## Exercise 1.1 Page No: 5

**1. Determine whether each of the following relations are reflexive, symmetric and transitive:**
**(i) Relation R in the set A = {1, 2, 3,…….., 13, 14} defined as**
**R = {(x, y) : 3x – y = 0}**

**(ii) Relation R in the set N of natural numbers defined as**
**R = {(x, y) : y = x + 5 and x < 4}**

**(iii) Relation R in the set A = {1, 2, 3, 4, 5, 6} as**
**R = {(x, y) : y is divisible by x}**

**(iv) Relation R in the set Z of all integers defined as**
**R = {(x, y) : x – y is an integer}**

**(v) Relation R in the set A of human beings in a town at a particular time given by**

**(a) R = {(x, y) : x and y work at the same place}**
**(b) R = {(x, y) : x and y live in the same locality}**
**(c) R = {(x, y) : x is exactly 7 cm taller than y}**
**(d) R = {(x, y) : x is wife of y}**
**(e) R = {(x, y) : x is father of y}**

**Solution:**
(i)R = {(x, y) : 3x - y = 0}

A = {1, 2, 3, 4, 5, 6, ……13, 14}

Therefore, **R = {(1, 3), (2, 6), (3, 9), (4, 12)}** …(1)

As per reflexive property: (x, x) ∈ R, then R is reflexive)
Since there is no such pair, so R is not reflexive.

As per symmetric property: (x, y) ∈ R and (y, x) ∈ R, then R is symmetric.
Since there is no such pair, R is not symmetric

As per transitive property: If (x, y) ∈ R and (y, z) ∈ R, then (x, z) ∈ R. Thus R is transitive.

From (1), (1, 3) ∈ R and (3, 9) ∈ R but (1, 9) ∉ R, R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

(ii) R = {(x, y) : y = x + 5 and x < 4} in set N of natural numbers.

Values of x are 1, 2, and 3

So, **R = {(1, 6), (2, 7), (3, 8)}**

As per reflexive property: (x, x) ∈ R, then R is reflexive)

Since there is not such pair, R is not reflexive.

As per symmetric property: (x, y) ∈ R and (y, x) ∈ R, then R is symmetric.

Since there is no such pair, so R is not symmetric

As per transitive property: If (x, y) ∈ R and (y, z) ∈ R, then (x, z) ∈ R. Thus R is transitive.

Since there is no such pair, so R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

(iii) R = {(x, y) : y is divisible by x} in A = {1, 2, 3, 4, 5, 6}

From above we have,

R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (5, 5), (6, 6)}

As per reflexive property: (x, x) ∈ R, then R is reflexive.

(1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6) ∈ R . Therefore, R is reflexive.

As per symmetric property: (x, y) ∈ R and (y, x) ∈ R, then R is symmetric.

(1, 2) ∈ R but (2, 1) ∉ R. So R is not symmetric.

As per transitive property: If (x, y) ∈ R and (y, z) ∈ R, then (x, z) ∈ R. Thus R is transitive.

Also (1, 4) ∈ R and (4, 4) ∈ R and (1, 4) ∈ R, So R is transitive.

Therefore, R is reflexive and transitive but nor symmetric.

(iv) R = {(x, y) : x - y is an integer} in set Z of all integers.

Now, (x, x), say (1, 1) = x – y = 1 – 1 = 0 $\in$ Z => R is reflexive.

(x, y) $\in$ R and (y, x) $\in$ R, i.e.,
x – y and y – x are integers => R is symmetric.

(x, y) $\in$ R and (y, z) $\in$ R, then (x, z) $\in$ R i.e.,

x – y and y – z and x - z are integers.

(x, z) $\in$ R => R is transitive

Therefore, R is reflexive, symmetric and transitive.

(v)
(a) R = {(x, y) : x and y work at the same place}
For reflexive: x and x can work at same place
(x, x) $\in$ R
R is reflexive.

