Exercise7.1

1. Find the distance between the following pairs of points:
(i) (2, 3), (4, 1)
(ii) (-5, 7), (-1, 3)
(iii) (a, b), (- a, - b)

Solution:

Distance formula to find the distance between two points (x_1, y_1) and (x_2, y_2) is, say d,

d = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ OR d = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

(i) Distance between (2, 3), (4, 1)

d = $\sqrt{(4-2)^2 + (1-3)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$

(ii) Distance between (-5, 7), (-1, 3)

d = $\sqrt{(-1+5)^2 + (3-7)^2} = \sqrt{(4)^2 + (-4)^2} = \sqrt{32} = 4\sqrt{2}$

(iii) Distance between (a, b), (- a, - b)

d = $\sqrt{(-a-a)^2 + (-b-b)^2} = \sqrt{(-2a)^2 + (-2b)^2} = \sqrt{4a^2 + 4b^2} = 2\sqrt{a^2 + b^2}$

2. Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed in Section 7.2.

Solution: Let us consider, town A at point (0, 0). Therefore, town B will be at point (36, 15).

Distance between points (0, 0) and (36, 15)

d = $\sqrt{(36-0)^2 + (15-0)^2} = \sqrt{(36)^2 + (15)^2} = \sqrt{1296 + 225} = \sqrt{1521} = 39$

The distance between town A and B will be 39 km.

3. Determine if the points (1, 5), (2, 3) and (-2, -11) are collinear.

Solution: The sum of the lengths of any two line segments is equal to the length of the third line segment then all three points are collinear.

Consider, A = (1, 5) B = (2, 3) and C = (-2, -11)

Find the distance between points; say AB, BC and CA

AB = $\sqrt{(2-1)^2 + (3-5)^2} = \sqrt{(1)^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$

BC = $\sqrt{(-2-2)^2 + (-11-3)^2} = \sqrt{(-4)^2 + (-14)^2} = \sqrt{16+196} = \sqrt{212}$

CA = $\sqrt{(-2-1)^2 + (-11-5)^2} = \sqrt{(-3)^2 + (-16)^2} = \sqrt{9+256} = \sqrt{265}$

Since AB + BC ≠ CA
Therefore, the points
(1, 5), (2, 3), and (- 2, - 11) are not collinear.

4. Check whether (5, - 2), (6, 4) and (7, - 2) are the vertices of an isosceles triangle.
Solution:
Since two sides of any isosceles triangle are equal. To check whether given points are vertices of an isosceles triangle, we will find the distance between all the points.
Let the points (5, - 2), (6, 4), and (7, - 2) are representing the vertices A, B, and C respectively.

$$AB = \sqrt{(6-5)^2 + (4+2)^2} = \sqrt{(-1)^2 + (6)^2} = \sqrt{37}$$

$$BC = \sqrt{(7-6)^2 + (-2-4)^2} = \sqrt{(-1)^2 + (6)^2} = \sqrt{37}$$

$$CA = = \sqrt{(7-5)^2 + (-2+2)^2} = \sqrt{(-2)^2 + (0)^2} = 2$$

Here AB = BC = $\sqrt{37}$
This implies, whether given points are vertices of an isosceles triangle.

5. In a classroom, 4 friends are seated at the points A, B, C and D as shown in Fig. 7.8. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.

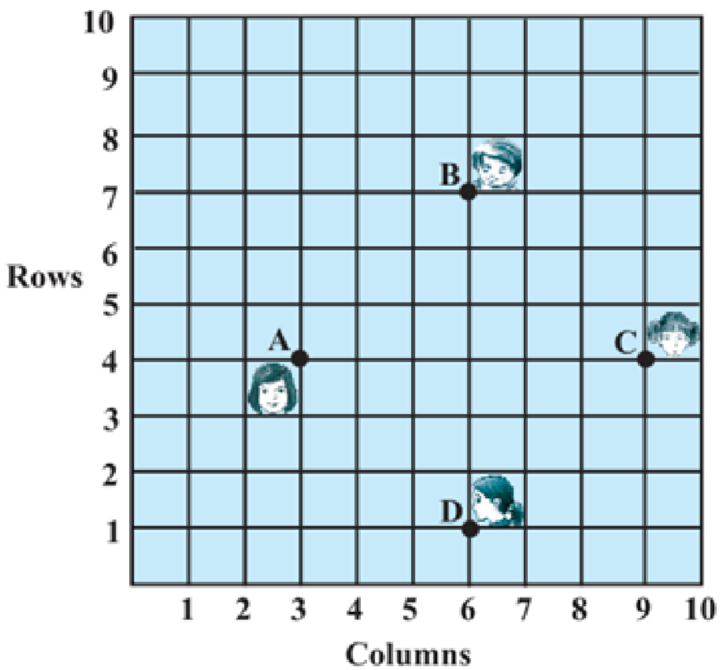

Fig. 7.8

Solution:

