

Exercise 4.6

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Examine the consistency of the system of equations in Exercises 1 to 6.

1. $x + 2y = 2$: and $2x + 3y = 3$

Solution:

Given set of equations is : $x + 2y = 2$: and $2x + 3y = 3$

This set of equation can be written in the form of matrix as $AX = B$, where

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \text{ and}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix}$$

So, $AX = B$ is

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1 \neq 0$$

Inverse of matrix exists. So system of equations is consistent.

2. $2x - y = 5$ and $x + y = 4$

Solution:

Given set of equations is : $2x - y = 5$ and $x + y = 4$

This set of equation can be written in the form of matrix as $AX = B$, where

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

So, $AX = B$ is

$$\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 3 \neq 0$$

Inverse of matrix exists. So system of equations is consistent.

3. $x + 3y$ and $2x + 6y = 8$

Solution:

Given set of equations is : $x + 3y$ and $2x + 6y = 8$

This set of equation can be written in the form of matrix as $AX = B$.

$$\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

Where,

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix} = 0$$

$$\text{adj. } A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$$

And

$$(\text{adj. } A)B = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix} \neq 0$$

The given equations are inconsistent.

4. $x + y + z = 1$; $2x + 3y + 2z = 2$ and $ax + ay + 2az = 4$

Solution:

Given set of equations is : $x + y + z = 1$; $2x + 3y + 2z = 2$ and $ax + ay + 2az = 4$

This set of equation can be written in the form of matrix as $AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{vmatrix} = 1(6a - 2a) - 1(4a - 2a) + 1(2a - 3a)$$

$$= 4a - 2a - a = a \neq 0$$

System of equations is consistent.

5. $3x - y - 2z = 2$; $2y - z = -1$ and $3x - 5y = 3$

Solution:

Given set of equations is : $3x - y - 2z = 2$; $2y - z = -1$ and $3x - 5y = 3$

This set of equation can be written in the form of matrix as $AX = B$

$$\begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$$

$$= 3(-5) + (3) - 2(-6)$$

$$= 15 - 15$$

$$= 0$$

Now,

$$(\text{adj. } A) = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}$$

$$(\text{adj. } A)B = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ -6 \end{bmatrix} \neq 0$$

6. Given set of equations is :

$$5x - y + 4z = 5$$

$$2x + 3y + 5z = 2$$

$$5x - 2y + 6z = -1$$

Solution:

Given set of equations is: $5x - y + 4z = 5$; $2x + 3y + 5z = 2$; $5x - 2y + 6z = -1$

This set of equation can be written in the form of matrix as $AX = B$

$$\begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{vmatrix}$$

$$= 5(18 + 10) + 1(12 - 25) + 4(-4 - 15)$$

$$= 140 - 13 - 76$$

$$= 140 - 89$$

$$= 51$$

$$\neq 0$$

System of equations is consistent.

Solve system of linear equations, using matrix method, in Exercises 7 to 14.

7. $5x + 2y = 4$ and $7x + 3y = 5$

Solution:

Given set of equations is : $5x + 2y = 4$ and $7x + 3y = 5$

This set of equation can be written in the form of matrix as $AX = B$

$$\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

Where,

$$A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$$

And $|A| = 1 \neq 0$

System is consistent.

Now,

$$X = A^{-1}B = \frac{1}{|A|}(\text{adj. } A)B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1} \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 12-10 \\ -28+25 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$$\Rightarrow x = 2 \text{ and } y = -3$$

8. $2x - y = -2$ and $3x + 4y = 3$

Solution:

Given set of equations is : $2x - y = -2$ and $3x + 4y = 3$

This set of equation can be written in the form of matrix as $AX = B$

$$\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$$

$$|A| = 11 \neq 0$$

System is consistent.

So,

$$X = A^{-1}B = \frac{1}{|A|}(\text{adj. } A)B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \frac{1}{11} \begin{bmatrix} -8+3 \\ 6+6 \end{bmatrix} = \frac{1}{11} \begin{bmatrix} -5 \\ 12 \end{bmatrix}$$

Therefore, $x = -5/11$ and $y = 12/11$

9. $4x - 3y = 3$ and $3x - 5y = 7$

Solution:

Given set of equations is : $4x - 3y = 3$ and $3x - 5y = 7$

This set of equation can be written in the form of matrix as $AX = B$

$$\begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

Where,

$$A = \begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}$$

And $|A| = -20 + 9 = -11 \neq 0$

System is consistent.

So,

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-11} \begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$$= \frac{1}{-11} \begin{bmatrix} -15+21 \\ -9+28 \end{bmatrix} = \frac{1}{-11} \begin{bmatrix} 6 \\ 19 \end{bmatrix}$$

Therefore, $x = 6/-11$ and $y = 19/-11$

10. $5x + 2y = 3$ and $3x + 2y = 5$

Solution:

Given set of equations is : $5x + 2y = 3$ and $3x + 2y = 5$

This set of equation can be written in the form of matrix as $AX = B$

$$\begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

Where

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}$$

And $|A| = 4 \neq 0$

System is consistent.

So,

$$X = A^{-1}B = \frac{1}{|A|}(\text{adj. } A)B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

Therefore, $x = -1$ and $y = 4$.

