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1. Using differentials, find the approximate value of each of the following up to 3 places of decimal:

(i) $\sqrt{25.3}$

(ii) $\sqrt{49.5}$

(iii) $\sqrt{0.6}$

(iv) $(0.009)^{1/3}$

(v) $(0.999)^{1/10}$

(vi) $(15)^{1/4}$

(vii) $(26)^{1/3}$

(viii) $(255)^{1/4}$

(ix) $(82)^{1/4}$

(x) $(401)^{1/2}$

(xi) $(0.0037)^{1/2}$

(xii) $(26.57)^{1/3}$

(xiii) $(81.5)^{1/4}$

(xiv) $(3.968)^{3/2}$

(xv) $(32.15)^{1/5}$

Solution:

(i) $\sqrt{25.3}$

Consider, $y=\sqrt{x}$ ……….(1) and then

$$y+\Delta y=\sqrt{x+\Delta x}$$

On differentiating equation (1) w.r.t. x, we get

$\frac{dy}{dx} = \frac{1}{2}x^{\frac{-1}{2}} = \frac{1}{2\sqrt{x}}$

$\Rightarrow dy = \frac{dx}{2\sqrt{x}}$ ……….(2)

Now, given expression can be written as,

$\sqrt{25.3} = \sqrt{25 + 0.3}$

Here, $x = 25$ and $\Delta x = 0.3$, then $\Delta y = \sqrt{x + \Delta x} - \sqrt{x}$

$= \sqrt{25.3} - \sqrt{25} = \sqrt{25.3} - 5$

$\Rightarrow \sqrt{25.3} = \Delta y + 5$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (2),

$dy = \frac{0.3}{2\sqrt{25}}$

= 0.03

Hence, approximately value of $\sqrt{25.3}$ is 5 + 0.03 = 5.03.

(ii) $\sqrt{49.5}$

Consider, $y = \sqrt{x}$ ……….(1)

On differentiating equation (1) w.r.t. x, we get

$\frac{dy}{dx} = \frac{1}{2}x^{\frac{-1}{2}} = \frac{1}{2\sqrt{x}}$

$\Rightarrow dy = \frac{dx}{2\sqrt{x}}$ ……….(2)

Now, from equation (1), $y + \Delta y = \sqrt{x + \Delta x}$

= $\sqrt{49.5} = \sqrt{49 + 0.5}$

Here, $x = 49$ and $\Delta x = 0.5$, then

$\Delta y = \sqrt{x + \Delta x} - \sqrt{x}$

= $\sqrt{49.5} - \sqrt{49} = \sqrt{49.5} - 7$

$\Rightarrow \sqrt{49.5} = \Delta y + 7$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (2), $dy = \dfrac{0.5}{2\sqrt{49}}$

= 0.0357

So, approximately value of $\sqrt{49.5}$ is 7 + 0.0357 = 7.0357.

(iii) $\sqrt{0.6}$

Consider, $y = \sqrt{x}$ ……….(1)

On differentiating equation (1) w.r.t. x, we get

$\dfrac{dy}{dx} = \dfrac{1}{2}x^{\frac{-1}{2}}$ = $\dfrac{1}{2\sqrt{x}}$

$dy = \dfrac{dx}{2\sqrt{x}}$ ……….(2)

Now, from equation (1), $y + \Delta y = \sqrt{x + \Delta x}$

= $\sqrt{0.6} = \sqrt{0.64 - 0.04}$

Here, $x = 0.64$ and $\Delta x = -0.04$, then

$\Delta y = \sqrt{x + \Delta x} - \sqrt{x}$

= $\sqrt{0.6}-\sqrt{0.64}=\sqrt{0.6}-0.8$

$\sqrt{0.6}=\Delta y+0.8$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (2), $dy=\frac{-0.04}{2\sqrt{0.64}}$ = -0.025

Therefore, approximately value of $\sqrt{0.6}$ is 0.8 – 0.025 = 0.775.

