Exercise 5.3

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Find $\frac{dy}{dx}$ in the following Exercise 1 to 15.

1. $2x+3y=\sin x$

Solution: Given function is $2x+3y=\sin x$

Derivate function with respect to x, we have

$$\frac{d}{dx}(2x)+\frac{d}{dx}(3y)=\frac{d}{dx}\sin x$$

$$2+3\frac{dy}{dx}=\cos x$$

$$3\frac{dy}{dx}=\cos x-2$$

$$\frac{dy}{dx}=\frac{\cos x-2}{3}$$

2. $2x+3y=\sin y$

Solution: Given function is $2x+3y=\sin y$

Derivate function with respect to x, we have

$$\frac{d}{dx}(2x)+\frac{d}{dx}(3y)=\frac{d}{dx}\sin y$$

$$2+3\frac{dy}{dx}=\cos y\frac{dy}{dx}$$

$$-\frac{dy}{dx}(\cos y-3)=-2$$

$$\frac{dy}{dx}=\frac{2}{\cos y-3}$$

3. $ax+by^2=\cos y$

Solution: Given function is $ax+by^2=\cos y$

Derivate function with respect to x, we have

$$\frac{d}{dx}(ax)+\frac{d}{dx}(by^2)=\frac{d}{dx}\cos y$$

$$a+b.2y\frac{dy}{dx}=-\sin y\frac{dy}{dx}$$

$$2by\frac{dy}{dx}+\sin y\frac{dy}{dx}=-a$$

$$-\frac{dy}{dx}(2by+\sin y)=-a$$

$$\frac{dy}{dx}=\frac{-a}{2by+\sin y}$$

4. $xy+y^2=\tan x+y$

Solution: Given function is $xy+y^2=\tan x+y$

Derivate function with respect to x, we have

$$\frac{d}{dx}(xy)+\frac{d}{dx}(y^2)=\frac{d}{dx}\tan x+\frac{d}{dx}y$$

$$x\frac{d}{dx}y+y\frac{d}{dx}x+2y\frac{dy}{dx}=\sec^2 x+\frac{dy}{dx}$$

[Solving first term using Product Rule]

$$x\frac{dy}{dx}+y+2y\frac{dy}{dx}=\sec^2 x+\frac{dy}{dx}$$

$$x\frac{dy}{dx}+2y\frac{dy}{dx}-\frac{dy}{dx}=\sec^2 x-y$$

$$(x+2y-1)\frac{dy}{dx}=\sec^2 x-y$$

$$\frac{dy}{dx}=\frac{\sec^2 x-y}{x+2y-1}$$

5. $x^2 + xy + y^2 = 100$

Solution: Given function is $x^2 + xy + y^2 = 100$

Derivate function with respect to x, we have

$$\frac{d}{dx}x^2 + \frac{d}{dx}xy + \frac{d}{dx}y^2 = \frac{d}{dx}100$$

$$2x + \left(x\frac{d}{dx}y + y\frac{d}{dx}x \right) + 2y\frac{dy}{dx} = 0$$

$$2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$$

$$(x+2y)\frac{dy}{dx} = -2x - y$$

6. $x^3 + x^2y + xy^2 + y^3 = 81$

Solution: Given function is $x^3 + x^2y + xy^2 + y^3 = 81$

Derivate function with respect to x, we have

$$\frac{d}{dx}x^3 + \frac{d}{dx}x^2y + \frac{d}{dx}xy^2 + \frac{d}{dx}y^3 = \frac{d}{dx}81$$

$$3x^2 + \left(x^2\frac{dy}{dx} + y.\frac{d}{dx}x^2 \right) + x\frac{d}{dx}y^2 + y^2\frac{d}{dx}x + 3y^2\frac{dy}{dx} = 0$$ (using product rule)

$$3x^2 + x^2\frac{dy}{dx} + y.2x + x.2y\frac{dy}{dx} + y^2.1 + 3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(x^2 + 2xy + 3y^2\right) = -3x^2 - 2xy - y^2$$

$$\frac{dy}{dx} = \frac{\left(3x^2 + 2xy + y^2\right)}{x^2 + 2xy + 3y^2}$$

7. $\sin^2 y + \cos xy = \pi$

Solution: Given function is $\sin^2 y + \cos xy = \pi$

Derivate function with respect to x, we have

$$\frac{d}{dx}(\sin y)^2 + \frac{d}{dx}\cos xy = \frac{d}{dx}(\pi)$$

$$2\sin y\frac{d}{dx}\sin y - \sin xy\frac{d}{dx}(xy) = 0$$

$$2\sin y\cos y\frac{dy}{dx} - \sin xy\left(x\frac{dy}{dx} + y.1\right) = 0$$

$$\sin 2y\frac{dy}{dx} - x\sin xy\frac{dy}{dx} - y\sin xy = 0$$

$$(\sin 2y - x\sin xy)\frac{dy}{dx} = y\sin xy$$

$$\frac{dy}{dx} = \frac{y\sin xy}{\sin 2y - x\sin xy}$$

8. $\sin^2 x + \cos^2 y = 1$

Solution: Given function is $\sin^2 x + \cos^2 x = 1$
Derivate function with respect to x, we have