From figure, the coordinates of points A, B, C and D are (3, 4), (6, 7), (9, 4) and (6,1).
Find distance between points using distance formula, we get

$AB = \sqrt{(6-3)^2 + (7-4)^2} = \sqrt{9+9} = 3\sqrt{2}$

$BC = \sqrt{(9-6)^2 + (4-7)^2} = \sqrt{9+9} = 3\sqrt{2}$

$CD = \sqrt{(6-9)^2 + (1-4)^2} = \sqrt{9+9} = 3\sqrt{2}$

$DA = \sqrt{(6-3)^2 + (1-4)^2} = \sqrt{9+9} = 3\sqrt{2}$

AB = BC = CD = DA = $3\sqrt{2}$

All sides are of equal length. Therefore, ABCD is a square and hence, Champa was correct.

6. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

(i) (- 1, - 2), (1, 0), (- 1, 2), (- 3, 0)
(ii) (- 3, 5), (3, 1), (0, 3), (- 1, - 4)
(iii) (4, 5), (7, 6), (4, 3), (1, 2)

Solution:

(i) Let the points (- 1, - 2), (1, 0), (- 1, 2), and (- 3, 0) be representing the vertices A, B, C, and D of the given quadrilateral respectively.
$AB = \sqrt{(1+1)^2 + (0+2)^2} = \sqrt{4+4} = 2\sqrt{2}$
$BC = \sqrt{(-1-1)^2 + (2-0)^2} = \sqrt{4+4} = 2\sqrt{2}$
$CD = \sqrt{(-3+1)^2 + (0-2)^2} = \sqrt{4+4} = 2\sqrt{2}$
$DA = \sqrt{(-3+1)^2 + (0-2)^2} = \sqrt{4+4} = 2\sqrt{2}$
$AC = \sqrt{(-1+1)^2 + (2+2)^2} = \sqrt{0+16} = 4$
$BD = \sqrt{(-3-1)^2 + (0-0)^2} = \sqrt{16+0} = 4$

Side length = AB = BC = CD = DA = $2\sqrt{2}$
Diagonal Measure = AC = BD = 4
Therefore, the given points are the vertices of a square.

(ii) Let the points (- 3, 5), (3, 1), (0, 3), and (- 1, - 4) be representing the vertices A, B, C, and D of the given quadrilateral respectively.

AB = $\sqrt{(-3-3)^2 + (1-5)^2}$ = $\sqrt{36+16}$ = $2\sqrt{13}$
BC = $\sqrt{(0-3)^2 + (3-1)^2}$ = $\sqrt{9+4}$ = $\sqrt{13}$
CD = $\sqrt{(-1-0)^2 + (-4-3)^2}$ = $\sqrt{1+49}$ = $5\sqrt{2}$
AD = $\sqrt{(-1+3)^2 + (-4-5)^2}$ = $\sqrt{4+81}$ = $\sqrt{85}$

All sides of this quadrilateral are of different lengths. Therefore, it can be said that it is a general quadrilateral.

(iii) Let the points (4, 5), (7, 6), (4, 3), and (1, 2) be representing the vertices A, B, C, and D of the given quadrilateral respectively.

AB = $\sqrt{(7-4)^2 + (6-5)^2}$ = $\sqrt{9+1}$ = $\sqrt{10}$
BC = $\sqrt{(4-7)^2 + (3-6)^2}$ = $\sqrt{9+9}$ = $\sqrt{18}$
CD = $\sqrt{(1-4)^2 + (2-3)^2}$ = $\sqrt{9+1}$ = $\sqrt{10}$
AD = $\sqrt{(1-4)^2 + (2-5)^2}$ = $\sqrt{9+9}$ = $\sqrt{18}$
AC (diagonal) = $\sqrt{(4-4)^2 + (3-5)^2}$ = $\sqrt{0+4}$ = 2
BD (diagonal) = $\sqrt{(1-7)^2 + (2-6)^2}$ = $\sqrt{36+16}$ = $2\sqrt{13}$

Opposite sides of this quadrilateral are of the same length. However, the diagonals are of different lengths. Therefore, the given points are the vertices of a parallelogram.