11. $2x + y + z = 1$ and $x - 2y - z = 3/2$ and $3y - 5z = 9$

Solution:

Given set of equations is : $2x + y + z = 1$ and $x - 2y - z = 3/2$ and $3y - 5z = 9$

This set of equation can be written in the form of matrix as $AX = B$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$$

Where

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}$$

And

$$|A| = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{vmatrix}$$

$$= 34 \neq 0$$

System is consistent.

So,

$$X = A^{-1}B = \frac{1}{|A|}(\text{adj. } A)B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34} \begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix} \begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix}$$

$$= \frac{1}{34} \begin{bmatrix} 13+12+9 \\ 5-15+27 \\ 3-9-45 \end{bmatrix} = \frac{1}{34} \begin{bmatrix} 34 \\ 17 \\ -51 \end{bmatrix}$$

Therefore, $x = 1$, $y = \frac{1}{2}$ and $z = \frac{3}{2}$

12. $x - y + z = 4$ and $2x + y - 3z = 0$ and $x + y + z = 2$

Solution:

Given set of equations is : $x - y + z = 4$ and $2x + y - 3z = 0$ and $x + y + z = 2$

This set of equation can be written in the form of matrix as $AX = B$

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

Where,

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$$

And

$$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 10 \neq 0$$

System is consistent.

So,

$$X = A^{-1}B = \frac{1}{|A|}(\text{adj. } A)B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 16+0+4 \\ -20+0+10 \\ 4-0+6 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Therefore, $x = 2$, $y = -1$ and $z = 1$

13.

$$2x + 3y + 3z = 5$$

$$x - 2y + z = -4$$

$$3x - y - 2z = 3$$

Solution:

Given set of equations is :

$$2x + 3y + 3z = 5$$

$$x - 2y + z = -4$$

$$3x - y - 2z = 3$$

This set of equation can be written in the form of matrix as $AX = B$

$$\begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

Where

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}$$

And,

$$|A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix}$$

$$= 40 \neq 0$$

System is consistent.

So,

$$X = A^{-1}B = \frac{1}{|A|}(\text{adj. } A)B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix} \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$= \frac{1}{40} \begin{bmatrix} 25 - 12 + 27 \\ 25 + 52 + 3 \\ 25 - 44 - 21 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Therefore, $x = 1$, $y = 2$ and $z = -1$.

14.

$$x - y + 2z = 7$$

$$3x + 4y - 5z = -5$$

$$2x - y + 3z = 12$$

Solution:

Given set of equations is :

$$x - y + 2z = 7$$

$$3x + 4y - 5z = -5$$

$$2x - y + 3z = 12$$

This set of equation can be written in the form of matrix as $AX = B$

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

Where,

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$$

And

$$|A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{vmatrix}$$

$$= 4 \neq 0$$

System is consistent.

So,

$$X = A^{-1}B = \frac{1}{|A|}(\text{adj. } A)B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix} \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 49 - 5 - 36 \\ -133 + 5 + 132 \\ -77 + 5 + 84 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Therefore, $x = 2$, $y = 1$ and $z = 3$.

15. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1} . Using A^{-1} solve the system of equations.

$$2x - 3y + 5z = 11$$

$$3x + 2y - 4z = -5$$

$$x + y - 2z = -3$$

Solution:

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$$

$= -1 \neq 0$; Inverse of matrix exists.

Find the inverse of matrix:

Cofactors of matrix:

$$A_{11} = 0, A_{12} = 2, A_{13} = 1$$

$$A_{21} = -1, A_{22} = -9, A_{23} = -5$$

$$A_{31} = 2, A_{32} = 23, A_{33} = 13$$

$$\text{adj. } A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

So,

$$A^{-1} = \frac{1}{-1} \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

Now, matrix of equation can be written as:

$$AX = B$$

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

And, $X = A^{-1} B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Therefore, $x = 1$, $y = 2$ and $z = 3$.

16. The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs. 60. The cost of 2 kg onion, 4 kg wheat and 2 kg rice is Rs. 90. The cost of 6 kg onion, 2 kg wheat and 3 kg rice is Rs. 70. Find cost of each item per kg by matrix method.

Solution:

Let x , y and z be the per kg. prices of onion, wheat and rice respectively.

According to given statement, we have following equations,

$$4x + 3y + 2z = 60$$

$$2x + 4y + 6z = 90$$

$$6x + 2y + 3z = 70$$

The above system of equations can be written in the form of matrix as, $AX = B$

$$\begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

Where,

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}$$

And

$$|A| = \begin{vmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{vmatrix}$$

$$= 4(0) - 3(-30) + 2(-20)$$

$$= 50 \neq 0$$

System is consistent, and $X = A^{-1} B$

First find invers of A.

Cofactors of all the elements of A are:

$$A_{11} = 0, A_{12} = 30, A_{13} = -20$$

$$A_{21} = -5, A_{22} = 0, A_{23} = 10$$

$$A_{31} = 10, A_{32} = -20, A_{33} = 10$$

$$\text{adj. } A = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

Again,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix} \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$= \frac{1}{50} \begin{bmatrix} -450 + 700 \\ 1800 - 1400 \\ -1200 + 900 + 700 \end{bmatrix}$$

$$= \frac{1}{50} \begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

Therefore, $x = 5$, $y = 8$ and $z = 8$.

The cost of onion, wheat and rice per kg are Rs. 5, Rs. 8 and Rs. 8 respectively.