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Mathematics is a unique symbolic language in which the whole world works and acts accordingly. This text book is an attempt to make learning of Mathematics easy for the students community.

Mathematics is not about numbers, equations, computations or algorithms; it is about understanding

— William Paul Thurston

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The main goal of Mathematics in School Education is to mathematise the child's thought process. It will be useful to know how to mathematise than to know a lot of Mathematics.

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Chapter 1

NUMBER SYSTEM

Learning Objectives

- To recall the decimal notation and to understand the place value in decimals.
- To learn the concepts of decimals as fractions with denominators of tens and its powers.
- To represent decimal numbers on number line.

X2W4X3

Recap

Decimal Numbers

Kala and Kavin went to a stationery shop to buy pencils. Their conversation is given below.

Kala : What is the price of a pencil?

Shopkeeper : The price of a pencil is four rupees and fifty paise.

Kala : Ok sir. Give me a pencil.

Kavin : We usually express the bill amount in rupees and paise as decimals. So the price of a pencil can be expressed as ₹4.50. Here 4 is the integral part and 50 is the decimal part. The dot represents the decimal point.

(After a week of time in the class room)

Teacher : We have studied about fractions and decimal numbers in earlier classes. Let us recall decimals now. Kala and kavin, can you measure the length of your pencils?

Kavin : Both the pencils seem to be of same length. Shall we measure and check?

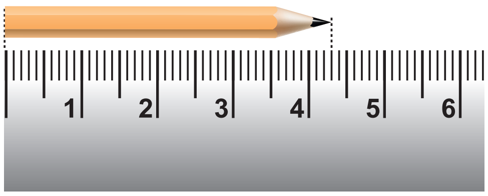


Fig. 1.1

Kala : Ok Kavin. The length of my pencil is 4 cm 3 mm (Fig. 1.1).

Kavin : Length of my pencil is 4 cm 5 mm (Fig. 1.2).

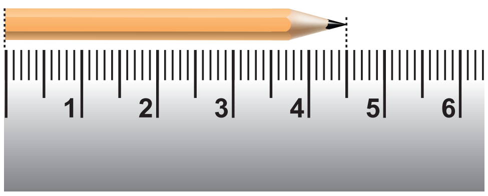


Fig. 1.2

Kala : Can we express these lengths in terms of centimetres?

Kavin : Each centimetre is divided into ten equal parts known as millimetres. Do you remember that we have studied about tenths. I can say the length of my pencil as 4 and 5 tenths of a *cm*.

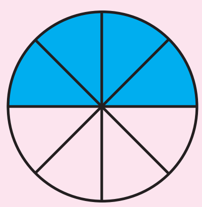
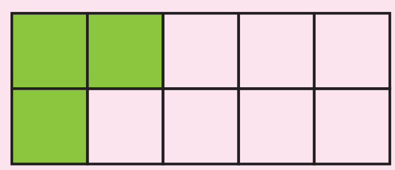
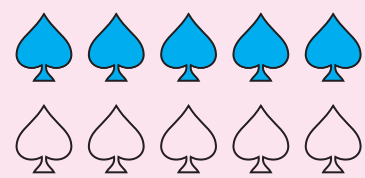
Kala : Since 1 *mm* = $\frac{1}{10}$ *cm* or one tenth of a *cm*, it can be represented as 4.5 *cm*.

Kavin : Then the length of your pencil is 4.3 *cm*. Is it right?

Teacher : Both of you are right. Now we will further study about decimal numbers.

Try these

1. Observe the following and write the fraction of the shaded portion and mention in decimal form also.

(i)  (ii)  (iii) 

2. Represent the following fractions in decimal form by converting denominator into ten or powers of 10.

S.No.	Fraction	Decimal Form
(i)	$\frac{3}{5}$	
(ii)	$\frac{4}{10}$	
(iii)	$\frac{2}{4}$	
(iv)	$\frac{4}{20}$	
(v)	$\frac{7}{10}$	

3. Give any two life situations where we use decimal numbers.

1.1 Introduction

Consider the following situation. Ravi has planned to celebrate pongal festival in his native place Kanthapuram. He has purchased dress materials and groceries for the celebration. The details are furnished below.

Bill-1

ABC Textiles

S.No.	Particulars	Rate (in ₹) per m	Length of the material	Price(₹)
1.	Pant material	120	4.75 m	570.00
2.	Shirt material	108	5.25 m	567.00
3.	Churidar material	150	4.50 m	675.00
4.	Saree	960 per saree	5.50 m	960.00

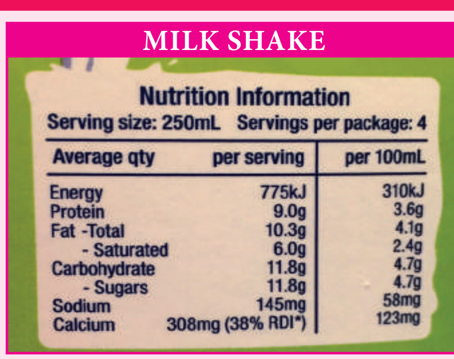
Bill-2

XYZ Groceries

S.No.	Item	Rate (in ₹)	Quantity	Price(₹)
1.	Rice	60/kg	1.000 kg	60.00
2.	Dhall	85/kg	0.500 kg	42.50
3.	Jaggery	40/kg	1.750 kg	70.00
4.	Ghee	420/kg	0.250 kg	105.00
5.	Nuts	800/kg	0.100 kg	80.00
6.	Coconuts	25	5	125.00
7.	Banana	60/dozen	1dozen	60.00
8.	Sugarcane	50	2	100.00
				642.50

What do you observe in the bills shown above? The prices are usually represented in decimals. But the quantities of length are represented in terms of metre and centimetre and that of weight are represented in terms of kilograms and grams. To express the quantities in terms of higher units, we use the concept of decimals.

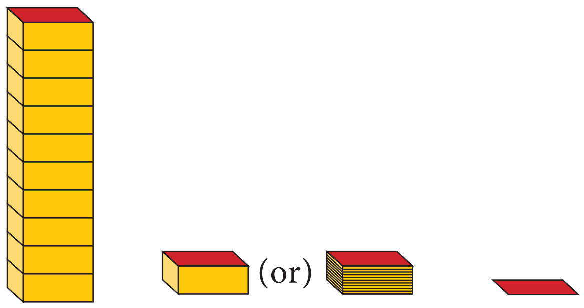
MATHEMATICS ALIVE - Number System in Real Life



Healthy Food-Nutritional values

1.2 Representing a Decimal Number

(i) Observe the following pictures which represent tens, ones and tenths



Tens Ones Tenths

For example, the decimal number 3.2 is 3 ones and 2 tenths that can be represented using the pictorial representations as follows.

3 ones 2 tenths

Similarly any decimal number can be represented as above

Try this

Represent the following decimal numbers pictorially.

(i) 5 ones and 3 tenths
(ii) 6 tenths
(iii) 7 ones and 9 tenths
(iv) 6 ones and 4 tenths
(v) Seven tenths

We have studied about place value of digits of a number in primary classes. Now let us extend our learning of place values to decimal digits of a number. Let us recall the expanded form of a number. Consider the number 3768.

The expanded form of 3768 is given by $3\times1000+7\times100+6\times10+8$.

Now, let us consider a decimal number 235.68

Expressing the decimal number in the expanded form, we get

$$235.68 = 200+30+5+\frac{6}{10}+\frac{8}{100}$$
$$= 2\times100+3\times10+5\times1+6\times\frac{1}{10}+8\times\frac{1}{100}$$

Think

The above expression can also be expressed in terms of powers of 10 as follows $235.68 = 2\times10^2+3\times10^1+5\times10^0+6\times10^{-1}+8\times10^{-2}$. Hence, in any number, when we move towards right from one digit to the next, the place value of a digit is divided by 10^1.

Representing the two numbers 3768 and 25.6 in the place value grid, we get

3768	Th	H	T	O
	3	7	6	8

25.6	Tens	Ones	Tenths
	2	5	6

In the beginning of this chapter, we came across the discussion about the length of the pencils of Kavin and Kala. Those lengths can also be represented in the place value grid as follows.

Length of Kavin's pencil is

4 and 5 tenths of a cm

Place value		
4.5	Ones	Tenths
	4	5

Length of Kala's pencil is

4 and 3 tenths of a cm

Place value		
4.3	Ones	Tenths
	4	3

We know that the digit to the right of ones place is known as tenths and the point is called decimal point which separates the integral part and decimal part of a number.

In the above situation, we have expressed numbers with one decimal digit in place value grid. Now to express numbers with two decimal digits in place value grid, let us consider the example already discussed in the introduction part.

Ravi has purchased dress materials for pongal festival. The length of the pant material is 4 m 75 cm. To express the centimetre in terms of metres, we proceed as follows.

We know that

$$100\ cm = 1\ m$$

$$1\ cm = \frac{1}{100}\ m$$

X4Z7B9

Therefore one centimetre can be represented as one hundredth of a metre.

Similarly, $75\ cm = \frac{75}{100} = 0.75\ m$

So the length of the pant material is $4 + 0.75\ m$

That is 4.75 m. It can be read as four and seventy five hundredth of a metre or four point seven five metres.

Note

The decimal digits of a number have to be read as separate digits.

Try these

1. Express the following decimal numbers in an expanded form and place value grid form.

   (i) 56.78 (ii) 123.32 (iii) 354.56

2. Express the following measurements in terms of metre and in decimal form. One is done for you.

S.No.	Measurements	In metre	Decimal form
1.	7 *m* 36 *cm*	7 and 36 hundredths of a *m*	7.36
2.	26 *m* 50 *cm*		
3.	93 *cm*		
4.	36 *m* 60 *cm*		
5.	126 *m* 45 *cm*		

3. Write the following numbers in the place value grid and find the place value of the underlined digits.

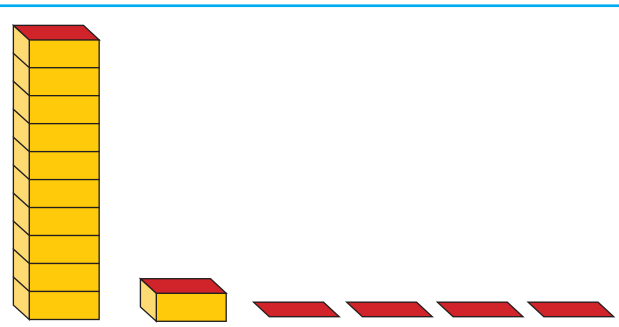
(i) 36.$\underline{37}$ (ii) 267.0$\underline{6}$ (iii) 0.$\underline{2}$3 (iv) 27.6$\underline{9}$ (v) 53.$\underline{2}$7

Note that the tenth and hundredth place values of a number can be written as $\frac{1}{10}$ and $\frac{1}{100}$ respectively.

Example 1.1 Represent the following decimal numbers pictorially.

(i) 0.3 (ii) 3.6 (iii) 2.7 (iv) 11.4

Solution

S.No.	Decimal Number	Pictorial representation
(i)	0.3	
(ii)	3.6	
(iii)	2.7	
(iv)	11.4	

Example 1.2 Write the following in the place value grid and find the place value of the underlined digits.

(i) 0.3$\underline{7}$ (ii) 2.$\underline{7}$3 (iii) 28.2$\underline{7}$1

Solution

S.No	Tens	Ones	Tenths	Hundredths	Thousandths
1	-	0	3	7	-
2	-	2	7	3	-
3	2	8	2	7	1

(i) The place value of 7 in 0.37 is Hundredth.

(ii) The place value of 7 in 2.73 is Tenth.

(iii) The place value of 7 in 28.271 is Hundredth.

Example 1.3 The height of a person is 165 *cm*. Express this height in metre.

Solution

Given, height of the person = 165 *cm*.

Hence, the height of the person = $\frac{165}{100}$ = 1.65 *m*.

Since, 1 $cm = \frac{1}{100} m = 0.01\ m$

Example 1.4 Praveen goes trekking with his friends. He has to record the distance in kilometres in his sports book. Can you help him?

The trekking record for four days are given below

(i) 4 *m* (ii) 28 *m* (iii) 537 *m* (iv) 3983 *m*

Solution

(i) $4\ m = \frac{4}{1000} km = 0.004\ km$

(ii) $28\ m = \frac{28}{1000} km = 0.028\ km$

(iii) $537\ m = \frac{537}{1000} km = 0.537\ km$

(iv) $3983\ m = \frac{3983}{1000} km = 3.983\ km$

Since, $1 m = \frac{1}{1000} km = 0.001\ km$

So, Praveen's trekking record is 0.004 *km*, 0.028 *km*, 0.537 *km*, 3.983 *km*.

Example 1.5 Express the numbers given in expanded form in the place value grid. Also write its decimal representation.