For symmetric: x and y work at same place so y and x also work at same place.
(x, y) $\in$ R and (y, x) $\in$ R
R is symmetric.

For transitive: x and y work at same place and y and z work at same place, then x and z also work at same place.

(x, y) $\in$ R and (y, z) $\in$ R then (x, z) $\in$ R
R is transitive

Therefore, R is reflexive, symmetric and transitive.

(b) R = {(x, y) : x and y live in the same locality}

(x, x) $\in$ R => R is reflexive.

(x, y) $\in$ R and (y, x) $\in$ R => R is symmetric.

Again,
(x, y) $\in$ R and (y, z) $\in$ R then (x, z) $\in$ R => R is transitive.

Therefore, R is reflexive, symmetric and transitive.

(c) R = {(x, y) : x is exactly 7 cm taller than y}

x can not be taller than x, so R is not reflexive.

x is taller than y then y can not be taller than x, so R is not symmetric.

Again, x is 7 cm taller than y and y is 7 cm taller than z, then x can not be 7 cm taller than z, so R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

(d) R = {(x, y) : x is wife of y}

x is not wife of x, so R is not reflexive.

x is wife of y but y is not wife of x, so R is not symmetric.

Again, x is wife of y and y is wife of z then x can not be wife of z, so R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

(e) R = {(x, y) : x is father of y}

x is not father of x, so R is not reflexive.

x is father of y but y is not father of x, so R is not symmetric.

Again, x is father of y and y is father of z then x cannot be father of z, so R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

**2. Show that the relation R in the set R of real numbers, defined as R = {(a, b) : $a \le b^2$} is neither reflexive nor symmetric nor transitive.**

**Solution:**
R = {(a, b) : $a \le b^2$} , Relation R is defined as the set of real numbers.

(a, a) ∈ R then $a \le a^2$ , which is false. R is not reflexive.

(a, b)=(b, a) ∈ R then $a \le b^2$ and $b \le a^2$, it is false statement. R is not symmetric.

Now, $a \le b^2$ and $b \le c^2$,then $a \le c^2$ , which is false. R is not transitive

Therefore, R is neither reflexive, nor symmetric and nor transitive.

**3. Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as R = {(a, b) : b = a + 1} is reflexive, symmetric or transitive.**

**Solution:** R = {(a, b) : b = a + 1}

R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)}

When b = a, a = a + 1: which is false, So R is not reflexive.

If (a, b) = (b,a), then b = a+1 and a =b+1: Which is false, so R is not symmetric.

Now, if (a, b), (b,c) and (a, c) belongs to R then
b = a+1 and c =b+1 which implies c = a + 2: Which is false, so R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

**4. Show that the relation R in R defined as R = {(a, b) : a ≤ b}, is reflexive and transitive but not symmetric.**

**Solution:**
a ≤ a: which is true, (a, a) ∈ R, So R is reflexive.

a ≤ b but b ≤ a (false): (a, b) ∈ R but (b, a) ∉ R, So R is not symmetric.

Again, a ≤ b and b ≤ c then a ≤ c : (a, b) ∈ R and (b, c) and (a, c) ∈ R, So R is transitive.

Therefore, R is reflexive and transitive but not symmetric.

**5. Check whether the relation R in R defined by R = {(a, b) : $a \le b^3$} is reflexive, symmetric or transitive.**

**Solution:** R = {(a, b) : $a \le b^3$}

$a \le a^3$: which is true, (a, a) ∉ R, So R is not reflexive.

$a \le b^3$ but $b \le a^3$ (false): (a, b) ∈ R but (b, a) ∉ R, So R is not symmetric.

Again, $a \le b^3$ and $b \le c^3$ then $a \le c^3$ (false) : (a, b) ∈ R and (b, c) ∈ R and (a, c) ∉ R, So R is transitive.

Therefore, R is neither reflexive, nor transitive and nor symmetric.

**6. Show that the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} is symmetric but neither reflexive nor transitive.**

**Solution:**

R = {(1, 2), (2, 1)}

(x, x) ∉ R. R is not reflexive.

