## Exercise 5.6 Page No: 181

**If x and y are connected parametrically by the equations given in Exercise 1 to 10, without eliminating the parameter, find dy/dx.**

**1.** $x = 2at^2, y = at^4$

**Solution:** Given functions are $x = 2at^2$ and $y = at^4$

$$\frac{dx}{dt} = \frac{d}{dt}\left(2at^2\right)$$

$$\frac{dx}{dt} = 2a\frac{d}{dt}\left(t^2\right)$$

= $2a.2t = 4at$ and

$$\frac{dy}{dt} = \frac{d}{dt}\left(at^4\right)$$

$$\frac{dy}{dt} = a\frac{d}{dt}\left(t^4\right) = a.4t^3 = 4at^3$$

Now,

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4at^3}{4at} = t^2$$

**2.** $x = a\cos\theta, y = b\cos\theta$

**Solution:** Given functions are $x = a\cos\theta$ and $y = b\cos\theta$

$$\frac{dx}{d\theta} = \frac{d}{d\theta}\left(a\cos\theta\right)$$

$$\frac{dx}{d\theta} = a\frac{d}{d\theta}\left(\cos\theta\right)$$

$$\frac{dx}{d\theta} = -a\sin\theta$$

and

$$\frac{dy}{d\theta} = \frac{d}{d\theta}\left(b\cos\theta\right)$$

$$\frac{dy}{d\theta} = b\frac{d}{d\theta}(\cos\theta)$$

$$\frac{dy}{d\theta} = -b\sin\theta$$

Now,

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-a\sin\theta}{-b\sin\theta} = \frac{b}{a}$$

**3.** $x = \sin t, y = \cos 2t$

**Solution:** Given functions are $x = \sin t$ and $y = \cos 2t$

$\frac{dx}{dt} = \cos t$ and

$$\frac{dy}{dt} = -\sin 2t\frac{d}{dt}(2t) = -2\sin 2t$$

Now,

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2\sin 2t}{\cos t} = \frac{-2\times 2\sin t\cos t}{\cos t} = -4\sin t$$

**4.** $x = 4t, y = \frac{4}{t}$

**Solution:** Given functions are $x = 4t$ and $y = \frac{4}{t}$

$$\frac{dx}{dt} = \frac{d}{dt}(4t) = 4\frac{d}{dt}t = 4$$

and

$$\frac{dy}{dt} = \frac{d}{dt}\left(\frac{4}{t}\right)$$

$$= \frac{t\frac{d}{dt}4 - 4\frac{d}{dt}t}{t^2}$$

$$\Rightarrow \frac{dy}{dt} = \frac{t\times 0 - 4\times 1}{t^2} = -\frac{4}{t^2}$$

Now,

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{-\frac{4}{t^2}}{4}=\frac{-1}{t^2}$$

**5.** $x=\cos\theta-\cos 2\theta, y=\sin\theta-\sin 2\theta$

**Solution:** Given functions are $x=\cos\theta-\cos 2\theta$ and $y=\sin\theta-\sin 2\theta$

$$\frac{dx}{d\theta}=\frac{d}{d\theta}\cos\theta-\frac{d}{d\theta}\cos 2\theta$$

$$\frac{dx}{d\theta}=-\sin\theta-(-\sin 2\theta)\frac{d}{d\theta}2\theta$$

$$\frac{dx}{d\theta}=-\sin\theta+(-\sin 2\theta)2$$

$$\frac{dx}{d\theta}=2\sin 2\theta-\sin\theta$$

And

$$\frac{dy}{d\theta}=\frac{d}{d\theta}\sin\theta-\frac{d}{d\theta}\sin 2\theta$$

$$\frac{dy}{d\theta}=\cos\theta-\cos 2\theta\frac{d}{d\theta}2\theta$$

$$\frac{dy}{d\theta}=\cos\theta-\cos 2\theta\times 2$$

$$\frac{dy}{d\theta}=\cos\theta-2\cos 2\theta$$

Now $\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{\cos\theta-2\cos 2\theta}{2\sin 2\theta-\sin\theta}$

