

Exercise: 12.3

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1. Find the area of the shaded region in Fig. 12.19, if $PQ = 24$ cm, $PR = 7$ cm and O is the centre of the circle.

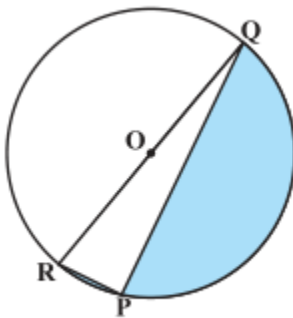


Fig. 12.19

Solution:

Here, $\angle P$ is in the semi-circle and so,

$$\angle P = 90^\circ$$

So, it can be concluded that QR is hypotenuse of the circle and is equal to the diameter of the circle.

$$\therefore QR = D$$

Using Pythagorean theorem,

$$QR^2 = PR^2 + PQ^2$$

$$\text{Or, } QR^2 = 7^2 + 24^2$$

$$\Rightarrow QR = 25 \text{ cm} = \text{Diameter}$$

Hence, the radius of the circle = $25/2$ cm

Now, the area of the semicircle = $(\pi R^2)/2$

$$= (22/7 \times 25/2 \times 25/2)/2 \text{ cm}^2$$

$$= 13750/56 \text{ cm}^2 = 245.54 \text{ cm}^2$$

Also, area of the $\Delta PQR = \frac{1}{2} \times PR \times PQ$

$$= \frac{1}{2} \times 7 \times 24 \text{ cm}^2$$

$$= 84 \text{ cm}^2$$

Hence, the area of the shaded region = $245.54 \text{ cm}^2 - 84 \text{ cm}^2$

$$= 161.54 \text{ cm}^2$$

2. Find the area of the shaded region in Fig. 12.20, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and $\angle AOC = 40^\circ$.

Solution:

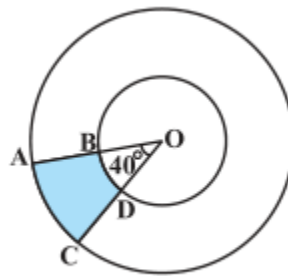


Fig. 12.20

Given,

Angle made by sector = 40° ,

Radius the inner circle = $r = 7 \text{ cm}$, and

Radius of the outer circle = $R = 14 \text{ cm}$

We know,

$$\text{Area of the sector} = \left(\frac{\theta}{360^\circ}\right) \times \pi r^2$$

$$\text{So, Area of OAC} = \left(\frac{40^\circ}{360^\circ}\right) \times \pi r^2 \text{ cm}^2$$

$$= 68.44 \text{ cm}^2$$

$$\text{Area of the sector OBD} = (40^\circ/360^\circ) \times \pi r^2 \text{ cm}^2$$

$$= 1/9 \times 22/7 \times 7^2 = 17.11 \text{ cm}^2$$

Now, area of the shaded region ABDC = Area of OAC - Area of the OBD

$$= 68.44 \text{ cm}^2 - 17.11 \text{ cm}^2 = 51.33 \text{ cm}^2$$

3. Find the area of the shaded region in Fig. 12.21, if ABCD is a square of side 14 cm and APD and BPC are semicircles.

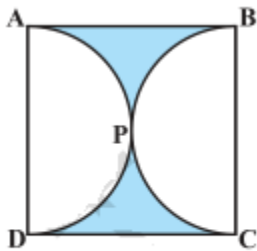


Fig. 12.21

Solution:

Side of the square ABCD (as given) = 14 cm

So, Area of ABCD = a^2

$$= 14 \times 14 \text{ cm}^2 = 196 \text{ cm}^2$$

We know that the side of the square = diameter of the circle = 14 cm

So, side of the square = diameter of the semicircle = 14 cm

\therefore Radius of the semicircle = 7 cm

Now, area of the semicircle = $(\pi R^2)/2$

$$= (22/7 \times 7 \times 7)/2 \text{ cm}^2 =$$

$$= 77 \text{ cm}^2$$

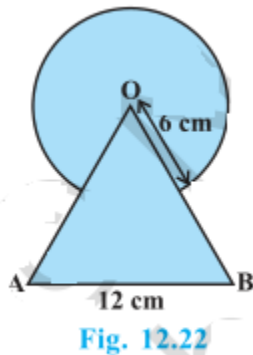
$$\therefore \text{Area of two semicircles} = 2 \times 77 \text{ cm}^2 = 154 \text{ cm}^2$$