(i) $3+\frac{5}{10}+\frac{3}{100}+\frac{4}{1000}$ (ii) $40+6+\frac{7}{10}+\frac{2}{100}+\frac{6}{1000}$

Solution

(i)

Tens	Ones	Tenths	Hundredths	Thousandths
0	3	5	3	4

$$3+\frac{5}{10}+\frac{3}{100}+\frac{4}{1000}=3.534$$

(ii)

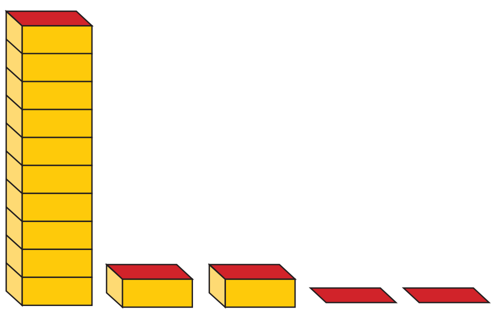
Tens	Ones	Tenths	Hundredths	Thousandths
4	6	7	2	6

$$40+6+\frac{7}{10}+\frac{2}{100}+\frac{6}{1000}=46.726$$

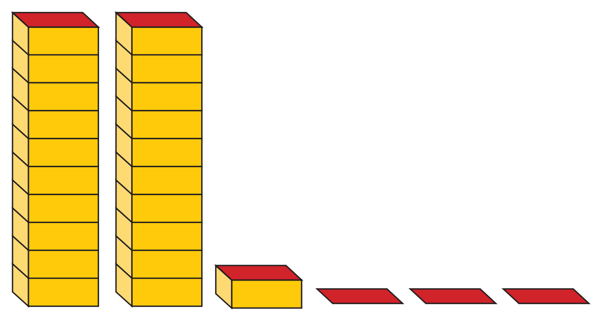
Exercise 1.1

1. Write the decimal numbers for the following pictorial representation of numbers.

(i)



(ii)



2. Express the following in *cm* using decimals.

(i) 5 *mm* (ii) 9 *mm* (iii) 42 *mm*

(iv)8 *cm* 9 *mm* (v) 375 *mm*

3. Express the following in metres using decimals.

(i) 16 *cm* (ii) 7 *cm* (iii) 43 *cm*

(iv) 6 *m* 6 *cm* (v) 2 *m* 54 *cm*

4. Expand the following decimal numbers.

(i) 37.3 (ii) 658.37 (iii) 237.6 (iv) 5678.358

5. Express the following decimal numbers in place value grid and write the place value of the underlined digit.

(i) $53.\underline{6}1$ (ii) $263.\underline{2}71$ (iii) $17.3\underline{9}$ (iv) $9.6\underline{5}7$ (v) $4972.06\underline{8}$

Objective type questions

6. The place value of 3 in 85.073 is ________

(i) tenths (ii) hundredths (iii) thousands (iv) thousandths

7. To convert grams into kilograms, we have to divide it by

(i) 10000 (ii) 1000 (iii) 100 (iv) 10

8. The decimal representation of 30 *kg* and 43 *g* is ____ *kg*.

(i) 30.43 (ii) 30.430 (iii) 30.043 (iv) 30.0043

9. A cricket pitch is about 264 cm wide. It is equal to ____ *m*.

(i) 26.4 (ii) 2.64 (iii) 0.264 (iv) 0.0264

1.3 Fractions and Decimals

Let us see the relationship between fractions and decimals.

1.3.1 Conversion of Fractions to Decimals

We are familiar with fraction as a part of a whole. The place value of the decimal digits of a number are tenths $\left(\frac{1}{10}\right)$, hundredths $\left(\frac{1}{100}\right)$, thousandths $\left(\frac{1}{1000}\right)$ and so on.

If the denominator of a fraction is any of $10, 10^2, 10^3, \ldots$ we can express them as decimals. Consider the example of distributing a box of 10 pencils to ten students. The portion of pencils given to 6 students will be $\frac{6}{10}$ which can be expressed as 0.6.

If the denominator of a fraction is any number that can be made as powers of 10 using the concept of equivalent fractions, then it can also be expressed as decimals. Consider the example of sharing 5 peanut cakes among five friends. The share of one person is $\frac{1}{5}$. To represent this fraction as a decimal number, we first convert the denominator into 10. This can be done by writing the equivalent fraction of $\frac{1}{5}$, namely $\frac{2}{10}$. Now the decimal representation of $\frac{2}{10}$ is 0.2

Think

Can you express the denominators of all fractions as powers of 10?

1.3.2 Conversion of Decimals to Fractions

As we convert fractions into a decimal number, the decimal numbers can also be expressed as fractions.

For example, let the price of brand '*x*' slippers be ₹ 399.95

Expanding the above price, we get,

$$399.95 = 3 \times 100 + 9 \times 10 + 9 \times 1 + 9 \times \frac{1}{10} + 5 \times \frac{1}{100}$$

$$= 399 + \frac{95}{100} = \frac{39995}{100} = \frac{7999}{20}$$

Similarly, if the price of brand '*y*' slipper is ₹ 159.95, then it can be expressed in terms of fractions as below.

$$159.95 = 159 + \frac{95}{100} = \frac{15995}{100} = \frac{3199}{20}$$

Try these

1. Convert the following fractions into the decimal numbers.

   (i) $\frac{16}{1000}$ (ii) $\frac{638}{10}$ (iii) $\frac{1}{20}$ (iv) $\frac{3}{50}$

2. Write the fraction for each of the following:

   (i) 6 hundreds + 3 tens + 3 ones + 6 hundredths + 3 thousandths.

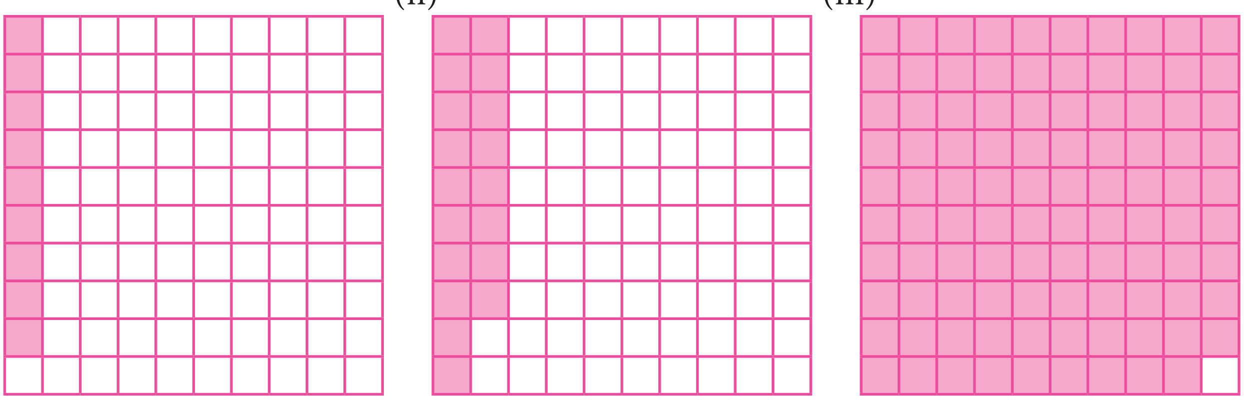
   (ii) 3 thousands + 3 hundreds + 4 tens + 9 ones + 6 tenths.

3. Convert the following decimals into fractions.

   (i) 0.0005 (ii) 6.24

Example 1.6 Write the shaded portion of the figures given below as a fraction and as a decimal number.

(i) (ii) (iii)



Solution

No.	Shaded portions	Fraction	Decimal number
(i)	9 squares out of 100 squares	$\frac{9}{100}$	0.09
(ii)	18 squares out of 100 squares	$\frac{18}{100}$	0.18
(iii)	99 squares out of 100 squares	$\frac{99}{100}$	0.99

Example 1.7 Express the following fractions as decimal numbers.

(i) $\frac{3}{5}$ (ii) $\frac{5}{100}$

Solution

(i) $\frac{3}{5} = \frac{6}{10} = 0.6$

(ii) $\frac{5}{100} = 0.05$

Example 1.8 Write the following fractions as decimals.

(i) $\frac{2}{5}$ (ii) $\frac{3}{4}$ (iii) $\frac{9}{1000}$ (iv) $\frac{1}{50}$ (v) $3\frac{1}{5}$

Solution

(i) We have to find a fraction equivalent to $\frac{2}{5}$ whose denominator is 10.

$$\frac{2}{5}=\frac{2\times2}{5\times2}=\frac{4}{10}=0.4$$

(ii) We have to find a fraction equivalent to $\frac{3}{4}$ with denominator 100. Since there is no whole number that gives 10 when multiplied by 4, let us make the denominator as 100.

So, $\frac{3}{4}=\frac{3\times25}{4\times25}=\frac{75}{100}=0.75$

(iii) In $\frac{9}{1000}$, tenth and hundredth place is zero.

Therefore, $\frac{9}{1000}=0.009$

(iv) We have to find a fraction equivalent to $\frac{1}{50}$ whose denominator is 100.

$$\frac{1}{50}=\frac{1\times2}{50\times2}=\frac{2}{100}=0.02$$

(v) In $3\frac{1}{5}$, keep the whole number 3 as such, we can find a fraction equivalent to $\frac{1}{5}$ with denominator 10.

$$3+\frac{1}{5}=3+\frac{1\times2}{5\times2}=3+\frac{2}{10}=3.2$$

Example 1.9 Convert the following into simplest fractions.

(i) 0.04 (ii) 3.46 (iii) 0.862

Solution

(i) $0.04=\frac{4}{100}=\frac{1}{25}$

(ii) $3.46=3+\frac{46}{100}$

$$=3+\frac{46\div2}{100\div2}$$

$$=3+\frac{23}{50}$$

$$=3\frac{23}{50}$$

(iii) $0.862 = \frac{862}{1000}$

$= \frac{862 \div 2}{1000 \div 2} = \frac{431}{500}$

Example 1.10 Find the decimal form of the following fractions.

(i) $153 + 96 + 7 + \frac{5}{10} + \frac{2}{1000}$ (ii) $999 + 99 + 9 + \frac{9}{10} + \frac{9}{100}$ (iii) $23 + \frac{6}{10} + \frac{8}{1000}$

Solution

(i) $153 + 96 + 7 + \frac{5}{10} + \frac{2}{1000} = 256 + 5 \times \frac{1}{10} + 0 \times \frac{1}{100} + 2 \times \frac{1}{1000}$

$= 256.502$ (since hundredths place is not there, it is taken as '0')

(ii) $999 + 99 + 9 + \frac{9}{10} + \frac{9}{100} = 1107 + 9 \times \frac{1}{10} + 9 \times \frac{1}{100}$

$= 1107.99$

(iii) $23 + \frac{6}{10} + \frac{8}{1000} = 23 + 6 \times \frac{1}{10} + 0 \times \frac{1}{100} + 8 \times \frac{1}{1000}$

$= 23.608$ (since hundredths place is not there, it is taken as '0')

Example 1.11 Write each of the following as decimals.

(i) Four hundred four and five hundredths

(ii) Two and twenty five thousandths.

Solution

(i) Four hundred four and five hundredths.

$= 404 + \frac{5}{100}$

$= 404 + 0 \times \frac{1}{10} + 5 \times \frac{1}{100} = 404.05$

(ii) Two and twenty five thousandths

$= 2 + \frac{25}{1000}$

$= 2 + \frac{2}{100} + \frac{5}{1000}$ $\left[\text{since, } \frac{25}{1000} = \frac{20+5}{1000} = \frac{20}{1000} + \frac{5}{1000} = \frac{2}{100} + \frac{5}{1000}\right]$

$= 2 + \frac{0}{10} + \frac{2}{100} + \frac{5}{1000} = 2.025$ [as there is no tenth we take it as 0 tenth]

Note

For any decimal number, number of zeroes in the denominator and number of decimal digits are equal.

Example 1.12 Express the following as fractions (i) A capsule contains 0.85 mg of medicine.

(ii) A juice container has 4.5 litres of mango juice.

Solution

(i) $0.85 = 0 + \frac{8}{10} + \frac{5}{100}$

$= \frac{85}{100} = \frac{17}{20}$

A capsule holds $\frac{17}{20}$ mg of medicine.

(ii) $4.5 = 4 + \frac{5}{10}$

$= 4\frac{5}{10} = 4\frac{1}{2}$

A juice container has $4\frac{1}{2}$ litres of mango juice.

DO YOU KNOW?

A decimal is a fraction written in a special form. Decimal comes from the Latin word 'decimus' which means tenth. It comes from the root word 'decem'.

Exercise 1.2

1. Fill in the following place value table.

S. No.	Decimal form	Hundreds (100)	Tens (10)	Ones (1)	Tenths $\left(\frac{1}{10}\right)$	Hundredths $\left(\frac{1}{100}\right)$	Thousandths $\left(\frac{1}{1000}\right)$
1.	320.157	3	___	0	1	5	7
2.	103.709	1	0	3	___	0	9
3.	4.003	0	0	4	0	___	___
4.	360.805	3	___	___	8	0	___

2. Write the decimal numbers from the following place value table.

S. No.	Hundreds (100)	Tens (10)	Ones (1)	Tenths $\left(\frac{1}{10}\right)$	Hundredths $\left(\frac{1}{100}\right)$	Thousandths $\left(\frac{1}{1000}\right)$	Decimal form
1.	8	0	1	5	6	2	___
2.	9	3	2	0	5	6	___
3.	0	4	7	5	0	9	___
4.	5	0	3	0	0	7	___
5.	6	8	0	3	1	0	___
6.	1	0	9	9	0	8	___

3. Write the following decimal numbers in the place value table.

(i) 25.178 (ii) 0.025 (iii) 428.001 (iv) 173.178 (v) 19.54

4. Write each of the following as decimal numbers.

(i) $20 + 1 + \frac{2}{10} + \frac{3}{100} + \frac{7}{1000}$ (ii) $3 + \frac{8}{10} + \frac{4}{100} + \frac{5}{1000}$

(iii) $6 + \frac{0}{10} + \frac{0}{100} + \frac{9}{1000}$ (iv) $900 + 50 + 6 + \frac{3}{100}$

(v) $\frac{6}{10} + \frac{3}{100} + \frac{1}{1000}$

5. Convert the following fractions into decimal numbers.

(i) $\frac{3}{10}$ (ii) $3\frac{1}{2}$ (iii) $3\frac{3}{5}$ (iv) $\frac{3}{2}$ (v) $\frac{4}{5}$ (vi) $\frac{99}{100}$ (vii) $3\frac{19}{25}$

6. Write the following decimals as fractions.

(i) 2.5 (ii) 6.4 (iii) 0.75

7. Express the following decimals as fractions in lowest form.

(i) 2.34 (ii) 0.18 (iii) 3.56

Objective type questions

8. $3 + \frac{4}{100} + \frac{9}{1000} = ?$

(i) 30.49 (ii) 3049 (iii) 3.0049 (iv) 3.049

9. $\frac{3}{5} =$ ________

(i) 0.06 (ii) 0.006 (iii) 6 (iv) 0.6

10. The simplest form of 0.35 is

(i) $\frac{35}{1000}$ (ii) $\frac{35}{10}$ (iii) $\frac{7}{20}$ (iv) $\frac{7}{100}$

1.4 Comparison of Decimals

Let us see Bob Beamon's Long Jump record of 1968 summer Olympics which stood for 23 years. His record was 8.90 *m*. This record was broken by Carl Lewis and Mike Powell in 1991,World championship held at Tokyo, Japan. Carl Lewis jumped 8.91 *m* and Powell jumped 8.95 *m*. Can you compare these distances?

X7C5R3

We follow these steps to compare decimals.

