points P and Q have been taken on opposite sides AB and CD respectively of a parallelogram ABCD such that AP = CQ. Show that AC and PQ bisect each other.





Solution:

(a) Given: ABCD is parallelogram and X is mid-point of BC. The line AX produced meets DC produced at Q and ABPQ is a || gm.
To prove: (i) ΔABX ≅ ΔQCX
(ii) DC = CQ = QP
Proof:
In ΔABX and ΔQCX, we have
BX = XC [X is the mid-point of BC]
∠AXB = ∠CXQ [Vertically opposite angles]
∠XCQ = ∠XBA [Alternate angle, since AB || CQ]
So, ABX ≅ ΔQCX by A.S.A axiom of congruence
Now, by C.P.C.T
CQ = AB
But,
AB = DC and AB = QP [As ABCD and ABPQ are || gms]
Hence,
DC = CQ = QP

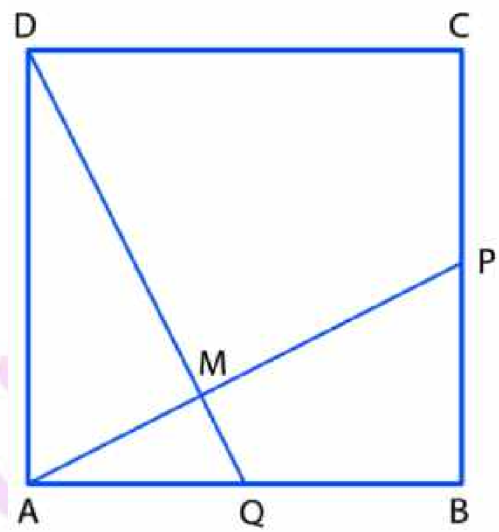
(b) In || gm ABCD, P and Q are points on AB and CD respectively, PQ and AC intersect each other at O and AP = CQ
To prove: AC and PQ bisect each other i.e. AO = OC and PO = OQ
Proof:
In ΔAOP and ΔCOQ
AP = CQ [Given]
∠AOP = ∠COQ [Vertically opposite angles]
∠OAP = ∠OCP [Alternate angles]
So, ΔAOP ≅ ΔCOQ by A.A.S axiom of congruence
Now, by C.P.C.T
OP = OQ and OA = OC
Hence proved.

15. ABCD is a square. A is joined to a point P on BC and D is joined to a point Q on AB. If AP =

DQ, prove that AP and DQ are perpendicular to each other.
Solution:

Given: ABCD is a square. P is any point on BC and Q is any point on AB and these points are taken such that AP = DQ

To prove: AP ⊥ DQ



Proof:

In ΔABP and ΔADQ, we have

AP = DQ [Given]

AD = AB [Sides of square ABCD]

∠DAQ = ∠ABP [Each 90°]

So, ΔABP ≅ ΔADQ by R.H.S axiom of congruency

Now, by C.P.C.T

∠BAP = ∠ADQ

But, ∠BAD = 90°

∠BAD = ∠BAP + ∠PAD … (i)

90° = ∠BAP + ∠PAD

∠BAP + ∠PAD = 90°

∠BAP + ∠ADQ = 90°

Now, in ΔADM we have

(∠MAD + ∠ADM) + ∠AMD = 180° [Angles sum property of a triangle]

90° + ∠AMD = 180° [From (i)]

∠AMD = 180° - 90° = 90°

So, DM ⊥ AP

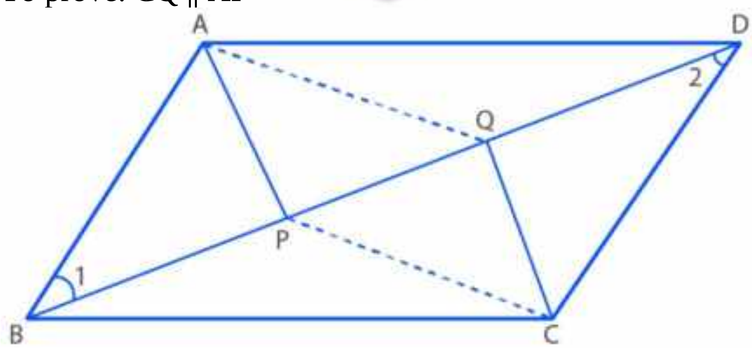
⇒ DQ ⊥ AP

Hence, AP ⊥ DQ

16. If P and Q are points of trisection of the diagonal BD of a parallelogram ABCD, prove that CQ || AP.
Solution:

Given: ABCD is a || gm is which BP = PQ = QD

To prove: CQ || AP



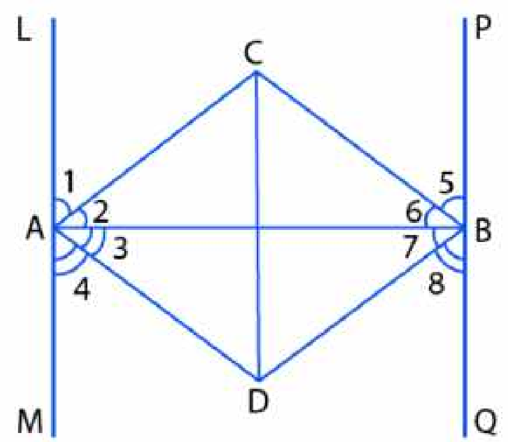
Proof:

In || gm ABCD, we have

AB = CD [Opposite sides of a || gm are equal]

And BD is the transversal
So, ∠1 = ∠2 [Alternate interior angles] … (i)
Now, in ΔABP and ΔDCQ
AB = CD [Opposite sides of a || gm are equal]
∠1 = ∠2 [From (i)]
BP = QD [Given]
So, ΔABP ≅ ΔDCQ by S.A.S axiom of congruency
Then by C.P.C.T, we have
AP = QC
Also, ∠APB = ∠DQC [By C.P.C.T]
-∠APB = -∠DQC [Multiplying both sides by -1]
180° - ∠APB = 180° - ∠DQC [Adding 180° both sides]
∠APQ = ∠CQP
But, these are alternate angles
Hence, AP || QC ⇒ CQ || AP.

17. A transversal cuts two parallel lines at A and B. The two interior angles at A are bisected and so are the two interior angles at B ; the four bisectors form a quadrilateral ABCD. Prove that
(i) ABCD is a rectangle.
(ii) CD is parallel to the original parallel lines.
18. In a parallelogram ABCD, the bisector of ∠A meets DC in E and AB = 2 AD. Prove that
(i) BE bisects ∠B
(ii) ∠AEB = a right angle.



Solution:

Given: LM || PQ and AB is the transversal line cutting ∠M at A and PQ at B
AC, AD, BC and BD is the bisector of ∠LAB, ∠BAM, ∠PAB and ∠ABQ respectively.
AC and BC intersect at C and AD and BD intersect at D.
A quadrilateral ABCD is formed.
To prove: (i) ABCD is a rectangle
(ii) CD || LM and PQ
Proof:
(1) ∠LAB + ∠BAM = 180° [LAM is a straight line]

½ (∠LAB + ∠BAM) = 90^o
½ ∠LAB + ½ ∠BAM = 90^o
∠2 + ∠3 = 90^o [Since, AC and AD is bisector of ∠LAB & ∠BAM respectively]
∠CAD = 90^o
∠A = 90^o

(2) Similarly, ∠PBA + ∠QBA = 180^o [PBQ is a straight line]
½ (∠PBA + ∠QBA) = 90^o
½ ∠PBA + ½ ∠QBA = 90^o
∠6 + ∠7 = 90^o [Since, BC and BD is bisector of ∠PAB & ∠QBA respectively]
∠CBD = 90^o
∠B = 90^o

(3) ∠LAB + ∠ABP = 180^o [Sum of co-interior angles is 180^o and given LM || PQ]
½ ∠LAB + ½ ∠ABP = 90^o
∠2 + ∠6 = 90^o [Since, AC and BC is bisector of ∠LAB & ∠PBA respectively]

(4) In ΔACB,
∠2 + ∠6 + ∠C = 180^o [Angles sum property of a triangle]
(∠2 + ∠6) + ∠C = 180^o
90^o + ∠C = 180^o [using (3)]
∠C = 180^o – 90^o
∠C = 90^o

(5) ∠MAB + ∠ABQ = 180^o [Sum of co-interior angles is 180^o and given LM || PQ]
½ ∠MAB + ½ ∠ABQ = 90^o
∠3 + ∠7 = 90^o [Since, AD and BD is bisector of ∠MAB & ∠ABQ respectively]

(6) In ΔADB,
∠3 + ∠7 + ∠D = 180^o [Angles sum property of a triangle]
(∠3 + ∠7) + ∠D = 180^o
90^o + ∠D = 180^o [using (5)]
∠D = 180^o – 90^o
∠D = 90^o

(7) ∠LAB + ∠BAM = 180^o
∠BAM = ∠ABP [From (1) and (2)]
½ ∠BAM = ½ ∠ABP
∠3 = ∠6 [Since, AD and BC is bisector of ∠BAM and ∠ABP respectively]
Similarly, ∠2 = ∠7

(8) In ΔABC and ΔABD,
∠2 = ∠7 [From (7)]
AB = AB [Common]
∠6 = ∠3 [From (7)]

So, ΔABC ≅ ΔABD by A.S.A axiom of congruency
Then, by C.P.C.T we have
AC = DB
Also, CB = AD

(9) ∠A = ∠B = ∠C =∠D = 90° [From (1), (2), (3) and (4)]
AC = DB [Proved in (8)]
CB = AD [Proved in (8)]
Hence, ABCD is a rectangle.

(10) Since, ABCD is a rectangle [From (9)]
OA = OD [Diagonals of rectangle bisect each other]

(11) In ΔAOD, we have
OA = OD [From (10)]
∠9 = ∠3 [Angles opposite to equal sides are equal]

(12) ∠3 = ∠4 [AD bisects ∠MAB]

(13) ∠9 = ∠4 [From (11) and (12)]
But these are alternate angles.
OD || LM ⇒ CD || LM
Similarly, we can prove that
∠10 = ∠8
But these are alternate angles,
So, OD || PQ ⇒ CD || PQ

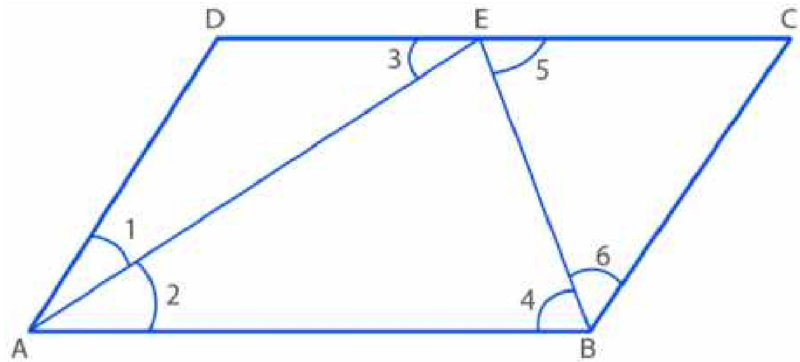
(14) CD || LM [Proved in (13)]
CD || PQ [Proved in (13)]

18. In a parallelogram ABCD, the bisector of ∠A meets DC in E and AB = 2 AD. Prove that:
(i) BE bisects ∠B
(ii) ∠AEB is a right angle
Solution:

Given: ABCD is a || gm in which bisectors of angle A and B meets in E and AB = 2 AD
To prove: (i) BE bisects ∠B
(ii) ∠AEB = 90°
Proof:
(1) In || gm ABCD
∠1 = ∠2 [AD bisects angles ∠A]

(2) AB || DC and AE is the transversal
∠2 = ∠3 [Alternate angles]

(3) ∠1 = ∠2 [From (1) and (2)]



(4) In ΔADE, we have
∠1 = ∠3 [Proved in (3)]
DE = AD [Sides opposite to equal angles are equal]
⇒ AD = DE

(5) AB = 2 AD [Given]
AB/2 = AD
AB/2 = DE [using (4)]
DC/2 = DE [AB = DC, opposite sides of a || gm are equal]
So, E is the mid-point of D.
⇒ DE = EC

(6) AD = BC [Opposite sides of a || gm are equal]
(7) DE = BC [From (4) and (6)]
(8) EC = BC [From (5) and (7)]

(9) In ΔBCE, we have
EC = BC [Proved in (8)]
∠6 = ∠5 [Angles opposite to equal sides are equal]

(10) AB || DC and BE is the transerval
∠4 = ∠5 [Alternate angles]

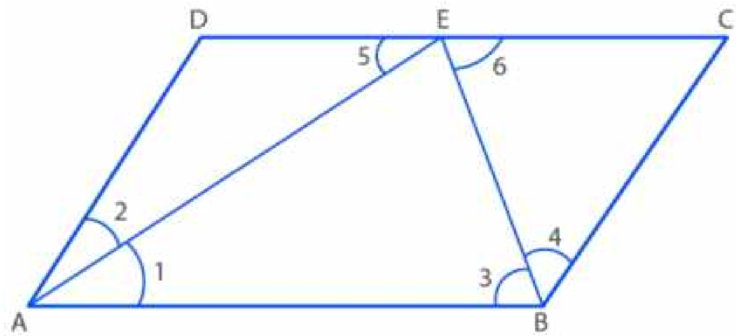
(11) ∠4 = ∠6 [From (9) and (10)]
So, BE is bisector of ∠B

(12) ∠A + ∠B = 180° [Sum of co-interior angles is equal to 180°, AD || BC]
½ ∠A + ½ ∠B = 180°/ 2
∠2 + ∠4 = 90° [AE is bisector of ∠A and BE is bisector of ∠B]

(13) In ΔAPB,
∠AEB + ∠2 + ∠4 = 180°
∠AEB + 90° = 180°
Hence, ∠AEB = 90°

19. ABCD is a parallelogram, bisectors of angles A and B meet at E which lie on DC. Prove that AB.
Solution:

Given: ABCD is a parallelogram in which bisector of ∠A and ∠B meets DC in E
To prove: AB = 2 AD
Proof:
In parallelogram ABCD, we have
AB || DC



∠1 = ∠5 [Alternate angles, AE is transversal]
∠1 = ∠2 [AE is bisector of ∠A, given]
Thus, ∠2 = ∠5 … (i)
Now, in ΔAED
DE = AD [Sides opposite to equal angles are equal]
∠3 = ∠6 [Alternate angles]
∠3 = ∠4 [Since, BE is bisector of ∠B (given)]
Thus, ∠4 = ∠6 … (ii)
In ΔBCE, we have
BC = EC [Sides opposite to equal angles are equal]
AD = BC [Opposite sides of || gm are equal]
AD = DE = EC [From (i) and (ii)]
AB = DC [Opposite sides of a || gm are equal]
AB = DE + EC
= AD + AD
Hence,
AB = 2 AD

20. ABCD is a square and the diagonals intersect at O. If P is a point on AB such that AO =AP, prove that 3 ∠POB = ∠AOP.
Solution:

Given: ABCD is a square and the diagonals intersect at O. P is the point on AB such that AO = AP
To prove: 3 ∠POB = ∠AOP
Proof:
(1) In square ABCD, AC is a diagonal
So, ∠CAB = 45^o
∠OAP = 45^o

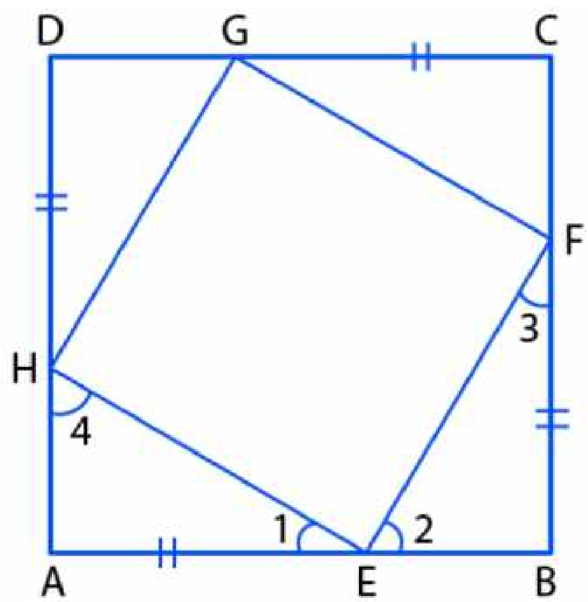
(2) In ΔAOP,
∠OAP = 45^o [From (1)]
AO = AP [Sides opposite to equal angles are equal]
Now,
∠AOP + ∠APO + ∠OAP = 180^o [Angles sum property of a triangle]
∠AOP + ∠AOP + 45^o = 180^o
2 ∠AOP = 180^o – 45^o
∠AOP = 135^o/2

(3) ∠AOB = 90^o [Diagonals of a square bisect at right angles]
So, ∠AOP + ∠POB = 90^o
135^o/2 + ∠POB = 90^o [From (2)]
∠POB = 90^o – 135^o/2
= (180^o – 135^o)/2
= 45^o/2

3 ∠POB = 135º/2 [Multiplying both sides by 3]
Hence,
∠AOP = 3 ∠POB [From (2) and (3)]

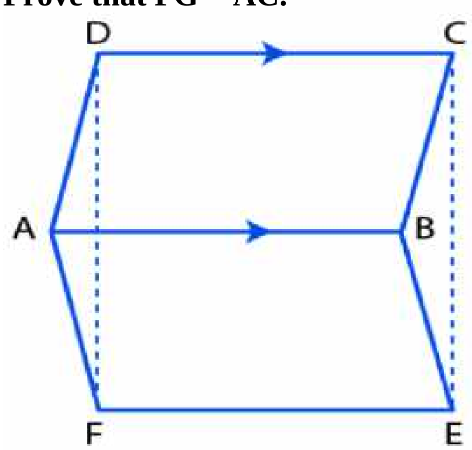
21. ABCD is a square. E, F, G and H are points on the sides AB, BC, CD and DA respectively such that AE = BF = CG = DH. Prove that EFGH is a square.
Solution:

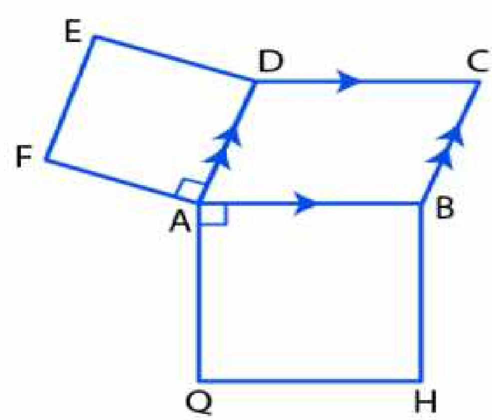
Given: ABCD is a square in which E, F, G and H are points on AB, BC, CD and DA
Such that AE = BF = CG = DH
EF, FG, GH and HE are joined
To prove: EFGH is a square



Proof:
Since, AE = BF = CG = DH
So, EB = FC = GD = HA
Now, in ΔAEH and ΔBFE
AE = BF [Given]
AH = EB [Proved]
∠A = ∠B [Each 90º]
So, ΔAEH ≅ ΔBFE by S.A.S axiom of congruency
Then, by C.P.C.T we have
EH = EF
And ∠4 = ∠2
But ∠1 + ∠4 = 90º
∠1 + ∠2 = 90º
Thus, ∠HEF = 90º
Hence, EFGH is a square.

22. (a) In the Figure (1) given below, ABCD and ABEF are parallelograms. Prove that
(i) CDFE is a parallelogram
(ii) FD = EC
(iii) Δ AFD = ΔBEC.
(b) In the figure (2) given below, ABCD is a parallelogram, ADEF and AGHB are two squares. Prove that FG = AC.





Solution:

Given: ABCD and ABEF are || gms
To prove: (i) CDFE is a parallelogram
(ii) FD = EC
(iii) Δ AFD = ΔBEC
Proof:
(1) DC || AB and DC = AB [ABCD is a || gm]
(2) FE || AB and FE = AB [ABEF is a || gm]
(3) DC || FE and DC = FE [From (1) and (2)]
Thus, CDFE is a || gm
(4) CDEF is a || gm
So, FD = EC
(5) In ΔAFD and ΔBEC, we have
AD = BC [Opposite sides of || gm ABCD are equal]
AF = BE [Opposite sides of || gm ABEF are equal]
FD = BE [From (4)]
Hence, ΔAFD ≅ ΔBEC by S.S.S axiom of congruency

(b) Given: ABCD is a || gm, ADEF and AGHB are two squares
To prove: FG = AC
Proof:
(1) $\angle FAG + 90^o + 90^o + \angle BAD = 360^o$ [At a point total angle is 360^o]
$\angle FAG = 360^o – 90^o – 90^o – \angle BAD$
$\angle FAG = 180^o - \angle BAD$

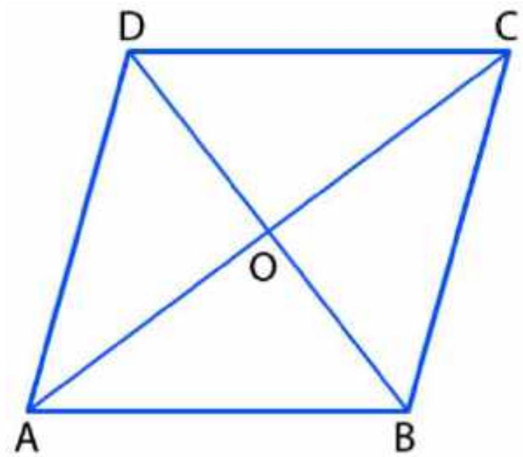
(2) $\angle B + \angle BAD = 180^o$ [Adjacent angle in || gm is equal to 180^o]
$\angle B = 180^o – \angle BAD$

(3) $\angle FAG = \angle B$ [From (1) and (2)]

(4) In ΔAFG and ΔABC, we have
AF = BC [FADE and ABCD both are squares on the same base]
Similarly, AG = AB
$\angle FAG = \angle B$ [From (3)]
So, ΔAFG ≅ ΔABC by S.A.S axiom of congruency
Hence, by C.P.C.T
FG = AC

23. ABCD is a rhombus in which ∠A = 60°. Find the ratio AC : BD.
Solution:

Let each side of the rhombus ABCD be a
$\angle A = 60^o$
So, ABD is an equilateral triangle
⇒ BD = AB = a