Hence, area of the shaded region = Area of the Square - Area of two semicircle

$$= 196 \text{ cm}^2 - 154 \text{ cm}^2$$

$$= 42 \text{ cm}^2$$

4. Find the area of the shaded region in Fig. 12.22, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.



Solution:

It is given that OAB is an equilateral triangle having each angle as 60°

Area of the sector is common in both.

Radius of the circle = 6 cm.

Side of the triangle = 12 cm.

$$\text{Area of the equilateral triangle} = \frac{\sqrt{3}}{4} \times (OA)^2 = \frac{\sqrt{3}}{4} \times 12^2 = 36\sqrt{3} \text{ cm}^2$$

$$\text{Area of the circle} = \pi R^2 = \frac{22}{7} \times 6^2 = \frac{792}{7} \text{ cm}^2$$

$$\text{Area of the sector making angle } 60^\circ = \left(\frac{60^\circ}{360^\circ}\right) \times \pi r^2 \text{ cm}^2$$

$$= \frac{1}{6} \times \frac{22}{7} \times 6^2 \text{ cm}^2 = \frac{132}{7} \text{ cm}^2$$

Area of the shaded region = Area of the equilateral triangle + Area of the circle - Area of the sector

$$= 36\sqrt{3} \text{ cm}^2 + 792/7 \text{ cm}^2 - 132/7 \text{ cm}^2$$

$$= (36\sqrt{3} + 660/7) \text{ cm}^2$$

5. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in Fig. 12.23. Find the area of the remaining portion of the square.

Solution:

Side of the square = 4 cm

Radius of the circle = 1 cm

Four quadrant of a circle are cut from corner and one circle of radius are cut from middle.

$$\text{Area of square} = (\text{side})^2 = 4^2 = 16 \text{ cm}^2$$

$$\text{Area of the quadrant} = (\pi R^2)/4 \text{ cm}^2 = (22/7 \times 1^2)/4 = 11/14 \text{ cm}^2$$

$$\therefore \text{Total area of the 4 quadrants} = 4 \times (11/14) \text{ cm}^2 = 22/7 \text{ cm}^2$$

$$\text{Area of the circle} = \pi R^2 \text{ cm}^2 = (22/7 \times 1^2) = 22/7 \text{ cm}^2$$

$$\text{Area of the shaded region} = \text{Area of square} - (\text{Area of the 4 quadrants} + \text{Area of the circle})$$

$$= 16 \text{ cm}^2 - (22/7 + 22/7) \text{ cm}^2$$

$$= 68/7 \text{ cm}^2$$

6. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in Fig. 12.24. Find the area of the design.



Fig. 12.24

Solution:

Radius of the circle = 32 cm

Draw a median AD of the triangle passing through the centre of the circle.

$$\Rightarrow BD = AB/2$$

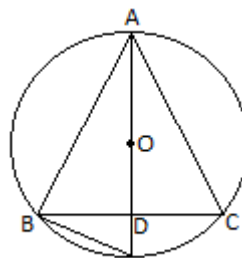
Since, AD is the median of the triangle

$$\therefore AO = \text{Radius of the circle} = \frac{2}{3} AD$$

$$\Rightarrow \frac{2}{3} AD = 32 \text{ cm}$$

$$\Rightarrow AD = 48 \text{ cm}$$

In $\triangle ADB$,



By Pythagoras theorem,

$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow AB^2 = 48^2 + (AB/2)^2$$

$$\Rightarrow AB^2 = 2304 + AB^2/4$$

$$\Rightarrow \frac{3}{4} (AB^2) = 2304$$

$$\Rightarrow AB^2 = 3072$$

$$\Rightarrow AB = 32\sqrt{3} \text{ cm}$$

$$\text{Area of } \triangle ADB = \frac{\sqrt{3}}{4} \times (32\sqrt{3})^2 \text{ cm}^2 = 768\sqrt{3} \text{ cm}^2$$

$$\text{Area of circle} = \pi R^2 = \frac{22}{7} \times 32 \times 32 = \frac{22528}{7} \text{ cm}^2$$