7. Find the point on the x-axis which is equidistant from (2, - 5) and (- 2, 9).

Solution:

To find a point on x-axis. Therefore, its y-coordinate will be 0. Let the point on x-axis be (x,0).
Consider A = (x, 0); B = (2, - 5) and C = (- 2, 9).
AB = $\sqrt{(2-x)^2 + (-5-0)^2}$ = $\sqrt{(2-x)^2 + 25}$
AC = $\sqrt{(-2-x)^2 + (9-0)^2}$ = $\sqrt{(-2-x)^2 + 81}$

Since both the distance are equal in measure, so AB = AC

$$\sqrt{(2-x)^2 + 25} = \sqrt{(-2-x)^2 + 81}$$

Simply the above equation,
Remove square root by taking square both the sides, we get
$(2-x)^2 + 25 = (-2-x)^2 + 81$

$x^2 + 4 - 4x + 25 = x^2 + 4 + 4x + 81$
$8x = 25 - 81 = -56$
$x = -7$
Therefore, the point is (- 7, 0).

8. Find the values of y for which the distance between the points P (2, - 3) and Q (10, y) is 10 units.

Solution:
Given: Distance between (2, - 3) and (10, y) is 10.
Using distance formula,

$PQ = \sqrt{(10-2)^2 + (y+3)^2} = \sqrt{(8)^2 + (y+3)^2}$
Since PQ = 10
$\sqrt{(8)^2 + (y+3)^2} = 10$
Simplify the above equation and find the value of y.
Squaring both sides,
$64 + (y + 3)^2 = 100$
$(y + 3)^2 = 36$
$y + 3 = \pm 6$
$y + 3 = +6$ or $y + 3 = -6$
Therefore, y = 3 or -9.

9. If Q (0, 1) is equidistant from P (5, - 3) and R (x, 6), find the values of x. Also find the distance QR and PR.
Solution:
Given: Q (0, 1) is equidistant from P (5, - 3) and R (x, 6), which means PQ = QR
Step 1: Find the distance between PQ and QR using distance formula,
PQ = $\sqrt{(5-0)^2 + (-3-1)^2} = \sqrt{(-5)^2 + (-4)^2} = \sqrt{25+16} = \sqrt{41}$
QR = $\sqrt{(0-x)^2 + (1-6)^2} = \sqrt{(-x)^2 + (-5)^2} = \sqrt{x^2+25}$
Step 2: Use PQ = QR
$\sqrt{41} = \sqrt{x^2+25}$
Squaring both the sides, to omit square root
$41 = x^2 + 25$
$x^2 = 16$
$x = \pm 4$
x = 4 or x = -4

Coordinates of Point R will be R (4, 6) OR R (-4, 6),
If R (4, 6), then QR
QR = $\sqrt{(0-4)^2 + (1-6)^2}$ = $\sqrt{(4)^2 + (-5)^2}$ = $\sqrt{16+25}$ = $\sqrt{41}$
PR = $\sqrt{(5-4)^2 + (-3-6)^2}$ = $\sqrt{(1)^2 + (9)^2}$ = $\sqrt{1+81}$ = $\sqrt{82}$
If R (-4, 6), then
QR = $\sqrt{(0+4)^2 + (1-6)^2}$ = $\sqrt{(4)^2 + (-5)^2}$ = $\sqrt{16+25}$ = $\sqrt{41}$
PR = $\sqrt{(5+4)^2 + (-3-6)^2}$ = $\sqrt{(9)^2 + (9)^2}$ = $\sqrt{81+81}$ = $9\sqrt{2}$

10. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (- 3, 4).

Solution:
Point (x, y) is equidistant from (3, 6) and (- 3, 4).
$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x-(-3))^2 + (y-4)^2}$
$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x+3)^2 + (y-4)^2}$
Squaring both sides, $(x - 3)^2 + (y - 6)^2 = (x + 3)^2 + (y - 4)^2$
$x^2 + 9 - 6x + y^2 + 36 - 12y = x^2 + 9 + 6x + y^2 + 16 - 8y$
36 - 16 = 6x + 6x + 12y - 8y
20 = 12x + 4y
3x + y = 5
3x + y - 5 = 0