**6.** $x=a(\theta-\sin\theta), y=a(1+\cos\theta)$

**Solution:** Given functions are $x=a(\theta-\sin\theta)$ and $y=a(1+\cos\theta)$

$$\frac{dx}{d\theta}=a\frac{d}{d\theta}(\theta-\sin\theta)$$

$$\frac{dx}{d\theta}=a\left[\frac{d}{d\theta}\theta-\frac{d}{d\theta}\sin\theta\right]$$

$$\frac{dx}{d\theta}=a(1-\cos\theta)$$

$$\frac{dy}{d\theta}=a\frac{d}{d\theta}(1+\cos\theta)$$

$$\frac{dy}{d\theta}=a\left[\frac{d}{d\theta}(1)+\frac{d}{d\theta}\cos\theta\right]$$

$$\frac{dy}{d\theta}=a(0-\sin\theta)$$

$= -a\sin\theta$

$$\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{-a\sin\theta}{a(1-\cos\theta)}=\frac{-\sin\theta}{1-\cos\theta}$$

$$=-\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}$$

$$=-\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$$

$$=-\cot\frac{\theta}{2}$$

**7.** $x=\frac{\sin^3 t}{\sqrt{\cos 2t}}, y=\frac{\cos^3 t}{\sqrt{\cos 2t}}$

**Solution:** Given functions are $x=\frac{\sin^3 t}{\sqrt{\cos 2t}}$ and $y=\frac{\cos^3 t}{\sqrt{\cos 2t}}$

$$\frac{dx}{dt}=\frac{\sqrt{\cos 2t}.\frac{d}{dt}\left(\sin^3 t\right)-\sin^3 t.\frac{d}{dt}\left(\sqrt{\cos 2t}\right)}{\left(\sqrt{\cos 2t}\right)^2}$$ [By quotient rule]

$$=\frac{\sqrt{\cos 2t}.3\sin^2 t\frac{d}{dt}(\sin t)-\sin^3 t.\frac{1}{2}(\cos 2t)^{\frac{-1}{2}}\frac{d}{dt}(\cos 2t)}{\cos 2t}$$

$$=\frac{\sqrt{\cos 2t}.3\sin^2 t\cos t-\frac{\sin^3 t}{2\sqrt{\cos 2t}}(-2\sin 2t)}{\cos 2t}$$

$$=\frac{3\sin^2 t\cos t\cos 2t+\sin^3 t.\sin 2t}{(\cos 2t)^{\frac{3}{2}}}$$

$$=\frac{\sin^2 t\cos t\left(3\cos 2t+2\sin^2 t\right)}{(\cos 2t)^{\frac{3}{2}}}$$

And $$\frac{dy}{dt}=\frac{\sqrt{\cos 2t}.\frac{d}{dt}\left(\cos^3 t\right)-\cos^3 t.\frac{d}{dt}\left(\sqrt{\cos 2t}\right)}{\left(\sqrt{\cos 2t}\right)^2}$$ [By quotient rule]

$$=\frac{\sqrt{\cos 2t}.3\cos^2 t\frac{d}{dt}(\cos t)-\cos^3 t.\frac{1}{2}(\cos 2t)^{\frac{-1}{2}}\frac{d}{dt}(\cos 2t)}{\cos 2t}$$

$$=\frac{\sqrt{\cos 2t}.3\cos^2 t(-\sin t)-\frac{\cos^3 t}{2\sqrt{\cos 2t}}(-2\sin 2t)}{\cos 2t}$$

$$=\frac{-3\cos^2 t\sin t\cos 2t+\cos^3 t.\sin 2t}{(\cos 2t)^{\frac{3}{2}}}$$

$= \dfrac{-3\cos^2 t \sin t \cos 2t + \cos^3 t . 2\sin t \cos t}{(\cos 2t)^{\frac{3}{2}}}$

$= \dfrac{\sin t \cos^2 t\left(2\cos^2 t - 3\cos 2t\right)}{(\cos 2t)^{\frac{3}{2}}}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\dfrac{\sin t \cos^2 t\left(2\cos^2 t - 3\cos 2t\right)}{(\cos 2t)^{\frac{3}{2}}}}{\dfrac{\sin^2 t \cos t\left(3\cos 2t + 2\sin^2 t\right)}{(\cos 2t)^{\frac{3}{2}}}}$$

$= \dfrac{\cos t\left[2\cos^2 t - 3\left(2\cos^2 t - 1\right)\right]}{\sin t\left[3\left(1 - 2\sin^2 t\right) + 2\sin^2 t\right]}$

