Exercise 14.2

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1. Factorise the following expressions.

(i) $a^2 + 8a + 16$

(ii) $p^2 – 10 p + 25$

(iii) $25m^2 + 30m + 9$

(iv) $49y^2 + 84yz + 36z^2$

(v) $4x^2 – 8x + 4$

(vi) $121b^2 – 88bc + 16c^2$

(vii) $(l + m)^2 – 4lm$ (Hint: Expand $(l + m)^2$ first)

(viii) $a^4 + 2a^2b^2 + b^4$

Solution:

(i) $a^2 + 8a + 16$

$= a^2 + 2 \times 4 \times a + 4^2$

$= (a + 4)^2$

$Using\ identity: (x+y)^2 = x^2 + 2xy + y^2$

(ii) $p^2 – 10 p + 25$

$= p^2 - 2 \times 5 \times p + 5^2$

$= (p - 5)^2$

$Using\ identity: (x-y)^2 = x^2 - 2xy + y^2$

(iii) $25m^2 + 30m + 9$

$= (5m)^2 - 2 \times 5m \times 3 + 3^2$

$= (5m + 3)^2$

$Using\ identity: (x+y)^2 = x^2 + 2xy + y^2$

(iv) $49y^2 + 84yz + 36z^2$

$= (7y)^2 + 2 \times 7y \times 6z + (6z)^2$

$= (7y + 6z)^2$

$Using\ identity: (x+y)^2 = x^2 + 2xy + y^2$

(v) $4x^2 – 8x + 4$

$= (2x)^2 - 2 \times 4x + 2^2$

$= (2x - 2)^2$

$Using\ identity: (x-y)^2 = x^2 - 2xy + y^2$

(vi) $121b^2 - 88bc + 16c^2$

$= (11b)^2 - 2 \times 11b \times 4c + (4c)^2$

$= (11b - 4c)^2$

$Using\ identity: (x - y)^2 = x^2 - 2xy + y^2$

(vii) $(l + m)^2 - 4lm$ (Hint: Expand $(l + m)^2$ first)

Expand $(l + m)^2$ $using\ identity: (x + y)^2 = x^2 + 2xy + y^2$

$(l + m)^2 - 4lm = l^2 + m^2 + 2ml - 4lml$

$= l^2 + m^2 - 2ml$

$= (l - m)^2$

$Using\ identity: (x - y)^2 = x^2 - 2xy + y^2$

(viii) $a^4 + 2a^2b^2 + b^4$

$= (a^2)^2 + 2 \times a^2 \times b^2 + (b^2)^2$

$= (a^2 + b^2)^2$

$Using\ identity: (x + y)^2 = x^2 + 2xy + y^2$

2. Factorise.

(i) $4p^2 – 9q^2$

(ii) $63a^2 – 112b^2$

(iii) $49x^2 – 36$

(iv) $16x^5 – 144x^3$ differ

(v) $(l + m)^2 - (l - m)^2$

(vi) $9x^2 y^2 – 16$

(vii) $(x^2 – 2xy + y^2) – z^2$

(viii) $25a^2 – 4b^2 + 28bc – 49c^2$

Solution:

(i) $4p^2 – 9q^2$

$= (2p)^2 - (3q)^2$

$= (2p - 3q)(2p + 3q)$

Using Identity: $x^2 - y^2 = (x + y)(x - y)$

(ii) $63a^2 – 112b^2$

$= 7(9a^2 – 16b^2)$
$= 7((3a)^2 – (4b)^2)$
$= 7(3a + 4b)(3a - 4b)$

Using Identity: $x^2 - y^2 = (x + y)(x - y)$

(iii) $49x^2 – 36$

$= (7a)^2 - 6^2)$

$= (7a + 6)(7a – 6)$

Using Identity: $x^2 - y^2 = (x + y)(x - y)$

(iv) $16x^5 – 144x^3$
$= 16x^3 (x^2 – 9)$

$= 16x^3 (x^2 – 9)$

$= 16x^3 (x – 3)(x + 3)$

Using Identity: $x^2 - y^2 = (x + y)(x - y)$

(v) $(l + m)^2 - (l - m)^2$

$= \{(l + m) - (l – m)\} \{(l + m) + (l – m)\}$

Using Identity: $x^2 - y^2 = (x + y)(x - y)$

$= (l + m – l + m)(l + m + l – m)$

$= (2m)(2l)$

$= 4 ml$

(vi) $9x^2 y^2 – 16$

$= (3xy)^2 - 4^2$

$= (3xy – 4)(3xy + 4)$

Using Identity: $x^2 - y^2 = (x + y)(x - y)$

(vii) $(x^2 - 2xy + y^2) - z^2$

$= (x - y)^2 - z^2$

Using Identity: $(x-y)^2 = x^2 - 2xy + y^2$

$= \{(x - y) - z\}\{(x - y) + z\}$

$= (x - y - z)(x - y + z)$

Using Identity: $x^2 - y^2 = (x+y)(x-y)$

(viii) $25a^2 - 4b^2 + 28bc - 49c^2$

$= 25a^2 - (4b^2 - 28bc + 49c^2)$

$= (5a)^2 - \{(2b)^2 - 2(2b)(7c) + (7c)^2\}$

$= (5a)^2 - (2b - 7c)^2$

Using Identity: $x^2 - y^2 = (x+y)(x-y), we\ have$

$= (5a + 2b - 7c)\ (5a - 2b - 7c)$

3. Factorise the expressions.
(i) $ax^2 + bx$
(ii) $7p^2 + 21q^2$
(iii) $2x^3 + 2xy^2 + 2xz^2$
(iv) $am^2 + bm^2 + bn^2 + an^2$
(v) $(lm + l) + m + 1$
(vi) $y (y + z) + 9 (y + z)$
(vii) $5y^2 - 20y - 8z + 2yz$
(viii) $10ab + 4a + 5b + 2$
(ix) $6xy - 4y + 6 - 9x$

Solution:

(i) $ax^2 + bx = x(ax + b)$

(ii) $7p^2 + 21q^2 = 7(p^2 + 3q^2)$

(iii) $2x^3 + 2xy^2 + 2xz^2 = 2x(x^2 + y^2 + z^2)$

(iv) $am^2 + bm^2 + bn^2 + an^2 = m^2(a+b) + n^2(a+b) = (a + b)(m^2 + n^2)$

(v) (lm + l) + m + 1 = lm + m + l + 1 = m(l + 1) + (l + 1) = (m + 1) (l + 1)

(vi) y (y + z) + 9 (y + z) = (y + 9) (y + z)

(vii) $5y^2$ – 20y – 8z + 2yz = 5y (y – 4) + 2z (y – 4) = (y – 4)(5y + 2z)

(viii) 10ab + 4a + 5b + 2 = 5b (2a +1) + 2 (2a + 1) = (2a + 1) (5b + 2)

(ix) 6xy – 4y + 6 – 9x = 6xy – 9x – 4y + 6 = 3x (2y – 3) – 2(2y – 3) = (2y – 3) (3x – 2)

4. Factorise.
(i) $a^4 - b^4$
(ii) $p^4 - 81$
(iii) $x^4 - (y + z)^4$
(iv) $x^4 - (x - z)^4$
(v) $a^4 - 2a^2b^2 + b^4$

Solution:
(i) $a^4 - b^4$

$= (a^2)^2 - (b^2)^2$
$= (a^2 - b^2)(a^2 + b^2)$
= (a – b)(a + b) $(a^2 + b^2)$

(ii) $p^4 - 81$

$= (p^2)^2 - (9)^2$
$= (p^2 - 9)(p^2 + 9)$
$= (p^2 - 3^2)(p^2 + 9)$
= (p - 3)(p + 3) $(p^2 + 9)$

(iii) $x^4 - (y + z)^4$ $= (x^2)^2 - [(y + z)^2]$^2

$= \{x^2 - (y + z)^2\}\{x^2 + (y + z)^2\}$

= {(x – (y + z) (x + (y + z)} $\{x^2 + (y + z)^2\}$

= (x – y – z)(x + y + z) $\{x^2 + (y + z)^2\}$

(iv) $x^4 - (x - z)^4 = (x^2)^2 - \{(x - 2)^2\}^2$

$= \{x^2 - (x - z)^2\}\{x^2 + (x - z)^2\}$

$= \{x - (x - z)\}\{x + (x - z)\}\{x^2 + (x - z)^2\}$

$= z(2x - z)(x^2 + x^2 - 2xz + z^2)$

$= z(2x - z)(2x^2 - 2xz + z^2)$

(v) $a^4 - 2a^2b^2 + b^4 = (a^2)^2 - 2a^2b^2 + (b^2)^2$

$= (a^2 - b^2)^2$

$= ((a - b)(a+b))^2$

5. Factorise the following expressions.
(i) $p^2 + 6p + 8$
(ii) $q^2 - 10q + 21$
(iii) $p^2 + 6p - 16$

Solution:

(i) $p^2 + 6p + 8$

We observed that, 8 = 4 x 2 and 4 + 2 = 6

$p^2 + 6p + 8$ can be written as $p^2 + 2p + 4p + 8$
Taking Common terms, we get

$p^2 + 6p + 8 = p^2 + 2p + 4p + 8 = p (p + 2) + 4 (p + 2)$

Again p + 2 is common in both the terms.

= (p + 2) (p + 4)

This implies: $p^2 + 6p + 8 = (p + 2) (p + 4)$

(ii) $q^2 - 10q + 21$
Observed that, 21 = -7 x -3 and -7 + (-3) = -10

$q^2 - 10q + 21 = q^2 - 3q - 7q + 21$

= q (q – 3) – 7 (q – 3)

= (q – 7) (q – 3)

This implies $q^2 - 10q + 21 = (q - 7) (q - 3)$

(iii) $p^2 + 6p – 16$

We observed that, 16 = -2 x 8 and 8 + (-2) = 6

$p^2 + 6p – 16 = p^2 – 2p + 8p – 16$

$= p(p – 2) + 8(p – 2)$

$= (p + 8)(p – 2)$

So, $p^2 + 6p – 16 = (p + 8)(p – 2)$