(1, 2) ∈ R and (2,1) ∈ R. R is symmetric.

Again, (x, y) ∈ R and (y, z) ∈ R then (x, z) does not imply to R. R is not transitive.

Therefore, R is symmetric but neither reflexive nor transitive.

**7. Show that the relation R in the set A of all the books in a library of a college, given by R = {(x, y) : x and y have same number of pages} is an equivalence relation.**

**Solution:**
Books x and x have same number of pages. (x, x) ∈ R. R is reflexive.

If (x, y) ∈ R and (y, x) ∈ R, so R is symmetric.
Because, Books x and y have same number of pages and Books y and x have same number of pages.

Again, (x, y) ∈ R and (y, z) ∈ R and (x, z) ∈ R. R is transitive.

Therefore, R is an equivalence relation.

**8. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is even}, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}.**

**Solution:**
A = {1, 2, 3, 4, 5} and R = {(a, b) : |a – b| is even}

We get, R = {(1, 3), (1, 5), (3, 5), (2, 4)}

For (a, a), |a – b| = |a – a| = 0 is even. Therfore, R is reflexive.

If |a – b| is even, then |b – a| is also even. R is symmetric.

Again, if |a – b| and |b – c| is even then |a – c| is also even. R is transitive.

Therefore, R is an equivalence relation.

(b) We have to show that, Elements of {1, 3, 5} are related to each other.

$|1 – 3| = 2$
$|3 – 5| = 2$
$|1 - 5| = 4$
All are even numbers.

Elements of {1, 3, 5} are related to each other.

Similarly, $|2 - 4| = 2$ (even number), elements of (2, 4) are related to each other.

Hence no element of {1, 3, 5} is related to any element of {2, 4}.

**9. Show that each of the relation R in the set A = {x ∈ Z : 0 ≤ x ≤ 12}, given by**
**(i) R = {(a, b) : |a – b| is a multiple of 4}**
**(ii) R = {(a, b) : a = b}**
**is an equivalence relation. Find the set of all elements related to 1 in each case.**

**Solution:**

(i) A = {x ∈ Z : 0 ≤ x ≤ 12}
So, A = {0, 1, 2, 3, ………, 12}

Now R = {(a, b) : |a – b| is a multiple of 4}

R = {(4, 0), (0, 4), (5, 1), (1, 5), (6, 2), (2, 6), ....., (12, 9), (9, 12),...., (8, 0), (0, 8), ...., (8, 4), (4, 8),....., (12, 12)}

Here, (x, x) = |4-4| = |8-8|= |12-12| = 0 : multiple of 4.

R is reflexive.

|a – b| and |b – a| are multiple of 4. (a, b) ∈ R and (b, a) ∈ R.

R is symmetric.

And |a – b| and |b – c| then |a - c| are multiple of 4. (a, b) ∈ R and (b, c) ∈ R and (a, c) ∈ R
R is transitive.

Hence R is an equivalence relation.

(ii) Here, (a, a) = a = a.

(a, a) $\in$ R . So R is reflexive.

a = b and b = a. (a, b) $\in$ R and (b, a) $\in$ R.

R is symmetric.

And a = b and b = c then a = c. (a, b) $\in$ R and (b, c) $\in$ R and (a, c) $\in$ R
R is transitive.

Hence R is an equivalence relation.

Now set of all elements related to 1 in each case is

(i) Required set = {1, 5, 9}
(ii) Required set = {1}

**10. Give an example of a relation. Which is**
**(i) Symmetric but neither reflexive nor transitive.**
**(ii) Transitive but neither reflexive nor symmetric.**
**(iii) Reflexive and symmetric but not transitive.**
**(iv) Reflexive and transitive but not symmetric.**
**(v) Symmetric and transitive but not reflexive.**

**Solution:**
(i)Consider a relation R = {(1, 2), (2, 1)} in the set {1, 2, 3}

(x, x) $\notin$ R. R is not reflexive.