Area of the design = Area of circle - Area of $\triangle ADB$

$$= \left(\frac{22528}{7} - 768\sqrt{3} \right) \text{ cm}^2$$

7. In Fig. 12.25, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.

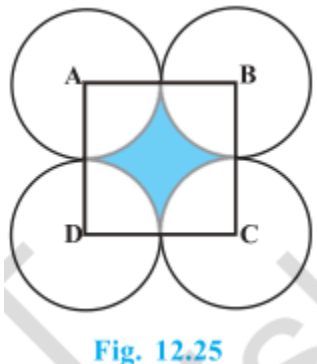


Fig. 12.25

Solution:

Side of square = 14 cm

Four quadrants are included in the four sides of the square.

$$\therefore \text{Radius of the circles} = \frac{14}{2} \text{ cm} = 7 \text{ cm}$$

$$\text{Area of the square ABCD} = 14^2 = 196 \text{ cm}^2$$

$$\text{Area of the quadrant} = \frac{(\pi R^2)}{4} \text{ cm}^2 = \frac{(22/7 \times 7^2)}{4} \text{ cm}^2$$

$$= \frac{77}{2} \text{ cm}^2$$

$$\text{Total area of the quadrant} = 4 \times 77/2 \text{ cm}^2 = 154 \text{ cm}^2$$

Area of the shaded region = Area of the square ABCD - Area of the quadrant

$$= 196 \text{ cm}^2 - 154 \text{ cm}^2$$

$$= 42 \text{ cm}^2$$

8. Fig. 12.26 depicts a racing track whose left and right ends are semicircular.

The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If

the track is 10 m wide, find :

(i) the distance around the track along its inner edge

(ii) the area of the track.

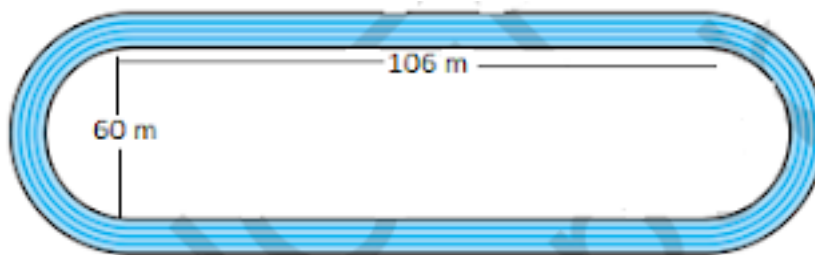


Fig. 12.26

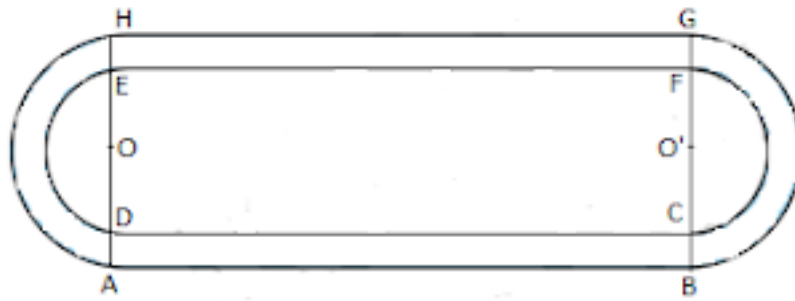
Solution:

Width of the track = 10 m

Distance between two parallel lines = 60 m

Length of parallel tracks = 106 m

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$$DE = CF = 60 \text{ m}$$

Radius of inner semicircle, $r = OD = O'C$

$$= 60/2 \text{ m} = 30 \text{ m}$$

Radius of outer semicircle, $R = OA = O'B$

$$= 30 + 10 \text{ m} = 40 \text{ m}$$

Also, $AB = CD = EF = GH = 106 \text{ m}$

Distance around the track along its inner edge = $CD + EF + 2 \times (\text{Circumference of inner semicircle})$

$$= 106 + 106 + (2 \times \pi r) \text{ m} = 212 + (2 \times 22/7 \times 30) \text{ m}$$

$$= 212 + 1320/7 \text{ m} = 2804/7 \text{ m}$$

Area of the track = Area of ABCD + Area EFGH + $2 \times (\text{area of outer semicircle}) - 2 \times (\text{area of inner semicircle})$

$$= (AB \times CD) + (EF \times GH) + 2 \times (\pi r^2/2) - 2 \times (\pi R^2/2) \text{ m}^2$$

$$= (106 \times 10) + (106 \times 10) + 2 \times \pi/2 (r^2 - R^2) \text{ m}^2$$

$$= 2120 + 22/7 \times 70 \times 10 \text{ m}^2$$

$$= 4320 \text{ m}^2$$

9. In Fig. 12.27, AB and CD are two diameters of a circle (with centre O) perpendicular to each other

and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.

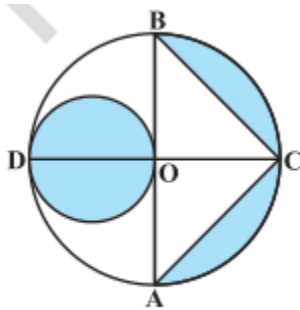


Fig. 12.27

Solution:

Radius of larger circle, $R = 7$ cm

Radius of smaller circle, $r = 7/2$ cm

Height of $\triangle BCA = OC = 7$ cm

Base of $\triangle BCA = AB = 14$ cm

Area of $\triangle BCA = \frac{1}{2} \times AB \times OC = \frac{1}{2} \times 7 \times 14 = 49 \text{ cm}^2$

Area of larger circle = $\pi R^2 = \frac{22}{7} \times 7^2 = 154 \text{ cm}^2$

Area of larger semicircle = $\frac{154}{2} \text{ cm}^2 = 77 \text{ cm}^2$

Area of smaller circle = $\pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2} \text{ cm}^2$

Area of the shaded region = Area of larger circle - Area of triangle - Area of larger semicircle +
Area of smaller circle

Area of the shaded region = $(154 - 49 - 77 + \frac{77}{2}) \text{ cm}^2$

$= \frac{133}{2} \text{ cm}^2 = 66.5 \text{ cm}^2$

10. The area of an equilateral triangle ABC is 17320.5 cm^2 . With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see Fig. 12.28). Find the area of the shaded region. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73205$)

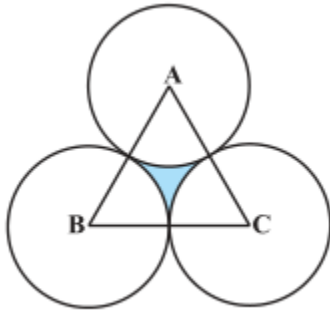


Fig. 12.28

Solution:

ABC is an equilateral triangle.

$$\therefore \angle A = \angle B = \angle C = 60^\circ$$

There are three sectors each making 60° .

$$\text{Area of } \triangle ABC = 17320.5 \text{ cm}^2$$

$$\Rightarrow \frac{\sqrt{3}}{4} \times (\text{side})^2 = 17320.5$$

$$\Rightarrow (\text{side})^2 = 17320.5 \times \frac{4}{1.73205}$$

$$\Rightarrow (\text{side})^2 = 4 \times 10^4$$

$$\Rightarrow \text{side} = 200 \text{ cm}$$

$$\text{Radius of the circles} = \frac{200}{2} \text{ cm} = 100 \text{ cm}$$

$$\text{Area of the sector} = \left(\frac{60^\circ}{360^\circ}\right) \times \pi r^2 \text{ cm}^2$$

$$= \frac{1}{6} \times 3.14 \times (100)^2 \text{ cm}^2$$

$$= \frac{15700}{3} \text{ cm}^2$$

$$\text{Area of 3 sectors} = 3 \times \frac{15700}{3} = 15700 \text{ cm}^2 =$$

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Area of the shaded region = Area of equilateral triangle ABC - Area of 3 sectors

$$= 17320.5 - 15700 \text{ cm}^2 = 1620.5 \text{ cm}^2$$

11. On a square handkerchief, nine circular designs each of radius 7 cm are made (see Fig. 12.29). Find the area of the remaining portion of the handkerchief.