$$\frac{d}{dx}(\sin x)^2 + \frac{d}{dx}(\cos x)^2 = \frac{d}{dx}(1)$$

$$2\sin x\frac{d}{dx}\sin x + 2\cos y\frac{d}{dx}\cos y = 0$$

$$2\sin x\cos x + 2\cos y\left(-\sin y\frac{dy}{dx}\right) = 0$$

$$\sin 2x - \sin 2y\frac{dy}{dx} = 0$$

$$-\sin 2y \frac{dy}{dx} = -\sin 2x$$

$$\frac{dy}{dx} = \frac{\sin 2x}{\sin 2y}$$

9. $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Solution: Given function is

$$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

Step 1: Simplify the given function,

Put $x = \tan\theta$, we have

$$y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$= \sin^{-1}(\sin 2\theta) = 2\theta$

Result in terms of x, we get

$$y = 2\tan^{-1} x$$

Step 2: Derivative the function

$$\frac{dy}{dx} = 2.\frac{1}{1+x^2} = \frac{2}{1+x^2}$$

10. $y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), \frac{-1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

Solution: Given function is

$$y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), \frac{-1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$$

Step 1: Simplify the given function,

$$y = \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right)$$

$$y = \tan^{-1}(\tan 3\theta) = 3\theta$$

Result in terms of x, we get

$$y = 3\tan^{-1} x$$

Step 2: Derivative the function

$$\frac{dy}{dx} = 3.\frac{1}{1+x^2} = \frac{3}{1+x^2}$$

11. $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$

Solution: Given function is

$$y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$$

Step 1: Simplify the given function,

Put $x = \tan\theta$

$$y = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$= \cos^{-1}(\cos 2\theta) = 2\theta = 2\tan^{-1} x$

Step 2: Derivative the function

$$\frac{dy}{dx} = 2.\frac{1}{1+x^2} = \frac{2}{1+x^2}$$

12. $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$

Solution: Given function is $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$

Step 1: Simplify the given function,

Put $x = \tan\theta$

$$y = \sin^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$= \sin^{-1}(\cos 2\theta)$

$= \sin^{-1}\sin\left(\frac{\pi}{2}-2\theta\right) = \frac{\pi}{2}-2\theta$

$= \frac{\pi}{2}-2\tan^{-1}x$

Step 2: Derivative the function

$\frac{dy}{dx} = 0-2.\frac{1}{1+x^2} = \frac{-2}{1+x^2}$ (Derivative of a constant is always revert a value zero)

13. $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right), -1 < x < 1$

Solution:

Given function is:
y = cos-1 (2x/(1+x2)), -1 < x < 1
let x = tan θ, we get
y = cos-1 [(2 tan θ)/(1+ tan2 θ)]
= cos-1 (sin 2 θ)
= cos-1 {cos (π/2) - 2θ}
= (π/2) - 2θ
[Since, -1 < x < 1 => **-1** < tan θ < 1
So, - π/4 < θ < π/4]
= (π/2) - 2tan-1 x
Now, differentiating both sides with respect to x, we get
dy/dx = 0 - 2/(1+x2)
= -2/(1+x2)

14. $y=\sin^{-1}\left(2x\sqrt{1-x^2}\right), \frac{-1}{\sqrt{2}}<x<\frac{1}{\sqrt{2}}$

Solution: Given function is $y=\sin^{-1}\left(2x\sqrt{1-x^2}\right), -\frac{1}{\sqrt{2}}<x<\frac{1}{\sqrt{2}}$

Step 1: Simplify the given function,

Put $x=\sin\theta$

$y=\sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$

= $\sin^{-1}\left(2\sin\theta\sqrt{\cos^2\theta}\right)$

= $\sin^{-1}\left(2\sin\theta\cos\theta\right)$

= $\sin^{-1}\left(\sin 2\theta\right)=2\theta=2\sin^{-1}x$

Step 2: Derivative the function

$\frac{dy}{dx}=2.\frac{1}{\sqrt{1-x^2}}=\frac{2}{\sqrt{1-x^2}}$

15. $y=\sec^{-1}\left(\frac{1}{2x^2-1}\right), 0<x<\frac{1}{\sqrt{2}}$

Solution: Given function is $y=\sec^{-1}\left(\frac{1}{2x^2-1}\right), 0<x<\frac{1}{\sqrt{2}}$

Step 1: Simplify the given function,

Put $x = \cos\theta$

$y = y = \sec^{-1}\left(\frac{1}{2\cos^2\theta - 1}\right)$

= $y = \sec^{-1}\left(\frac{1}{\cos 2\theta}\right)$

= $\sec^{-1}(\sec 2\theta)$

= $2\theta = 2\cos^{-1}x$

Step 2: Derivative the function

$\frac{dy}{dx} = 2.\frac{-1}{\sqrt{1-x^2}} = \frac{-2}{\sqrt{1-x^2}}$