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Find adjoint of each of the matrices in Exercises 1 and 2.

1.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

2.

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$$

Solution:

1. Let A =

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Cofactors of the above matrix are

$A_{11} = 4$

$A_{12} = -3$

$A_{21} = -2$

$A_{22} = 1$

$$\text{adj. A} = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

2.

$$\text{Let A} = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$$

Cofactors of the above matrix are

$$A_{11} = +\begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 \qquad A_{21} = -\begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = 1 \qquad A_{31} = +\begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} = -11$$

$$A_{12} = -\begin{vmatrix} 2 & 5 \\ -2 & 1 \end{vmatrix} = -12 \qquad A_{22} = +\begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} = 5 \qquad A_{32} = -\begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} = -1$$

$$A_{13} = +\begin{vmatrix} 2 & 3 \\ -2 & 0 \end{vmatrix} = 6 \qquad A_{23} = -\begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = 2 \qquad A_{33} = +\begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 5$$

Therefore,

$$\text{adj. A} = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}$$

Verify A (adj A) = (adj A) A = |A| I in Exercises 3 and 4

3. $\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$ 4. $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

Solution:

3.

$$\text{Let A} = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$$

$$\text{adj. A} = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$$

$$\text{A.(adj. A)} = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}\begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Again, $|A| = \begin{vmatrix} 2 & 3 \\ -4 & -6 \end{vmatrix} = -12 + 12 = 0$

$$|A|I = (0)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

LHS = RHS

Verified.

4.

$$\text{Let A} = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

Cofactors of A,

$A_{11} = +\begin{vmatrix} 0 & -2 \\ 0 & 3 \end{vmatrix} = 0$ $A_{21} = -\begin{vmatrix} -1 & 2 \\ 0 & 3 \end{vmatrix} = 3$ $A_{31} = +\begin{vmatrix} -1 & 2 \\ 0 & -2 \end{vmatrix} = 2$

$A_{12} = -\begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} = -11$ $A_{22} = +\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 1$ $A_{32} = -\begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = 8$

$A_{13} = +\begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} = 0$ $A_{23} = -\begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = -1$ $A_{33} = +\begin{vmatrix} 1 & -1 \\ 3 & 0 \end{vmatrix} = 3$

Now,

$$\text{adj. } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

Now, Verify A (adj A) = (adj A) A = |A| I

$$A(\text{adj. } A) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$(\text{adj. } A)A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix} = 1(0) - (-1)(11) + 2(0) = 11$$

$$|A|I = 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Verified.

Find the inverse of each of the matrices (if it exists) given in Exercises 5 to 11.

5.

$$\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$$

Solution:

$$\text{Let } A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$$

$$|A| = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix} = 14 \neq 0$$

Since determinant of the matrix is not zero, so inverse of this matrix is possible.

As we know, formula to find matrix inverse is:

$$A^{-1} = \frac{1}{|A|} \text{adj.A}$$

$$\text{adj. A} = \begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$$

This implies,

$$A^{-1} = \frac{1}{14}\begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$$

6.

$$\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$$

Solution:

$$\text{Let } A = \begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$$

$$|A| = \begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix} = 13 \neq 0$$

Since determinant of the matrix is not zero, so inverse of this matrix is possible.

As we know, formula to find matrix inverse is:

$$A^{-1} = \frac{1}{|A|} \text{adj.A}$$

$$\text{adj. A} = \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$$

This implies,

$$A^{-1} = \frac{1}{13}\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$$

7.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

Solution:

$$\text{Let } A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{vmatrix} = 1(10) - 2(0) + 3(0) = 10 \neq 0$$

Therefore,

A^{-1} exists

Find adj A:

$$A_{11} = +\begin{vmatrix} 2 & 4 \\ 0 & 5 \end{vmatrix} = 10 \quad A_{21} = -\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix} = -10 \quad A_{31} = +\begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} = 2$$

$$A_{12} = -\begin{vmatrix} 0 & 4 \\ 0 & 5 \end{vmatrix} = 0 \quad A_{22} = +\begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix} = 5 \quad A_{32} = -\begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix} = -4$$

$$A_{13} = +\begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix} = 0 \quad A_{23} = -\begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} = 0 \quad A_{33} = +\begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = 2$$

As we know, formula to find matrix inverse is:

$$A^{-1} = \frac{1}{|A|}\text{adj.A}$$

$$A^{-1} = \frac{1}{10}\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

8.