(iv) $(0.009)^{\frac{1}{3}}$

Consider, $y=\sqrt[3]{x}$ ……….(1)

On differentiating equation (1) w.r.t. x, we get

$\frac{dy}{dx}=\frac{1}{3}x^{\frac{-2}{3}}$ = $\frac{1}{3x^{2/3}}$

$dy=\frac{dx}{3x^{2/3}}=\frac{dx}{3\left(x^{1/3}\right)^2}$ ……….(2)

Now, from equation (1), $y+\Delta y=\sqrt{x+\Delta x}$

= $(0.009)^{\frac{1}{3}}=(0.008+0.001)^{\frac{1}{3}}$

Here, $x=0.008$ and $\Delta x=0.001$,

then $\Delta y=\sqrt{x+\Delta x}-\sqrt{x}$

= $(0.009)^{\frac{1}{3}}-(0.008)^{\frac{1}{3}}=(0.009)^{\frac{1}{3}}-0.2$

$(0.009)^{\frac{1}{3}}=\Delta y+0.2$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (2), $dy = \frac{0.001}{3\left((0.008)^{\frac{1}{3}}\right)^2}$

$= \frac{0.001}{3 \times 0.04}$ = 0.0083

Therefore, approximately value of $(0.009)^{\frac{1}{3}}$ is 0.2 + 0.0083 = 0.2083.

(v) $(0.999)^{\frac{1}{10}}$

Consider, $y = x^{\frac{1}{10}}$ ……….(1)

On differentiating equation (1) w.r.t. x, we get

$\frac{dy}{dx} = \frac{1}{10} x^{\frac{-9}{10}} = \frac{1}{10x^{\frac{9}{10}}}$

$dy = \frac{dx}{10\left(x^{\frac{1}{10}}\right)^9}$ ……….(2)

Now, from equation (1), $y + \Delta y = \sqrt{x + \Delta x}$

$= (0.999)^{\frac{1}{10}} = (1 - 0.001)^{\frac{1}{10}}$ ……….(3)

Here $x = 1$ and $\Delta x = -0.001$

Then $\Delta y = (x + \Delta x)^{\frac{1}{10}} - x^{\frac{1}{10}}$

$= (0.999)^{\frac{1}{10}} - 1$

$\Rightarrow (0.999)^{\frac{1}{10}} = 1 + \Delta y$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (2),

$$dy = \frac{-0.001}{10\left(1^{\frac{1}{10}}\right)^9} = -0.0001$$

Therefore, approximate value of $(0.999)^{\frac{1}{10}}$ is 1 – 0.0001 = 0.9999.

(vi) $(15)^{\frac{1}{4}}$

Consider, $y = x^{\frac{1}{4}}$ ……….(1)

On differentiating equation (1) w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{4}x^{\frac{-3}{4}} = \frac{1}{4x^{\frac{3}{4}}}$$

$$\Rightarrow dy = \frac{dx}{4\left(x^{\frac{1}{4}}\right)^3}$$ ……….(2)

Now, from equation (1), $y + \Delta y = \sqrt{x + \Delta x}$

= $(15)^{\frac{1}{4}}$ = $(16-1)^{\frac{1}{4}}$ ……….(3)

Here $x = 16$ and $\Delta x = -1$

Then $\Delta y = (x + \Delta x)^{\frac{1}{4}} - x^{\frac{1}{4}}$

= $(15)^{\frac{1}{4}} - 16^{\frac{1}{4}} = (15)^{\frac{1}{4}} - 2$

$\Rightarrow (15)^{\frac{1}{4}} = 2 + \Delta y$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (2), $dy = \frac{-1}{4\left(16^{\frac{1}{4}}\right)^3} = \frac{-1}{32}$

Therefore, approximate value of $(15)^{\frac{1}{4}}$ is $2 - \frac{1}{32} = \frac{63}{32}$ = 1.96875.

(vii) $(26)^{\frac{1}{3}}$

Consider, $y = \sqrt[3]{x}$ ……….(1)

On differentiating equation (1) w.r.t. x, we get

$\frac{dy}{dx} = \frac{1}{3}x^{\frac{-2}{3}} = \frac{1}{3x^{2/3}}$

$dy = \frac{dx}{3x^{2/3}} = \frac{dx}{3\left(x^{1/3}\right)^2}$ ……….(2)

Now, from equation (1), $y + \Delta y = \sqrt{x + \Delta x}$

$= (26)^{\frac{1}{3}} = (27-1)^{\frac{1}{3}}$

Here, $x = 27$ and $\Delta x = -1$,

then $\Delta y = \sqrt{x+\Delta x} - \sqrt{x} = (26)^{\frac{1}{3}} - (27)^{\frac{1}{3}} = (26)^{\frac{1}{3}} - 3$

$(26)^{\frac{1}{3}} = \Delta y + 3$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (2),

$dy = \frac{-1}{3\left((27)^{\frac{1}{3}}\right)^2}$

$= \frac{-1}{27}$

Therefore, approximately value of $(26)^{\frac{1}{3}}$ is $3 - \frac{1}{27} = \frac{80}{27}$ = 2.9629.

