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Write Minors and Cofactors of the elements of following determinants:

1.

(i) $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$ (ii) $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

Solution:

Find Minors of elements:

Say, M_{ij} is minor of element a_{ij}

M_{11} = Minor of element a_{11} = 3

M_{12} = Minor of element a_{12} = 0

M_{21} = Minor of element a_{21} = -4

M_{22} = Minor of element a_{22} = 2

Find cofactor of a_{ij}

Let cofactor of a_{ij} is A_{ij}, which is $(-1)^{i+j} M_{ij}$

$A_{11} = (-1)^{1+1} M_{11} = (-1)^2 (3) = 3$

$A_{12} = (-1)^{1+2} M_{12} = (-1)^3 (0) = 0$

$A_{21} = (-1)^{2+1} M_{21} = (-1)^3 (-4) = 4$

$A_{22} = (-1)^{2+2} M_{22} = (-1)^4 (2) = 2$

(ii)

$\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

Solution:

Find Minors of elements:

Say, M_{ij} is minor of element a_{ij}

M_{11} = Minor of element a_{11} = d

M_{12} = Minor of element a_{12} = b

M_{21} = Minor of element a_{21} = c

M_{22} = Minor of element a_{22} = a

Find cofactor of a_{ij}

Let cofactor of a_{ij} is A_{ij}, which is $(-1)^{i+j} M_{ij}$

$A_{11} = (-1)^{1+1} M_{11} = (-1)^2 (d) = d$

$A_{12} = (-1)^{1+2} M_{12} = (-1)^3 (b) = -b$

$A_{21} = (-1)^{2+1} M_{21} = (-1)^3 (c) = -c$

$A_{22} = (-1)^{2+2} M_{22} = (-1)^4 (a) = a$

2.

(i) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

(ii) $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -5 \\ 0 & 2 & 2 \end{vmatrix}$

Solution:

(i) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

Find Minors and cofactors of elements:

Say, M_{ij} is minor of element a_{ij} and A_{ij} is cofactor of a_{ij}

M_{11} = Minor of element a_{11} = $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$ and $A_{11} = 1$

M_{12} = Minor of element a_{12} = $\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 - 0 = 0$ and $A_{12} = 0$

M_{13} = Minor of element a_{13} = $\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$ and A_{13} =0

M_{21} = Minor of element a_{21} = $\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 - 0 = 0$ and A_{21} = 0

M_{22} = Minor of element a_{22} = $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$ and A_{22} = 1

M_{23} = Minor of element a_{23} = $\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$ and A_{23} = 0

M_{31} = Minor of element a_{21} = $\begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0 - 0 = 0$ and A_{31} = 0

M_{32} = Minor of element a_{32} = $\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$ and A_{32} = 0

M_{33} = Minor of element a_{33} = $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$ and A_{33} = 1

(ii) $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -5 \\ 0 & 2 & 2 \end{vmatrix}$

Find Minors and cofactors of elements:

Say, M_{ij} is minor of element a_{ij} and A_{ij} is cofactor of a_{ij}

M_{11} = Minor of element a_{11} = $\begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 10 - (-1) = 11$ and A_{11} = 11

M_{12} = Minor of element a_{12} = $\begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = 6 - 0 = 6$ and A_{12} =-6

M_{13} = Minor of element a_{13} = $\begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$ and A_{13} =3

M_{21} = Minor of element a_{21} = $\begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = 0 - 4 = -4$ and $A_{21} = 4$

M_{22} = Minor of element a_{22} = $\begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2$ and $A_{22} = 2$

M_{23} = Minor of element a_{23} = $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$ and $A_{23} = -1$

M_{31} = Minor of element a_{31} = $\begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = 0 - 20 = -20$ and $A_{31} = -20$

M_{32} = Minor of element a_{32} = $\begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = -1 - 12 = -13$ and $A_{32} = 13$

M_{33} = Minor of element a_{33} = $\begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5 - 0 = 5$ and $A_{33} = 5$

3. Using Cofactors of elements of second row, evaluate Δ.

$$\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

Solution:
Find Cofactors of elements of second row:

A_{21} = Cofactor of element a_{21} = $(-1)^{2+1}\begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} = (-1)^3(9-16) = 7$

A_{22} = Cofactor of element a_{22} = $(-1)^{2+2}\begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix} = (-1)^4(15-8) = 7$

A_{23} = Cofactor of element a_{23} = $(-1)^{2+3}\begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = (-1)^5(10-3) = -7$

Now, $\Delta = a_{21} A_{21} + a_{22} A_{22} + a_{23} A_{23} = 14 + 0 - 7 = 7$

4. Using Cofactors of elements of third column, evaluate Δ.

$$\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$$

Solution:

Find Cofactors of elements of third column:

A_{13} = Cofactor of element a_{13} = $(-1)^{1+3}\begin{vmatrix} 1 & y \\ 1 & z \end{vmatrix} = (-1)^4(z-y) = z-y$

A_{23} = Cofactor of element a_{23} = $(-1)^{2+3}\begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix} = (-1)^5(z-x) = x-z$

A_{33} = Cofactor of element a_{33} = $(-1)^{3+3}\begin{vmatrix} 1 & x \\ 1 & y \end{vmatrix} = (-1)^6(y-x) = y-x$

Now, $\Delta = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$

$= yz(z-y) + zx(x-z) + xy(y-x)$

$= (yz^2 - y^2z) + (xy^2 - xz^2) + (xz^2 - x^2y)$

$= (y-z)\left[-yz + x(y+z) - x^2\right]$

$= (y-z)\left[-y(z-x) + x(z-x)\right]$

= (x – y)(y – x)(z – x)

5. If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ **and A_{ij} is cofactor of a_{ij} then value of Δ is given by:**

(A) $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$

(B) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$

(C) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$

(D) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Solution: Option (D) is correct.