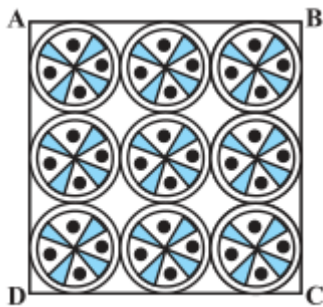


Fig. 12.29

Solution:

Number of circular design = 9

Radius of the circular design = 7 cm

There are three circles in one side of square handkerchief.

\therefore Side of the square = $3 \times \text{diameter of circle} = 3 \times 14 = 42 \text{ cm}$

Area of the square = $42 \times 42 \text{ cm}^2 = 1764 \text{ cm}^2$

Area of the circle = $\pi r^2 = \frac{22}{7} \times 7 \times 7 = 154 \text{ cm}^2$

Total area of the design = $9 \times 154 = 1386 \text{ cm}^2$

Area of the remaining portion of the handkerchief = Area of the square - Total area of the design

$$= 1764 - 1386 = 378 \text{ cm}^2$$

12. In Fig. 12.30, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the

(i) quadrant OACB,

(ii) shaded region.

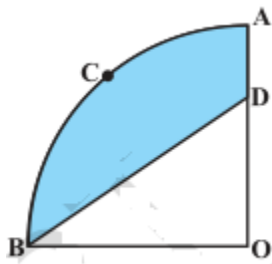


Fig. 12.30

Solution:

Radius of the quadrant = 3.5 cm = $\frac{7}{2}$ cm

(i) Area of quadrant OACB = $\frac{(\pi R^2)}{4} \text{ cm}^2$

$$= \frac{(22/7 \times 7/2 \times 7/2)}{4} \text{ cm}^2$$

$$= 77/8 \text{ cm}^2$$

(ii) Area of triangle BOD = $\frac{1}{2} \times \frac{7}{2} \times 2 \text{ cm}^2$

$$= 7/2 \text{ cm}^2$$

Area of shaded region = Area of quadrant - Area of triangle BOD

$$= (77/8 - 7/2) \text{ cm}^2 = 49/8 \text{ cm}^2$$

$$= 6.125 \text{ cm}^2$$

13. In Fig. 12.31, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. (Use $\pi = 3.14$)

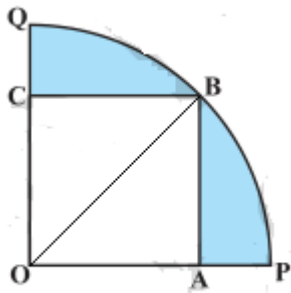


Fig. 12.31

Solution:

Side of square = OA = AB = 20 cm

Radius of the quadrant = OB

OAB is right angled triangle

By Pythagoras theorem in $\triangle OAB$,

$$OB^2 = AB^2 + OA^2$$

$$\Rightarrow OB^2 = 20^2 + 20^2$$

$$\Rightarrow OB^2 = 400 + 400$$

$$\Rightarrow OB^2 = 800$$

$$\Rightarrow OB = 20\sqrt{2} \text{ cm}$$

$$\text{Area of the quadrant} = \frac{(\pi R^2)}{4} \text{ cm}^2 = \frac{3.14}{4} \times (20\sqrt{2})^2 \text{ cm}^2 = 628 \text{ cm}^2$$

$$\text{Area of the square} = 20 \times 20 = 400 \text{ cm}^2$$

$$\text{Area of the shaded region} = \text{Area of the quadrant} - \text{Area of the square}$$

$$= 628 - 400 \text{ cm}^2 = 228 \text{ cm}^2$$

14. AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see Fig. 12.32). If $\angle AOB = 30^\circ$, find the area of the shaded region.