1.4.1 Decimal Numbers with Equal Decimal Digits

Step:1 Compare the whole number part of the two numbers.
The decimal number that has the greater whole number part is greater.

Step:2 If the whole number part is equal, then compare the digits at the tenths place. The decimal number that has the larger tenths digit is greater.

Step:3 If the whole number part and the digits at the tenths place are equal, compare the digits at the hundredths place. The decimal number that has the larger hundredth digit is greater. The same procedure can be extended to any number of decimal digits.

1.4.2 Decimal Numbers with Unequal Decimal Digits

Let us now compare the numbers 45.55 and 45.5. In this case, we first compare the whole number part. We see that the whole number part for both the numbers are equal. So, we now compare the tenths place. We find that for 45.55 and 45.5, the tenth place is also equal. Now we proceed to compare hundredth place. The hundredth place of 45.5 is 0 and that of 45.55 is 5. Comparing the hundredths place, we get $0 < 5$.

Therefore, $45.50 < 45.55$

Note

Zeros added to the right end of decimal digits do not change the value of that decimal number.

Example 1.13 Velan bought 8.36 *kg* of potato and Sekar bought 6.29 *kg* of potato. Which is heavier?

Solution

Compare 8.36 and 6.29

Comparing the whole number part, we get $8 > 6$.

Therefore, $8.36 > 6.29$

Example 1.14 Two automatic ice cream machines A and B designed to fill cups with 100 *ml* of ice cream were tested. Two cups of ice creams, one from machine A and one from machine B are weighed and found to be 99.56 *ml* from machine A and 99.65 *ml* from machine B. Can you say which machine gives more quantity of ice creams?

Solution

Compare 99.56 and 99.65

Here the whole number parts of the given two numbers are equal.

Comparing the digits at tenths place, we get 5 < 6.

Therefore, 99.56 < 99.65

Example 1.15 A standard art paper is about 0.05 *mm* thick and matte coated paper is 0.09 *mm* thick. Can you say which paper is more thick?

Solution

Compare 0.05 and 0.09

By using the steps given above, the integral parts and tenths places are equal. By comparing the hundredth place, we get 5 < 9. Therefore, 0.05 < 0.09.

So far we discussed about the comparison of two decimal numbers. Extending this, we can arrange the given decimal numbers in ascending or descending order.

Example 1.16 Now let us arrange the long jump records of students in a school for 3 years in ascending order

(i) 1 year -4.90 *m* (ii) 2 year-4.91 *m* (iii) 3 year-4.95 *m*

Solution

The whole number parts of the three decimal numbers are equal.

The digits at tenths place are also equal.

The digits at hundredths place are 0, 1 and 5. Here $0<1<5$

Therefore, the ascending order is 4.90, 4.91, 4.95.

> **Note**
>
> The descending order is 4.95,4.91,4.90

Example 1.17 Megala and Mala bought two watermelons weighing 13.523 kg and 13.52 kg. Which is a heavier one?

Solution

$$13.523 = 10 + 3 + \frac{5}{10} + \frac{2}{100} + \frac{3}{1000}$$

$$13.52 = 10 + 3 + \frac{5}{10} + \frac{2}{100} + \frac{0}{1000}$$

> **Note**
>
> 3.300 and 3.3 are the same.
>
> $3.300 = 3.3$

In this case, the two numbers have same digits upto hundredths place. But the thousandth place of 13.523 is 3, which is greater than 0, which is the thousandth place of 15.32.

So, 13.523 > 13.520

Exercise 1.3

1. Compare the following decimal numbers and find out the smaller number.

   (i) 2.08, 2.086 (ii) 0.99, 1.9 (iii) 3.53, 3.35

   (iv) 5.05, 5.50 (v) 123.5, 12.35

2. Arrange the following in ascending order.

(i) 2.35, 2.53, 5.32, 3.52, 3.25 (ii) 123.45, 123.54, 125.43, 125.34, 125.3

3. Compare the following decimal numbers and find the greater number.

(i) 24.5, 20.32 (ii) 6.95, 6.59 (iii) 17.3, 17.8

(iv) 235.42, 235.48 (v) 0.007, 0.07 (vi) 4.571, 4.578

4. Arrange the given decimal numbers in descending order.

(i) 17.35, 71.53, 51.73, 73.51, 37.51 (ii) 456.73, 546.37, 563.47, 745.63 457.71

Objective type questions

5. 0.009 is equal to

(i) 0.90 (ii) 0.090 (iii) 0.00900 (iv) 0.900

6. 37.70 ☐ 37.7

(i) = (ii) < (iii) > (iv) ≠

7. 78.56 ☐ 78.57

(i) < (ii) > (iii) = (iv) ≠

1.5 Representing Decimal Numbers on the Number Line

We have already learnt to represent fractions on a number line. Now let us see the representation of decimal numbers on a number line. Consider the decimal number 0.8. That is there are 8 tenths $\left(\frac{8}{10}\right)$. We know that 0.8 is more than 0 and less than 1. Let us divide the unit length into ten equal parts and take 8 parts as shown below.

Draw a number line and divide the intervals into ten equal parts. Now 0.8 lies between 0 and 1.

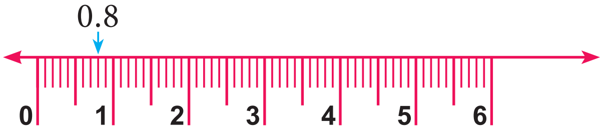


Fig. 1.3

Can we represent 1.4 on a number line? Let us see now. 1.4 has one and four tenths in it. So it lies between 1 and 2. It is marked on the number line as shown below.

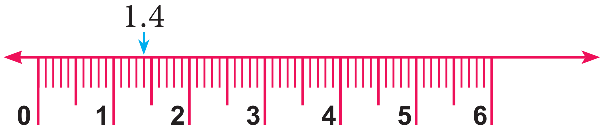


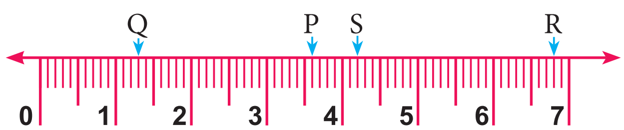
Fig. 1.4

Try these

1. Mark the following decimal numbers on the number line.
   (i) 0.3 (ii) 1.7 (iii) 2.3
2. Identify any two decimal numbers between 2 and 3.
3. Write any decimal number which is greater than 1 and less than 2.

Exercise 1.4

1. Write the decimal numbers represented by the points P, Q, R and S on the given number line.



2. Represent the following decimal numbers on the number line.
   (i) 1.7 (ii) 0.3 (iii) 2.1
3. Between which two whole numbers, the following decimal numbers lie?
   (i) 3.3 (ii) 2.5 (iii) 0.9
4. Find the greater decimal number in the following.
   (i) 2.3, 3.2 (ii) 5.6, 6.5 (iii) 1.2, 2.1
5. Find the smaller decimal number in the following.
   (i) 25.3, 25.03 (ii) 7.01, 7.3 (iii) 5.6, 6.05

Objective type questions

6. Between which two whole numbers 1.7 lie?
   (i) 2 and 3 (ii) 3 and 4 (iii) 1 and 2 (iv) 1 and 7
7. The decimal number which lies between 4 and 5 is _____
   (i) 4.5 (ii) 2.9 (iii) 1.9 (iv) 3.5

Exercise 1.5

Miscellaneous Practice problems

1. Write the following decimal numbers in the place value table.
   (i) 247.36 (ii) 132.105
2. Write each of the following as decimal number.
   (i) $300+5+\frac{7}{10}+\frac{9}{100}+\frac{2}{100}$ (ii) $1000+400+30+2+\frac{6}{10}+\frac{7}{100}$

3. Which is greater?

(i) 0.888 (or) 0.28 (ii) 23.914 (or) 23.915

4. In a 25 *m* swimming competition, the time taken by 5 swimmers A, B, C, D and E are 15.7 seconds, 15.68 seconds, 15.6 seconds, 15.74 seconds and 15.67 seconds respectively. Identify the winner.

5. Convert the following decimal numbers into fractions.

(i) 23.4 (ii) 46.301

L8J3Z2

6. Express the following in kilometres using decimals.

(i) 256 *m* (ii) 4567 *m*

7. There are 26 boys and 24 girls in a class. Express the fractions of boys and girls as decimal numbers.

Challenge Problems

8. Write the following amount using decimals.

(i) 809 rupees 99 paise (ii) 147 rupees 70 paise

9. Express the following in metres using decimals.

(i) 1328 *cm* (ii) 419 *cm*

10. Express the following using decimal notation.

(i) 8 *m* 30 *cm* in metres (ii) 24 *km* 200 *m* in kilometres

11. Write the following fractions as decimal numbers.

(i) $\frac{23}{10000}$ (ii) $\frac{421}{100}$ (iii) $\frac{37}{10}$

12. Convert the following decimals into fractions and reduce them to the lowest form.

(i) 2.125 (ii) 0.0005

13. Represent the decimal numbers 0.07 and 0.7 on a number line.

14. Write the following decimal numbers in words.

(i) 4.9 (ii) 220.0 (iii) 0.7 (iv) 86.3

15. Between which two whole numbers the given numbers lie?

(i) 0.2 (ii) 3.4 (iii) 3.9

(iv) 2.7 (v) 1.7 (vi) 1.3

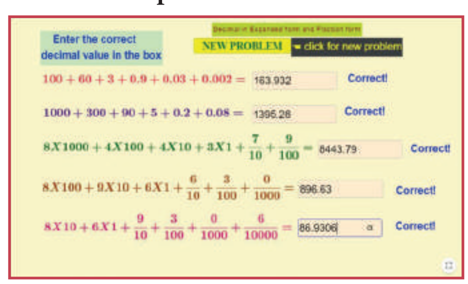
16. By how much is $\frac{9}{10}$ *km* less than 1 *km*. Express the same in decimal form.

Summary

- $\frac{1}{10}$ (one-tenth) of a unit can be written as 0.1 in decimal notation.
- The dot represents the decimal point and it comes between ones place and tenths place.
- The place value of the decimal digits of a number are tenths $\left(\frac{1}{10}\right)$, hundredths $\left(\frac{1}{100}\right)$, thousandths $\left(\frac{1}{1000}\right)$ and so on.
- In any number, when we move towards right from one digit to the next, the place value of a digit is divided by 10^1.
- If the denominator of a fraction is any of 10, 10^2, 10^3,... we can express them as decimals.
- If the denominator of a fraction is any number that can be made as powers of 10 using the concepts of equivalent fractions, then it can also be expressed as decimals.
- To compare two decimal numbers, we compare the digits from left to right.

ICT Corner

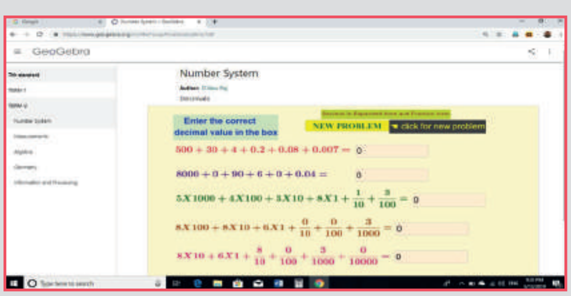
Expected outcome



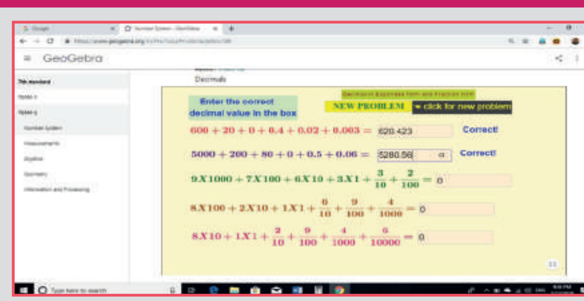
Step-1 : Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "Number System" will open. Click on "NEW PROBLEM"

Step-2 : Type the correct Decimal in the Input box and click enter. If the answer is correct it will show "Correct" else it will show "Try Again". Enter all the correct answers and click for "New Problem"

Step 1



Step 2



Browse in the link

Number system : https://www.geogebra.org/m/f4w7csup#material/p8mc7dfr

or Scan the QR Code.

B350_7_MATHS_EM

Chapter

2

MEASUREMENTS

Learning Objectives

- To understand the concepts of area and circumference of circle.
- To understand the area of circular and rectangular pathways.

T4I2W4

2.1 Introduction

We have already studied that closed shapes such as rectangle and square having area and perimeter. Pasting of tiles on a wall, paving a parking lot with stones, fencing a field or ground etc., are some of the places where the knowledge of area and perimeter of rectangle are essential. In this chapter, we extend this concept to circles. The best example of a circle is the wheel. The invention of wheel is perhaps the greatest achievement of mankind.

The teacher shows the following pictures of wheel and asks questions as given below:

Fig. 2.1

Fig. 2.2

Teacher : Bharath, can you tell me what is the name of the picture given in Fig 2.1?

Bharath : Yes Sir/Madam, wheel of a bicycle.

Teacher : Sathish, would you tell me what is in Fig.2.2?

Sathish : Yes Sir/Madam, it is a wheel of a car.

Teacher : Suresh, will you name the shape of both figures?

Suresh : Yes Sir/Madam, they are circular in shape.

Teacher : Yes, you are right. Now Mary, will you tell me the distance covered by the wheel when it rotates once.

Mary : I don't know sir/madam.

Teacher : Ok, How do we measure the distance around the circle? We cannot measure the curves with the help of a ruler, as these shapes do not have straight edges. But there is a way to measure the distance around the circle. Mark a point on its boundary. Place the wheel on the floor in such a way that the marked point coincides with the floor. Take it as the initial point. Rotate the wheel once on the floor along a straight line till the marked point again touches the floor. The distance covered by it is the distance around the outer edge of a circle. That is, circumference.

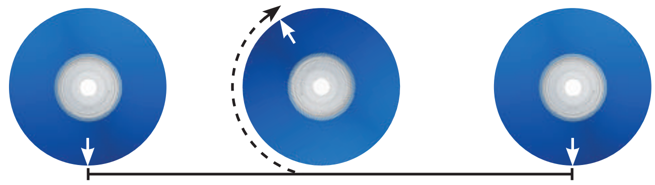


Fig. 2.3

Suresh : Teacher, is there any other way to find the distance?

