Exercise 2.4 Page: 36

1. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3 + x^2 - 5x + 2; \frac{1}{2}, 1, -2$

Solutions: Given, p(x) = $2x^3 + x^2 - 5x + 2$
And zeroes for p(x) are = $\frac{1}{2}, 1, -2$
∴ $p(1/2) = 2(\frac{1}{2})^3 + (\frac{1}{2})^2 - 5(1/2) + 2 = ¼ + ¼ - 5/2 + 2 = 0$

$p(1) = 2.1^3 + 1^2 - 5.1 + 2 = 0$

$p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2 = 0$

Hence, proved $\frac{1}{2}, 1, -2$ are the zeroes of $2x^3 + x^2 - 5x + 2$.

Now, comparing the given polynomial with general expression, we get;

∴ $ax^3 + bx^2 + cx + d = 2x^3 + x^2 - 5x + 2$
a=3, b=1, c= -5 and d = 2

As we know, if α, β, γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$, then;

$\alpha + \beta + \gamma = -b/a$

$\alpha\beta + \beta\gamma + \gamma\alpha = c/a$

$\alpha \beta \gamma = -d/a$.

Therefore, putting the values of zeroes of the polynomial,

$\alpha + \beta + \gamma = ½+1+(-2) = -1/2 = -b/a$

$\alpha\beta + \beta\gamma + \gamma\alpha = (1/2 \times 1) + (1 \times -2) + (-2 \times 1/2) = -5/2 = c/a$

$\alpha \beta \gamma = ½ \times 1 \times (-2) = -2/2 = -d/a$

Hence, the relationship between the zeroes and the coefficients are satisfied.

(ii) $x^3 - 4x^2 + 5x + 2; 2, 1, 1$

Solutions: Given, p(x) = $x^3 - 4x^2 + 5x + 2$
And zeroes for p(x) are 2, 1, 1.

∴ p(2)= $2^3 - 4.2^2 + 5.2 + 2 = 0$

p(1) = $1^3 - 4.1^2 + 5.1 + 2 = 0$

Hence proved, 2, 1, 1 are the zeroes of $x^3 - 4x^2 + 5x + 2$.

Now, comparing the given polynomial with general expression, we get;

∴ $ax^3 + bx^2 + cx + d$ = $\mathbf{x^3 - 4x^2 + 5x + 2}$

a=1, b = -4, c = 5 and d = 2

As we know, if α, β, γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$, then;

α + β + γ = –b/a
αβ + βγ + γα = c/a
α β γ = – d/a.

Therefore, putting the values of zeroes of the polynomial,

α + β + γ = 2+1+1 = 4 = -(-4)/1 = –b/a

αβ + βγ + γα = 2.1+1.1+1.2 = 5 = 5/1= c/a

α β γ = 2 × 1 × 1 = 2 = -(-2)/1 = -d/a

Hence, the relationship between the zeroes and the coefficients are satisfied.

2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, –7, –14 respectively.

Solutions: Let us consider the cubic polynomial is $ax^3 + bx^2 + cx + d$ and the values of the zeroes of the polynomials be α, β, γ.

As per the given question,

α + β + γ = -b/a = 2/1

αβ + βγ + γα = c/a = -7/1

α β γ = -d/a = -14/1

Thus, from above three expressions we get the values of coefficient of polynomial.
a = 1, b = -2, c = -7, d = 14

Hence, the cubic polynomial is $x^3 - 2x^2 - 7x + 14$.

3. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are a – b, a, a + b, find a and b.

Solutions: We are given with the polynomial here,
p(x) = $x^3 - 3x^2 + x + 1$

And zeroes are given as a – b, a, a + b

Now, comparing the given polynomial with general expression, we get;

∴ $px^3 + qx^2 + rx + s = x^3 - 3x^2 + x + 1$

p = 1, q = -3, r = 1 and s = 1

Sum of zeroes = a – b + a + a + b

-q/p = 3a

Putting the values q and p.

-(-3)/1 = 3a

a=1

Thus, the zeroes are 1-b, 1, 1+b.

Now, product of zeroes = 1(1-b)(1+b)

-s/p=1-b^2

-1/1=1-b^2

$b^2 = 1 + 1$ =2

b=$\sqrt{2}$

Hence, 1-$\sqrt{2}$, 1, 1+$\sqrt{2}$ *are the zeroes of* $x^3 - 3x^2 + x + 1$.

4. If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are 2 ± $\sqrt{3}$, find other zeroes.

Solutions: Since this is a polynomial equation of degree 4, hence there will be total 4 roots.

Let f(x) = $x^4 - 6x^3 - 26x^2 + 138x - 35$

Since 2 + $\sqrt{3}$ and 2 - $\sqrt{3}$ are zeroes of given polynomial f(x).

∴ [x−(2 + $\sqrt{3}$)] [x−2 - $\sqrt{3}$] = 0

$(x-2-\sqrt{3})(x-2+\sqrt{3}) = 0$

On multiplying the above equation we get,

$x^2 - 4x + 1$, this is a factor of a given polynomial f(x).

Now, if we will divide f(x) by g(x), the quotient will also be a factor of f(x) and the remainder will be 0.

$$
\begin{array}{r|l}
 & x^2 - 2x - 35 \\
\hline
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 - 4x^3 + x^2 \\
 & (-)\ (+)\ (-) \\
\hline
 & -2x^3 - 27x^2 + 138x - 35 \\
 & -2x^3 + 8x^2 - 2x \\
 & (+)\ (-)\ (+) \\
\hline
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & (+)\ (-)\ (+) \\
\hline
 & 0
\end{array}
$$

So, $x^4 - 6x^3 - 26x^2 + 138x - 35 = (x^2 - 4x + 1)(x^2 - 2x - 35)$

Now, on further factorizing $(x^2 - 2x - 35)$ we get,

$x^2 - (7-5)x - 35 = x^2 - 7x + 5x + 35 = 0$

$x(x - 7) + 5(x-7) = 0$

$(x+5)(x-7) = 0$

So, its zeroes are given by:

$x = -5$ and $x = 7$.

Therefore, all four zeroes of given polynomial equation are: $2 + \sqrt{3}$, $2 - \sqrt{3}$, -5 and 7.