(viii) $(255)^{\frac{1}{4}}$

Consider, $y = x^{\frac{1}{4}}$ ……….(1)

On differentiating equation (1) w.r.t. x, we get

$\frac{dy}{dx} = \frac{1}{4}x^{\frac{-3}{4}}$ = $\frac{1}{4x^{\frac{3}{4}}}$

$dy = \frac{dx}{4\left(x^{\frac{1}{4}}\right)^3}$ ……….(2)

Now, from equation (1), $y + \Delta y = \sqrt{x + \Delta x}$

= $(15)^{\frac{1}{4}}$ = $(256-1)^{\frac{1}{4}}$ ……….(3)

Here $x = 256$ and $\Delta x = -1$

Then $\Delta y = (x + \Delta x)^{\frac{1}{4}} - x^{\frac{1}{4}}$

= $(255)^{\frac{1}{4}} - (256)^{\frac{1}{4}} = (255)^{\frac{1}{4}} - 4$

$(255)^{\frac{1}{4}} = 4 + \Delta y$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (2), $dy = \frac{-1}{4\left(256^{\frac{1}{4}}\right)^3} = \frac{-1}{256}$

Therefore, approximate value of $(255)^{\frac{1}{4}}$ is $4 - \frac{1}{256} = \frac{1023}{256}$ = 3.9961.

(ix) $(82)^{\frac{1}{4}}$

Consider, $y = x^{\frac{1}{4}}$ ……….(1)

On differentiating equation (1) w.r.t. x, we get

$\frac{dy}{dx} = \frac{1}{4} x^{\frac{-3}{4}}$

$= \frac{1}{4x^{\frac{3}{4}}}$

$dy = \frac{dx}{4\left(x^{\frac{1}{4}}\right)^3}$ ……….(2)

Now, from equation (1), $y + \Delta y = \sqrt{x + \Delta x} = (82)^{\frac{1}{4}} = (81+1)^{\frac{1}{4}}$ ……….(3)

Here $x = 81$ and $\Delta x = 1$

then $\Delta y = (x + \Delta x)^{\frac{1}{4}} - x^{\frac{1}{4}}$

$= (82)^{\frac{1}{4}} - (81)^{\frac{1}{4}} = (82)^{\frac{1}{4}} - 3$

$(82)^{\frac{1}{4}} = 3 + \Delta y$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (2), $dy = \frac{1}{4\left(81^{\frac{1}{4}}\right)^3} = \frac{1}{108}$

Therefore, approximate value of $(82)^{\frac{1}{4}}$ is $3 + \frac{1}{108} = \frac{325}{108}$ = 3.0092.

(x) $\sqrt{401}$

Consider, $y = \sqrt{x}$ ……….(1)

On differentiating equation (1) w.r.t. x, we get

$\frac{dy}{dx} = \frac{1}{2}x^{\frac{-1}{2}} = \frac{1}{2\sqrt{x}}$

$dy = \frac{dx}{2\sqrt{x}}$ ……….(2)

Now, from equation (1), $y + \Delta y = \sqrt{x + \Delta x}$

$= \sqrt{401} = \sqrt{400 + 1}$

Here, $x = 400$ and $\Delta x = 1$, then $\Delta y = \sqrt{x + \Delta x} - \sqrt{x}$

$= \sqrt{401} - \sqrt{400} = \sqrt{401} - 20$

$\Rightarrow \sqrt{401} = \Delta y + 20$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (2), $dy = \frac{1}{2\sqrt{400}} = \frac{1}{40}$

Therefore, approximately value of $\sqrt{401}$ is $20+\frac{1}{40}=\frac{801}{40}$ = 20.025.