$= \dfrac{\cos t\left(3 - 4\cos^2 t\right)}{\sin t\left(3 - 4\sin^2 t\right)}$

$= \dfrac{-\left(4\cos^2 t - 3\cos t\right)}{3\sin t - 4\sin^3 t}$

$= \dfrac{-\cos 3t}{\sin 3t} = -\cot 3t$

**8.** $x = a\left(\cos t + \log \tan \frac{t}{2}\right), y = a \sin t$

**Solution:** Given functions are $x = a\left(\cos t + \log \tan \frac{t}{2}\right)$ and $y = a \sin t$

$$\frac{dx}{dt} = a\left[-\sin t + \frac{1}{\tan \frac{t}{2}} \cdot \frac{d}{dt}\left(\tan \frac{t}{2}\right)\right]$$

$= a\left[-\sin t + \frac{1}{\tan\frac{t}{2}}.\sec^2\frac{t}{2}.\frac{1}{2}\right]$

$= a\left[-\sin t + \frac{\cos\frac{t}{2}}{\sin\frac{t}{2}}.\frac{1}{\cos^2\frac{t}{2}}.\frac{1}{2}\right]$

$= a\left[-\sin t + \frac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}}\right]$

$= a\left[-\sin t + \frac{1}{\sin t}\right]$

$= a\left[\frac{1}{\sin t} - \sin t\right] = a\left(\frac{1-\sin^2 t}{\sin t}\right) = \frac{a\cos^2 t}{\sin t}$

and $\frac{dy}{dt} = a\cos t$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{a\cos t}{\left(\frac{a\cos^2 t}{\sin t}\right)}$

$= \frac{\sin t}{\cos t} = \tan t$

**9.** $x = a\sec\theta, y = b\tan\theta$

**Solution:** Given functions are $x = a\sec\theta$ and $y = b\tan\theta$

$\frac{dx}{d\theta} = a\sec\theta\tan\theta$ and

$\frac{dy}{d\theta} = \sec^2\theta$

Now,

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{b\sec^2\theta}{a\sec\theta\tan\theta}$$

$= \frac{b\sec\theta}{a\tan\theta}$

$= \frac{b.\frac{1}{\cos\theta}}{a.\frac{\sin\theta}{\cos\theta}}$

$= \frac{b}{\cos\theta}.\frac{\cos\theta}{\sin\theta}$

$= \frac{b}{a\sin\theta}$

$= \frac{b}{a}\operatorname{cosec}\theta$

**10.** $x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$

**Solution:** Given functions are $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$

$$\frac{dx}{d\theta} = a(-\sin\theta + \theta\cos\theta + \sin\theta.1)$$

$= a\theta\cos\theta$

and

$$\frac{dy}{d\theta} = a\left[\cos\theta - \{\theta(-\sin\theta) + \cos\theta.1\}\right]$$

$= a[\cos\theta + \theta\sin\theta - \cos\theta]$

$= a\theta\sin\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$$

**11.**

**If** $x=\sqrt{a^{\sin^{-1}t}}, y=\sqrt{a^{\cos^{-1}t}}$, **show that** $\frac{dy}{dx}=\frac{-y}{x}$.

**Solution:**

$$x=\sqrt{a^{\sin^{-1}t}}=\left(a^{\sin^{-1}t}\right)^{\frac{1}{2}}$$

$= a^{\frac{1}{2}\sin^{-1}t}$

and

$$y=\sqrt{a^{\cos^{-1}t}}=\left(a^{\cos^{-1}t}\right)^{\frac{1}{2}}$$

$= a^{\frac{1}{2}\cos^{-1}t}$

Now,

$$\frac{dx}{dt}=a^{\frac{1}{2}\sin^{-1}t}\log a\frac{d}{dt}\left(\frac{1}{2}\sin^{-1}t\right)$$

$= a^{\frac{1}{2}\sin^{-1}t}\log a\frac{1}{2}\frac{1}{\sqrt{1-t^2}}$

And $\frac{dy}{dt}=a^{\frac{1}{2}\cos^{-1}t}\log a\frac{d}{dt}\left(\frac{1}{2}\cos^{-1}t\right)$

$= a^{\frac{1}{2}\cos^{-1}t}\log a\frac{1-}{2}\frac{1}{\sqrt{1-t^2}}$

Now,

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{a^{\frac{1}{2}\cos^{-1}t}\log a\frac{1}{2}\frac{-1}{\sqrt{1-t^2}}}{a^{\frac{1}{2}\sin^{-1}t}\log a\frac{1}{2}\frac{1}{\sqrt{1-t^2}}}$$

$= \frac{-a^{\frac{1}{2}\cos^{-1}t}}{a^{\frac{1}{2}\sin^{-1}t}} = \frac{-y}{x}$