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1. Carry out the following divisions.
(i) 28x⁴ ÷ 56x
(ii) –36y³ ÷ 9y²
(iii) 66pq²r³ ÷ 11qr²
(iv) 34x³y³z³ ÷ 51xy²z³
(v) 12a⁸b⁸ ÷ (– 6a⁶b⁴)

Solution:

(i)
$28x^4 = 2 \times 2 \times 7 \times x \times x \times x \times x \times x$
$56x = 2 \times 2 \times 2 \times 7 \times x$

$28x^4 \div 56x = \dfrac{2 \times 2 \times 7 \times x \times x \times x \times x \times x}{2 \times 2 \times 2 \times 7 \times x} = \dfrac{x^3}{2} = \dfrac{1}{2}x^3$

(ii)

$-36y^3 \div 9y^2 = \dfrac{-2 \times 2 \times 3 \times 3 \times y \times y \times y}{3 \times 3 \times y \times y} = $ -4y

(iii)

$66pq^2r^3 \div 11qr^2 = \dfrac{2 \times 3 \times 11 \times p \times q \times q \times r \times r \times r}{11 \times q \times r \times r} = $ 6pqr

(iv)

$34x^3y^3z^3 \div 51xy^2z^3 = \dfrac{2 \times 17 \times x \times x \times x \times y \times y \times y \times z \times z \times z}{3 \times 17 \times x \times y \times y \times z \times z \times z} = $ 2/3 x² y

(v)
$12a^8b^8 \div (-6a^6b^4) = \dfrac{2 \times 2 \times 3 \times a^8 \times b^8}{-2 \times 3 \times a^6 \times b^4} = $ -2 a² b⁴

2. Divide the given polynomial by the given monomial.

(i)(5x² – 6x) ÷ 3x

(ii)(3y⁸ – 4y⁶ + 5y⁴) ÷ y⁴

(iii) 8(x³ y² z² + x² y³ z² + x² y² z³) ÷ 4x² y² z²

(iv)(x³ + 2x² + 3x) ÷ 2x

(v) (p³q⁶ – p⁶q³) ÷ p³ q³

Solution:

(i) $5x^2 - 6x = x(5x - 6)$

$(5x^2 - 6x) \div 3x = \dfrac{x(5x - 6)}{3x} = \dfrac{1}{3}(5x - 6)$

(ii) $3y^8 - 4y^6 + 5y^4 = y^4(3y^4 - 4y^2 + 5)$

$(3y^8 - 4y^6 + 5y^4) \div y^4 = \dfrac{y^4(3y^4 - 4y^2 + 5)}{y^4} = 3y^4 - 4y^2 + 5$

(iii) $8(x^3 y^2 z^2 + x^2 y^3 z^2 + x^2 y^2 z^3) = 8x^2 y^2 z^2 (x + y + z)$

$8(x^3 y^2 z^2 + x^2 y^3 z^2 + x^2 y^2 z^3) \div 4x^2 y^2 z^2 = \dfrac{8x^2 y^2 z^2 (x + y + z)}{4x^2 y^2 z^2} = 2(x + y + z)$

(iv) $x^3 + 2x^2 + 3x = x(x^2 + 2x + 3)$

$(x^3 + 2x^2 + 3x) \div 2x = \dfrac{x(x^3 + 2x^2 + 3)}{2x} = \dfrac{1}{2}(x^2 + 2x + 3)$

(v) $p^3 q^6 - p^6 q^3 = p^3 q^3 (q^3 - p^3)$

$(p^3 q^6 - p^6 q^3) \div p^3 q^3 = \dfrac{p^3 q^3 (q^3 - p^3)}{p^3 q^3} = q^3 - p^3$

3. Work out the following divisions.

(i) (10x – 25) ÷ 5

(ii) (10x – 25) ÷ (2x – 5)

(iii) 10y(6y + 21) ÷ 5(2y + 7)

(iv) $9x^2y^2(3z - 24) ÷ 27xy(z - 8)$

(v) $96abc(3a - 12) (5b - 30) ÷ 144(a - 4) (b - 6)$

Solution:

(i) $(10x - 25) ÷ 5 = \frac{5(2x-5)}{5} = 2x - 5$

(ii) $(10x - 25) ÷ (2x - 5) = \frac{5(2x-5)}{2x-5} = 5$

(iii) $10y(6y + 21) ÷ 5(2y + 7) = \frac{10y \times 3(2y+7)}{5(2y+7)} = 6y$

(iv) $9x^2y^2(3z - 24) ÷ 27xy(z - 8) = \frac{9x^2y^2 \times 3(z-8)}{27xy(z-8)} = xy$

(v) $96 \, abc(3a - 12) (5b - 30) ÷ 144(a - 4) (b - 6) = \frac{96 \, abc \times 3(a-4) \times 5(b-6)}{144(a-4)(b-6)} = 10abc$

4. Divide as directed.

(i) $5(2x + 1) (3x + 5) ÷ (2x + 1)$

(ii) $26xy(x + 5)(y - 4) ÷ 13x(y - 4)$

(iii) $52pqr (p + q) (q + r) (r + p) ÷ 104pq(q + r) (r + p)$

(iv) $20(y + 4) (y2 + 5y + 3) ÷ 5(y + 4)$

(v) $x(x + 1) (x + 2) (x + 3) ÷ x(x + 1)$

Solution:

(i) $5(2x + 1)(3x + 5) \div (2x + 1) = \dfrac{5(2x + 1)(3x + 1)}{(2x + 1)}$

$= 5(3x + 1)$

(ii) $26\,xy\,(x + 5)(y - 4) \div 13\,x\,(y - 4) = \dfrac{2 \times 13 \times xy\,(x + 5)(y - 4}{13\,x(y - 4)}$

$= 2\,y(x + 5)$

(iii) $52\,pqr\,(p + q)(q + r)(r + p) \div 104\,pq\,(q + r)(r + p)$

$= \dfrac{2 \times 2 \times 13 \times p \times q \times r \times (p + q) \times (q + r) \times (r + p)}{2 \times 2 \times 2 \times 13 \times p \times q \times (q + r) \times (r + p)}$

$= \dfrac{1}{2} r(p + q)$

(iv) $20\,(y + 4)(y^2 + 5y + 3) = 2 \times 2 \times 5 \times (y + 4)(y^2 + 5y + 3)$

$20\,(y + 4)(y^2 + 5y + 3) \div 5\,(y + 4) = \dfrac{2 \times 2 \times 5 \times (y + 4) \times (y^2 + 5y + 3)}{5 \times (y + 4)}$

$= 4(y^2 + 5y + 3)$

(v) $x\,(x + 1)(x + 2)(x + 3) \div x\,(x + 1) = \dfrac{x(x + 1)(x + 2)(x + 3)}{x(x + 1)}$

$= (x + 2)(x + 3)$

5. Factorise the expressions and divide them as directed.

(i) $(y^2 + 7y + 10) \div (y + 5)$

(ii) $(m^2 - 14m - 32) \div (m + 2)$

(iii) $(5p^2 - 25p + 20) \div (p - 1)$

(iv) $4yz(z^2 + 6z - 16) \div 2y(z + 8)$

(v) $5pq(p^2 - q^2) \div 2p(p + q)$

(vi) $12xy(9x^2 - 16y^2) \div 4xy(3x + 4y)$

(vii) $39y^3(50y^2 - 98) \div 26y^2(5y + 7)$

Solution:

(i) **$(y^2 + 7y + 10) ÷ (y + 5)$**

First solve for equation, $(y^2 + 7y + 10)$
$(y^2 + 7y + 10) = y^2 + 2y + 5y + 10 = y (y + 2) + 5(y + 2) = (y + 2)(y + 5)$

Now, $(y^2 + 7y + 10) ÷ (y + 5) = (y + 2)(y + 5) / (y + 5) = y + 2$

(ii) **$(m^2 – 14m – 32) ÷ (m + 2)$**

Solve for $m^2 – 14m – 32$, we have

$m^2 – 14m – 32 = m^2 + 2m - 16m – 32 = m(m + 2) – 16(m + 2) = (m – 16)(m + 2)$

Now, $(m^2 – 14m – 32) ÷ (m + 2) = (m – 16)(m + 2) /)(m + 2) = m - 16$

(iii) **$(5p^2 – 25p + 20) ÷ (p – 1)$**

Step 1: Take 5 common from the equation, $5p^2 – 25p + 20$, we get

$5p^2 – 25p + 20 = 5(p^2 – 5p + 4)$

Step 2: Factorize $p^2 – 5p + 4$

$p^2 – 5p + 4 = p^2 – p - 4p + 4 = (p – 1) (p – 4)$

Step 3: Solve original equation

$(5p^2 – 25p + 20) ÷ (p – 1) = 5(p – 1) (p – 4) / (p-1) = 5(p – 4)$

(iv) **$4yz(z^2 + 6z – 16) ÷ 2y(z + 8)$**

Factorize $z^2 + 6z – 16$,

$z^2 + 6z – 16 = z^2 - 2z + 8z – 16 = (z – 2)(z + 8)$

Now, $4yz(z^2 + 6z – 16) ÷ 2y(z + 8) = 4yz(z – 2)(z + 8) / 2y(z + 8 = 2z(z - 2)$

(v)   **5pq(p² – q²) ÷ 2p(p + q)**

$p^2 - q^2$ can be written as $(p - q)(p + q)$ using identity.

$5pq(p^2 - q^2) \div 2p(p + q) = 5pq(p - q)(p + q) / 2p(p + q) = 5/2\ q\ (p - q)$

(vi)   12xy(9x² – 16y²) ÷ 4xy(3x + 4y)

Factorize $9x^2 - 16y^2$, we have

$9x^2 - 16y^2 = (3x)^2 - (4y)^2 = (3x + 4y)\ (3x - 4y)$ using identity: $p^2 - q^2 = (p - q)(p + q)$

Now, $12xy(9x^2 - 16y^2) \div 4xy(3x + 4y) = 12xy(3x + 4y)\ (3x - 4y) / 4xy(3x + 4y) = 3\ (3x - 4y)$

(vii)   39y³(50y² – 98) ÷ 26y²(5y + 7)

First solve for $50y^2 - 98$, we have

$50y^2 - 98 = 2(25y^2 - 49) = 2((5y)^2 - 7^2) = 2(5y - 7)\ (5y + 7)$

Now, $39y^3(50y^2 - 98) \div 26y^2(5y + 7) = \dfrac{3 \times 13 \times y^3 \times 2(5y - 7)\ (5y + 7)}{2 \times 13 \times y^2 (5y + 7)} = 3y(5y - 7)$