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{vmatrix} = 1(-3) - 0 + 0 = -3 \neq 0$$

Therefore,

A^{-1} exists

Find adj A:

$A_{11} = +\begin{vmatrix} 3 & 0 \\ 2 & -1 \end{vmatrix} = -3 \quad A_{21} = -\begin{vmatrix} 0 & 0 \\ 2 & -1 \end{vmatrix} = 0 \quad A_{31} = +\begin{vmatrix} 0 & 0 \\ 3 & 0 \end{vmatrix} = 0$

$A_{12} = -\begin{vmatrix} 3 & 0 \\ 5 & -1 \end{vmatrix} = 3 \quad A_{22} = +\begin{vmatrix} 1 & 0 \\ 5 & -1 \end{vmatrix} = -1 \quad A_{32} = -\begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix} = 0$

$A_{13} = +\begin{vmatrix} 3 & 3 \\ 5 & 2 \end{vmatrix} = -9 \quad A_{23} = -\begin{vmatrix} 1 & 0 \\ 5 & 2 \end{vmatrix} = -2 \quad A_{33} = +\begin{vmatrix} 1 & 0 \\ 3 & 3 \end{vmatrix} = 3$

$$\text{adj. } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

As we know, formula to find matrix inverse is:

$$A^{-1} = \frac{1}{|A|}adj.A$$

$$A^{-1} = \frac{-1}{3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

9.

$$\begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$$

Solution:

$$\text{Let } A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{vmatrix}$$

$=2(-1)-1(4) + 3(1) = -3$

$\neq 0$

Therefore,

A^{-1} exists

Find adj A:

$A_{11} = +\begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix} = -1$ $A_{21} = -\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = 5$ $A_{31} = +\begin{vmatrix} 1 & 3 \\ -1 & 0 \end{vmatrix} = 3$

$A_{12} = -\begin{vmatrix} 4 & 0 \\ -7 & 1 \end{vmatrix} = -4$ $A_{22} = +\begin{vmatrix} 2 & 3 \\ -7 & 1 \end{vmatrix} = 23$ $A_{32} = -\begin{vmatrix} 2 & 3 \\ 4 & 0 \end{vmatrix} = 12$

$A_{13} = +\begin{vmatrix} 4 & -1 \\ -7 & 2 \end{vmatrix} = 1$ $A_{23} = -\begin{vmatrix} 2 & 1 \\ -7 & 2 \end{vmatrix} = -11$ $A_{33} = +\begin{vmatrix} 2 & 1 \\ 4 & -1 \end{vmatrix} = -6$

$$\text{adj. A} = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$$

As we know, formula to find matrix inverse is:

$$A^{-1} = \frac{1}{|A|}\text{adj.A}$$

$$A^{-1} = \frac{-1}{3}\begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$$

10.

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$$

Solution:

$$\text{Let } A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{vmatrix}$$

$$=1(2)+1(9)+2(-6) = -1$$

$$\neq 0$$

Therefore,

A^{-1} exists

Find adj A:

$A_{11} = +\begin{vmatrix} 2 & -3 \\ -2 & 4 \end{vmatrix} = 2 \quad A_{21} = -\begin{vmatrix} -1 & 2 \\ -2 & 4 \end{vmatrix} = 0 \quad A_{31} = +\begin{vmatrix} -1 & 2 \\ 2 & -3 \end{vmatrix} = -1$

$A_{12} = -\begin{vmatrix} 0 & -3 \\ 3 & 4 \end{vmatrix} = -9 \quad A_{22} = +\begin{vmatrix} 2 & 2 \\ 3 & 4 \end{vmatrix} = -2 \quad A_{32} = -\begin{vmatrix} 1 & 2 \\ 0 & -3 \end{vmatrix} = 3$

