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Differentiate the functions with respect to x in Exercise 1 to 10.

1. $\frac{e^x}{\sin x}$

Solution: Let $y = \frac{e^x}{\sin x}$

Differentiate the functions with respect to x, we get

$$\frac{dy}{dx} = \frac{\sin x \frac{d}{dx} e^x - e^x \frac{d}{dx} \sin x}{\sin^2 x}$$

[Using quotient rule]

$$= \frac{\sin x e^x - e^x \cos x}{\sin^2 x}$$

$$= e^x \frac{(\sin x - \cos x)}{\sin^2 x}$$

2. $e^{\sin^{-1} x}$

Solution: Let $y = e^{\sin^{-1} x}$

Differentiate the functions with respect to x, we get

$$\frac{dy}{dx} = e^{\sin^{-1} x} \cdot \frac{d}{dx} \sin^{-1} x$$

$$= e^{\sin^{-1} x} \cdot \frac{1}{\sqrt{1-x^2}}$$

$$\left[\because \frac{d}{dx} e^{f(x)} = e^{f(x)} \frac{d}{dx} f(x)\right]$$

3. e^{x^3}

Solution: Let $y = e^{x^3} = e^{(x^3)}$

Differentiate the functions with respect to x, we get

$$\frac{dy}{dx}=e^{\left(x^3\right)}\frac{d}{dx}x^3$$

$= e^{\left(x^3\right)}.3x^2=3x^2.e^{\left(x^3\right)}$

$$\left[\because \frac{d}{dx}e^{f(x)}=e^{f(x)}\frac{d}{dx}f\left(x\right)\right]$$

4. $\sin\left(\tan^{-1}e^{-x}\right)$

Solution: Let $y=\sin\left(\tan^{-1}e^{-x}\right)$

Differentiate the functions with respect to x, we get

$$\frac{dy}{dx}=\cos\left(\tan^{-1}e^{-x}\right)\frac{d}{dx}\left(\tan^{-1}e^{-x}\right)$$

$$\left[\because \frac{d}{dx}\sin f\left(x\right)=\cos f\left(x\right)\frac{d}{dx}f\left(x\right)\right]$$

$= \cos\left(\tan^{-1}e^{-x}\right)\frac{1}{1+\left(e^{-x}\right)^2}\frac{d}{dx}e^{-x}$

$$\left[\because \frac{d}{dx}\tan^{-1}f\left(x\right)=\frac{1}{\left(f\left(x\right)\right)^2}\frac{d}{dx}f\left(x\right)\right]$$

$= \cos\left(\tan^{-1}e^{-x}\right)\frac{1}{1+e^{-2x}}e^{-x}\frac{d}{dx}\left(-x\right)$

$= -\frac{e^{-x}\cos\left(\tan^{-1}e^{-x}\right)}{1+e^{-2x}}$

5. $\log\left(\cos e^{x}\right)$

Solution: Let $y=\log\left(\cos e^{x}\right)$

Differentiate the functions with respect to x, we get

$$\frac{dy}{dx}=\frac{1}{\cos e^x}\frac{d}{dx}\left(\cos e^x\right)\quad\left[\because \frac{d}{dx}\log f(x)=\frac{1}{f(x)}\frac{d}{dx}f(x)\right]$$

$$=\frac{1}{\cos e^x}\left(-\sin e^x\right)\frac{d}{dx}e^x\quad\left[\because \frac{d}{dx}\cos f(x)=-\sin f(x)\frac{d}{dx}f(x)\right]$$

$$=-\left(\tan e^x\right)e^x=-e^x\left(\tan e^x\right)$$

6. $e^x+e^{x^2}+\ldots\ldots+e^{x^5}$

Solution: Let $y=e^x+e^{x^2}+\ldots..+e^{x^5}$
Define the given function for 5 terms,

Let us say, y = $e^x+e^{x^2}+e^{x^3}+e^{x^4}+e^{x^5}$

Differentiate the functions with respect to x, we get

$$\frac{dy}{dx}=\frac{d}{dx}e^x+\frac{d}{dx}e^{x^2}+\frac{d}{dx}e^{x^3}+\frac{d}{dx}e^{x^4}+\frac{d}{dx}e^{x^5}$$

$$=e^x+e^{x^2}\frac{d}{dx}x^2+e^{x^3}\frac{d}{dx}x^3+e^{x^4}\frac{d}{dx}x^4+e^{x^5}\frac{d}{dx}x^5$$

$$=e^x+e^{x^2}.2x+e^{x^3}.3x^2+e^{x^4}.4x^3+e^{x^5}.5x^4$$

$$=e^x+2xe^{x^2}+3x^2e^{x^3}+4x^3.e^{x^4}+5x^4.e^{x^5}$$

7. $\sqrt{e^{\sqrt{x}}},x>0$

Solution: Let $y=\sqrt{e^{\sqrt{x}}}$

or y = $\left(e^{\sqrt{x}}\right)^{\frac{1}{2}}$

Differentiate the functions with respect to x, we get

$$\frac{dy}{dx}=\frac{1}{2}\left(e^{\sqrt{x}}\right)^{\frac{-1}{2}}\frac{d}{dx}e^{\sqrt{x}}$$

$$\left[\because \frac{d}{dx}\left(f(x)\right)^n = n\left(f(x)\right)^{n-1}\frac{d}{dx}f(x)\right]$$

$= \dfrac{1}{2\sqrt{e^{\sqrt{x}}}}e^{\sqrt{x}}\dfrac{d}{dx}\sqrt{x}$

$= \dfrac{1}{2\sqrt{e^{\sqrt{x}}}}e^{\sqrt{x}}\dfrac{1}{2\sqrt{x}}$

$= \dfrac{e^{\sqrt{x}}}{4\sqrt{x}\sqrt{e^{\sqrt{x}}}}$

8. $\log(\log x), x > 1$

Solution: Let $y = \log(\log x)$

Differentiate the functions with respect to x, we get

$\dfrac{dy}{dx} = \dfrac{1}{\log x}\dfrac{d}{dx}(\log x)$

$= \dfrac{1}{\log x}.\dfrac{1}{x} = \dfrac{1}{x\log x}$

9. $\dfrac{\cos x}{\log x}, x > 0$

Solution: Let $y = \dfrac{\cos x}{\log x}$

Differentiate the functions with respect to x, we get

$\dfrac{dy}{dx} = \dfrac{\log x\dfrac{d}{dx}(\cos x) - \cos x\dfrac{d}{dx}(\log x)}{(\log x)^2}$ [By quotient rule]

$= \dfrac{\log x(-\sin x) - \cos x.\dfrac{1}{x}}{(\log x)^2}$

$= \dfrac{-\left(\sin x \log x + \dfrac{\cos x}{x}\right)}{(\log x)^2}$

$= \dfrac{-(x \sin x \log x + \cos x)}{x(\log x)^2}$

10. $\cos\left(\log x + e^x\right), x > 0$

Solution: Let $y = \cos\left(\log x + e^x\right)$

Differentiate the functions with respect to x, we get

$\dfrac{dy}{dx} = -\sin\left(\log x + e^x\right)\dfrac{d}{dx}\left(\log x + e^x\right)$

$= -\sin\left(\log x + e^x\right).\left(\dfrac{1}{x} + e^x\right)$

$= \left(\dfrac{1}{x} + e^x\right)\sin\left(\log x + e^x\right)$