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1. Find area of the triangle with vertices at the point given in each of the following:
(i) (1, 0), (6, 0), (4, 3)
(ii) (2, 7), (1, 1), (10, 8)
(iii) (–2, –3), (3, 2), (–1, –8)
Solution:
Formula for Area of triangle:

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

(i)

$$\frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[1(0-3)-0(6-4)+1(18-0)\right]$$

$$= \frac{15}{2} \text{ sq. units}$$

(ii)

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[2(1-8)-7(1-10)+1(8-10)\right]$$

$$= \frac{47}{2} \text{ sq. unit}$$

(iii)

$$\text{Area} = \frac{1}{2}\begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[-2(10)+3(4)-22\right]$$

= 15 sq. Units

2. Show that points: A (a, b + c), B (b, c + a), C (c, a + b) are collinear.

Solution:
Points are collinear if area of triangle is equal to zero.
i.e. Area of triangle = 0

$$\text{Area of Triangle} = \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[a(c+a-a-b)-(b+c)(b-c)+1\{b(a+b)-c(c+a)\}\right]$$

$$= \frac{1}{2}\left(ac-ab-b^2+c^2+ab+b^2-c^2-ac\right)$$

= 0
Therefore, points are collinear.

3. Find values of k if area of triangle is 4 sq. units and vertices are

(i) (k, 0), (4, 0), (0, 2)
(ii) (–2, 0), (0, 4), (0, k)

Solution:
(i)
Area of triangle = ±4 (Given)

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 4$$

$$\frac{1}{2}\left[k(0-2)-0+1(8-0)\right] = 4$$

½(-2k + 4) = 4

-k + 4 = 4
Now: -k + 4 = ±4
-k + 4 = 4 and –k + 4 = -4
k = 0 and k = 8

(ii) $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 4$

$$\frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix} = 4$$

½(-8+2k) = 4

or -k + 4 = 4
Now: -k + 4 = ±4
-k + 4 = 4 and –k + 4 = -4
k = 0 and k = 8

4. (i) Find equation of line joining (1, 2) and (3, 6) using determinants.
(ii) Find equation of line joining (3, 1) and (9, 3) using determinants.

Solution:
Let A(x, y) be any vertex of a triangle.
All points are on one line if area of triangle is zero.

$$\frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\frac{1}{2}\left[x(2-6)-y(1-3)+1(6-6)\right]=0$$

-4x + 2y = 0

y = 2x

Which is equation of line.

(ii) Let A(x, y) be any vertex of a triangle.
All points are on one line if area of triangle is zero.

$$\frac{1}{2}\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$$

$$\frac{1}{2}\left[x(1-3)-y(3-9)+1(9-9)\right]=0$$

-2x + 6y = 0

x – 3y = 0

Which is equation of line.

12. If area of triangle is 35 sq units with vertices (2, – 6), (5, 4) and (k, 4). Then k is

(A) 12 (B) –2 (C) –12, –2 (D) 12, –2

Solution:
Option (D) is correct.

Explanation:

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 35$$

$$\frac{1}{2}\left[2(4-4)-(-6)(5-k)+1(20-4k)\right] = 35$$

Solving above expression, we have

25 – 5k = ±35

25 – 5k = 35 and 25 – 5k = -35

k = -2 and k = 12.