Teacher : Yes, mark a point on its boundary and using a thread and scale we can measure its length.

We are now going to discuss the circumference and area of the circle.

MATHEMATICS ALIVE - Measurements in Real Life


Cement Pipes


Cup and Saucer

2.2 Circle

In our daily life, we come across circular shapes in various places. For understanding of circular shapes, first let us see how to trace a circle through an activity.

Activity

Put a pin on a board, put a loop of string around it, and insert a pencil into the loop. Keep the string stretched and draw with the pencil. The pencil traces out a circle.

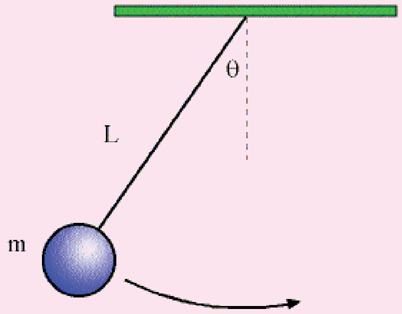
What happens if we change the position of the pin? Do we get the same circle or a different circle? Can we make the string longer? Do we get a circle of the same size? Can we change the pencil to a pen? What happens to the circle? Yes, changing the pen to pencil or sketch pen will only change the colour of the circle but changing the position of the pin and length of the string will change the place where the circle is drawn and also the size of the circle. These two quantities namely, the position of the pin and the length of the string are important to specify the circle.

The position of the pin on the board is the centre (O) of the circle. The length of the string is the radius (r) of the circle.

While tracing a circle, the two positions of the string which falls on a straight line is the diameter (d) of the circle. It is twice the radius ($d = 2r$).

Try these

1. A few real life examples of circular shapes are given below.



Give three more examples.

2. Find the diameter of your bicycle wheel?
3. If the diameter of the circle is 14cm, what will be it's radius?
4. If the radius of a bangle is 2 inches then find the diameter.

Activity

Calculating the perimeter of a circle:

Make the students to draw five circles with different radii on a paper and instruct them to measure the radius, diameter and circumference of each of the circle using thread and scale. Note down the measurements in the following table.

Circle	Radius (r)	Diameter (d)	Circumference(C)	Ratio of Circumference to diameter $\left(\frac{C}{d}\right)$

What do you infer from the above table? Can you conclude that the circumference of a circle is always greater than three times its diameter?

2.3 Circumference of a Circle

All circles are similar to one another. So, the ratio of the circumference to that of diameter is a constant, that is $\frac{\text{Circumference}}{\text{Diameter}} = \text{constant [say } \pi \text{ (pi)]}$

Its approximate value is 3.14. Therefore $\frac{C}{d} = \pi$.

The diameter is twice the radius ($2r$), so the above equation can be written as $\frac{C}{2r} = \pi$.

Therefore, the circumference of circle, $C = 2\pi r$ units.

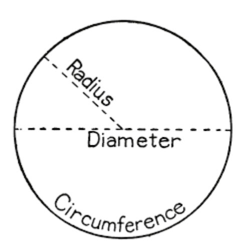


Fig. 2.4

Obviously now, we see that

Circumference, $C = \pi d$ and $d = 2r$.

Thus for any circle with a given 'r' or 'd', we can find 'C' and vice-versa.

DO YOU KNOW?

1. Calculation of decimal digits of π is an interesting task among mathematicians.
2. The constant π was used in the construction of popular pyramids of Egypt.
3. Computers helped mathematicians to calculate the value of π with more than 12 trillion decimal digits.

Think

A circle has the shortest perimeter of all closed figures with the same area. Justify with an example.

Example 2.1 Calculate the circumference of the bangle shown in Fig. 2.5 (Take $\pi = 3.14$).

Solution

Given, $d = 6\ cm$, $d = 2r = 6\ cm$, $r = 3\ cm$

Circumference of a circle $= 2\pi r$ *units*

$= 2\pi \times 3$

$= 18.84$

The circumference is 18.84 *cm*.

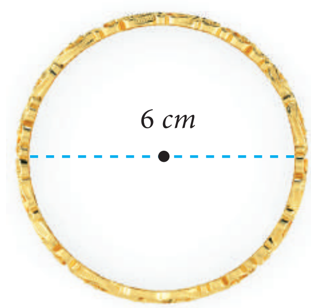


Fig. 2.5

Example 2.2 What is the circumference of the circular disc of radius 14 *cm*? (use $\pi = \frac{22}{7}$)

Solution

Radius of circular disc $(r) = 14\ cm$

Circumference of the disc $= 2\pi r$ *units*

$$= 2 \times \frac{22}{7} \times 14$$

$= 88\ cm$

Example 2.3 If the circumference of the circle is 132 *m*. Then calculate the radius and diameter (Take $\pi = \frac{22}{7}$).

Solution

Circumference of the circle, $C = 2\pi r$ *units*

The circumference of the given circle = 132 *m*

$$\frac{C}{2\pi} = r$$

$$r = \frac{132}{2 \times \frac{22}{7}}$$

$$= \frac{132}{2} \times \frac{7}{22}$$

$$= 21\, m$$

$$d = 2r$$

$$= 2 \times 21$$

$$= 42\ m$$

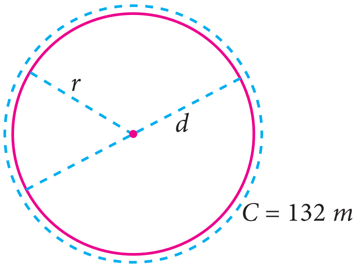


Fig. 2.6

Example 2.4 What is the distance travelled by the tip of the seconds hand of a clock in 1 minute, if the length of the hand is 56 *mm* (use $\pi = \frac{22}{7}$).

Fig. 2.7

Solution

Here the distance travelled by the tip of the seconds hand of a clock in 1 minute is the circumference of the circle and the length of the seconds hand is the radius of the circle. So, $r = 56\ mm$

Circumference of the circle, $C = 2\pi r$ *units*

$$= 2 \times \frac{22}{7} \times 56$$

$$= 2 \times 22 \times 8$$

$$= 352\ mm$$

Therefore, distance travelled by the tip of the seconds hand of a clock in 1 minute is 352 *mm.*

Example 2.5 The radius of a tractor wheel is 77 *cm*. Calculate the distance covered by it in 35 rotations? (use $\pi = \frac{22}{7}$)

Fig. 2.8

Solution

The distance covered in one rotation

= the circumference of the circle

$= 2\pi r$ *units*

$$= 2 \times \frac{22}{7} \times 77$$

$$= 2 \times 22 \times 11$$

$$= 484\ cm$$

The distance covered in one rotation $= 484\ cm$

The distance covered in 35 rotations $= 484 \times 35$

$= 16940\ cm$

Example 2.6 A farmer wants to fence his circular poultry farm with barbed wire whose radius is 420 m. The cost of fencing is ₹12 per metre. He has ₹30,000 with him. How much more amount will be needed to fence his farm? (Here $\pi = \frac{22}{7}$)

Solution

The radius of the poultry farm is $= 420\ m$

The length of the barbed wire for fencing the poultry farm is equal to the circumference of the circle.

We know that the circumference of the circle $= 2\pi r$ *units*

$= 2 \times \frac{22}{7} \times 420$

$= 2 \times 22 \times 60$

The length of the barbed wire to fence the poultry farm $= 2640\ m$

The cost of fencing the poultry farm at the rate of ₹12 per metre $= 2640 \times 12$

$=$ ₹31,680

Given that he has ₹30,000 with him.

The excess amount required $=$ ₹31,680 $-$ ₹30,000 $=$ ₹1,680.

Example 2.7 Find the perimeter of the given shape (Fig.2.9) (Take $\pi = \frac{22}{7}$).

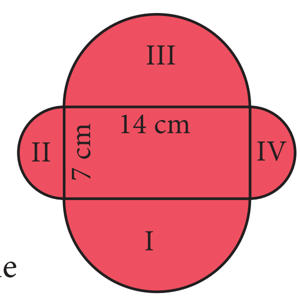


Fig. 2.9

Solution

In this shape, we have to calculate the circumference of semicircle on each side of the rectangle. The outer boundary of this figure is made up of semicircles of two different sizes. Diameters of each of the semicircles are 7 cm and 14 cm. We know that the circumference of the circle $= \pi d$ *units*.

Circumference of the semicircular part $= \frac{1}{2}\pi d$ *units*

Hence, the circumference of the semicircle having diameter 7 cm is,

$$= \frac{1}{2} \times \frac{22}{7} \times 7 = 11\ cm$$

Circumference of the pair of semicircular parts (II and IV) $= 2 \times 11 = 22\ cm$

Similarly, circumference of the semicircle having diameter 14 cm is,

$$= \frac{1}{2} \times \frac{22}{7} \times 14 = 22\ cm$$

Circumference of the pair of semicircular parts (I and III) $= 2 \times 22 = 44\ cm$.

Perimeter of the given shape $= 22 + 44 = 66\ cm$.

Example 2.8 Kannan divides a circular disc of radius 14 cm into four equal parts. What is the perimeter of a quadrant shaped disc? (use $\pi = \frac{22}{7}$)

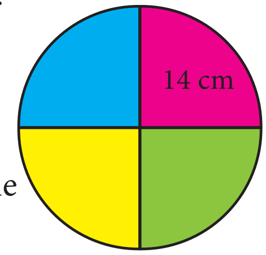


Fig. 2.10

Solution

To find the perimeter of the quadrant disc, we need to find the circumference of quadrant shape.

Given that radius (r) = 14 *cm*.

We know that the circumference of circle $= 2\pi r$ *units*.

So, the circumference of the quadrant arc $= \frac{1}{4} \times 2\pi r$

$= \frac{\pi r}{2}$

$= \frac{22}{7} \times \frac{14}{2}$

$= 22$ *cm*

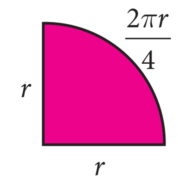


Fig. 2.11

Given, the radius of the circle $= 14$ *cm*

Thus, perimeter of required quadrant shaped disc $= 14 + 14 + 22$

$= 50$ *cm*.

Think

(i) Is the circumference of the semicircular arc and semicircular shaped disc same? Discuss.

(ii) The traffic lights are circular. Why?

(iii) When you throw a stone on still water in pond, ripples are circular. Why?

Exercise 2.1

1. Find the missing values in the following table for the circles with radius (r), diameter (d) and Circumference (C).

S.No.	radius (r)	diameter (d)	Circumference (C)
(i)	15 cm		
(ii)			1760 cm
(iii)		24 m	

2. Diameters of different circles are given below. Find their circumference (Take $\pi = \frac{22}{7}$).

   (i) $d = 70$ *cm* (ii) $d = 56$ *m* (iii) $d = 28$ *mm*

3. Find the circumference of the circles whose radii are given below.

   (i) 49 *cm* (ii) 91 *mm*

4. The diameter of a circular well is 4.2 *m*. What is its circumference?

5. The diameter of the bullock cart wheel is 1.4 m. Find the distance covered by it in 150 rotations?

6. A ground is in the form of a circle whose diameter is 350 m. An athlete makes 4 revolutions. Find the distance covered by the athlete.

7. A wire of length 1320 cm is made into circular frames of radius 7 cm each. How many frames can be made?

8. A Rose garden is in the form of circle of radius 63 m. The gardener wants to fence it at the rate of ₹150 per metre. Find the cost of fencing?

Objective type questions

9. Formula used to find the circumference of a circle is

   (i) $2\pi r$ *units* (ii) $\pi r^2 + 2r$ *units* (iii) πr^2 *sq. units* (iv) πr^3 *cu. units*

10. In the formula, $C = 2\pi r$, 'r' refers to

   (i) circumference (ii) area (iii) rotation (iv) radius

11. If the circumference of a circle is 82π, then the value of 'r' is

   (i) 41 cm (ii) 82 cm (iii) 21 cm (iv) 20 cm

12. Circumference of a circle is always

   (i) three times of its diameter
   (ii) more than three times of its diameter
   (iii) less than three times of its diameter
   (iv) three times of its radius

2.4 Area of the Circle

Let us consider the following situation.

A bull is tied with a rope to a pole. The bull goes round to eat grass. What will be the portion of grass that the bull can graze?

Can you tell what is needed to be found in the above situation, Area or Perimeter? In this situation we need to find the area of the circular region.

Let us find a way to calculate the area (A) of a circle in terms of known area, that is the area of a rectangle.

1. Draw a circle on a sheet of paper.

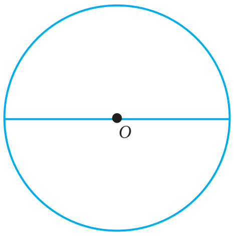


Fig. 2.12

2. Fold it once along its diameter to obtain two semicircles. Shade one half of the circle (Fig. 2.13)

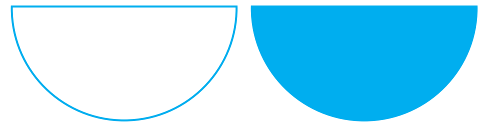


Fig. 2.13

3. Again fold the semicircles to get 4 sectors. Fig. 2.14 shows a circle divided into four sectors. The sectors are re-arranged and made into a shape as shown in Fig. 2.15.

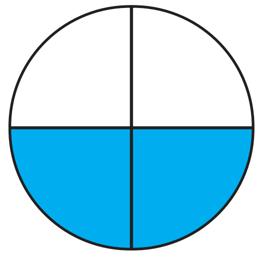


Fig. 2.14

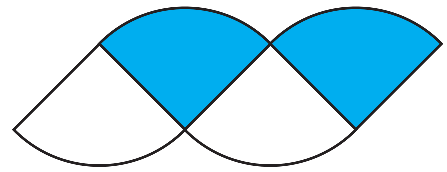


Fig. 2.15

4. Repeat this process of folding to eight folds, then it looks like a small sectors as shown in the Fig. 2.16. The sectors are re-arranged and made into a shape as shown in Fig. 2.17.

