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1. Find the rate of change of the area of a circle with respect to its radius r when (a) r = 3 cm (b) r = 4 cm

Solution: Consider x denote the area of the circle of radius r.
Area of circle, x = π r^2

And, Rate of change of area x w.r.t. r is

$$\frac{dx}{dr} = \pi(2r) = 2\pi r$$

(a) r = 3 cm

$$\frac{dx}{dr} = 2\pi(3) = 6\pi$$ sq. cm

(b) r = 4 cm

$$\frac{dx}{dr} = 2\pi(4) = 8\pi$$ sq. cm

2. The volume of a cube is increasing at the rate of 8 cm^3/sec. How fast is the surface area increasing when the length of an edge is 12 cm?

Solution: Consider a side of the cube be x cm.

Rate of increase of volume of cube = 8 cm^3/sec

$$\Rightarrow \frac{d}{dt}(x.x.x) = \frac{d}{dt}(x^3)$$

$$\Rightarrow 3x^2 \frac{d}{dt} x = 8$$

$$\Rightarrow \frac{dx}{dt} = \frac{8}{3x^2}$$ ……….(1)

Consider y be the surface area of the cube, i.e., $y = 6x^2$

Rate of change of surface area of the cube = $\frac{dy}{dt} = 6\frac{d}{dt}x^2$

= $6\left(2x\frac{dx}{dt}\right)$ = $12x\left(\frac{8}{3x^2}\right)$

= $4\left(\frac{8}{x}\right) = \frac{32}{x}$ cm^2/sec

Put x = 12 cm (Given)

$\frac{dy}{dt} = \frac{32}{12} = \frac{8}{3}$ cm^2/sec

As, $\frac{dy}{dt}$ is positive, therefore surface area is increasing at the rate of $\frac{8}{3}$ cm^2/sec.

3. The radius of the circle is increasing uniformly at the rate of 3 cm per second. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

Solution: Consider x cm be the radius of the circle at time t.

Rate of increase of radius of circle = 3 cm/sec

$\Rightarrow \frac{dx}{dt}$ is positive and equal to 3 cm/sec

Consider y be the area of the circle.

$\Rightarrow y = \pi r^2$

$\therefore$ Rate of change of area of circle = $\frac{dy}{dt}$ = $\pi\frac{d}{dt}x^2$

= $\pi.2x\frac{dx}{dt} = 2\pi x(3)$

= $6\pi x$

Put x = 10 cm (given),

$\frac{dy}{dt} = 6\pi(10) = 60\pi$ cm^2/sec

As, $\frac{dy}{dt}$ is positive, therefore surface area is increasing at the rate of 60π cm^2/sec.

4. An edge of a variable cube is increasing at the rate of 3 cm per second. How fast is the volume of the cube increasing when the edge if 10 cm long?

Solution: Consider x cm be the edge of variable cube at time t.

Rate of increase of edge = 3 cm/sec

$\Rightarrow \frac{dx}{dt}$ is positive and = 3 cm/sec

Consider y be the volume of the cube.

$\Rightarrow y = x^3$

Therefore, Rate of change of volume of cube = $\frac{dy}{dt} = \frac{d}{dt}x^3$

$= 3x^2\frac{dx}{dt} = 3x^2(3)$

$= 9x^2$ cm^3/sec

Put x = 10 cm (Given)

$\frac{dy}{dt} = 9(10)^2 = 900$ cm^3/sec

As, $\frac{dy}{dt}$ is positive, therefore volume of cube is increasing at the rate of 900 cm^3/sec.

5. A stone is dropped into a quite lake and waves move in circles at the rate of 5 cm/sec. At the instant when radius of the circular wave is 8 cm, how fast is the enclosed area increasing?

Solution: Consider x cm be the radius of the circular wave at time t.

Rate of increase of radius of circular wave = 5 cm/sec

$\frac{dx}{dt}$ is positive and = 5 cm/sec

Consider y be the enclosed area of the circular wave.

$y = \pi x^2$

Rate of change of area = $\frac{dy}{dt}$ = $\pi \frac{d}{dt} x^2$

= $\pi .2x \frac{dx}{dt} = 2\pi x(5)$ = $10\pi x$

Put x = 8 cm (Given)

$\frac{dy}{dt}$ = $10\pi(8) = 80\pi$ cm^2/sec

As, $\frac{dy}{dt}$ is positive, therefore area of circular wave is increasing at the rate of 80π cm^2/sec.

6. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of its circumference?