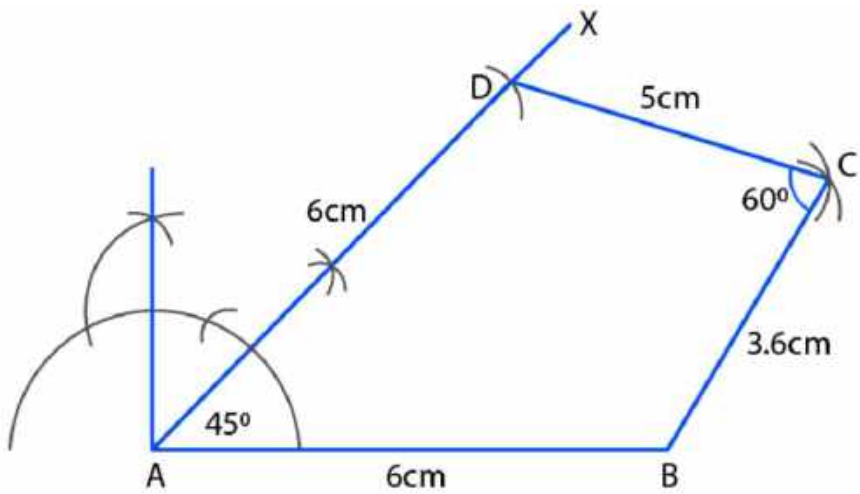


We know that, the diagonals of a rhombus bisect each other at right angles
So, in right triangle AOB, we have
$AO^2 + OB^2 = AB^2$ [By Pythagoras Theorem]
$AO^2 = AB^2 - OB^2$
$= a^2 - (½ a)^2$
$= a^2 - a^2/4$
$= 3a^2/4$
$AO = \sqrt{(3a^2/4)} = \sqrt{3}a/2$
But, AC = 2 AO = 2 x 3a/2 = 3a
Hence,
AC : BD = $\sqrt{3}a : a = \sqrt{3} : 1$

Exercise 13.2

1. Using ruler and compasses only, construct the quadrilateral ABCD in which ∠ BAD = 45°, AD = AB = 6cm, BC = 3.6cm, CD = 5cm. Measure ∠ BCD.

Solution:



Steps of construction:

(i) Draw a line segment AB = 6cm

(ii) At A, draw a ray AX making an angle of 45^o and cut off AD = 6cm

(iii) With centre B and radius 3.6 cm and with centre D and radius 5 cm, draw two arcs intersecting each other at C.

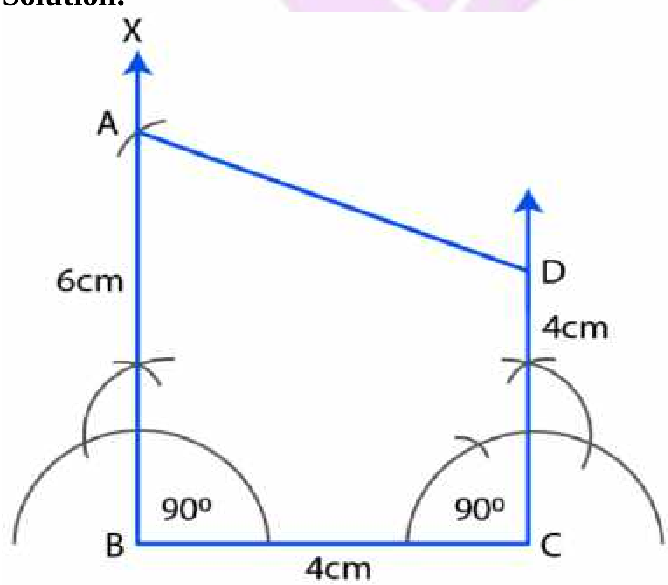
(iv) Join BC and DC.

Thus, ABCD is the required quadrilateral.

On measuring ∠BCD, it is 60^o.

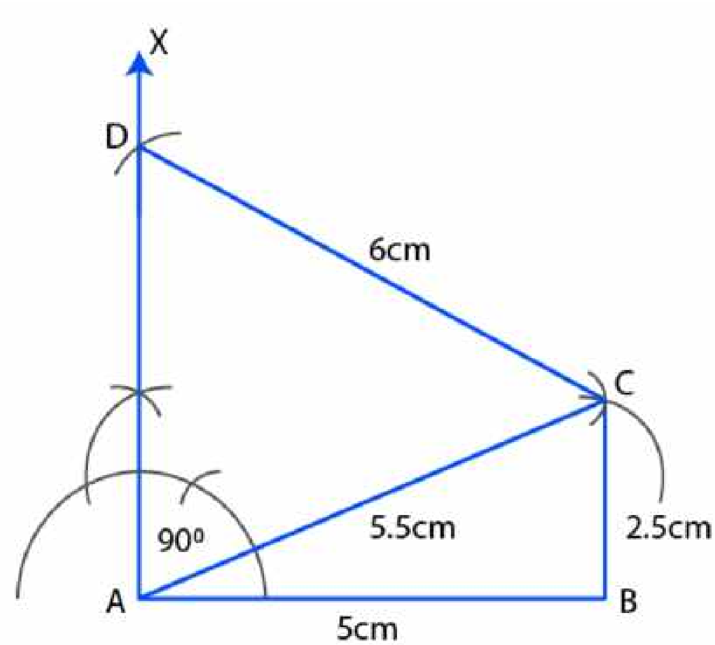
2. Draw a quadrilateral ABCD with AB = 6cm, BC = 4cm, CD = 4 cm and ∠ BC = ∠ BCD = 90°.

Solution:



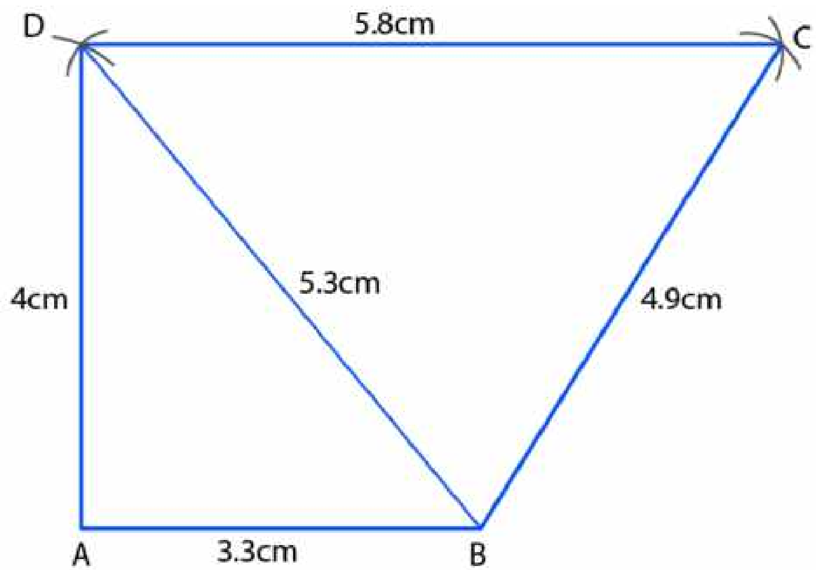
Steps of construction:
(i) Draw a line segment BC = 4 cm
(ii) At B and C draw rays BX and CY making an angle of 90º each
(iii) From BX, cut off BA = 6 cm and from CY, cut off CD = 4cm
(iv) Join AD.
Thus, ABCD is the required quadrilateral.

3. Using ruler and compasses only, construct the quadrilateral ABCD given that AB = 5 cm, BC = 2.5 cm, CD = 6 cm, ∠BAD = 90° and the diagonal AC = 5.5 cm.
Solution:



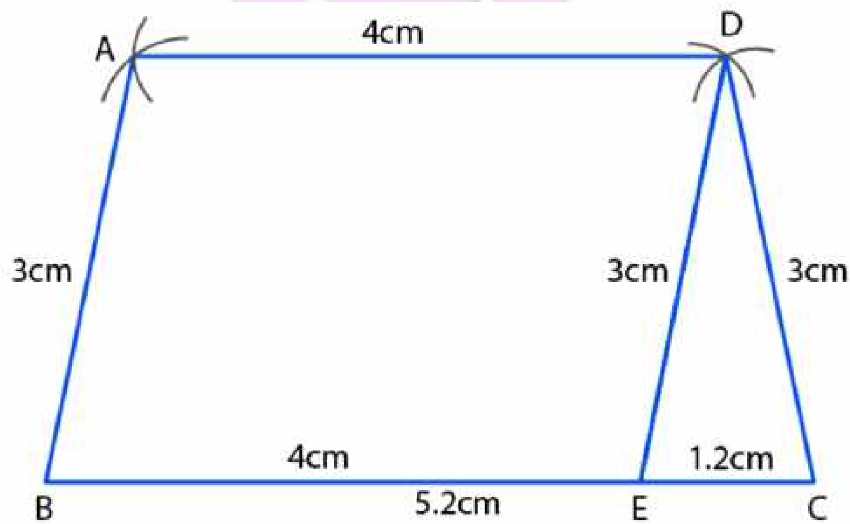
Steps of construction:
(i) Draw a line segment AB = 5cm
(ii) With centre A and radius 5.5cm and with centre B and radius 2.5cm draw arcs which intersect each other at C.
(iii) Join AC and BC.
(iv) At A, draw a ray AX making an angle of 90º.
(v) With centre C and radius 6cm, draw an arc intersecting AX at D
(vi) Join CD.
Thus, ABCD is the required quadrilateral.

4. Construct a quadrilateral ABCD in which AB = 3.3 cm, BC = 4.9 cm, CD = 5.8 cm, DA = 4 cm and BD = 5.3 cm.
Solution:



Steps of construction:
(i) Draw a line segment AB = 3.3cm
(ii) with centre A and radius 4cm, and with centre B and radius 5.3cm, draw arcs intersecting each other at D.
(iii) Join AD and BD.
(iv) With centre B and radius 4.9 cm and with centre D and radius 5.8cm, draw arcs intersecting each other at C.
(v) Join BC and DC.
Thus, ABCD is the required quadrilateral.