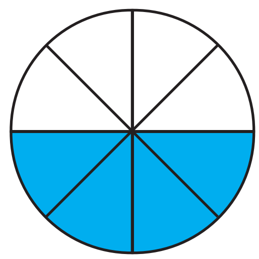


Fig. 2.16

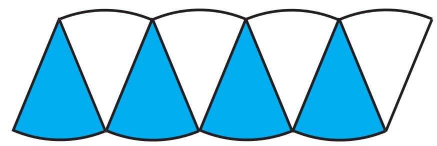


Fig. 2.17

5. Press and unfold the circle. It is then divided into 16 equal sectors and then into 32 equal sectors. As the number of sectors increase the assembled shape begins to look more or less like a rectangle, as shown in Fig. 2.19.

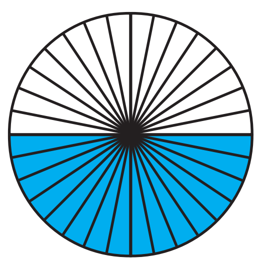


Fig. 2.18

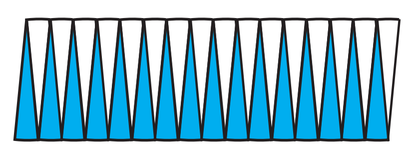


Fig. 2.19

6. The top and bottom of the rectangle is more or less equivalent to the circumference of the circle. Hence the top of the rectangle is half of the circumference $= \pi r$. The height of the rectangle is nearly equivalent to the radius of the circle. When the number of sectors is increased infinitely, the circle can be rearranged to form a rectangle of length 'πr' and breadth 'r'.

We know that the area of the rectangle $= l \times b$

$= \pi r \times r$

$= \pi r^2$

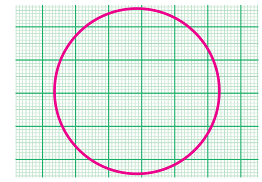
= Area of the circle

T8Y3Q4

So, the area of the circle, $A = \pi r^2$ *sq.units*.

Activity

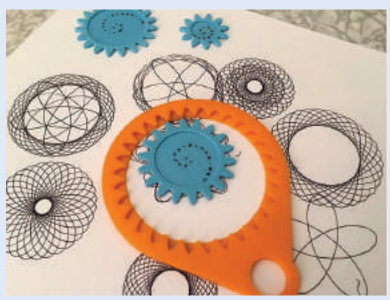
Draw circles of different measurements on a graph sheet. Find the area by counting the number of squares enclosed by the circle. So we get a rough estimate of the area of the circle.

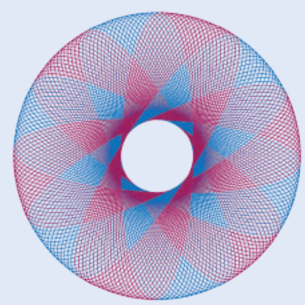


DO YOU KNOW?

Designs obtained by spirograph.

Shown below are some shapes using a spirograph. We can observe that each of the different designs are circular.





Think

If the area and circumference of the circle are equal then what will be the value of the radius?

Example 2.9 Find the area of the circle of radius 21 *cm* (Use $\pi = 3.14$).

Solution

$$\text{Radius } (r) = 21\ cm$$

$$\text{Area of a circle} = \pi r^2\ sq.\ units$$

$$= 3.14 \times 21 \times 21$$

$$= 1384.74$$

$$\text{Area of the circle} = 1384.74\ cm^2$$

Example 2.10 Find the area of a hula loop whose diameter is 28 *cm* (use $\pi = \frac{22}{7}$).

Solution

$$\text{Given the diameter } (d) = 28\ cm$$

$$\text{Radius } (r) = \frac{28}{2} = 14\ cm$$

$$\text{Area of a circle} = \pi r^2\ sq.units$$

$$= \frac{22}{7} \times 14 \times 14$$

$$\text{So, the area of the circle} = 616\ cm^2.$$

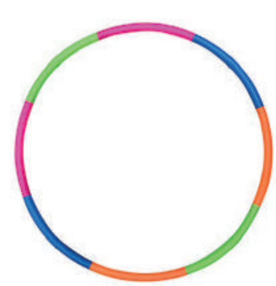


Fig. 2.20

Example 2.11 The area of the circular region is 2464 cm^2. Find its radius and diameter. (use $\pi = \frac{22}{7}$)

Solution

Given that the area of the circular region $= 2464\ cm^2$

$$\pi r^2 = 2464$$

$$\frac{22}{7} \times r^2 = 2464$$

$$r^2 = 2464 \times \frac{7}{22}$$

$$r^2 = 112 \times 7 = 784$$

$$r^2 = 2 \times 2 \times 2 \times 2 \times 7 \times 7$$

$$= 4 \times 4 \times 7 \times 7$$

$$= 4^2 \times 7^2$$

$$r^2 = (4 \times 7)^2 \quad [r \times r = (4\times7)\times(4\times7)]$$

$$r = 4 \times 7$$

$$= 28\ cm$$

2	784
2	392
2	196
2	98
7	49
	7

Diameter $(d) = 2 \times r = 2 \times 28 = 56\ cm.$

Example 2.12 A gardener walks around a circular park of distance 154 m. If he wants to level the park at the rate of ₹25 per $sq.m$, how much amount will he need? (use $\pi = \frac{22}{7}$)

Solution

The distance covered by the man is nothing but the circumference of the circle.

Given that the distance covered $= 154\ m$

Therefore circumference of the circle $= 154\ m$

That is, $2\pi r = 154$

$$2 \times \frac{22}{7} \times r = 154$$

$$r = 154 \times \frac{7}{44}$$

$$r = 3.5 \times 7$$

$$= 24.5$$

Area of the park $= \pi r^2\ sq.units$

$$= \frac{22}{7} \times 24.5 \times 24.5$$

$$= 22 \times 3.5 \times 24.5$$

$$= 1886.5\ m^2$$

Cost of levelling the park per sq.m = ₹ 25.

Cost of levelling the park of $1886.5 m^2$ $= 1886.5 \times 25 =$ ₹ 47,162.50

Example 2.13 Find the length of the rope by which a cow must be tethered in order that it may be able to graze an area of 9856 *sq.m* (use $\pi = \frac{22}{7}$)

Solution

2	3136
2	1568
2	784
2	392
2	196
2	98
7	49
	7

Given that, the area of the circle $= 9856\ sq.m$

$$\pi r^2 = 9856$$

$$\frac{22}{7} \times r^2 = 9856$$

$$r^2 = 9856 \times \frac{7}{22}$$

$$r^2 = 448 \times 7 = 3136$$

$$r^2 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7$$

$$r^2 = 8 \times 8 \times 7 \times 7 = 8^2 \times 7^2 = (8 \times 7)^2$$

$$r = 8 \times 7 = 56\ m$$

Therefore the length of the rope should be 56 *m*.

Example 2.14 A garden is made up of a rectangular portion and two semicircular regions on either sides. If the length and width of the rectangular portion are 16 *m* and 8 *m* respectively, calculate $(\pi = 3.14)$

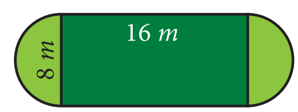


Fig. 2.21

(i) the perimeter of the garden (ii) the total area of the garden

Solution

(i) The perimeter of the rectangular garden

Perimeter include two lengths each of 16 *m* and two semi circular arcs of diameter 8 *m*.

$$\text{Circumference of the semicircle} = \frac{\pi d}{2}\ units$$

$$= \frac{\pi \times 8}{2} = 4\pi$$

$$= 4 \times 3.14$$

$$= 12.56\ m$$

Therefore, circumference of two semicircles $= 2 \times 12.56$

$$= 25.12\ m$$

Total perimeter = length + length + circumference of two semicircles

$$= 16 + 16 + 25.12$$

$$= 32 + 25.12$$

$$= 57.12\ m$$

(ii) The total area of the garden

Total area of the garden = Area of rectangle + Area of 2 semicircles

= Area of rectangle + Area of the circle

Here, the area of the rectangle $= l \times b$ *sq. units*

$= 16 \times 8$

$= 128\ m^2$ ...(1)

S9Q8V4

Area of the circle $= \pi r^2$ *sq.units*

$= 3.14 \times 4 \times 4$

$= 3.14 \times 16$

$= 50.24\ m^2$ ...(2)

From (1) and (2), the total area of garden $= 128 + 50.24$

$= 178.24\ m^2$

Try these

Draw circles of different radii on a graph paper. Find the area by counting the number of squares covered by the circle. Also find the area by using the formula.

(i) Find the area of the circle, if the radius is 4.2 *cm*.

(ii) Find the area of the circle if the diameter is 28 *cm*.

Exercise 2.2

1. Find the area of the dining table whose diameter is 105 *cm*.
2. Calculate the area of the shotput circle whose radius is 2.135 *m*.
3. A sprinkler placed at the centre of a flower garden sprays water covering a circular area. If the area watered is 1386 cm^2, find its radius and diameter.
4. The circumference of a circular park is 352 *m*. Find the area of the park.

5. In a grass land, a sheep is tethered by a rope of length 4.9 *m*. Find the maximum area that the sheep can graze.
6. Find the length of the rope by which a bull must be tethered in order that it may be able to graze an area of 2464 m^2.
7. Lalitha wants to buy a round carpet of radius is 63 *cm* for her hall. Find the area that will be covered by the carpet.
8. Thenmozhi wants to level her circular flower garden whose diameter is 49 *m* at the rate of ₹150 per m^2. Find the cost of levelling.

9. The floor of the circular swimming pool whose radius is 7 m has to be cemented at the rate of ₹18 per m^2. Find the total cost of cementing the floor.

Objective type questions

10. The formula used to find the area of the circle is ________ *sq.units*.

   (i) $4\pi r^2$ (ii) πr^2 (iii) $2\pi r^2$ (iv) $\pi r^2 + 2r$

11. The ratio of the area of a circle to the area of its semicircle is

   (i) 2:1 (ii) 1:2 (iii) 4:1 (iv) 1:4

12. Area of a circle of radius 'n' units is

   (i) $2\pi r^p$ *sq. units* (ii) πm^2 *sq. units* (iii) πr^2 *sq. units* (iv) πn^2 *sq. units*

2.5 Area of Pathways

We come across pathways in different shapes. Here we restrict ourselves to two kinds of pathways namely circular and rectangular.

2.5.1. Circular Pathways

We observe around us circular shapes where we need to find the area of the pathway. The area of pathway is the difference between the area of outer circle and inner circle. Let 'R' be the radius of the outer circle and 'r' be the radius of inner circle.

Fig. 2.22

Therefore, the area of the circular pathway $= \pi R^2 - \pi r^2$

$= \pi(R^2 - r^2)$ *sq. units.*

2.5.2 Rectangular Pathways

Consider a rectangular park as shown in Fig 2.23. A uniform path is to be laid outside the park. How do we find the area of the path? The uniform path including the park is also a rectangle. If we consider the path as outer rectangle, then the park will be the inner rectangle. Let l and b be the length and breadth of the park. Area of the park (inner rectangle) $= l\,b$ *sq.units*. Let w be the width of the path. If L, B are the length and breadth of the outer rectangle, then $L = l + 2w$ and $B = b + 2w$.

Fig. 2.23

Here, the area of the rectangular pathway

= Area of the outer rectangle – Area of the inner rectangle

= $(LB - lb)$ *sq. units*

Example 2.15 A park is circular in shape. The central portion has playthings for kids surrounded by a circular walking pathway. Find the walking area whose outer radius is 10 *m* and inner radius is 3 *m*.

Solution

The radius of the outer circle, $R = 10\ m$

The radius of the inner circle, $r = 3\ m$

The area of the circular path $=$ Area of outer circle $-$ Area of inner circle

$$= \pi R^2 - \pi r^2$$

$$= \pi\left(R^2 - r^2\right)\ sq.units$$

$$= \frac{22}{7} \times (10^2 - 3^2)$$

$$= \frac{22}{7} \times \left((10\times 10) - (3\times 3)\right)$$

$$= \frac{22}{7} \times (100 - 9)$$

$$= \frac{22}{7} \times 91$$

$$= 286\ m^2$$

Z5P7S6

Try these

(i) If the outer radius and inner radius of the circles are respectively 9 *cm* and 6 *cm*, find the width of the circular pathway.

(ii) If the area of the circular pathway is 352 *sq.cm* and the outer radius is 16 *cm*, find the inner radius.

(iii) If the area of the inner rectangular region is 15 *sq.cm* and the area covered by the outer rectangular region is 48 *sq.cm*, find the area of the rectangular pathway.

Example 2.16 The radius of a circular flower garden is 21 *m*. A circular path of 14 *m* wide is laid around the garden. Find the area of the circular path.

Fig. 2.24

Solution

The radius of the inner circle $r = 21\ m$

The path is around the inner circle.

Therefore, the radius of the outer circle, $R = 21 + 14 = 35\ m$

The area of the circular path $= \pi\left(R^2 - r^2\right)\ sq.units$

$$= \frac{22}{7} \times (35^2 - 21^2)$$

$$= \frac{22}{7} \times \left((35\times 35) - (21\times 21)\right)$$

$$= \frac{22}{7} \times (1225 - 441)$$

$$= \frac{22}{7} \times 784$$

$$= 22 \times 112 = 2464\ m^2$$

Example 2.17 The radius of a circular cricket ground is 76 *m*. A drainage 2 *m* wide has to be constructed around the cricket ground for the purpose of draining the rain water. Find the cost of constructing the drainage at the rate of ₹180/- per *sq.m*.

Solution

The radius of the inner circle (cricket ground), $r = 76\ m$

A drainage is constructed around the cricket ground.

Therefore, the radius of the outer circle, $R = 76 + 2 = 78\ m$

We have, area of the circular path $= \pi\left(R^2 - r^2\right)$ *sq. units*

$$= \frac{22}{7} \times (78^2 - 76^2)$$

$$= \frac{22}{7} \times (6084 - 5776)$$

$$= \frac{22}{7} \times 308$$

$$= 22 \times 44 = 968\ m^2$$

Given, the cost of constructing the drainage per sq.m is ₹180.

Therefore, the cost of constructing the drainage $= 968 \times 180 =$ ₹1,74,240.

Example 2.18 A floor is 10 *m* long and 8 *m* wide. A carpet of size 7 *m* long and 5 *m* wide is laid on the floor. Find the area of the floor that is not covered by the carpet.