Solution: Consider x cm be the radius of the circle at time t.

Rate of increase of radius of circle = 0.7 cm/sec

$\Rightarrow \frac{dx}{dt}$ is positive and = 0.7 cm/sec

Consider y be the circumference of the circle.

$\Rightarrow y = 2\pi x$

Rate of change of circumference of circle = $\frac{dy}{dt}$

= $2\pi \frac{d}{dt} x$ = $2\pi(0.7)$

= 1.4π cm/sec

7. The length x of a rectangle is decreasing at the rate of 5 cm/minute. When x = 8 cm and y = 6 cm, find the rates of change of (a) the perimeter and (b) the area of the rectangle.

Solution: Given: Rate of decrease of length x of rectangle is 5 cm/minute.

$\frac{dx}{dt}$ is negative = –5 cm/minute

Also, Rate of increase of width y of rectangle is 4 cm/minute

$\Rightarrow \frac{dy}{dt}$ is positive

= 4 cm/minute

(a) Consider z denotes the perimeter of rectangle.

$x = 2x + 2y$

$\frac{dz}{dt} = 2\frac{dx}{dt} + 2\frac{dy}{dt}$

= $2(-5) + 2(4) = -2$ is negative.

Therefore, perimeter of the rectangle is decreasing at the rate of 2 cm/sec.

(b) Consider z denotes the area of rectangle.

$z = xy$

$\frac{dz}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$

= $8(4)+6(-5)=2$ is positive.

Therefore, Area of the rectangle is increasing at the rate of 2 cm^2/sec.

8. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.

Solution: Consider x cm be the radius of the spherical balloon at time t.

According to the question, $\frac{d}{dt}\left(\frac{4}{3}\pi x^3\right)=900$

$$\frac{4\pi}{3}\frac{d}{dt}x^3=900$$

$$\frac{4\pi}{3}.3x^2\frac{dx}{dt}=900$$

$$4\pi x^2=\frac{dx}{dt}=900$$

$$\frac{dx}{dt}=\frac{900}{4\pi x^2}$$

$$\frac{dx}{dt}=\frac{900}{4\pi(15)^2}$$

$$\frac{dx}{dt}=\frac{900}{4\pi(225)}$$

$$\frac{dx}{dt}=\frac{900}{900\pi}=\frac{1}{\pi}$$

Radius of balloon is increasing at the rate of $\frac{1}{\pi}$ cm sec.

9. A balloon, which always remains spherical has a variables radius. Find the rate at which its volume is increasing with the radius when the later is 10 cm.

Solution: As we know, Volume of sphere, V = $\frac{4}{3}\pi x^3$

$$\frac{dV}{dx} = \frac{d}{dx}\left(\frac{4}{3}\pi x^3\right)$$

= $\frac{4}{3}\pi .3x^2$

= $4\pi x^2$

$$\frac{dV}{dx} = 4\pi(10)^2 = 400\pi$$

Therefore, the volume is increasing at the rate of 400π cm^3/sec.

10. A ladder 5 cm long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?

Solution: Consider AB be the ladder and length of ladder is 5 cm. So AB = 5 cm.
Let C is the junction of wall and ground, let CA = x meters, CB = y meters

So, according to the equation: As x increases, y decreases

and $\frac{dx}{dt}$ = 2 cm/s

In ΔABC,
$AC^2 + BC^2 = AB^2$ [Using Pythagoras theorem]
$x^2 + y^2 = 25$ ……….(1)

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2x(2) + 2y\frac{dy}{dt} = 0$$

$2y\frac{dy}{dt}=-4x$

$\frac{dy}{dt}=\frac{-2x}{y}$ ……….(2)

When $x=4,\ 16+y^2=25$

$y=3$ [From equation(1)]

From equation (2), $\frac{dy}{dt}=\frac{-2\times 4}{3}=\frac{-8}{3}$ cm/s

11. A particle moves along the curve 6y = x^3 + 2 , Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

Solution: Equation of the curve, 6y = x^3 + 2 ……….(1)
Consider (x, y) be the required point on curve (1)

As per the given statements, $\frac{dy}{dx}=8$ ……….(2)

From equation (1), $6\frac{dy}{dx}=3x^2$

$6\times 8=3x^2$ [From equation (2)]

$x^2=\frac{6\times 8}{3}$

$x=\pm 4$ (two values of x)

When $x=4,$

$6y=64+2$

$y=11$

Required point is (4, 11).

When $x=-4,$

$6y=-64+2$

$y = \frac{-31}{3}$

Required point is $\left(-4, \frac{-31}{3}\right)$.