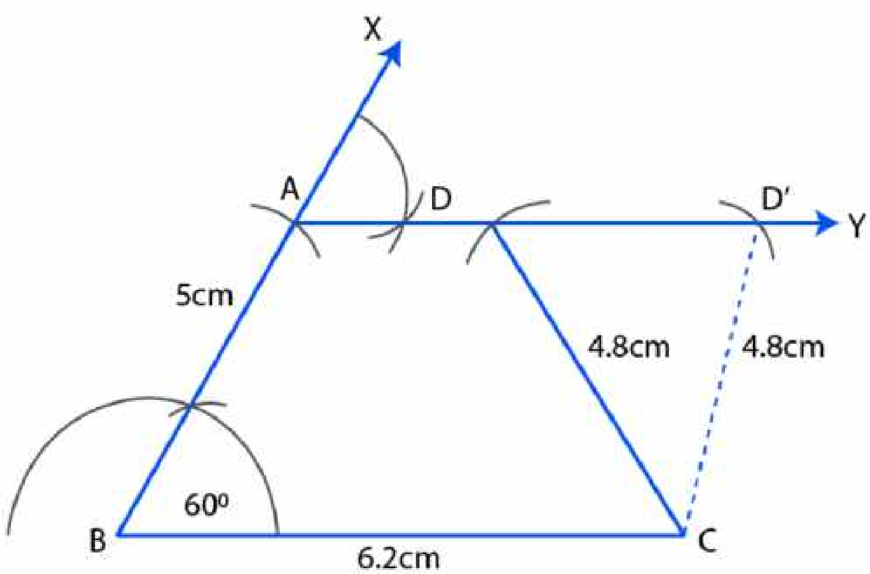
5. Construct a trapezium ABCD in which AD || BC, AB = CD = 3 cm, BC = 5.2cm and AD = 4 cm.
Solution:



Steps of construction:
(i) Draw a line segment BC = 5.2cm
(ii) From BC, cut off BE = AD = 4cm

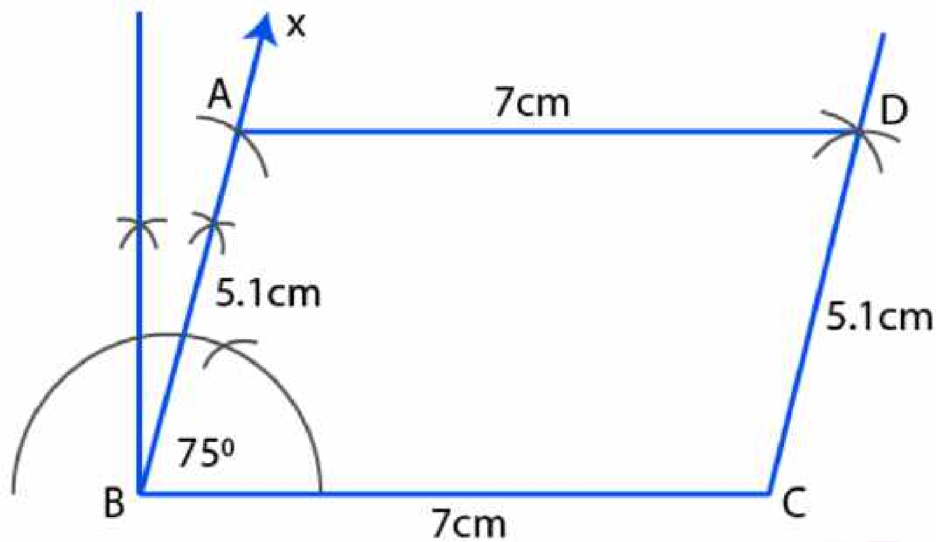
(iii) With centre E and C, and radius 3 cm, draw arcs intersecting each other at D.
(iv) Join ED and CD.
(v) With centre D and radius 4cm and with centre B and radius 3cm, draw arcs intersecting each other at A.
(vi) Join BA and DA
Thus, ABCD is the required trapezium.

6. Construct a trapezium ABCD in which AD || BC, ∠B= 60°, AB = 5 cm. BC = 6.2 cm and CD = 4.8 cm.
Solution:



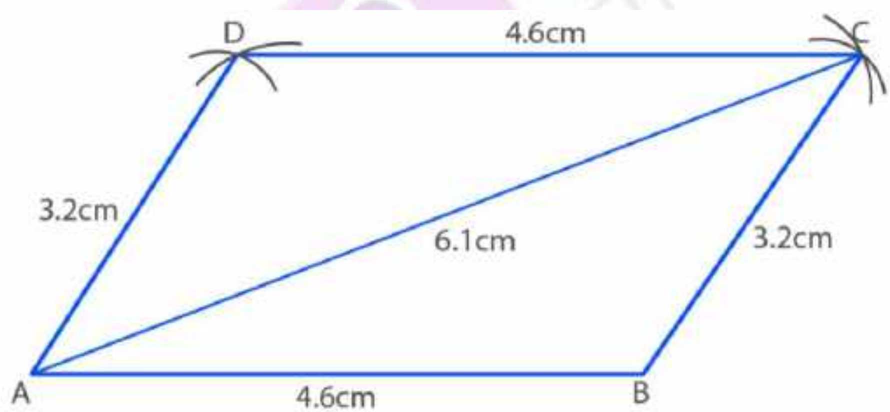
Steps of construction:
(i) Draw a line segment BC = 6.2cm
(ii) At B, draw a ray BX making an angle of 60^o and cut off AB = 5cm
(iii) From A, draw a line AY parallel to BC.
(iv) With centre C and radius 4.8cm, draw an arc which intersects AY at D and D'.
(v) Join CD and CD'
Thus, ABCD and ABCD' are the required two trapeziums.

7. Using ruler and compasses only, construct a parallelogram ABCD with AB = 5.1 cm, BC = 7 cm and ∠ABC = 75°.
Solution:



Steps of construction:
(i) Draw a line segment BC = 7cm
(ii) A to B, draw a ray Bx making an angle of 75º and cut off AB = 5.1cm
(iii) With centre A and radius 7cm with centre C and radius 5.1cm, draw arcs intersecting each other at D.
(iv) Join AD and CD.
Thus, ABCD is the required parallelogram.

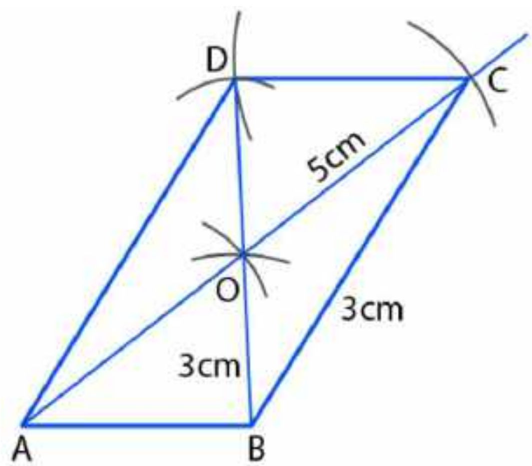
8. Using ruler and compasses only, construct a parallelogram ABCD in which AB = 4.6 cm, BC = 3.2 cm and AC = 6.1 cm.
Solution:



Steps of construction:
(i) Draw a line segment AB = 4.6cm
(ii) With centre A and radius 6.1cm and with centre B and radius 3.2cm, draw arcs intersecting each other at C.
(iii) Join AC and BC.
(iv) Again, with centre A and radius 3.2cm and with centre C and radius 4.6cm, draw arcs intersecting each other at C.
(v) Join AD and CD.

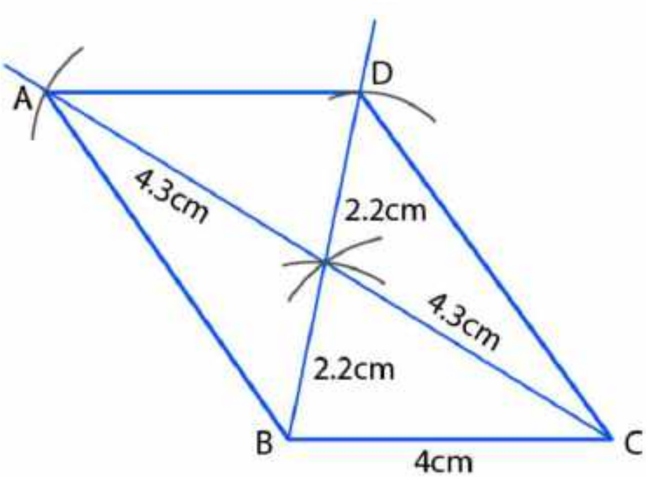
Thus, ABCD is the required parallelogram.

9. Using ruler and compasses, construct a parallelogram ABCD give that AB = 4 cm, AC = 10 cm, BD = 6 cm. Measure BC.
Solution:



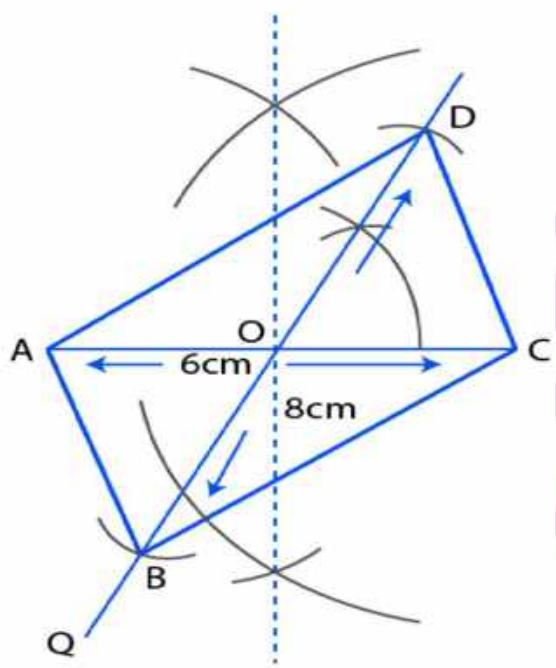
Steps of construction:
(i) Construct triangle OAB such that
OA = ½ x AC = ½ x 10cm = 5cm
OB = ½ x BD = ½ x 6cm = 3cm
As, diagonals of || gm bisect each other and AB = 4cm
(ii) Produce AO to C such that OA = OC = 5cm
(iii) Produce BO to D such that OB = OD = 3cm
(iv) Join AD, BC and CD
Thus, ABCD is the required parallelogram
(v) Measure BC which is equal to 7.2cm

10. Using ruler and compasses only, construct a parallelogram ABCD such that BC = 4 cm, diagonal AC = 8.6 cm and diagonal BD = 4.4 cm. Measure the side AB.
Solution:



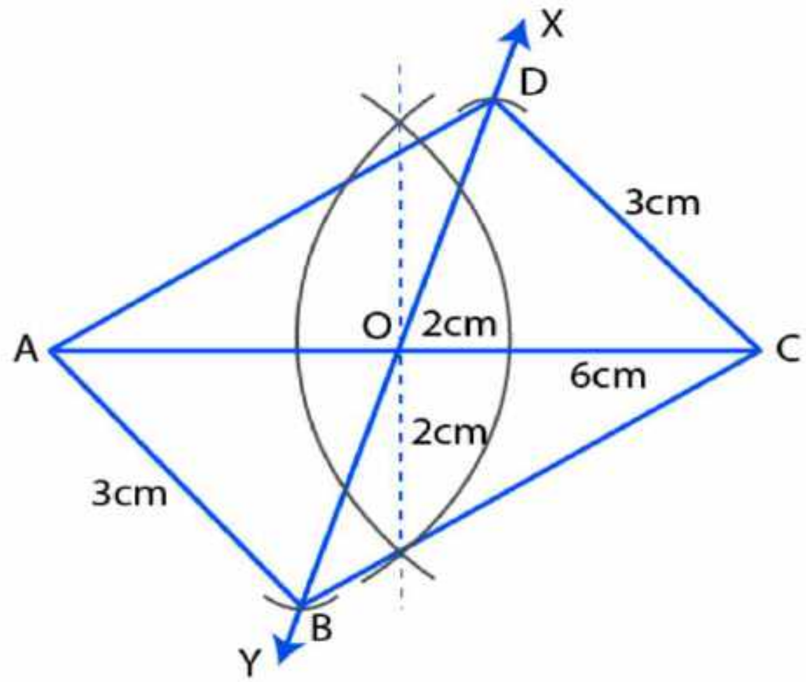
Steps of construction:
(i) Construct triangle OBC such that
OB = ½ x BD = ½ x 4.4cm = 2.2cm
OC = ½ x AC = ½ x 8.6cm = 4.3cm
Since, diagonals of || gm bisect each other and BC = 4cm
(ii) Produce BO to D such that BO = OD = 2.2cm
(iii) Produce CO to A such that CO = OA = 4.3cm
(iv) Join AB, AD and CD
Thus, ABCD is the required parallelogram.
(v) Measure the side AB, AB = 5.6cm

11. Use ruler and compasses to construct a parallelogram with diagonals 6 cm and 8 cm in length having given the acute angle between them is 60°. Measure one of the longer sides.
Solution:



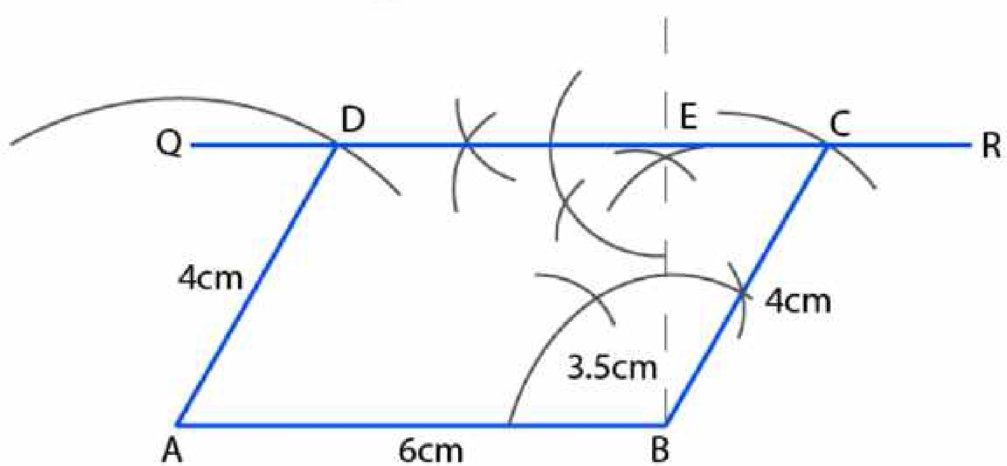
Steps of construction:
(i) Draw AC = 6cm
(ii) Find the mid-point O of AC. [Since, diagonals of || gm bisect each other]
(iii) Draw line POQ such that POC = 60º and OB = OD = ½ BD = ½ x 8cm = 4cm
So, from OP cut OD = 4cm and from OQ cut OB = 4cm
(iv) Join AB, BC, CD and DA.
Thus, ABCD is the required parallelogram.
(v) Measure the length of side AD = 6.1cm

12. Using ruler and compasses only, draw a parallelogram whose diagonals are 4 cm and 6 cm long and contain an angle of 75°. Measure and write down the length of one of the shorter sides of the parallelogram.
Solution:



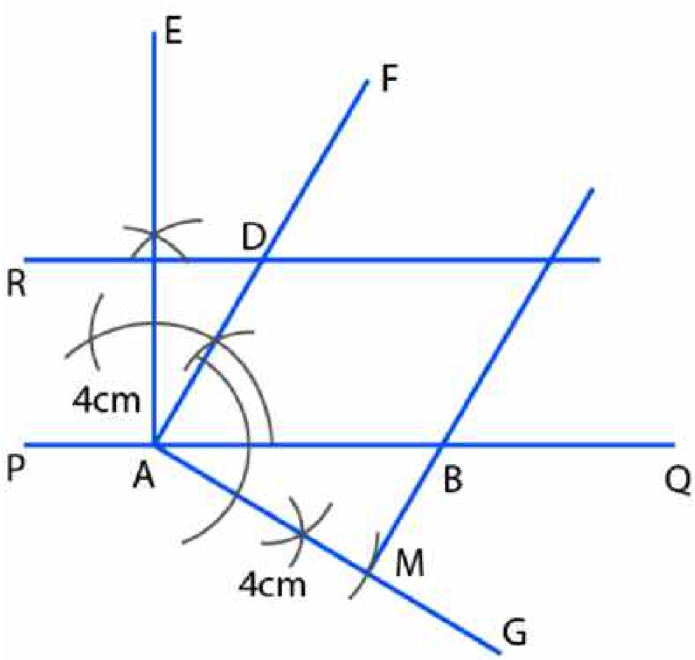
Steps of construction:
(i) Dra a line segment AC = 6cm
(ii) Bisect AC at O.
(iii) At O, draw a ray XY making an angle of 75° at O.
(iv) From OX and OY, cut off OD = OB = 4/2 = 2cm
(v) Join AB, BC, CD and DA.
Thus, ABCD is the required parallelogram.
On measuring one of the shorter sides, we get
AB = CD = 3cm

13. Using ruler and compasses only, construct a parallelogram ABCD with AB = 6 cm, altitude = 3.5 cm and side BC = 4 cm. Measure the acute angles of the parallelogram.
Solution:



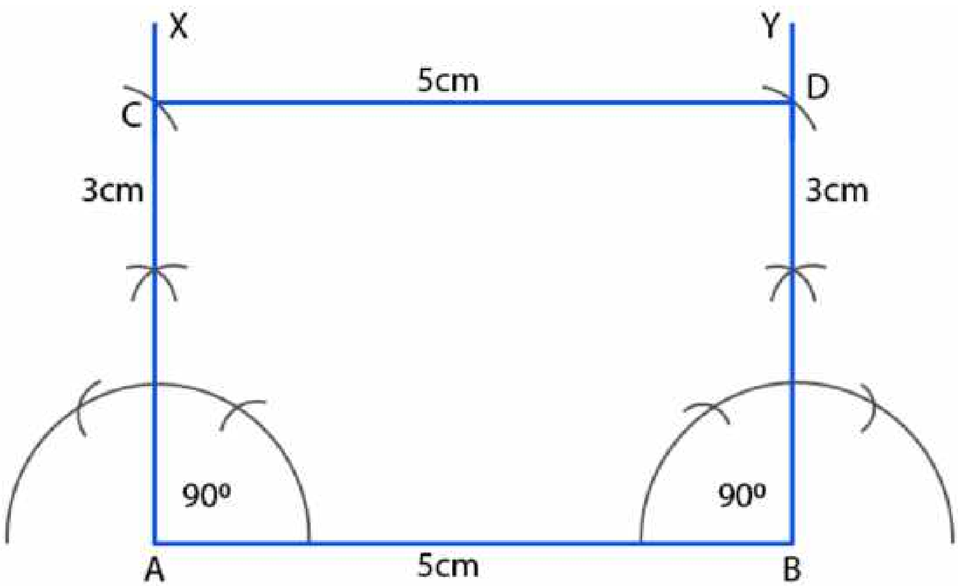
Steps of construction:
(i) Draw AB = 6cm
(ii) At B, draw BP ⊥ AB
(iii) From BP, cut BE = 3.5cm = height of || gm
(iv) Through E draw QR parallel to AB
(v) With B as centre and radius BC = 4cm draw an arc which cuts QR at C.
(vi) Since, opposite sides of || gm are equal
So, AD = BC = 4cm
(vii)With A as centre and radius = 4cm draw an arc which cuts QR at D.
Thus, ABCD is the required parallelogram.
(viii) To measure the acute angle of parallelogram which is equal to 61°.

14. The perpendicular distances between the pairs of opposite sides of a parallelogram ABCD are 3 cm and 4 cm and one of its angles measures 60°. Using ruler and compasses only, construct ABCD.
Solution:



Steps of construction:
(i) Draw a straight-line PQ, take a point A on it.
(ii) At A, construct ∠QAF = 60°
(iii) At A, draw AE ⊥ PQ from AE cut off AN = 3cm
(iv) Through N draw a straight line to PQ to meet AF at D.
(v) At D, draw AG ⊥ AD, from AG cut off AM = 4cm
(vi) Through M, draw at straight line parallel to AD to meet AQ in B and ND in C.
Then, ABCD is the required parallelogram

15. Using ruler and compasses, construct a rectangle ABCD with AB = 5cm and AD = 3 cm.
Solution:

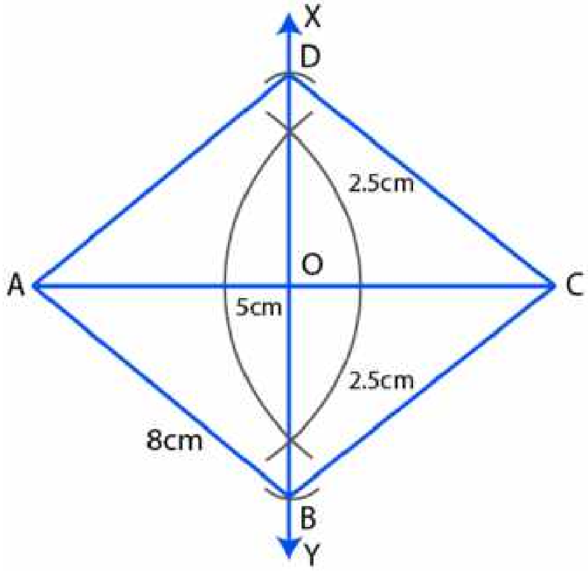


Steps of construction:
(i) Draw a straight-line AB = 5cm
(ii) At A and B construct ∠XAB and ∠YBA = 90º
(iii) From A and B cut off AC and BD = 3cm each
(iv) Join CD
Thus, ABCD is the required rectangle.

16. Using ruler and compasses only, construct a rectangle each of whose diagonals measures 6cm and the diagonals intersect at an angle of 45°.
Solution:

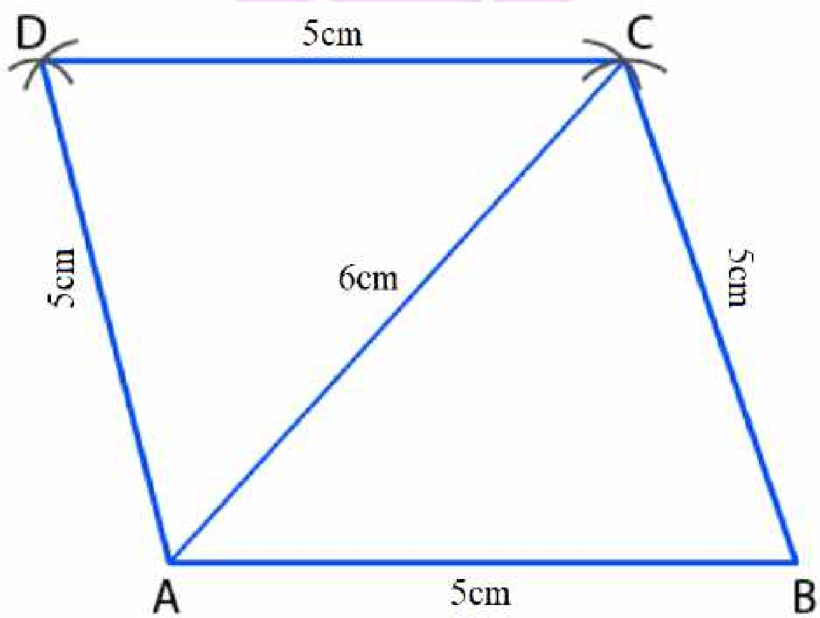
Steps of construction:
(i) Draw a line segment AC = 6cm
(ii) Bisect AC at O.
(iii) At O, draw a ray XY making an angle of 45º at O.
(iv) From XY, cut off OB = OD = 6/2 = 3cm each
(v) Join AB, BC CD and DA.
Thus, ABCD is the required rectangle.