Solution

Here, $L = 10\ m\ B = 8\ m$

Area of the floor $= L \times B$

$$= 10 \times 8$$

$$= 80\ m^2$$

Area of the carpet $= l \times b$

$$= 7 \times 5$$

$$= 35\ m^2$$

Therefore, the total area of the floor not covered by the carpet $= 80 - 35$

$$= 45\ m^2$$

Example 2.19 A picture of length 23 *cm* and breadth 11 *cm* is painted on a chart, such that there is a margin of 3 *cm* along each of its sides. Find the total area of the margin.

Solution

Here $L = 23\ cm\ B = 11\ cm$

Area of the chart $= L \times B$

$= 23 \times 11$

$= 253\ cm^2$

$l = L - 2w = 23 - 2(3) = 23 - 6 = 17\ cm$

$b = B - 2w = 11 - 2(3) = 11 - 6 = 5\ cm$

Area of the picture $17 \times 5 = 85\ cm^2$

Therefore, the area of the margin $= 253 - 85$

$= 168\ cm^2$.

Example 2.20 A verandah of width 3 *m* is constructed along the outside of a room of length 9 *m* and width 7 *m*. Find (a) the area of the verandah (b) the cost of cementing the floor of the verandah at the rate of ₹15 per *sq.m*.

Solution

Here, $l = 9\ m$, $b = 7\ m$

Area of the Room $= l \times b$

$= 9 \times 7$

$= 63\ m^2$

$L = l + 2w = 8 + 2(3) = 8 + 6 = 14\ m$

$B = b + 2w = 5 + 2(3) = 5 + 6 = 11\ m$

Area of the room including verandah $= L \times B$

$= 14 \times 11$

$= 154\ m^2$

The area of the verandah = Area of the room including verandah − Area of the room

$= 154 - 63$

$= 91\ m^2$

The cost of cementing the floor for 1*sq.m* = ₹15

Therfore, the cost of cementing the floor of the verandah $= 91 \times 15 =$ ₹1365.

Example 2.21 A Kho-Kho ground has dimensions $30\ m \times 19\ m$ which includes a lobby on all of its sides. The dimensions of the playing area is $27\ m \times 16\ m$. Find the area of the lobby.

Solution

From the dimensions of the ground we have,

$L = 30\ m;\ \ B = 19\ m;\ \ l = 27\ m;\ b = 16\ m$

Area of the kho kho ground $= L \times B$

$= 30 \times 19$

$= 570\ m^2$

Area of the play field $= l \times b$

$= 27 \times 16$

$= 432\ m^2$

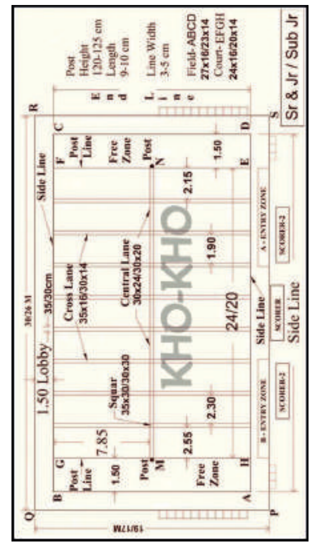


Fig. 2.25

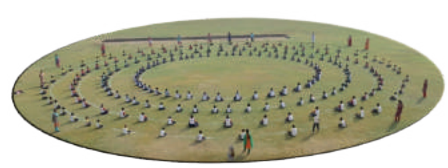
Area of the lobby $=$ Area of Kho-Kho ground $-$ Area of the play field

$= 570 - 432$

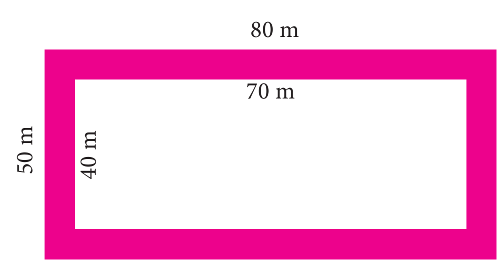
$= 148\ m^2$

Exercise 2.3

1. Find the area of a circular pathway whose outer radius is 32 *cm* and inner radius is 18 *cm*.
2. There is a circular lawn of radius 28 *m*. A path of 7 *m* width is laid around the lawn. What will be the area of the path?
3. A circular carpet whose radius is 106 *cm* is laid on a circular hall of radius 120 *cm*. Find the area of the hall uncovered by the carpet.
4. A school ground is in the shape of a circle with radius 103 *m*. Four tracks each of 3 *m* wide has to be constructed inside the ground for the purpose of track events. Find the cost of constructing the track at the rate of ₹50 per *sq.m*.



5. The figure shown is the aerial view of the pathway. Find the area of the pathway.



6. A rectangular garden has dimensions $11\ m \times 8\ m$. A path of 2 *m* wide has to be constructed along its sides. Find the area of the path.

7. A picture is painted on a ceiling of a marriage hall whose length and breadth are 18 *m* and 7 *m* respectively. There is a border of 10 *cm* along each of its sides. Find the area of the border.

8. A canal of width 1 *m* is constructed all along inside the field which is 24 *m* long and 15 *m* wide. Find (i) the area of the canal (ii) the cost of constructing the canal at the rate of ₹12 per *sq.m*.

Objective type questions

9. The formula to find the area of the circular path is

   (i) $\pi(R^2 - r^2)$ *sq. units* (ii) πr^2 *sq. units* (iii) $2\pi r^2$ *sq. units* (iv) $\pi r^2 + 2r$ *sq. units*

10. The formula used to find the area of the rectangular path is

    (i) $\pi(R^2 - r^2)$ *sq. units* (ii) $(L \times B) - (l \times b)$ *sq. units* (iii) *LB sq. units* (iv) *lb sq. units*

11. The formula to find the width of the circular path is

    (i) $(L - l)$ *units* (ii) $(B - b)$ *units* (iii) $(R - r)$ *units* (iv) $(r - R)$ *units*

Exercise 2.4

Miscellaneous Practice problems

1. A wheel of a car covers a distance of 3520 *cm* in 20 rotations. Find the radius of the wheel?

2. The cost of fencing a circular race course at the rate of ₹8 per metre is ₹2112. Find the diameter of the race course.

3. A path 2 *m* long and 1 *m* broad is constructed around a rectangular ground of dimensions 120 *m* and 90 *m* respectively. Find the area of the path.

4. The cost of decorating the circumference of a circular lawn of a house at the rate of ₹55 per metre is ₹16940. What is the radius of the lawn?

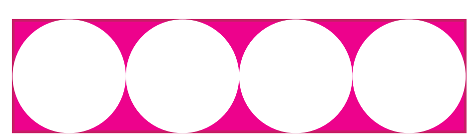
5. Four circles are drawn side by side in a line and enclosed by a rectangle as shown below.

   If the radius of each of the circles is 3 *cm*, then calculate:

   (i) The area of the rectangle.

   (ii) The area of each circle.

   (iii) The shaded area inside the rectangle.



Challenge Problems

6. A circular path has to be constructed around a circular lawn. If the outer and inner circumferences of the path are 88 *cm* and 44 *cm* respectively, find the width and area of the path.

7. A cow is tethered with a rope of length 35 m at the centre of the rectangular field of length 76 m and breadth 60 m. Find the area of the land that the cow cannot graze?

8. A path 5 m wide runs along the inside of the rectangular field. The length of the rectangular field is three times the breadth of the field. If the area of the path is 500 m^2 then find the length and breadth of the field.

9. A circular path has to be constructed around a circular ground. If the areas of the outer and inner circles are 1386 m^2 and 616 m^2 respectively, find the width and area of the path.

10. A goat is tethered with a rope of length 45 m at the centre of the circular grass land whose radius is 52 m. Find the area of the grass land that the goat cannot graze.

11. A strip of 4 cm wide is cut and removed from all the sides of the rectangular cardboard with dimensions $30\ cm \times 20\ cm$. Find the area of the removed portion and area of the remaining cardboard.

12. A rectangular field is of dimension $20\ m \times 15\ m$. Two paths run parallel to the sides of the rectangle through the centre of the field. The width of the longer path is 2 m and that of the shorter path is 1 m. Find (i) the area of the paths (ii) the area of the remaining portion of the field (iii) the cost of constructing the roads at the rate of ₹10 per $sq.m$.

Summary

- Circle is a round plane figure whose boundary (the circumference) consists of points equidistant from the fixed point (the centre).
- Distance around the circular region is called the circumference or perimeter of the circle.
- Circumference of a circle, $C = \pi d$ units, where 'd' is the diameter of a circle and $\pi = \frac{22}{7}$ or 3.14 approximately.
- Area of the circle is the region enclosed by the circle.
- The area of the circle, $(A) = \pi r^2$ $sq.\ units$, where 'r' is the radius of the circle.
- Area of the circular path = Area of the outer circle – Area of the inner circle
  $= \pi R^2 - \pi r^2 = \pi(R^2 - r^2)$ sq. units, where 'R' and 'r' are radius of the outer and inner circles respectively.
- Area of the rectangular path = Area of the outer rectangle – Area of the inner rectangle
  $= (LB - lb)$ $sq.units$, where L, B, and l, b are length and breadth of outer and inner rectangles respectively.

ICT Corner

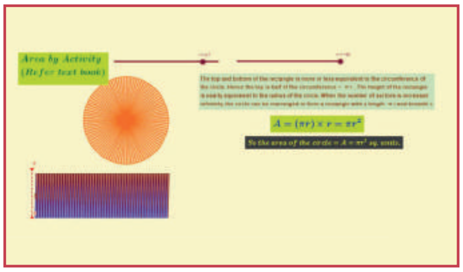
Step-1 :

Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named “Mensuration” will open. There are two activities named “Area by Activity” and “Circular path problem”

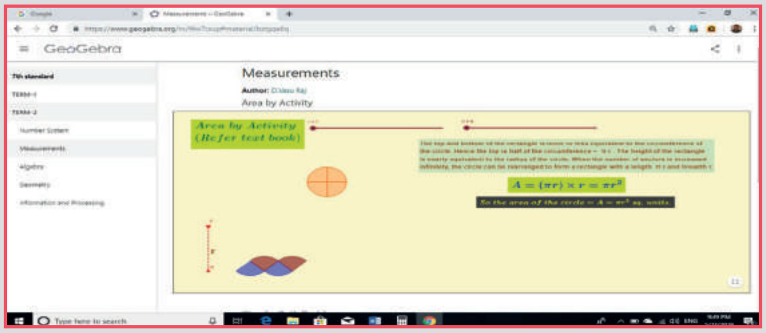
Step-2 :

In the first activity move the slider n (No. of divisions) and the slider r (to increase the radius). As the no. of division increases it becomes almost a rectangle with Length = half the circumference and breadth equal to the radius. Thus the area = Length × Breadth = πr^2. Also try the second activity.

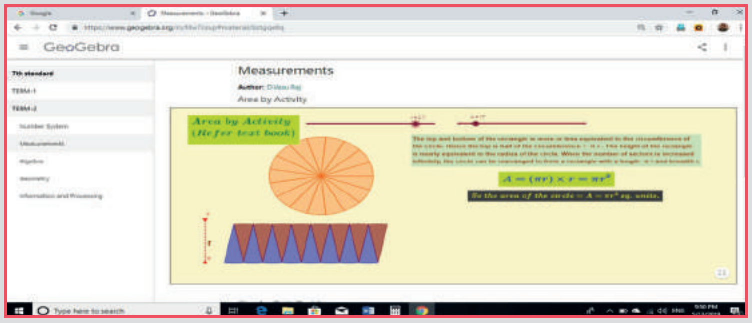
Expected Result is shown in this picture



Step 1



Step 2



Browse in the link

Measurements: https://www.geogebra.org/m/f4w7csup#material/bztgqe8q
or Scan the QR Code.

B350_7_MATHS_EM

Chapter

3

ALGEBRA

K8M8N1

Learning Objectives

- To express numbers in the exponential form.
- To understand the laws of exponents.
- To find the unit digit of numbers represented in exponential form.
- To know about the degree of expressions.

3.1 Introduction

Teacher asked students to tell the largest number known to them.

They called out numbers like, 'Thousand', 'Lakh', 'Million', 'Crore' and so on. Finally, Kumaran was declared as winner with his number 'Thousand lakh crore'.

All the students clapped. The teacher also honoured Kumaran and he was extremely happy.

But, everything came to an end as the teacher asked him to write his number on the blackboard. With great difficulty of counting the zeros several times, he wrote as 1000000000000000. Is it correct?

Now, the teacher wrote another five zeros on the right of the number and threw the open challenge to read the number. Of course, there was an absolute silence in the classroom.

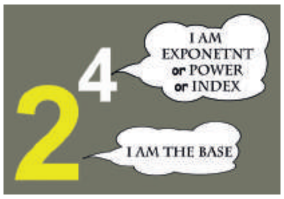
Fig. 3.1

Dealing with big numbers is not so easy. Isn't it? But, we do use very big numbers in reality? Here are few examples where we use large numbers in real life situations.

- The distance between the Earth and the Sun is 149600000000 m.
- Mass of the Earth is 5970000000000000000000000 kg.
- The speed of light is 299792000 m/sec.
- Average radius of the Sun is 695000 km.
- The distance between Moon and the Earth is 384467000 m.

There exists a simple and nice way to represent such numbers. To know that we need to learn about exponents.

3.2 Exponents and Powers



We can write large numbers in simplified form as given below.

For example, $16 = 8 \times 2 = 4 \times 2 \times 2 = 2 \times 2 \times 2 \times 2$

Instead of writing the factor 2 repeatedly 4 times, we can simply write it as 2^4. It can be read as 2 raised to the power of 4 or 2 to the power of 4 or simply 2 power 4.

This method of representing a number is called the exponential form. We say 2 is the base and 4 is the exponent.

Note

The exponent is usually written at the top right corner of the base and smaller in size when compared to the base.

Let us look at some more examples,

$64 = 4 \times 4 \times 4 = 4^3$ (base is 4 and exponent is 3)

Also, $64 = 8 \times 8 = 8^2$ (base is 8 and exponent is 2)

$243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$ (base is 3 and exponent is 5)

$125 = 5 \times 5 \times 5 = 5^3$ (base is 5 and exponent is 3)

Remember, when a number is expressed as a product of factors and when the factors are repeated, then it can be expressed in the exponential form. The repeated factor will be the base and the number of times the factor repeats will be its exponent.