12. The radius of an air bubble is increasing at the rate of 1/2 cm/s. At what rate is the volume of the bubble increasing when the radius is 1 cm?

Solution: Consider x cm be the radius of the air bubble at time t.

As per statement,

$\frac{dx}{dt}$ is positive = $\frac{1}{2}$ cm/sec ……….(1)

Volume of air bubble $(z) = \frac{4\pi}{3}x^3$

$\Rightarrow \frac{dz}{dt} = \frac{4\pi}{3}\frac{d}{dt}x^3$

$= \frac{4\pi}{3}.3x^2\frac{dx}{dt}$

$= 4\pi x^2\left(\frac{1}{2}\right)$

$\Rightarrow \frac{dz}{dt} = 2\pi x^2$

$= 2\pi(1)^2 = 2\pi$

Therefore, required rate of increase of volume of air bubble is 2π cm^3/sec.

13. A balloon which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$. Find the rate of change of its volume with respect to x.

Solution: Given: Diameter of the balloon = $\frac{3}{2}(2x+1)$

And, Radius of the balloon = $\frac{3}{4}(2x+1)$

So, Volume of the balloon = $\frac{4}{3}\pi\left(\frac{3}{4}(2x+1)\right)^3$

= $\frac{9\pi}{16}(2x+1)^3$ cubic units

Now, Rate of change of volume w.r.t. x = $\frac{dV}{dx}$

= $\frac{9\pi}{16}.3(2x+1)^2.\frac{d}{dx}(2x+x)$

= $\frac{27\pi}{16}(2x+1)^2.2$

= $\frac{27\pi}{8}(2x+1)^2$

14. Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

Solution: Consider the height and radius of the sand-cone formed at time t second be y cm and x cm respectively.

As per the given statement, $y=\frac{1}{6}x$

$\Rightarrow x=6y$

Volume of cone (V) = $\frac{1}{3}\pi x^2 y$

= $\frac{1}{3}\pi(6y)^2 y$

= $12\pi y^3$

$\Rightarrow \frac{dV}{dy} = 36\pi y^2$

Now, As, $\frac{dV}{dt} = 12$

$\Rightarrow \frac{dV}{dy} \times \frac{dy}{dt} = 12$

$\Rightarrow 36\pi y^2 \times \frac{dy}{dt} = 12$

$\Rightarrow \frac{dy}{dt} = \frac{1}{3\pi y^2}$

$\Rightarrow \frac{dy}{dt} = \frac{1}{3\pi 4^2} = \frac{1}{48\pi}$ cm/sec

15. The total cost C(x) in rupees associated with the production of x units of an item given by $C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000$.
Find the marginal cost when 17 units are produced.

Solution: Marginal cost = $\frac{dC}{dx}$

$= \frac{d}{dx}\left(0.007x^3 - 0.003x^2 + 15x + 4000\right)$

$= 0.021x^2 - 0.006x + 15$

Now, when $x = 17$, MC is

$= 0.021(17)^2 - 0.006 \times 17 + 15$

= 6.069 – 0.102 + 15 = 20.967

Therefore, required Marginal cost is Rs. 20.97.

16. The total revenue in rupees received from the sale of x units of a product is given by $R(x) = 13x^2 + 26x + 15$.
Find the marginal revenue when x = 7.

Solution: Marginal Revenue (MR) = $\frac{dR}{dx}$

= $\frac{d}{dx}\left(13x^2+26x+15\right)$

= $26x+26$

Now, when x = 7, MR is

= 26 x 7 + 26 = 208

Therefore, the required marginal revenue is Rs. 208.

Choose the correct answer in Exercises 17 and 18.

17. The rate of change of the area of a circle with respect to its radius r at **r** = **6 cm is:**

(A) 10π

(B) 12π

(C) 8π

(D) 11π

Solution:

Option (B) is correct.

Area of circle (A) = πr^2

$\Rightarrow \frac{dA}{dr} = 2\pi r$

= $2\pi \times 6 = 12\pi$

18. The total revenue in Rupees received from the sale of x units of a product is given by

R(x) = 3x^2 + 36x + 5 . The marginal revenue, when x = 15 is:

(A) 116 **(B) 96** **(C) 90** **(D) 126**

Solution: Option (D) is correct.

Total revenue R(x) = 3x^2 + 36x + 5

Marginal revenue = $\frac{d}{dx}R\left(x\right)=6x+36$ = 6 x 15 + 36 = 126