(xi) $\sqrt{0.0037}$

Consider, $y=\sqrt{x}$ ……….(1)

On differentiating equation (1) w.r.t. x, we get

$$\frac{dy}{dx}=\frac{1}{2}x^{\frac{-1}{2}} = \frac{1}{2\sqrt{x}}$$

$$dy=\frac{dx}{2\sqrt{x}}$$ ……….(2)

Now, from equation (1), $y+\Delta y=\sqrt{x+\Delta x}$

$= \sqrt{0.0037}=\sqrt{0.0036+0.0001}$

Here, $x=0.0036$ and $\Delta x=0.0001$, then

$$\Delta y=\sqrt{x+\Delta x}-\sqrt{x}$$

$= \sqrt{0.0037}-\sqrt{0.0036}=\sqrt{0.0037}-0.06$

$$\sqrt{0.0037}=\Delta y+0.06$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (2), $dy=\frac{0.0001}{2\sqrt{0.0036}}$

$= \frac{0.0001}{0.12}$

Therefore, approximately value of $\sqrt{0.0037}$ is 0.06 + $\frac{0.0001}{0.12}$ = 0.060833.

(xi) $(26.57)^{\frac{1}{3}}$

Consider, $y=\sqrt[3]{x}$ ……….(1)

On differentiating equation (1) w.r.t. x, we get

$$\frac{dy}{dx}=\frac{1}{3}x^{\frac{-2}{3}} = \frac{1}{3x^{2/3}}$$

$$dy=\frac{dx}{3x^{2/3}}=\frac{dx}{3\left(x^{1/3}\right)^2}$$ ……….(2)

Now, from equation (1), $y+\Delta y=\sqrt{x+\Delta x}$

$= (26.57)^{\frac{1}{3}}=(27-0.43)^{\frac{1}{3}}$

Here, $x=27$ and $\Delta x=-0.43$,

then $\Delta y=\sqrt{x+\Delta x}-\sqrt{x}$

$= (26.57)^{\frac{1}{3}}-(27)^{\frac{1}{3}}=(26.57)^{\frac{1}{3}}-3$

$(26.57)^{\frac{1}{3}}=\Delta y+3$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (2), $$dy=\frac{-0.43}{3\left((27)^{\frac{1}{3}}\right)^2}$$

$= \frac{-0.43}{27}$ = 0.0159

Therefore, approximately value of $(26.57)^{\frac{1}{3}}$ is 3 – 0.0159 = 2.9841.

(xii) $(81.5)^{\frac{1}{4}}$

Consider, $y = x^{\frac{1}{4}}$ ……….(1)

On differentiating equation (1) w.r.t. x, we get

$\frac{dy}{dx} = \frac{1}{4}x^{\frac{-3}{4}}$ = $\frac{1}{4x^{\frac{3}{4}}}$

$dy = \frac{dx}{4\left(x^{\frac{1}{4}}\right)^3}$ ……….(2)

Now, from equation (1), $y + \Delta y = \sqrt{x + \Delta x}$

= $(81.5)^{\frac{1}{4}}$ = $(81 + 0.5)^{\frac{1}{4}}$ ……….(3)

Here $x = 81$ and $\Delta x = 0.5$

Then $\Delta y = (x + \Delta x)^{\frac{1}{4}} - x^{\frac{1}{4}}$

= $(81.5)^{\frac{1}{4}} - (81)^{\frac{1}{4}} = (81.5)^{\frac{1}{4}} - 3$

$(81.5)^{\frac{1}{4}}$ = $3 + \Delta y$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (2), $dy = \frac{0.5}{4\left(81^{\frac{1}{4}}\right)^3} = \frac{0.5}{108}$

= 0.00462

Therefore, approximate value of $(82)^{\frac{1}{4}}$ is 3 + 0.00462= 3.00462.

(xiv) $(3.968)^{\frac{3}{2}}$

Consider, $y = x^{\frac{3}{2}} = x^{1+\frac{1}{2}} = x\sqrt{x}$ ……….(1)

$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$

$dy = \frac{3\sqrt{x}}{2}\ dx$ ……….(2)

Now, from equation (1), $y + \Delta y = \sqrt{x + \Delta x}$

= $(3.968)^{\frac{3}{2}} = (4 - 0.032)^{\frac{3}{2}}$

Here, $x = 4$ and $\Delta x = 0.032$, then $\Delta y = \sqrt{x + \Delta x} - \sqrt{x}$

= $(3.968)^{\frac{3}{2}} - (4)^{\frac{3}{2}} = (3.968)^{\frac{3}{2}} - 8$

$(3.968)^{\frac{3}{2}} = \Delta y + 8$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (2),

$dy = \frac{3}{2}\sqrt{4}(-0.032)$ = -0.096

Therefore, approximately value of $(3.968)^{\frac{3}{2}}$ is 8 - 0.096 = 7.904.