We can extend this notation to negative integers also. For example,

$-125 = (-5) \times (-5) \times (-5) = (-5)^3$ [base is '-5' and exponent is 3]

Hence, $(-5)^3$ is the exponential form of -125.

MATHEMATICS ALIVE - Algebra in Real Life

In a calculator, multiplication of two large numbers is displayed as 1.219326E17 which means 1.219326×10^{17} where, 'E' stands for exponent with base 10.

Numbers in Exponential Form

Now, we will see how to express numbers in exponential form.

Let us take any integer as 'a'.

Then, $a = a^1$ ['a power 1']

$a \times a = a^2$ ['a power 2' ; 'a' is multiplied by itself 2 times]

$a \times a \times a = a^3$ ['a power 3' ; 'a' is multiplied by itself 3 times]

$\vdots \quad \vdots \quad \vdots$

$a \times a \times \ldots \times a (n \text{ times}) = a^n$ ['a power n' ; 'a' multiplied by itself n times]

Thus we can generalize the exponential form as a^n, where the exponent is a positive integer $(n > 0)$.

Observe, the following examples.

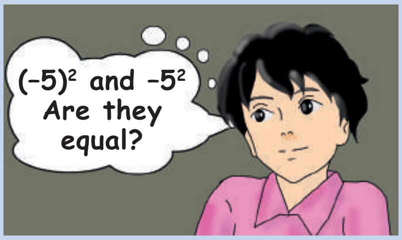
$100 = 10 \times 10 = 10^2$

This can also be expressed as the product of two different bases with the same exponent as, $100 = 25 \times 4 = (5 \times 5) \times (2 \times 2) = 5^2 \times 2^2$

We notice that 5 and 2 are the bases and 2 is the exponent.

In the same way, $a \times a \times a \times b \times b = a^3 \times b^2$

Think



Justify your answer.

Fig. 3.2

Consider, $35 = 7^1 \times 5^1$, where there is no repetition of factors. Thus, usually $7^1 \times 5^1$ is represented as 7×5. So, when the power is 1 the exponent will not be explicitly mentioned.

Note

1. The exponents 2 and 3 have special names 'squared' and 'cubed' respectively. For example, 4^2 is read as 'four squared' and 4^3 is read as 'four cubed'.
2. The other name of exponent is indices. Do you remember the word 'indices'? In Class VI, we heard of this word while applying BIDMAS rule in simplification. For example,

$6^3 + 4 \times 3 - 5 = (6 \times 6 \times 6) + 4 \times 3 - 5$ [BIDMAS]

$= 216 + (4 \times 3) - 5$ [BIDMAS]

$= 216 + 12 - 5$ [BIDMAS]

$= 228 - 5$ [BIDMAS]

$= 223$

Try these

Observe and complete the following table. First one is done for you.

Number	Expanded form	Exponential Form	Base	Exponent
216	6×6×6	6^3	6	3
144		12^2		
	(–5)×(–5)		–5	
		m^5		
343			7	3
15625	25×25×25			

Example 3.1 Express 729 in exponential form.

Solution

Dividing by 3, we get

3	729
3	243
3	81
3	27
3	9
	3

$729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

Also, $729 = 9 \times 9 \times 9 = 9^3$

Example 3.2 Express the following numbers in exponential form with the given base:

(i) 1000, base 10 (ii) 512, base 2 (iii) 243, base 3.

Solution

(i) $1000 = 10 \times 10 \times 10 = 10^3$

(ii) $512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^9$

(iii) $243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
	2

3	243
3	81
3	27
3	9
	3

Example 3.3 Find the value of (i) 13^2 (ii) $(-7)^2$ (iii) $(-4)^3$

Solution

(i) $13^2 = 13 \times 13 = 169$

(ii) $(-7)^2 = (-7) \times (-7) = 49$

(iii) $(-4)^3 = (-4) \times (-4) \times (-4)$

$= 16 \times (-4) = -64$

Note

$(-1)^n = -1$, if n is an odd natural number.

$(-1)^n = 1$, if n is an even natural number.

Think

Can you find two positive integers 'a' and 'b' such that $a^b = b^a$? $(a \neq b)$

Example 3.4 Find the value of $2^3 + 3^2$

Solution

$2^3 + 3^2 = (2 \times 2 \times 2) + (3 \times 3)$

$= 8 + 9 = 17$

Example 3.5 Which is greater 3^4 or 4^3?

Solution

$3^4 = 3 \times 3 \times 3 \times 3 = 81$

$4^3 = 4 \times 4 \times 4 = 64$

$81 > 64$ gives $3^4 > 4^3$

Therefore, 3^4 is greater.

Example 3.6 Expand a^3b^2 and a^2b^3. Are they equal?

Solution

$a^3b^2 = (a \times a \times a) \times (b \times b)$

$a^2b^3 = (a \times a) \times (b \times b \times b)$

Therefore, $a^3b^2 \neq a^2b^3$

DO YOU KNOW?

Ancient Tamilians have used lot of greater numbers in their day-to-day life. Refer to Kanakkathikaram written by a saint Karinayanar who lived in Tamilnadu during 10th century. More interestingly, they fixed a unique name for every big number. For example, 'Arpudham' means Ten crores, 'Padmam' means 10^{14}, 'Anantham' means 10^{29} and the word 'Avviyatham' denotes 10^{35}.

Pingalandai Nigandu Vaipaadu, an ancient Tamil Mathematics treatise, also confirms the usage of such big numbers. It is a book of multiplication table.

3.3 Laws of Exponents

Let us learn some rules to multiply and divide exponential numbers with the same base.

3.3.1. Multiplication of Numbers in Exponential form

Try these

Simplify and write the following in exponential form.

1. $2^3 \times 2^5$
2. $p^2 \times p^4$
3. $x^6 \times x^4$
4. $3^1 \times 3^5 \times 3^4$
5. $(-1)^2 \times (-1)^3 \times (-1)^5$

Let us calculate the value of $2^3 \times 2^2$

$$2^3 \times 2^2 = (2\times2\times2)\times(2\times2)$$
$$= 2\times2\times2\times2\times2$$
$$= 2^5$$
$$= 2^{3+2}$$

We observe that the base of 2^3 and 2^2 is the same 2 and the sum of the powers is 5. Now, let us consider negative integers as the base.

$$(-3)^3 \times (-3)^2 = \left[(-3)\times(-3)\times(-3)\right]\times\left[(-3)\times(-3)\right]$$
$$= (-3)\times(-3)\times(-3)\times(-3)\times(-3)$$
$$= (-3)^5$$
$$= (-3)^{3+2}$$

We observe that the base of $(-3)^3$ and $(-3)^2$ is the same as (-3) and the sum of the power is 5. Similarly, $p^4 \times p^2 = (p\times p\times p\times p)\times(p\times p) = p^6 = p^{4+2}$

Now, for any non-zero integer 'a' and whole number 'm' and 'n', consider a^m and a^n. That is, $a^m = a\times a\times a\times\ldots\times a$ (m times) and $a^n = a\times a\times a\times\ldots\times a$ (n times)

So, $a^m \times a^n = a\times a\times a\times\ldots\times a$ (m times) $\times\, a\times a\times a\times\ldots\times a$ (n times)

$= a\times a\times a\times\ldots\times a$ ($m+n$ times) $= a^{m+n}$

Therefore, $a^m \times a^n = a^{m+n}$

This is called Product Rule of exponents.

Fig. 3.3

Example 3.7 Simplify using Product Rule of exponents.

(i) $5^7 \times 5^3$ (ii) $3^3 \times 3^2 \times 3^4$ (iii) $25\times32\times625\times64$

Solution

(i) $5^7 \times 5^3 = 5^{7+3}$ [since, $a^m \times a^n = a^{m+n}$]

$= 5^{10}$

(ii) $3^3 \times 3^2 \times 3^4 = 3^{3+2} \times 3^4 = 3^5 \times 3^4$

$= 3^{5+4} = 3^9$

5	625
5	125
5	25
	5

2	64
2	32
2	16
2	8
2	4
	2

(iii) $25 \times 32 \times 625 \times 64 = (5 \times 5) \times (2 \times 2 \times 2 \times 2 \times 2)$
$\times (5 \times 5 \times 5 \times 5) \times (2 \times 2 \times 2 \times 2 \times 2 \times 2)$
$= 5^2 \times 2^5 \times 5^4 \times 2^6$
$= (5^2 \times 5^4) \times (2^5 \times 2^6)$ [grouping exponential numbers with the same base]
$= 5^{2+4} \times 2^{5+6} = 5^6 \times 2^{11}$

3.3.2 Division of Numbers in Exponential form

Let us calculate the value of $2^5 \div 2^2$

$$\frac{2^5}{2^2} = \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2} = 2 \times 2 \times 2$$
$$= 2^3$$
$$= 2^{5-2}$$

We observe that the base of 2^5 and 2^2 is the same '2' and the difference of powers is 3. Now, let us consider negative integers as the base.

Note

Can you find the value of $a^2 \times a^0$?

By Product Rule,

$$a^2 \times a^0 = a^{2+0}$$
$$a^2 \times a^0 = a^2$$
$$a^0 = \frac{a^2}{a^2} = 1$$

[dividing by a^2 on both sides]

Therefore, $a^0 = 1$, $a \neq 0$.

Consider $(-5)^3 \div (-5)^2$

$$\frac{(-5)^3}{(-5)^2} = \frac{(-5) \times (-5) \times (-5)}{(-5) \times (-5)}$$
$$= (-5)^1 = (-5)^{3-2}$$

We observe that the base of $(-5)^3$ and $(-5)^2$ is the same as (-5) and the difference of the power is 1.

Thus, we can observe that for any non-zero integer 'a' and for whole numbers 'm' and 'n', consider a^m and a^n, $m > n$.

That is, $a^m = a \times a \times a \times \ldots \times a$ (m times); $a^n = a \times a \times a \times \ldots \times a$ (n times)

$$\frac{a^m}{a^n} = \frac{a \times a \times a \ldots \times a (m \text{ times})}{a \times a \times a \ldots \times a (n \text{times})} = a \times a \times a \ldots \times a (m - n \text{ times}) = a^{m-n}$$

Therefore, $\frac{a^m}{a^n} = a^{m-n}$

This is called Quotient Rule of exponents.

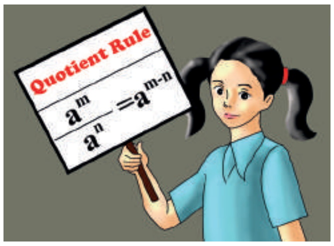


Fig. 3.4

Example 3.8 Simplify using quotient rule of exponents.

(i) $\frac{10^8}{10^6}$ (ii) $\frac{2^8 \times 3^5 \times 5^4}{3^3 \times 5^3 \times 2^4}$ (iii) $\frac{6^4}{6^0}$

Solution

(i) $\frac{10^8}{10^6} = 10^{8-6} = 10^2$

Think

What is Half of 2^{10}? Ragu claims the answer is 2^5. Is he correct? Discuss.

(ii) $\dfrac{2^8 \times 3^5 \times 5^4}{3^3 \times 5^3 \times 2^4} = \dfrac{2^8}{2^4} \times \dfrac{3^5}{3^3} \times \dfrac{5^4}{5^3}$ [grouping exponential numbers with the same base]

$= 2^{8-4} \times 3^{5-3} \times 5^{4-3} = 2^4 \times 3^2 \times 5^1$ $\left[\dfrac{a^m}{a^n} = a^{m-n}\right]$

(iii) $\dfrac{6^4}{6^0} = 6^{4-0} = 6^4$ (or) $\dfrac{6^4}{6^0} = \dfrac{6^4}{1} = 6^4$ [since $6^0 = 1$]

Try these

Simply the following.

1. $23^5 \div 23^2$
2. $11^6 \div 11^3$
3. $(-5)^3 \div (-5)^2$
4. $7^3 \div 7^3$
5. $15^4 \div 15$

3.3.3 Power of Exponential form

Let us find the value of $\left(2^2\right)^5$

$\left(2^2\right)^5 = 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 = 2^{2+2+2+2+2}$ (By Product rule)

$2^{10} = 2^{2\times5}$

Similarly, $\left(3^3\right)^4 = 3^3 \times 3^3 \times 3^3 \times 3^3$

$= 3^{3+3+3+3} = 3^{12} = 3^{3\times4}$

$\left(5^6\right)^2 = 5^6 \times 5^6 = 5^{6+6} = 5^{12} = 5^{6\times2}$

Try these

Simplify and write the following in exponent form.

1. $\left(3^2\right)^3$
2. $\left[\left(-5\right)^3\right]^2$
3. $\left(20^6\right)^2$
4. $\left(10^3\right)^5$

In general, for any non-zero integer 'a' and whole number 'm' and 'n',

$\left(a^m\right)^n = (a^m \times a^m \times a^m \ldots \times a^m)$ (n times)

$= a^{m+m+m\ldots+m \text{ (}n\text{ times)}}$

$= a^{m\times n}$

Hence, $\left(a^m\right)^n = a^{m\times n}$

This is called Power Rule of exponents.

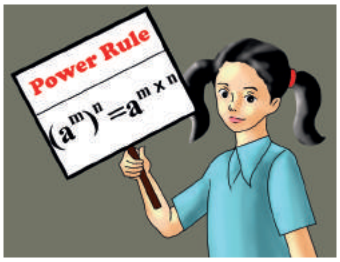


Fig. 3.5

Example 3.9 Simplify using power rule of exponents.

(i) $\left(8^3\right)^4$ (ii) $\left(11^5\right)^2$ (iii) $\left(2^6\right)^2 \times \left(2^4\right)^7$

Solution

(i) $\left(8^3\right)^4 = 8^{3\times4} = 8^{12}$ [since $\left(a^m\right)^n = a^{m\times n}$]

(ii) $\left(11^5\right)^2 = 11^{5\times2} = 11^{10}$ [since $\left(a^m\right)^n = a^{m\times n}$]

(iii) $\left(2^6\right)^2 \times \left(2^4\right)^7 = 2^{6\times2} \times 2^{4\times7}$ [since $\left(a^m\right)^n = a^{m\times n}$]

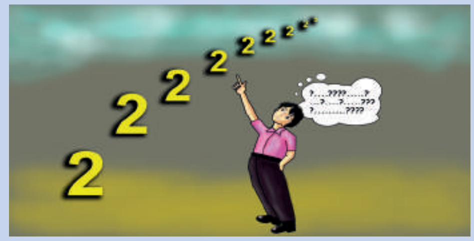
$= 2^{12} \times 2^{28}$

$= 2^{12+28} = 2^{40}$ [since $a^m \times a^n = a^{m+n}$]

Think

We learnt that $2^2 = 2\times 2$.

