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Find the principal values of the following:

1. $\sin^{-1}\left(-\frac{1}{2}\right)$

2. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

3. Cosec $^{-1}$ **(2)**

4. $\tan^{-1}\left(-\sqrt{3}\right)$

5. $\cos^{-1}\left(\frac{-1}{2}\right)$

6. tan^{-1}(-1)

7. $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

8. $\cot^{-1}\left(\sqrt{3}\right)$

9. $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

10. $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$

Solution 1: Consider y = $\sin^{-1}\left(-\frac{1}{2}\right)$

Solve the above equation, we have

sin y = -1/2

We know that sin π/6 = ½

So, sin y = - sin π/6

$\sin y = \sin\left(-\frac{\pi}{6}\right)$

Since range of principle value of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Principle value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is - π/6.

Solution 2:

Let y = $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Cos y = cos π/6 (as cos π/6 = $\sqrt{3}$ / 2)

y = π/6

Since range of principle value of $\cos^{-1}$ is [0, π]

Therefore, Principle value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is π/6

Solution 3: Cosec $^{-1}$ (2)

Let y = Cosec $^{-1}$ (2)

Cosec y = 2

We know that, cosec π /6 = 2

So Cosec y = cosec π /6

Since range of principle value of cosec^{-1} is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore, Principle value of Cosec $^{-1}$ (2) is Π/6.

Solution 4: $\tan^{-1}\left(-\sqrt{3}\right)$

Let y = $\tan^{-1}\left(-\sqrt{3}\right)$

tan y = - tan π/3

or tan y = tan (-π/3)

Since range of principle value of $\tan^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore, Principle value of $\tan^{-1}\left(-\sqrt{3}\right)$ is – π/3.

Solution 5: $\cos^{-1}\left(\frac{-1}{2}\right)$

y = $\cos^{-1}\left(\frac{-1}{2}\right)$

cos y = -1/2

$\cos y = -\cos\frac{\pi}{3}$

cos y = cos(π – π/3) = cos (2π/3)

Since principle value of $\cos^{-1}$ is [0, π]

Therefore, Principle value of $\cos^{-1}\left(\frac{-1}{2}\right)$ is 2π/3.

Solution 6: $\tan^{-1}(-1)$

Let y = $\tan^{-1}(-1)$

tan (y) = -1

tan y = -tan π/4

$\tan y = \tan\left(-\frac{\pi}{4}\right)$

Since principle value of $\tan^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore, Principle value of $\tan^{-1}(-1)$ is - π/4.

Solution 7: $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

y = $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

sec y = $2/\sqrt{3}$

$\sec y = \sec\frac{\pi}{6}$

Since principle value of $\sec^{-1}$ is [0, π]

Therefore, Principle value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is π/6

Solution 8: $\cot^{-1}\left(\sqrt{3}\right)$

y = $\cot^{-1}\left(\sqrt{3}\right)$

cot y = $\sqrt{3}$

cot y = π/6

Since principle value of $\cot^{-1}$ is [0, π]

Therefore, Principle value of $\cot^{-1}\left(\sqrt{3}\right)$ is π/6.

Solution 9: $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

Let y = $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

$\cos y = -\frac{1}{\sqrt{2}}$

$\cos y = -\cos\frac{\pi}{4}$

$\cos y = \cos\left(\pi - \frac{\pi}{4}\right) = \cos\frac{3\pi}{4}$

Since principle value of $\cos^{-1}$ is [0, π]

Therefore, Principle value of $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ is 3 π/ 4.

Solution 10. $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$

Ley y = $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$

$\operatorname{cosec} y = -\sqrt{2}$
$\operatorname{cosec} y = \operatorname{cosec}\frac{-\pi}{4}$

Since principle value of $\operatorname{cosec}^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore, Principle value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$ is – π/4

Find the values of the following:

11. $\tan^{-1}(1) + \cos^{-1}\ -\frac{1}{2} + \sin^{-1}\ -\frac{1}{2}$

12. $\cos^{-1}\ \frac{1}{2} + 2\sin^{-1}\ \frac{1}{2}$

13. If $\sin^{-1} x = y$, then

(A) $0 \le y \le \pi$

(B) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

(C) $0 < y < \pi$

(D) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

14. $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$ is equal to

(A) π

(B) - π/3

(C) π/3

(D) 2 π/3

Solution 11. $\tan^{-1}(1) + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$

$= \tan^{-1}\tan\frac{\pi}{4} + \cos^{-1}\left(-\cos\frac{\pi}{3}\right) + \sin^{-1}\left(-\sin\frac{\pi}{6}\right)$

$= \frac{\pi}{4} + \cos\left(\pi - \frac{\pi}{3}\right) + \sin^{-1}\sin\left(-\frac{\pi}{6}\right)$

$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$

$= \frac{3\pi + 8\pi - 2\pi}{12}$

$= \frac{9\pi}{12} = \frac{3\pi}{4}$

Solution 12:

Let $\cos^{-1}\left(\frac{1}{2}\right) = x$. Then, $\cos x = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$

$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Let $\sin^{-1}\left(\frac{1}{2}\right) = y$. Then, $\sin y = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$

$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

Now,

$$\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + \frac{2\pi}{6}$$

$$= \frac{\pi}{3} + \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

Solution 13: Option (B) is correct.

Given $\sin^{-1} x = y$,

The range of the principle value of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore, $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

Solution 14:

Option (B) is correct.

$\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) = \tan^{-1}(\tan \pi/3) - \sec^{-1}(-\sec \pi/3)$

$= \pi/3 - \sec^{-1}(\sec(\pi - \pi/3))$

$= \pi/3 - 2\pi/3 = -\pi/3$