17. Using ruler and compasses only, construct a square having a diagonal of length 5cm. Measure its sides correct to the nearest millimeter.
Solution:



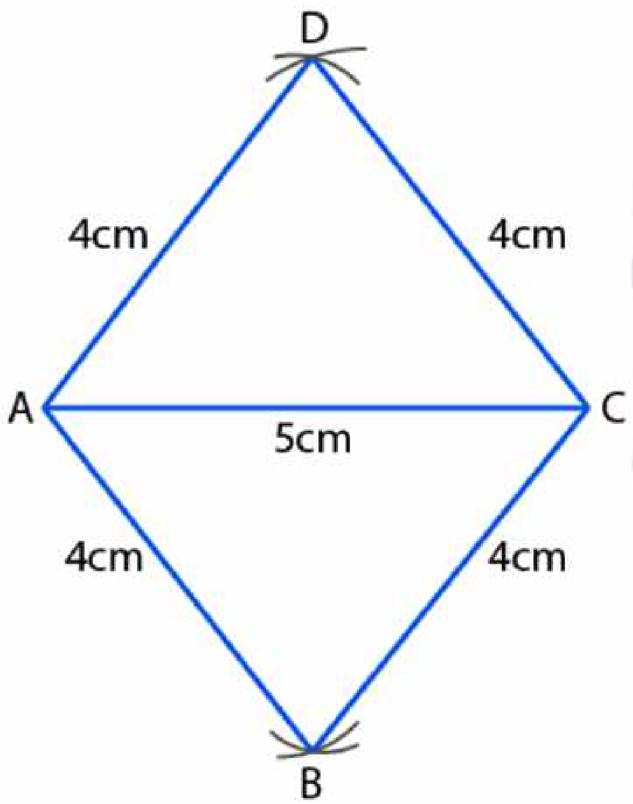
Steps of construction:
(i) Draw a line segment AC = 5cm
(ii) Draw its percentage bisector XY bisecting it at O
(iii) From XY, cut off
OB = OD = 5/2 = 2.5cm
(iv) Join AB, BC, CD and DA.
Thus, ABCD is the required square
On measuring its sides, each = 3.6cm (approximately)

18. Using ruler and compasses only construct A rhombus ABCD given that AB 5cm, AC = 6cm measure ∠BAD.
Solution:



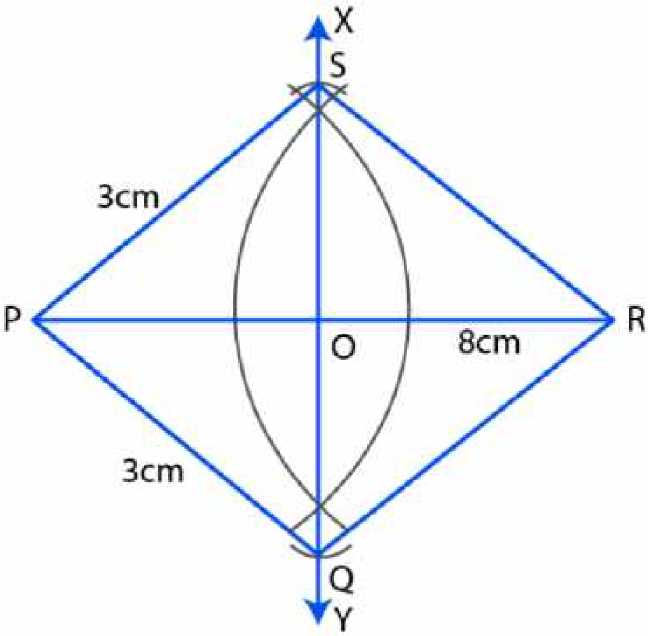
Steps of construction:
(i) Draw a line segment AB = 5cm
(ii) With centre A and radius 6cm, with centre B and radius 5cm, draw arcs intersecting each other at C.
(iii) Join AC and BC
(iv) With centre A and C and radius 5cm, draw arc intersecting each other 5cm, draw arcs intersecting each other at D
(v) Join AD and CD.
Thus, ABCD is a rhombus
On measuring, ∠BAD = 106°.

19. Using ruler and compasses only, construct rhombus ABCD with sides of length 4cm and diagonal AC of length 5 cm. Measure ∠ABC.
Solution:



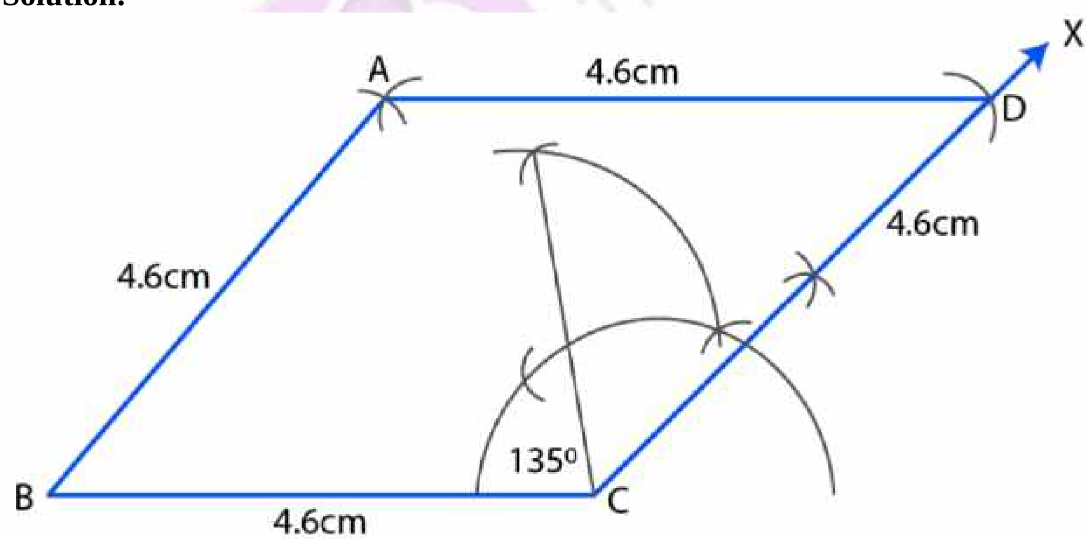
Steps of construction:
(i) Draw a line segment AC = 5cm
(ii) With centre A and C and radius 4cm, draw arcs intersecting each other above and below AC at D and B.
(iii) Join AB, BC, CD and DA.
Thus, ABCD is the required rhombus.

20. Construct a rhombus PQRS whose diagonals PR and QS are 8cip and 6cm respectively.
Solution:



Steps of construction:
(i) Draw a line segment PR = 8cm
(ii) Draw its perpendicular bisector XY intersecting it at O.
(iii) From XY, cut off OQ = OS = 6/2 = 3cm each
(iv) Join PQ, QR, RS and SP
Thus, PQRS is the required rhombus.

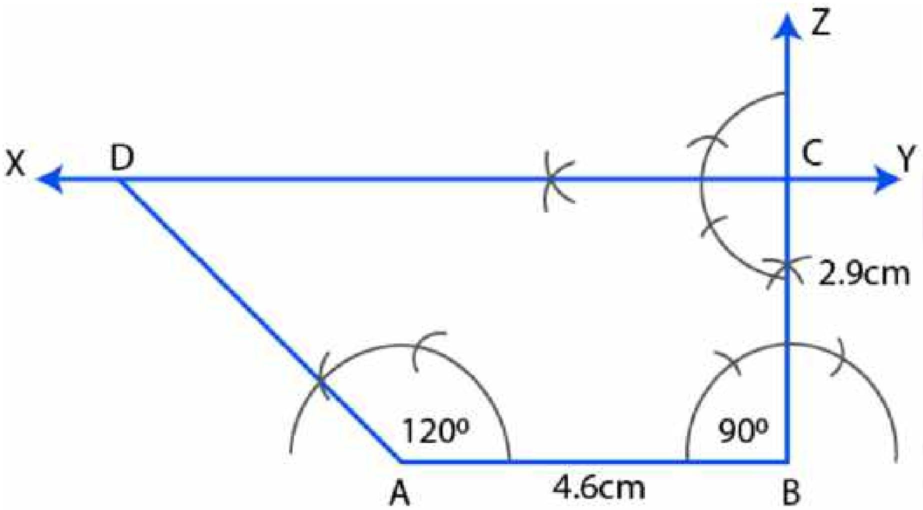
21. Construct a rhombus ABCD of side 4.6 cm and ∠BCD = 135°, by using ruler and compasses only.
Solution:



Steps of construction:
(i) Draw a line segment BC = 4.6cm
(ii) At C, draw a ray CX making an angle of 135° and cut off CD = 4.6cm

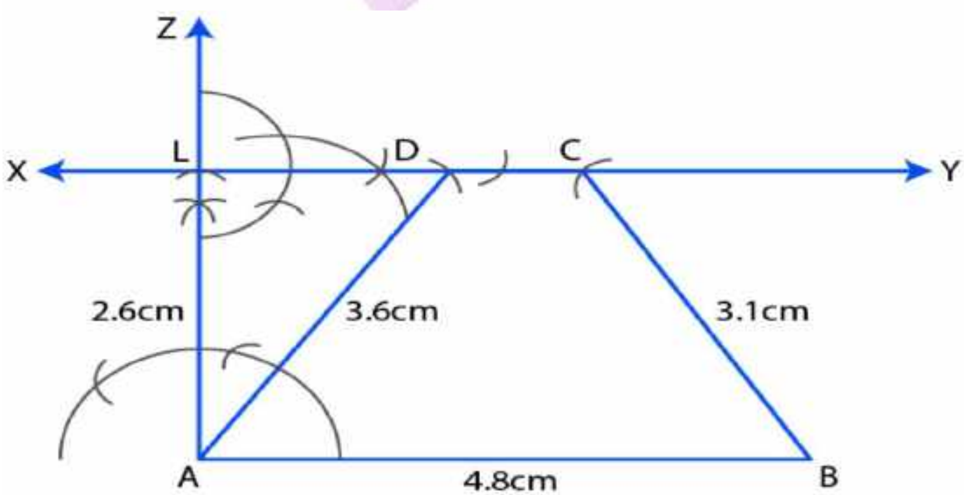
(iii) With centres B and D, and radius 4.6cm draw arcs intersecting each other at A.
(iv) Join BA and DA.
Thus, ABCD is the required rhombus.

22. Construct a trapezium in which AB || CD, AB = 4.6 cm, ∠ ABC = 90°, ∠ DAB = 120° and the distance between parallel sides is 2.9 cm.
Solution:



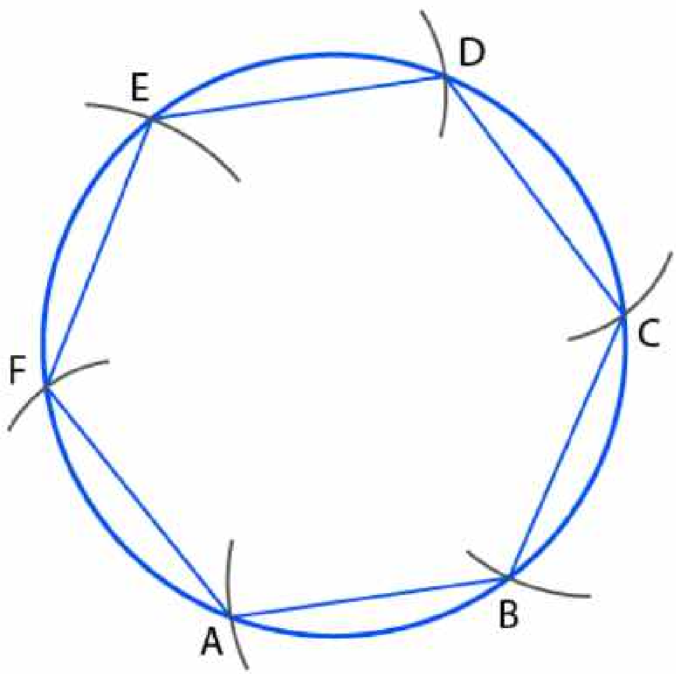
Steps of construction:
(i) Draw a line segment AB = 4.6cm
(ii) At B, draw a ray BZ making an angle of 90° and cut off BC = 2.9cm (distance between AB and CD)
(iii) At C, draw a parallel line XY to AB.
(iv) At A, draw a ray making an angle of 120° meeting XY at D.
Thus, ABCD is the required trapezium.