$A_{13} = +\begin{vmatrix} 0 & 2 \\ 3 & -2 \end{vmatrix} = -6 \quad A_{23} = -\begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} = -1 \quad A_{33} = +\begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} = 2$

$$\text{adj. } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

As we know, formula to find matrix inverse is:

$$A^{-1} = \frac{1}{|A|}\text{adj}.A$$

$$A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

11.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$$

Solution:

$$\text{Let } A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{vmatrix}$$

$$= \left(-\cos^2\alpha - \sin^2\alpha\right) - 0 + 0$$

$$= -\left(\cos^2\alpha + \sin^2\alpha\right) = -1 \neq 0$$

Therefore,

A^{-1} exists

Find adj A:

$$A_{11} = +\begin{vmatrix}\cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha\end{vmatrix} = -1 \quad A_{21} = -\begin{vmatrix}0 & 0 \\ \sin\alpha & -\cos\alpha\end{vmatrix} = 0 \quad A_{31} = +\begin{vmatrix}0 & 0 \\ \cos\alpha & \sin\alpha\end{vmatrix} = 0$$

$$A_{12} = -\begin{vmatrix}0 & \sin\alpha \\ 0 & -\cos\alpha\end{vmatrix} = 0 \quad A_{22} = +\begin{vmatrix}1 & 0 \\ 0 & -\cos\alpha\end{vmatrix} = -\cos\alpha \quad A_{32} = -\begin{vmatrix}1 & 0 \\ 0 & \sin\alpha\end{vmatrix} = -\sin\alpha$$

$$A_{13} = +\begin{vmatrix}0 & \cos\alpha \\ 0 & \sin\alpha\end{vmatrix} = 0 \quad A_{23} = -\begin{vmatrix}1 & 0 \\ 0 & \sin\alpha\end{vmatrix} = \sin\alpha \quad A_{33} = +\begin{vmatrix}1 & 0 \\ 0 & \cos\alpha\end{vmatrix} = \cos\alpha$$

$$\text{adj. } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33}\end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha\end{bmatrix}$$

As we know, formula to find matrix inverse is:

$$A^{-1} = \frac{1}{|A|}\text{adj.}A$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha\end{bmatrix}$$

12. Let A = $\begin{bmatrix} 3 & 7 \\ 2 & 5\end{bmatrix}$ **and B =** $\begin{bmatrix} 6 & 8 \\ 7 & 9\end{bmatrix}$ **verify that $(AB)^{-1} = B^{-1} A^{-1}$.**

Solution:

$$A = \begin{bmatrix} 3 & 7 \\ 2 & 5\end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 7 \\ 2 & 5\end{vmatrix} = 1 \neq 0$$

$$A^{-1} = \frac{1}{|A|}\text{ adj. } A$$

$$A^{-1} = \begin{bmatrix} 5 & -7 \\ -2 & 3\end{bmatrix}$$

Again,

$$B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$$

$$|B| = \begin{vmatrix} 6 & 8 \\ 7 & 9 \end{vmatrix} = -2 \neq 0$$

$$B^{-1} = \frac{1}{-2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$$

Now Multiply A and B,

$$AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$$

Find determinant of AB:

$$|AB| = \begin{vmatrix} 67 & 87 \\ 47 & 61 \end{vmatrix} = 4087 - 4089 = -2 \neq 0$$

Now, Verify $(AB)^{-1} = B^{-1} A^{-1}$

LHS:

$$|AB| = \begin{vmatrix} 67 & 87 \\ 47 & 61 \end{vmatrix} = 4087 - 4089 = -2 \neq 0$$

and

$$(AB)^{-1} = \frac{1}{-2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$$

RHS:

$$B^{-1}A^{-1} = \frac{1}{-2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$= \frac{1}{-2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$$

This implies, LHS = RHS (Verified)

13. If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$. Hence find A^{-1}.

Solution:
$A^2 = AA$

$$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

LHS = $A^2 - 5A + 7I$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

= RHS. (Proved)

To Find A^{-1}

Multiply $A^2 - 5A + 7I$ by A^{-1}, we have
(Consider I is 2x2 matrix)

$$A^2A^{-1} - 5A.A^{-1} + 7I\ A^{-1} = 0.A^{-1}$$

$$A - 5I\ + 7A^{-1} = 0$$

$$7A^{-1} = -A + 5I$$

$$= \begin{bmatrix} -3 & -1 \\ 1 & -2 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

14. For the matrix A = $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the numbers a and b such that $A^2 + aA + bI = O$.