(1, 2) $\in$ R and (2,1) $\in$ R. R is symmetric.

Again, (x, y) $\in$ R and (y, z) $\in$ R then (x, z) does not imply to R. R is not transitive.

Therefore, R is symmetric but neither reflexive nor transitive.

(ii) Relation R = {(a, b): a > b}

a > a (false statement).
Also a > b but b > a (false statement) and
If a > b but b > c, this implies a > c

Therefore, R is transitive, but neither reflexive nor symmetric.

(iii) R = {a, b): a is friend of b}

a is friend of a. R is reflexive.

Also a is friend of b and b is friend of a. R is symmetric.

Also if a is friend of b and b is friend of c then a cannot be friend of c. R is not transitive.

Therefore, R is reflexive and symmetric but not transitive.

(iv) Say R is defined in R as R = {(a, b) : a ≤ b}

a ≤ a: which is true, (a, a) ∈ R, So R is reflexive.

a ≤ b but b ≤ a (false): (a, b) ∈ R but (b, a) ∉ R, So R is not symmetric.

Again, a ≤ b and b ≤ c then a ≤ c : (a, b) ∈ R and (b, c) and (a, c) ∈ R, So R is transitive.

Therefore, R is reflexive and transitive but not symmetric.

(v)R = {(a, b): a is sister of b} (suppose a and b are female)

a is not sister of a. R is not reflexive.

a is sister of b and b is sister of a. R is symmetric.

Again, a is sister of b and b is sister of c then a is sister of c.

Therefore, R is symmetric and transitive but not reflexive.

**11. Show that the relation R in the set A of points in a plane given by R = {(P, Q) : distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further, show that the set of all points related to a point P ≠ (0, 0) is the circle passing through P with origin as centre.**

**Solution:** R = {(P, Q): distance of the point P from the origin is the same as the distance of the point Q from the origin}

Say "O" is origin Point.
Since the distance of the point P from the origin is always the same as the distance of the same point P from the origin.
OP = OP

So (P, P) R. R is reflexive.

Distance of the point P from the origin is the same as the distance of the point Q from the origin

OP = OQ then OQ = OP
R is symmetric.

Also OP = OQ and OQ = OR then OP = OR. R is transitive.

Therefore, R is an equivalent relation.

**12. Show that the relation R defined in the set A of all triangles as R = {($T_1$, $T_2$) : $T_1$ is similar to $T_2$}, is equivalence relation. Consider three right angle triangles $T_1$ with sides 3, 4, 5, $T_2$ with sides 5, 12, 13 and $T_3$ with sides 6, 8, 10. Which triangles among $T_1$, $T_2$ and $T_3$ are related?**

**Solution:**
**Case I:**
$T_1$, $T_2$ are triangle.

R = {($T_1$, $T_2$): $T_1$ is similar to $T_2$}

**Check for reflexive:**

As We know that each triangle is similar to itself, so ($T_1$, $T_1$) ∈ R
R is reflexive.

**Check for symmetric:**

Also two triangles are similar, then $T_1$ is similar to $T_2$ and $T_2$ is similar to $T_1$, so ($T_1$, $T_2$) ∈ R and ($T_2$, $T_1$) ∈ R
R is symmetric.

**Check for transitive:**

Again, if then $T_1$ is similar to $T_2$ and $T_2$ is similar to $T_3$, then $T_1$ is similar to $T_3$, so ($T_1$, $T_2$) ∈ R and ($T_2$, $T_3$) ∈ R and ($T_1$, $T_3$) ∈ R
R is transitive

Therefore, R is an equivalent relation.

**Case 2:** It is given that $T_1$, $T_2$ and $T_3$ are right angled triangles.

$T_1$ with sides 3, 4, 5
$T_2$ with sides 5, 12, 13 and
$T_3$ with sides 6, 8, 10

Since, two triangles are similar if corresponding sides are proportional.