(xv) $(32.15)^{\frac{1}{5}}$

Consider, $y = x^{\frac{1}{5}}$ ……….(1)

On differentiating equation (1) w.r.t. x, we get

$\frac{dy}{dx} = \frac{1}{5}x^{\frac{-4}{5}} = \frac{1}{5x^{\frac{4}{5}}}$

$dy = \frac{dx}{5\left(x^{\frac{1}{5}}\right)^4}$ ……….(2)

Now, from equation (1), $y + \Delta y = \sqrt{x + \Delta x}$

$= (32.15)^{\frac{1}{5}} = (32 + 0.15)^{\frac{1}{5}}$ ……….(3)

Here $x = 32$ and $\Delta x = 0.15$

Then $\Delta y = (x + \Delta x)^{\frac{1}{5}} - x^{\frac{1}{5}}$

$= (32.15)^{\frac{1}{5}} - (32)^{\frac{1}{5}} = (32.15)^{\frac{1}{5}} - 2$

$(32.15)^{\frac{1}{5}} = 2 + \Delta y$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (2),

$dy = \frac{0.15}{5\left(32^{\frac{1}{5}}\right)^4} = \frac{0.15}{80}$ = 0.001875

Therefore, approximate value of $(32.15)^{\frac{1}{5}}$ is 2 + 0.001875 = 2.001875.

2. Find the approximate value of $f(2.01)$ where $f(x) = 4x^2 + 5x + 2.$

Solution: Consider, $f(x) = y = x^3 + 5x + 2$ ……….(1)

On differentiating equation (1) w.r.t. x, we get

$f'(x)=\frac{dy}{dx}=8x+5$

$dy=(8x+5)\ dx$

= $(8x+5)\Delta x$ ……….(2)

Changing x to $x+\Delta x$ and y to $y+\Delta y$ in equation (1),

$y+\Delta y=f(x+\Delta x)$

= $f(2.01)=f(2+0.01)$ ……….(3)

Here, $x=2$ and $\Delta x=0.01$

From equation (3), $f(2.01)=y+\Delta y$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (1) and (2), we get

$f(2.01)=(4x^2+5x+2)+(8x+5)\Delta x$

Therefore,

$f(2.01)=4(4)+5(2)+2+(8\times 2+5)(0.01)$ = 28.21

Therefore, approximate value of $f(2.01)$ is 28.21.

3. Find the approximate value of $f(5.001)$ where $f(x)=x^3-7x^2+15.$

Solution: Consider, $f(x)=y=x^3-7x^2+15$ ……….(1)
On differentiating equation (1) w.r.t. x, we get

$f'(x)=\frac{dy}{dx}=3x^2-14x$

or $dy=\left(3x^2-14x\right)\ dx=\left(3x^2-14x\right)\Delta x$ ……….(2)

Changing x to $x+\Delta x$ and y to $y+\Delta y$ in equation (1),

$y+\Delta y=f\left(x+\Delta x\right)$

= $f\left(5.001\right)=f\left(5+0.001\right)$ ……….(3)

Here, $x=5$ and $\Delta x=0.001$

From equation (3), $f\left(5.001\right)=y+\Delta y$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (1) and (2),

$f\left(5.001\right)=\left(x^3-7x^2+15\right)+\left(3x^2-14x\right)\Delta x$

$f\left(5.001\right)=125-175+15+\left(75-70\right)\left(0.001\right)$

= $-35+0.005$ = -34.995

Therefore, approximate value of $f\left(5.001\right)$ is -34.995.