23. Construct a trapezium ABCD when one of parallel sides AB = 4.8 cm, height = 2.6cm, BC = 3.1 cm and AD = 3.6 cm.
Solution:



Step construction:
(i) Draw a line segment AB = 4.8cm
(ii) At A, draw a ray AZ making an angle of 90^{o} cut off AL = 2.6cm
(iii) At L, draw a line XY parallel to AB.
(iv) With centre A and radius 3.6cm and with centre B and radius 3.1cm, draw arcs intersecting XY at D and C respectively.
(v) Join AD and BC
Thus, ABCD is the required trapezium.

24. Construct a regular hexagon of side 2.5 cm.
Solution:

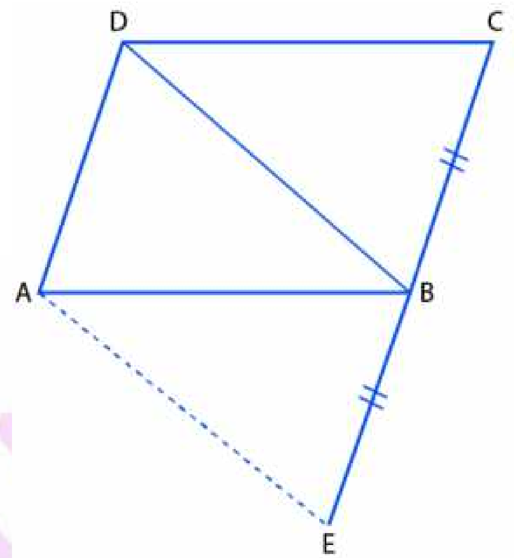


Steps of construction:
(i) With O as centre and radius = 2.5cm, draw a circle
(ii) take any point A on the circumference of circle.
(iii) With A as centre and radius = 2.5cm, draw an arc which cuts the circumference of circle at B.
(iv) With B as centre and radius = 2.5cm, draw an arc which cuts the circumference of circle at C.
(v) With C as centre and radius = 2.5cm, draw an arc which cuts the circumference of circle at D.
(vi) With D as centre and radius = 2.5cm, draw an arc which cuts the circumference of circle at E.
(vii) With E as centre and radius = 2.5cm, draw an arc which cuts the circumference of circle at F.
(viii) Join AB, BC, CD, DE, EF and FA.
(ix) ABCDEF is the required Hexagon.

Chapter Test

1. In the given figure, ABCD is a parallelogram. CB is produced to E such that BE=BC. Prove that AEBD is a parallelogram.

Solution:

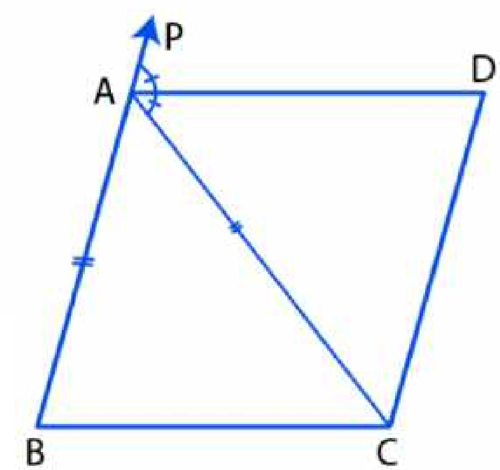


Given ABCD is a || gm in which CB is produced to E such that
BE = BC
BD and AE are joined
To prove: AEBD is a parallelogram
Proof:
In ΔAEB and ΔBDC
EB = BC [Given]
∠ABE = ∠DCB [Corresponding angles]
AB = DC [Opposite sides of || gm]
Thus, ΔAEB ≅ ΔBDC by S.A.S axiom
So, by C.P.C.T
But, AD = CB = BE [Given]
As the opposite sides are equal and ∠AEB = ∠DBC
But these are corresponding angles
Hence, AEBD is a parallelogram.

2. In the given figure, ABC is an isosceles triangle in which AB = AC. AD bisects exterior angle PAC and CD || BA. Show that (i) ∠DAC=∠BCA (ii) ABCD is a parallelogram.

Solution:

Given: In isosceles triangle ABC, AB = AC. AD is the bisector of ext. ∠PAC and CD || BA
To prove: (i) ∠DAC = ∠BCA
(ii) ABCD is a || gm
Proof:

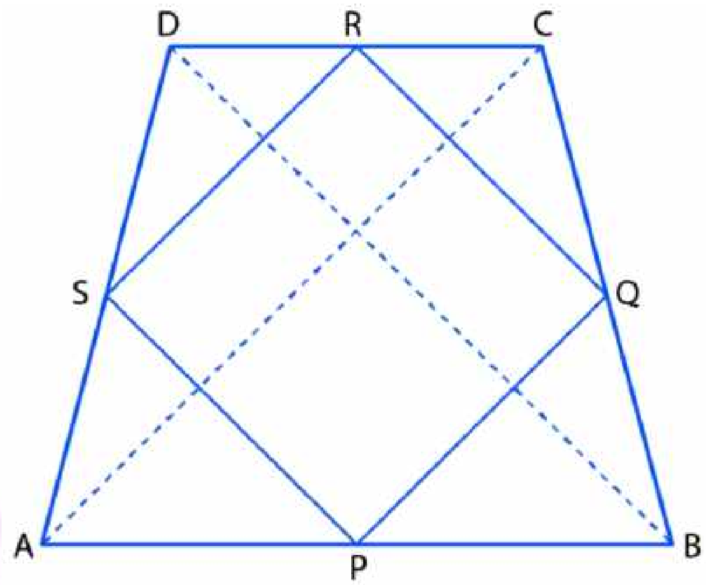


In ΔABC
AB = AC [Given]
∠C = ∠B [Angles opposite to equal sides]
Since, ext. ∠PAC = ∠B + ∠C
= ∠C + ∠C
= 2 ∠C
= 2 ∠BCA
So, ∠DAC = 2 ∠BCA
∠DAC = ∠BCA
But these are alternate angles
Thus, AD || BC
But, AB || AC
Hence, ABCD is a || gm.

3. Prove that the quadrilateral obtained by joining the mid-points of an isosceles trapezium is a rhombus.

Solution:

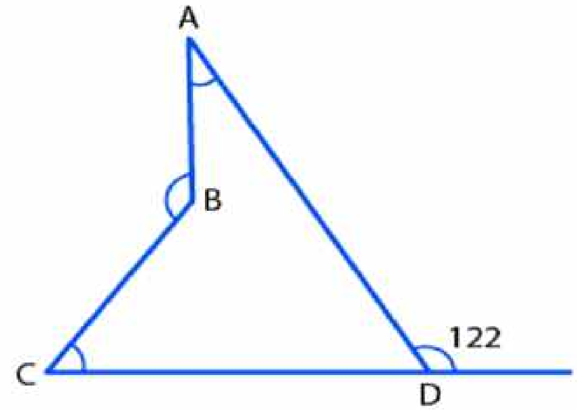
Given: ABCD is an isosceles trapezium in which AB || DC and AD = BC
P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively PQ, QR, RS and SP are joined.



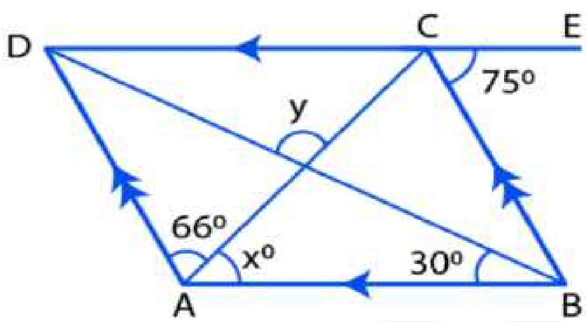
To prove: PQRS is a rhombus
Construction: Join AC and BD
Proof:
Since, ABCD is an isosceles trapezium
Its diagonals are equal
AC = BD
Now, in ΔABC
P and Q are the mid-points of AB and BC
So, PQ || AC and PQ = ½ AC … (i)
Similarly, in ΔADC
S and R mid-point of CD and AD
So, SR || AC and SR = ½ AC … (ii)
From (i) and (ii), we have
PQ || SR and PQ = SR
Thus, PQRS is a parallelogram.
Now, in ΔAPS and ΔBPQ
AP = BP [P is the mid-point]
AS = BQ [Half of equal sides]
∠A = ∠B [As ABCD is an isosceles trapezium]
So, ΔAPS ≅ ΔBPQ by SAS Axiom of congruency
Thus, by C.P.C.T we have
PS = PQ
But there are the adjacent sides of a parallelogram
So, sides of PQRS are equal
Hence, PQRS is a rhombus
- Hence proved

4. Find the size of each lettered angle in the following figures.

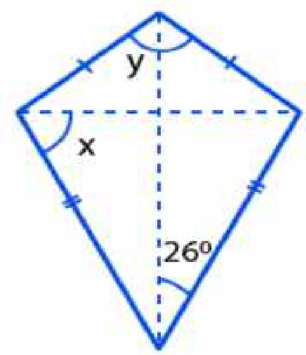
(i)



(ii)



(iii)



Solution:

(i) As CDE is a straight line

$\angle ADE + \angle ADC = 180^o$

$122^o + \angle ADC = 180^o$

$\angle ADC = 180^o - 122^o = 58^o$ … (i)

$\angle ABC = 360^o - 140^o = 220^o$ … (ii)

[At any point the angle is 360^o]

Now, in quadrilateral ABCD we have

$\angle ADC + \angle BCD + \angle BAD + \angle ABC = 360^o$

$58^o + 53^o + x + 220^o = 360^o$ [Using (i) and (ii)]

$331^o + x = 360^o$

$x = 360^o - 331^o$

$x = 29^o$

(ii) As DE || AB [Given]

$\angle ECB = \angle CBA$ [Alternate angles]