Therefore, 3/6 = 4/8 = 5/10 = 1/2

Therefore, $T_1$ and $T_3$ are related.

**13. Show that the relation R defined in the set A of all polygons as R = {($P_1$, $P_2$) :$P_1$ and $P_2$ have same number of sides}, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?**

**Solution:**

**Case I:**
R = {($P_1$, $P_2$) :$P_1$ and $P_2$ have same number of sides}
**Check for reflexive:**

$P_1$ and $P_1$ have same number of sides, So R is reflexive.

**Check for symmetric:**

$P_1$ and $P_2$ have same number of sides then $P_2$ and $P_1$ have same number of sides, so $(P_1, P_2) \in R$ and $(P_2, P_1) \in R$
R is symmetric.

**Check for transitive:**

Again, $P_1$ and $P_2$ have same number of sides, and $P_2$ and $P_3$ have same number of sides, then also $P_1$ and $P_3$ have same number of sides .
So $(P_1, P_2) \in R$ and $(P_2, P_3) \in R$ and $(P_1, P_3) \in R$
R is transitive

Therefore, R is an equivalent relation.

Since 3, 4, 5 are the sides of a triangle, the triangle is right angled triangle. Therefore, the set A is the set of right angled triangle.

**14. Let L be the set of all lines in XY plane and R be the relation in L defined as R = {($L_1$, $L_2$) : $L_1$ is parallel to $L_2$}. Show that R is an equivalence relation. Find the set of all lines related to the line y = 2x + 4.**

**Solution:**

$L_1$ is parallel to itself i.e., $(L_1, L_1) \in R$

R is reflexive

Now, let $(L_1, L_2) \in R$

$L_1$ is parallel to $L_2$ and $L_2$ is parallel to $L_1$

$(L_2, L_1) \in R$, Therefore, R is symmetric

Now, let $(L_1, L_2)$, $(L_2, L_3) \in R$

$L_1$ is parallel to $L_2$. Also, $L_2$ is parallel to $L_3$

$L_1$ is parallel to $L_3$

Therefore, R is transitive

Hence, R is an equivalence relation.

Again, The set of all lines related to the line y = 2x + 4, is the set of all its parallel lines.
Slope of given line is m = 2.
As we know slope of all parallel lines are same.
Hence, the set of all related to y = 2x + 4 is y = 2x + k, where k ∈ R.

**15. Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4,4), (1, 3), (3, 3), (3, 2)}. Choose the correct answer.**
**(A) R is reflexive and symmetric but not transitive.**
**(B) R is reflexive and transitive but not symmetric.**
**(C) R is symmetric and transitive but not reflexive.**
**(D) R is an equivalence relation.**

**Solution:**
Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4,4), (1, 3), (3, 3), (3, 2)}.

Step 1: (1, 1), (2, 2), (3, 3), (4, 4) ∈ R R.   R is reflexive.

Step 2:  (1, 2) ∈ R but (2, 1) ∉  R.  R is not symmetric.

Step 3:  Consider any set of points, (1, 3) ∈  R and (3, 2) ∈ R then (1, 2) ∈  R. So  R is transitive.

Option (B) is correct.

**16. Let R be the relation in the set N given by R = {(a, b) : a = b – 2, b > 6}. Choose the correct answer.**
**(A) (2, 4) ∈ R (B) (3, 8) ∈ R (C) (6, 8) ∈ R (D) (8, 7) ∈ R**

**Solution:** R = {(a, b) : a = b – 2, b > 6}

(A) Incorrect : Value of b = 4, not true.

(B) Incorrect : a = 3 and b = 8 > 6
a = b – 2 => 3 = 8 – 2 and 3 = 6, which is false.

(C) Correct: a = 6 and b = 8 > 6
a = b – 2 => 6 = 8 – 2 and 6 = 6, which is true.

(D) Incorrect : a = 8 and b = 7 > 6
a = b – 2 => 8 = 7 – 2 and 8 = 5, which is false.

Therefore, option (C) is correct.