4. Find the approximate change in the volume of a cube of side x **meters caused by increasing the side by 1%.**

Solution:
Side of a cube is x meters then cube volume (V) is x^3.

or V = x^3 ……….(1)

On differentiating equation (1) w.r.t. x, we get

$\frac{dV}{dx}=3x^2$ ……….(2)

According to the statement, increase in side = 1% = $\frac{x}{100}$

So, $\Delta x = \frac{x}{100}$ ……….(3)

Approximate change in volume, V, of cube = $\Delta V \sim dV$

$= \frac{dV}{dx} dx$

$= \frac{dV}{dx} \Delta x$

$\sim 3x^2\left(\frac{x}{100}\right) \sim \frac{3}{100}x^3 \sim 0.03x^3$ cubic meters

5. Find the approximate change in the surface area of a cube of side x meters caused by decreasing the side by 1%.

Solution:

Side of a cube is x meters then Surface area of a cube is (S) = $6x^2$

S = $6x^2$

On differentiating above equation w.r.t. x, we get

$\frac{dS}{dx} = 12x$

At per question, if decrease in side 1% is

= -1% of x

= -0.01 x

$\Delta x = -0.01x$

Since approximate change in surface area = $\Delta S \sim dS$ = $\frac{dS}{dx} dx$

We have,

$= \frac{dS}{dx}\Delta x \sim 12x(-0.01x) \sim -0.12x^2$ square meters (decreasing)

6. If the radius of a sphere is measured as 7 m with an error of 0.02 m, then find the approximate error in calculating its volume.

Solution: Consider, r be the radius of the sphere and Δr be the error.
then, as per question, r = 7 m and Δr = 0.02 m

We know that, Volume of sphere (V) = $\frac{4}{3}\pi r^3$

$$\frac{dV}{dr} = \frac{4}{3}\pi.3r^2$$

Approximate error in calculating the volume = Approximate value of ΔV

$$dV = \frac{dV}{dr}(dr)$$

$$= \left(\frac{4}{3}\pi.3r^2\right) dr$$

$$= 4\pi(7)^2(0.02)$$

$$= 3.92\times\frac{22}{7}$$

= 12.32 m^3

Therefore, the approximate error in calculating volume is 12.32 m^3.

7. If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating its surface area.

Solution: Consider, r be the radius of the sphere.
And, Surface area of the sphere (S) = $4\pi r^2$ (formula for SA)

$\frac{dS}{dr} = 8\pi r$

$dS = 8\pi r\ dr$

$dS = 8\pi r\ \Delta r$

$dS = 8\pi(9)(0.3)$

= 2.16π square meters

8. If $f(x) = 3x^2 + 15x + 5,$ then the approximate value of f(3.02) is:

(A) 47.66 (B) 57.66 (C) 67.66 (D) 77.66

Solution:

Option (D) is correct.

Explanation:

Consider, $f(x) = y = 3x^2 + 15x + 5$ ……….(1)

On differentiating equation (1) w.r.t. x, we get

$f'(x) = \frac{dy}{dx} = 6x + 15$

or $dy = (6x+15)\ dx = (6x+15)\Delta x$ ……….(2)

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in equation (1),

$y + \Delta y = f(x + \Delta x)$

= $f(3.02) = f(3 + 0.02)$ ……….(3)

Here, $x = 3$ and $\Delta x = 0.02$

So,

From equation (3), $f(3.02) = y + \Delta y$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (1) and (2),

$$f(3.02) = (3x^2 + 15x + 5) + (6x + 15)\Delta x$$

$$f(3.02) = 3(9) + 15(3) + 5 + (6 \times 3 + 15)(0.02)$$

= $77 + 0.66$ = 77.66

9. The approximate change in the volume of a cube of side x meters caused by increasing the side by 3% is:

(A) 0.06 x^3 m^3 (B) 0.6 x^3 m^3 (C) 0.09 x^3 m^3 (D) 0.9 x^3 m^3

Solution: option (C) is correct.

Explanation:

We know that, Volume (V) = x^3 ……….(1)

$\frac{dV}{dx} = 3x^2$ ……….(2)

As there is increase in side = 3% = $\frac{3x}{100}$

We have, $\Delta x = \frac{3x}{100}$ ……….(3)

Since approximate change in volume V of cube = $\Delta V \sim dV$ = $\frac{dV}{dx}dx$

= $\frac{dV}{dx}\Delta x$

$\sim$ $3x^2\left(\frac{3x}{100}\right) \sim \frac{9}{100}x^3 \sim 0.09x^3$ cubic meters