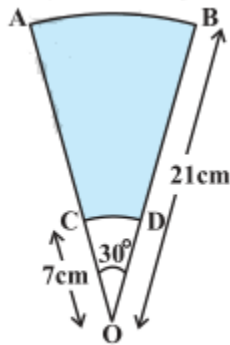


Fig. 12.32

Solution:

Radius of the larger circle, $R = 21$ cm

Radius of the smaller circle, $r = 7$ cm

Angle made by sectors of both concentric circles = 30°

Area of the larger sector = $(30^\circ/360^\circ) \times \pi R^2 \text{ cm}^2$

$$= \frac{1}{12} \times \frac{22}{7} \times 21^2 \text{ cm}^2$$

$$= \frac{231}{2} \text{ cm}^2$$

Area of the smaller circle = $(30^\circ/360^\circ) \times \pi r^2 \text{ cm}^2$

$$= \frac{1}{12} \times \frac{22}{7} \times 7^2 \text{ cm}^2$$

$$= \frac{77}{6} \text{ cm}^2$$

Area of the shaded region = $\frac{231}{2} - \frac{77}{6} \text{ cm}^2$

$$= \frac{616}{6} \text{ cm}^2 = \frac{308}{3} \text{ cm}^2$$

15. In Fig. 12.33, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.

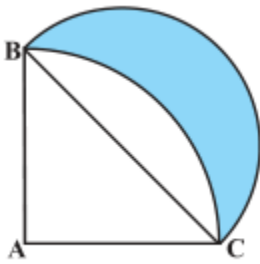


Fig. 12.33

Solution:

Radius of the the quadrant ABC of circle = 14 cm

$AB = AC = 14$ cm

BC is diameter of semicircle.

ABC is right angled triangle.

By Pythagoras theorem in ΔABC ,

$$BC^2 = AB^2 + AC^2$$

$$\Rightarrow BC^2 = 14^2 + 14^2$$

$$\Rightarrow BC = 14\sqrt{2} \text{ cm}$$

$$\text{Radius of semicircle} = \frac{14\sqrt{2}}{2} \text{ cm} = 7\sqrt{2} \text{ cm}$$

$$\text{Area of } \Delta ABC = \frac{1}{2} \times 14 \times 14 = 98 \text{ cm}^2$$

$$\text{Area of quadrant} = \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 = 154 \text{ cm}^2$$

$$\text{Area of the semicircle} = \frac{1}{2} \times \frac{22}{7} \times 7\sqrt{2} \times 7\sqrt{2} = 154 \text{ cm}^2$$

Area of the shaded region = Area of the semicircle + Area of ΔABC - Area of quadrant

$$= 154 + 98 - 154 \text{ cm}^2 = 98 \text{ cm}^2$$

16. Calculate the area of the designed region in Fig. 12.34 common between the two quadrants of circles of radius 8 cm each.

Solution:

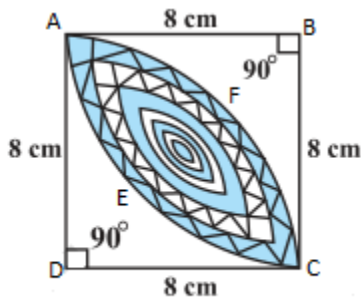


Fig. 12.34

$$AB = BC = CD = AD = 8 \text{ cm}$$

$$\text{Area of } \triangle ABC = \text{Area of } \triangle ADC = \frac{1}{2} \times 8 \times 8 = 32 \text{ cm}^2$$

$$\begin{aligned} \text{Area of quadrant AECB} &= \text{Area of quadrant AFCD} = \frac{1}{4} \times \frac{22}{7} \times 8^2 \\ &= \frac{352}{7} \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of shaded region} &= (\text{Area of quadrant AECB} - \text{Area of } \triangle ABC) + (\text{Area of quadrant AFCD} - \text{Area of } \triangle ADC) \end{aligned}$$

$$= \left(\frac{352}{7} - 32 \right) + \left(\frac{352}{7} - 32 \right) \text{ cm}^2$$

$$= 2 \times \left(\frac{352}{7} - 32 \right) \text{ cm}^2$$

$$= \frac{256}{7} \text{ cm}^2$$