Solution:

$$A^2 = AA = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$$

Since $A^2 + aA + bI = O$

$$\begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + a\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3a & 2a \\ a & a \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 11+3a+b & 8+2a+0 \\ 4+a+0 & 3+a+b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Equate corresponding elements, we get

11 + 3a + b = 0 …(1)
8 + 2a = 0 => a = -4

Substitute the value of a in equation (1),
11 + 3(-4) + b = 0
11 – 12 + b = 0
b = 1.

15. For the matrix A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$. Show that $A^3 - 6A^2 + 5A + 11I = O$. Hence, find A^{-1} .

Solution:

$$A^2 = AA = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+1 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$A^3 = A^2 A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2+2 & 4+4-1 & 4-6+3 \\ -3+8-28 & -3+16+14 & -3-24-42 \\ 7-3+28 & 7-6-14 & 7+9+42 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$

Now, LHS = $A^3 - 6A^2 + 5A + 11\,I$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 8-24+5+11 & 7-12+5 & 1-6+5 \\ -23+18+5 & 27-48+10+11 & -69+84-15 \\ 32-42+10 & -13+18-5 & 58-84+15+11 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

= 0

RHS (Proved)

Now**, find A^{-1}**

Multiply $A^3 - 6A^2 + 5A + 11I$ by A^{-1} , we have
(Consider I is 3x3 matrix)

$$A^3A^{-1} - 6A^2A^{-1} + 5AA^{-1} + 11I.A^{-1} = 0A^{-1}$$

$$A^2 - 6A + 5I + 11A^{-1} = 0$$

$$11A^{-1} = 6A - 5I - A^2$$

$$11A^{-1} = 6\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 6-5-4 & 6-2 & 6-1 \\ 6+3 & 12-5-8 & -18+14 \\ 12-7 & -6+3 & 18-5-14 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

Therefore,

$$A^{-1} = \frac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

16. If A = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Verify that $A^3 - 6A^2 + 9A - 4I = O$ and hence find A^{-1} .

Solution:

$A^2 = AA$

$$\begin{bmatrix} 4+1+1 & -2-2-1 & 1+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

Again, $A^3 = A^2A$

$$\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

Now, $\mathbf{A^3 - 6A^2 + 9A - 4I}$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22-36 & -21+30 & 21-30 \\ -21+30 & 22-36 & -21+30 \\ 21-30 & -21+30 & 22-36 \end{bmatrix} + \begin{bmatrix} 18-4 & -9-0 & 9-0 \\ -9-0 & 18-4 & -9-0 \\ 9-0 & -9-0 & 18-4 \end{bmatrix}$$

$$= \begin{bmatrix} -14+14 & 9-9 & -9+9 \\ 9-9 & -14+14 & 9-9 \\ -9+9 & 9-9 & -14+14 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

= 0 (RHS)

Multiply $A^3 - 6A^2 + 9A - 4I = O$ by A^{-1}, (here I is 3x3 matrix)

$$A^3A^{-1} - 6A^2A^{-1} + 9AA^{-1} - 4I.A^{-1} = 0.A^{-1}$$

$$A^2 - 6A + 9I - 4A^{-1} = 0$$

$$4A^{-1} = A^2 - 6A + 9I$$

Now Placing all the matrices,

$$4A^{-1}=\begin{bmatrix}6 & -5 & 5\\ -5 & 6 & -5\\ 5 & -5 & 6\end{bmatrix}-6\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}+9\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$4A^{-1}=\begin{bmatrix}6-12+9 & -5+6+0 & 5-6+0\\ -5+6+0 & 6-12+9 & -5+6+0\\ 5-6+0 & -5+6+0 & 6-12+9\end{bmatrix}$$

$$=\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}$$

Inverse of the matrix is :

$$A^{-1}=\frac{1}{4}\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}$$

17. Let A be a non-singular matrix of order 3 x 3. Then |adj. A| is equal to:
(A) |A| (B) $|A|^2$ (C) $|A|^3$ (D) 3|A|

Solution:

Option (B) is correct.
Explanation:
$|adj.\ A| = |A|^{n-1} = |A|^2$ (for n = 3)

18. If A is an invertible matrix of order 2, then det (A^{-1}) is equal to:

(A) det A (B) 1/ det A (C) 1 (D) 0

Solution:

Option (B) is correct.

Explanation:
$A A^{-1} = I$
det $(A A^{-1} = I)$
det(A) det(A^{-1}) = 1

det(A^{-1}) = 1/det A