$75^o = \angle CBA$

$\Rightarrow \angle CBA = 75^o$

Since, AD || BC we have

$(x + 66^o) + 75^o = 180^o$

$x + 141^o = 180^o$

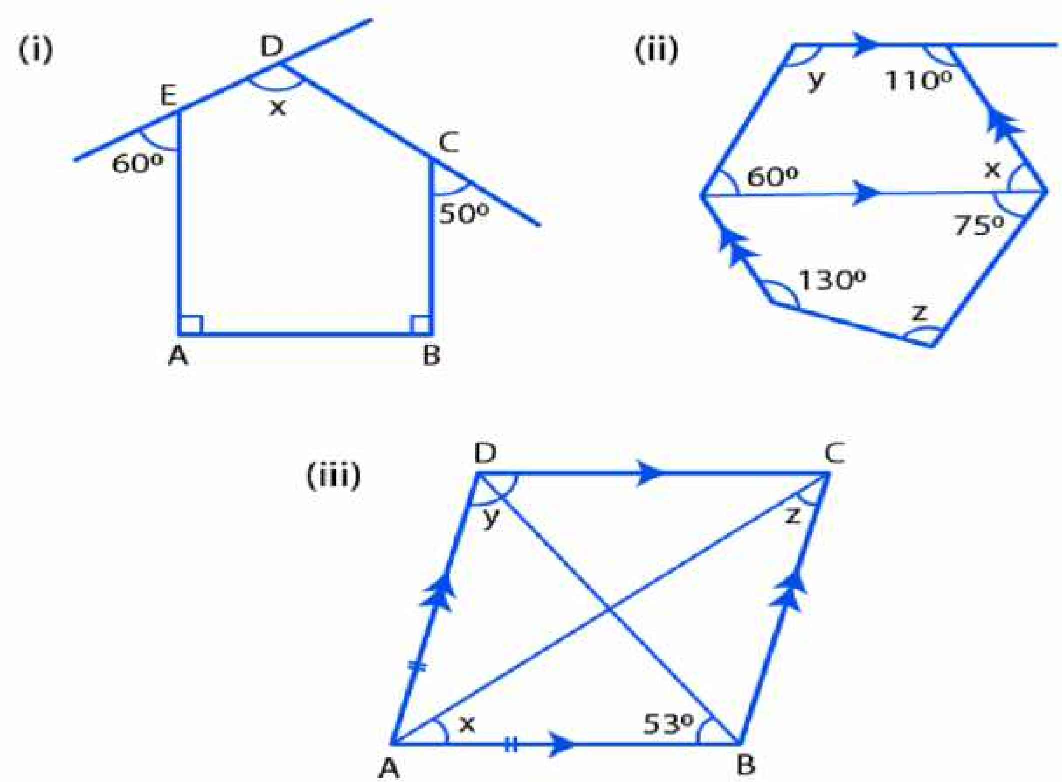
$x = 180^o - 141^o$

$x = 39^o$ … (i)

Now, in ΔAMB
x + 30° + ∠AMB = 180° [Angles sum property of a triangle]
39° + 30° + ∠AMB = 180° [From (i)]
69° + ∠AMB + 180°
∠AMB = 180° – 69° = 111° … (ii)
Since, ∠AMB = y [Vertically opposite angles]
⇒ y = 111°
Hence, x = 39° and y = 111°

(iii) In ΔABD
AB = AD [Given]
∠ABD = ∠ADB [Angles opposite to equal sides are equal]
∠ABD = 42° [Since, given ∠ADB = 42°]
And,
∠ABD + ∠ADB + ∠BAD = 180° [Angles sum property of a triangle]
42° + 42° + y = 180°
84° + y = 180°
y = 180° – 84°
y = 96°
∠BCD = 2 x 26° = 52°
In ΔBCD,
As BC = CD [Given]
∠CBD = ∠CDB = x [Angles opposite to equal sides are equal]
∠CBD + ∠CDB + ∠BCD = 180°
x + x + 52° = 180°
2x + 52° = 180°
2x = 180° – 52°
x = 128°/2
x = 64°
Hence, x = 64° and y = 90°.

5. Find the size of each lettered angle in the following figures:



Solution:

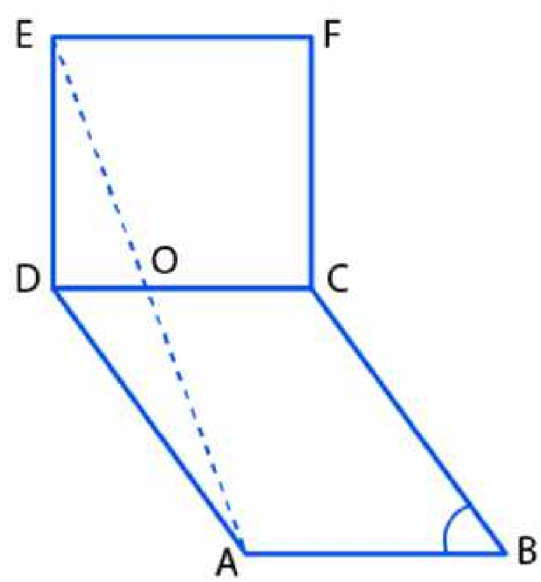
(i) Here, AB || CD and BC || AD
So, ABCD is a || gm
y = 2 x ∠ABD
y = 2 x 53° = 106° … (1)
Also, y + ∠DAB = 180°
∠DAB = 180° – 106°
= 74°
Thus, x = ½ ∠DAB [As AC bisects ∠DAB]
x = ½ x 74° = 37°
and ∠DAC = x = 37° … (ii)
Also, ∠DAC = z … (iii) [Alternate angles]
From (ii) and (iii),
z = 37°
Hence, x = 37°, y = 106° and z = 37°

(ii) As ED is a straight line, we have
60° + ∠AED = 180° [Linear pair]
∠AED = 180° – 60°
∠AED = 120° … (i)
Also, as CD is a straight line
50° + ∠BCD = 180° [Linear pair]
∠BCD = 180° - 50°

∠BCD = 130° … (ii)
In pentagon ABCDE, we have
∠A + ∠B+ ∠AED + ∠BCD + ∠x = 540° [Sum of interior angles in pentagon is 540°]
90° + 90° 120° + 130° + x = 540°
430° + x = 540°
x = 540° – 430°
x = 110°
Hence, value of x = 110°

(iii) In given figure, AD || BC [Given]
60° + y = 180° and x + 110° = 180°
y = 180° – 60° and x = 180° – 110°
y = 120° and x = 70°
Since, CD || AF [Given]
∠FAD = 70° … (i)
In quadrilateral ADEF,
∠FAD + 75° + z + 130° = 360°
70° + 75° + z + 130° = 360°
275° + z = 360°
z = 360° – 275° = 85°
Hence,
x = 70°, y = 120° and z = 85°

6. In the adjoining figure, ABCD is a rhombus and DCFE is a square. If ∠ABC = 56°, find (i) ∠DAG (ii) ∠FEG (iii) ∠GAC (iv) ∠AGC.
Solution:



Here ABCD and DCFE is a rhombus and square respectively.
So, AB = BC = DC = AD … (i)
Also, DC = EF = FC = EF … (ii)
From (i) and (ii), we have
AB = BC = DC = AD = EF = FC = EF … (iii)
∠ABC = 56° [Given]
∠ADC = 56° [Opposite angle in rhombus are equal]
So, ∠EDA = ∠EDC + ∠ADC = 90° + 56° = 146°
In ΔADE,
DE = AD [From (iii)]
∠DEA = ∠DAE [Equal sides have equal opposite angles]
∠DEA = ∠DAG = (180° - ∠EDA)/ 2
= (180° – 146°)/ 2
= 34°/ 2 = 17°
⇒ ∠DAG = 17°
Also, ∠DEG = 17°
∠FEG = ∠E - ∠DEG
= 90° – 17°

= 73°

In rhombus ABCD,

∠DAB = 180° – 56° = 124°

∠DAC = 124°/ 2 [Since, AC diagonals bisect the ∠A]

∠DAC = 62°

∠GAC = ∠DAC - ∠DAG

= 62° – 17°

= 45°

In ΔEDG,

∠D + ∠DEG + ∠DGE = 180° [Angles sum property of a triangle]

90° + 17° + ∠DGE = 180°

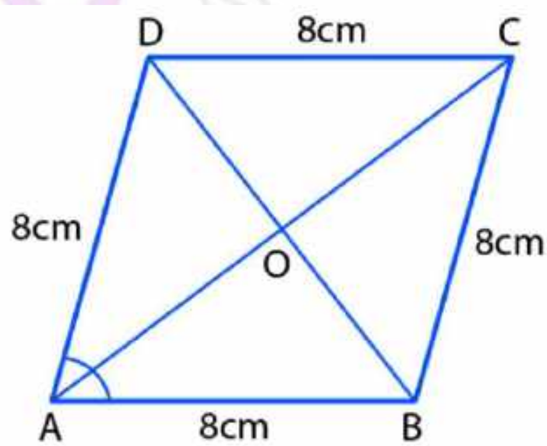
∠DGE = 180° – 107° = 73° … (iv)

Thus, ∠AGC = ∠DGE … (v) [Vertically opposite angles]

Hence from (iv) and (v), we have

∠AGC = 73°

7. If one angle of a rhombus is 60° and the length of a side is 8 cm, find the lengths of its diagonals.
Solution:



Each side of rhombus ABCD is 8cm

So, AB = BC = CD = DA = 8cm

Let ∠A = 60°

So, ΔABD is an equilateral triangle

Then,

AB = BD = AD = 8cm

As we know, the diagonals of a rhombus bisect each other at right angles

AO = OC, BO = OD = 4cm and ∠AOB = 90°

Now, in right ΔAOB

By Pythagoras theorem

$AB^2 = AO^2 + OB^2$

$8^2 = AO^2 + 4^2$

$64 = AO^2 + 16$

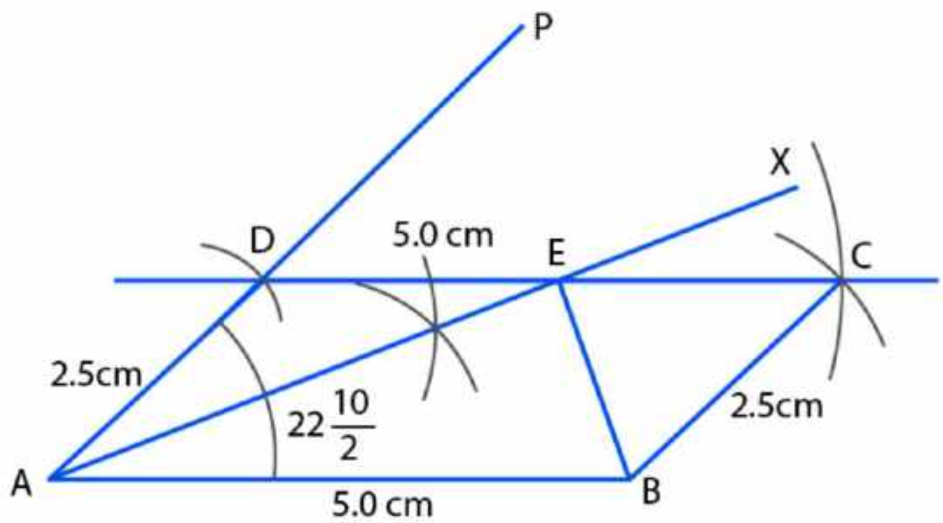
$AO^2 = 64 – 16 = 48$

AO = √48 = 4√3cm

But, AC = 2 AO

Hence, AC = 2 × 4√3 = 8√3cm.

8. Using ruler and compasses only, construct a parallelogram ABCD with AB = 5 cm, AD = 2.5 cm and ∠BAD = 45°. If the bisector of ∠BAD meets DC at E, prove that ∠AEB is a right angle.
Solution:



Steps of construction:
(i) Draw AB = 5.0cm
(ii) Draw BAP = 45^o on side AB
(iii) Take A as centre and radius 2.5cm cut the line AP at D
(iv) Take D as centre and radius 5.0cm draw an arc
(v) Take B as centre and radius equal to 2.5cm cut the arc of step (iv) at C
(vi) Join BC and CD
(vii) ABCD is the required parallelogram
(viii) Draw the bisector of ∠BAD, which cuts the DC at E
(ix) Join EB
(x) Measure ∠AEB which is equal to 90^o.