Exercise 5.2

Differentiate the functions with respect to x in Exercise 1 to 8.

1. $\sin\left(x^2+5\right)$

Solution: Let $y=\sin\left(x^2+5\right)$

Apply derivative both the sides with respect to x.

$\frac{dy}{dx}= \cos\left(x^2+5\right)\frac{d}{dx}\left(x^2+5\right)$

$= \cos\left(x^2+5\right)\left(2x+0\right)$

$= 2x\cos\left(x^2+5\right)$

2. $\cos\left(\sin x\right)$

Solution: Let $y=\cos\left(\sin x\right)$

Apply derivative both the sides with respect to x.

$\frac{dy}{dx}= -\sin\left(\sin x\right)\frac{d}{dx}\sin x$

$= -\sin\left(\sin x\right)\cos x$

3. $\sin\left(ax+b\right)$

Solution: Let $y=\sin\left(ax+b\right)$

Apply derivative both the sides with respect to x.

$\frac{dy}{dx}= \cos\left(ax+b\right)\frac{d}{dx}\left(ax+b\right)$

$= \cos\left(ax+b\right)\left(a+0\right)$ $= a\cos\left(ax+b\right)$

4. $\sec\left(\tan\sqrt{x}\right)$

Solution: Let $y=\sec\left(\tan\sqrt{x}\right)$

Apply derivative both the sides with respect to x.

$$\frac{dy}{dx}= \sec\left(\tan\sqrt{x}\right)\tan\left(\tan\sqrt{x}\right)\sec^2\sqrt{x}\frac{d}{dx}\sqrt{x}$$

$$=\sec\left(\tan\sqrt{x}\right)\tan\left(\tan\sqrt{x}\right)\sec^2\sqrt{x}.\frac{1}{2}x^{\frac{1}{2}-1}$$

$$=\sec\left(\tan\sqrt{x}\right)\tan\left(\tan\sqrt{x}\right)\sec^2\sqrt{x}.\frac{1}{2\sqrt{x}}$$

5. $\frac{\sin\left(ax+b\right)}{\cos\left(cx+d\right)}$

Solution: Let $y=\frac{\sin\left(ax+b\right)}{\cos\left(cx+d\right)}$

Using quotient rule,

$$\frac{dy}{dx}=\frac{\cos\left(cx+d\right)\frac{d}{dx}\sin\left(ax+b\right)-\sin\left(ax+b\right)\frac{d}{dx}\cos\left(cx+d\right)}{\cos^2\left(cx+d\right)}$$

$$=\frac{\cos\left(cx+d\right)\cos\left(ax+b\right)\frac{d}{dx}\left(ax+b\right)-\sin\left(ax+b\right)\left\{-\sin\left(cx+d\right)\right\}\frac{d}{dx}\left(cx+d\right)}{\cos^2\left(cx+d\right)}$$

$$=\frac{\cos\left(cx+d\right)\cos\left(ax+b\right)\left(a\right)+\sin\left(ax+b\right)\sin\left(cx+d\right)\left(c\right)}{\cos^2\left(cx+d\right)}$$

6. $\cos x^3\sin^2\left(x^5\right)$

Solution: Let $y=\cos x^3.\sin^2\left(x^5\right)$

Apply derivative both the sides with respect to x.

$$\frac{dy}{dx}= \cos x^3\frac{d}{dx}\sin^2\left(x^5\right)+\sin^2\left(x^5\right)\frac{d}{dx}\cos x^3$$

$= \cos x^3 .2\sin(x^5)\frac{d}{dx}\sin(x^5) + \sin^2(x^5)(-\sin x^3)\frac{d}{dx}x^3$

$= \cos x^3 .2\sin(x^5)\cos(x^5)(5x^4) - \sin^2(x^5)\sin x^3 .3x^2$

$= 10x^4\cos x^3\sin(x^5)\cos(x^5) - 3x^2\sin^2(x^5)\sin x^3$

7. $2\sqrt{\cot(x^2)}$

Solution: Let $y = 2\sqrt{\cos(x^2)}$

Apply derivative both the sides with respect to x.

$\frac{dy}{dx} = 2.\frac{1}{2}\{\cot(x^2)\}^{\frac{-1}{2}}.\frac{d}{dx}\cot(x^2)$

$= \frac{1}{\sqrt{\cot(x^2)}}.\{-\cos ec(x^2)\}\frac{d}{dx}x^2$

$= \frac{1}{\sqrt{\cot(x^2)}}.\{-\cos ec(x^2)\}(2x)$

$= \frac{-2x\cos ec(x^2)}{\sqrt{\cot(x^2)}}$

8. $\cos(\sqrt{x})$

Solution: Let $y = \cos(\sqrt{x})$

Apply derivative both the sides with respect to x.

$\frac{dy}{dx} = -\sin\sqrt{x}\frac{d}{dx}\sqrt{x}$

$= -\sin\sqrt{x}.\frac{1}{2}(x)^{\frac{-1}{2}} = \frac{-\sin\sqrt{x}}{2\sqrt{x}}$

9. Prove that the function f given by $f(x)=|x-1|, x\in\mathbb{R}$ is not differentiable at x = 1.

Solution: Given function: $f(x)=|x-1|$

$f(1)=|1-1|=0$

Right hand limit: $f'(1)=\lim\limits_{h\to 0}\frac{f(1+h)-f(1)}{h}$

$=\lim\limits_{h\to 0}\frac{|1+h-1|-0}{h}$

$=\lim\limits_{h\to 0}\frac{|h|}{h}=\lim\limits_{h\to 0}\frac{h}{h}=1$

and Left hand limit:

$f'(1)=\lim\limits_{h\to 0}\frac{f(1-h)-f(1)}{-h}$

$=\lim\limits_{h\to 0}\frac{|1-h-1|-0}{-h}$

$=\lim\limits_{h\to 0}\frac{|-h|}{-h}$

$=\lim\limits_{h\to 0}\frac{-h}{h}=-1$

Right hand limit ≠ Left hand limit

Therefore, f(x) is not differentiable at x = 1.

10. Prove that the greatest integer function defined by $f(x)=[x], 0<x<3$ is not differentiable at x = 1 and x = 2

Solution: Given function is

$f(x)=[x], 0<x<3$

Right hand limit:

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{|1+h| - 1}{h}$$

$$= \lim_{h \to 0} \frac{1-1}{h}$$

$$= \lim_{h \to 0} \frac{0}{h} = 0$$

and Left hand limit

$$f'(1) = \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h}$$

$$= \lim_{h \to 0} \frac{|1-h| - 1}{-h}$$

$$= \lim_{h \to 0} \frac{0-1}{-h} = \infty$$

Right hand limit ≠ Left hand limit

Therefore, $f(x) = [x]$ is not differentiable at x = 1.

In same way, $f(x) = [x]$ is not differentiable at x = 2.