What is the value of 2^{2^2}? Discuss.



3.3.4. Exponent Numbers with Different Base and Same Power

1. To understand the multiplication of exponent numbers with different base and same powers, let us consider the following example,

$$\begin{aligned}10^5 &= 10\times10\times10\times10\times10\\ &= (2\times5)\times(2\times5)\times(2\times5)\times(2\times5)\times(2\times5)\\ &= (2\times2\times2\times2\times2)\times(5\times5\times5\times5\times5)\\ 10^5 &= 2^5\times5^5\end{aligned}$$

But we know that, 10 = 2 × 5. Hence $10^5 = (2\times5)^5 = 2^5\times5^5$.

In general, for any non-zero integers '*a*' and '*b*' and for whole number '*m*' $(m>0)$,

$$\begin{aligned}a^m\times b^m &= a\times a\times a\times\ldots\times a\ (m\text{ times})\times\ b\times b\times b\times\ldots\times b\ (m\text{ times})\\ &= (a\times b)\times(a\times b)\times(a\times b)\times\ldots\times(a\times b)\ (m\text{ times}) = (a\times b)^m\end{aligned}$$

Therefore, $a^m\times b^m = (a\times b)^m$.

2. To understand the division of exponent numbers with different base and same powers, let us consider the follwing example,

$$\begin{aligned}10^5 &= 10\times10\times10\times10\times10\\ &= \left(\frac{20}{2}\right)\times\left(\frac{20}{2}\right)\times\left(\frac{20}{2}\right)\times\left(\frac{20}{2}\right)\times\left(\frac{20}{2}\right)\\ &= \frac{20\times20\times20\times20\times20}{2\times2\times2\times2\times2}\end{aligned}$$

Therefore, $10^5 = \frac{20^5}{2^5}$. But we know that, $10 = \left(\frac{20}{2}\right)$.

Hence, $10^5 = \left(\frac{20}{2}\right)^5 = \frac{20^5}{2^5}$.

Hence, for any two non-zero integers '*a*' and '*b*' and a whole number '*m*' $(m>0)$,

$$\begin{aligned}\left(\frac{a}{b}\right)^m &= \left(\frac{a}{b}\right)\times\left(\frac{a}{b}\right)\times\left(\frac{a}{b}\right)\times\ldots\times\left(\frac{a}{b}\right)\ (m\text{ times})\\ &= \frac{a\times a\times a\times\ldots\times a\,(m\text{ times})}{b\times b\times b\times\ldots\times b\,(m\text{ times})} = \frac{a^m}{b^m}\end{aligned}$$

Therefore, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

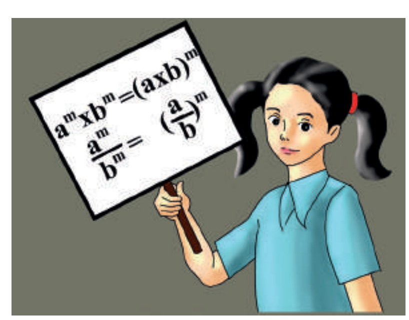


Fig. 3.6

Try these

1. Express the following exponent numbers using $a^m \times b^m = (a \times b)^m$.

   (i) $5^2 \times 3^2$

   (ii) $x^3 \times y^3$

   (iii) $7^4 \times 8^4$

2. Simplify the following exponent numbers by using $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

   (i) $5^3 \div 2^3$

   (ii) $(-2)^4 \div 3^4$

   (iii) $8^6 \div 5^6$

   (iv) $6^3 \div (-7)^3$

Example 3.10 Simplify by using the law of exponents.

(i) $7^6 \times 3^6$ (ii) $4^3 \times 2^3 \times 5^3$ (iii) $72^5 \div 9^5$ (iv) $6^{13} \times 48^{13} \div 12^{13}$

Solution

(i) $7^6 \times 3^6 = (7 \times 3)^6 = 21^6$ [Since, $a^m \times b^m = (a \times b)^m$]

(ii) $4^3 \times 2^3 \times 5^3 = (4 \times 2 \times 5)^3 = 40^3$ [Rule extended for 3 numbers]

(iii) $72^5 \div 9^5 = (72 \div 9)^5 = 8^5$ [Since, $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$]

(iv) $6^{13} \times 48^{13} \div 12^{13} = 6^{13} \times (48^{13} \div 12^{13})$ [BIDMAS]

$= 6^{13} \times \left(\frac{48}{12}\right)^{13}$ [Since, $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$]

$= 6^{13} \times 4^{13}$

$= (6 \times 4)^{13}$ [Since, $a^m \times b^m = (a \times b)^m$]

$= (24)^{13}$

G5T5T2

DO YOU KNOW?

1. All the 10 digits appear once in the expansion of 32043^2. That is, the value of $32043^2 = 1026753849$.

2. There are beautiful equations with same exponent consecutive natural numbers as the base.

$$3^2 + 4^2 = 5^2$$
$$10^2 + 11^2 + 12^2 = 13^2 + 14^2$$

Activity

Finding the pair

Divide the classroom into two groups. Each group will have a set of cards. Each member of Group 1 has to pair with one suitable member of Group 2 by stating the reason.

Group 1	Group 2
$3^6 \times 3^5$	100^3
$200^{30} \times 200^{14}$	$20^{15} \times 30^{15}$
$\frac{45^6}{45^2}$	3^{11}
$\frac{100^{52}}{100^{49}}$	70^{240}
$(6 \times 7)^3$	12^8
$(20 \times 30)^{15}$	45^4
$\left(12^4\right)^2$	200^{44}
$\left(70^{16}\right)^{15}$	$6^3 \times 7^3$

This activity can be extended till all the children in the class are familiarise with the laws of exponents.

Exercise 3.1

1. Fill in the blanks.
   (i) The exponential form 14^9 should be read as_______________
   (ii) The expanded form of p^3q^2 is_______________
   (iii) When base is 12 and exponent is 17, its exponential form is_____________
   (iv) The value of $(14 \times 21)^0$ is_______________
2. Say True or False.
   (i) $2^3 \times 3^2 = 6^5$
   (ii) $2^9 \times 3^2 = (2 \times 3)^{9 \times 2}$
   (iii) $3^4 \times 3^7 = 3^{11}$

(iv) $2^0 = (1000)^0$

(v) $2^3 < 3^2$

3. Find the value of the following.

(i) 2^6 (ii) 11^2 (iii) 5^4 (iv) 9^3

4. Express the following in exponential form.

(i) $6\times6\times6\times6$ (ii) $t\times t$ (iii) $5\times5\times7\times7\times7$ (iv) $2\times2\times a\times a$

5. Express each of the following numbers using exponential form.

(i)512 (ii)343 (iii)729 (iv)3125

6. Identify the greater number in each of the following.

(i) 6^3 or 3^6 (ii) 5^3 or 3^5 (iii) 2^8 or 8^2

7. Simplify the following.

(i) $7^2\times3^4$ (ii) $3^2\times2^4$ (iii) $5^2\times10^4$

8. Find the value of the following.

(i) $(-4)^2$ (ii) $(-3)\times(-2)^3$ (iii) $(-2)^3\times(-10)^3$

9. Simplify using laws of exponents.

(i) $3^5\times3^8$ (ii) $a^4\times a^{10}$ (iii) $7^x\times7^2$ (iv) $2^5\div2^3$

(v) $18^8\div18^4$ (vi) $(6^4)^3$ (vii) $(x^m)^0$ (viii) $9^5\times3^5$

(ix) $3^y\times12^y$ (x) $25^6\times5^6$

10. If $a=3$ and $b=2$, then find the value of the following.

(i) a^b+b^a (ii) a^a-b^b (iii) $(a+b)^b$ (iv) $(a-b)^a$

11. Simplify and express each of the following in exponential form:

(i) $4^5\times4^2\times4^4$ (ii) $(3^2\times3^3)^7$ (iii) $(5^2\times5^8)\div5^5$

(iv) $2^0\times3^0\times4^0$ (v) $\dfrac{4^5\times a^8\times b^3}{4^3\times a^5\times b^2}$

Objective type questions

12. $a\times a\times a\times a\times a$ is equal to

(i) a^5 (ii) 5^a (iii) $5a$ (iv) $a+5$

13. The exponential form of 72 is

(i) 7^2 (ii) 2^7 (iii) $2^2\times3^3$ (iv) $2^3\times3^2$

14. The value of x in the equation $a^{13}=x^3\times a^{10}$ is

(i) a (ii) 13 (iii) 3 (iv) 10

15. How many zeros are there in 100^{10}?

(i) 2 (ii) 3 (iii) 10 (iv) 20

16. $2^{40}+2^{40}$ is equal to

(i) 4^{40} (ii) 2^{80} (iii) 2^{41} (iv) 4^{80}

3.4 Unit Digit of Numbers in Exponential Form

Manipulating with exponent is very interesting and funny too.

We know that $9^3=9\times9\times9=729$, thus the unit digit (the last number of expanded form) of 9^3 is 9. Similarly, 4^4 is $4\times4\times4\times4=256$. Thus the unit digit of 4^4 is 6.

Can you guess the unit digit of 230^{116}, 181^{47}, 55^4, 56^{20} and 9^{29}?

It is very difficult to find by expanding the exponential form. But, we can try to tell the unit digit by observing some patterns.

Look at the following number pattern.

$$10^3=10\times10\times10=1000$$

$$10^4=10\times10\times10\times10=10000$$

$$10^5=10\times10\times10\times10\times10=100000$$

Thus, multiplying 10 by itself several times, we always get the unit digit as 0. In other words, 10 raised to the power of any number has the unit digit 0. That is, the unit digit of 10^x is always 0, for any positive integer x.

This is also true when the base is multiples of 10. Consider,

$$40^2=(4\times10)^2=4^2\times10^2$$
$$=16\times100=1600$$

Similarly, $230^{116}=(23\times10)^{116}=23^{116}\times10^{116}$

Thus, the unit digit of 230^{116} is 0.

Now, observe that,

$$1^5=1\times1\times1\times1\times1=1$$

$$1^6=1\times1\times1\times1\times1\times1=1$$

We learnt to expand 11 as 10+1.

So, $(10+1)^2=11^2=11\times11=121$

Similarly, $131=130+1=(13\times10)+1$

$$[(13\times10)+1]^2=131^2=131\times131=17161$$

Hence, if the exponential number is in the form 1^x or $[(\text{multiple of } 10)+1]^x$, then the unit digit is always 1, where x is a positive integer.

Therefore, the unit digit of 181^{47} is 1.

Similarly, by observing the following patterns, we can conclude that the unit digit of number with base ending with 5 is 5 and number with base ending with 6 is 6.

$5^1 = 5$	$6^1 = 6$
$5^2 = 5 \times 5 = 25$	$6^2 = 6 \times 6 = 36$
$5^3 = 25 \times 5 = 125$	$6^3 = 36 \times 6 = 216$

Therefore, the unit digit of $55^4 = (50+5)^4$ is 5 and the unit digit of $56^{20} = (50+6)^{20}$ is 6.

We conclude that, for the base number whose unit digits are 0,1,5 and 6 the unit digit of a number corresponding to any positive exponent remains unchanged.

Example 3.11 Find the unit digit of the following exponential numbers:

(i) 25^{23} (ii) 81^{100} (iii) 46^{31}

Solution

(i) 25^{23}

Unit digit of base 25 is 5 and power is 23

Thus, the unit digit of 25^{23} is 5.

(ii) 81^{100}

Unit digit of base 81 is 1 and power is 100

Thus, the unit digit of 81^{100} is 1.

(iii) 46^{31}

Unit digit of base 46 is 6 and power is 31

Thus, the unit digit of 46^{31} is 6.

Try these

Find the unit digit of the following exponential numbers:

(i) 106^{21} (ii) 25^8

(iii) 31^{18} (iv) 20^{10}

Look into the following example. Observe the pattern of unit digit when the base is 4.

$4^1 = 4$	(odd power)	$4^2 = 4 \times 4 = 16$	(even power)
$4^3 = 4 \times 4 \times 4 = 16 \times 4 = 64$	(odd power)	$4^4 = 64 \times 4 = 256$	(even power)
$4^5 = 256 \times 4 = 1024$	(odd power)	$4^6 = 1024 \times 4 = 4096$	(even power)

Note that for base ending with 4, the unit digit of the expanded form alternates between 4 and 6. Further we can notice when the power is odd its unit digit is 4 and when the power is even it is 6.

Similarly, when the base unit is 9,

$9^1 = 9$ (odd power) $9^2 = 9 \times 9 = 81$ (even power)

$9^3 = 9 \times 9 \times 9 = 81 \times 9 = 729$ (odd power) $9^4 = 729 \times 9 = 6561$ (even power)

$9^5 = 6561 \times 9 = 59049$ (odd power) $9^6 = 59049 \times 9 = 531441$ (even power)

Thus, for base ending with 9, the unit digit after the expansion is 9 for odd power and is 1 for even power.

As we have seen in the earlier case, this rule is applicable, when the base is in the form of [(multiple of 10)+4] or [(multiple of 10)+9].

For example consider, 24^{12}

In this, unit digit of base 24 is 4 and power is 12 (even power).

Therefore, unit digit of 24^{12} is 6.

Similarly consider, 89^{21}

Here unit digit of base 89 is 9 and power is 21 (odd power).

Therefore, unit digit of 89^{21} is 9.

We conclude that, for base ending with 4, the unit digit of the expanded form is 4 for odd power and is 6 for even power. Similarly, for base ending with 9, the unit digit of the expanded form is 9 for odd power and is 1 for even power. Remember, 4 and 6 are complements of 10. Also, 9 and 1 are complements of 10.

Example 3.12 Find the unit digit of the large numbers: (i) 4^7 (ii) 64^{10}

Solution

(i) 4^7

Unit digit of base 4 is 4 and power is 7 (odd power).

Therefore, unit digit of 4^7 is 4.

(ii) 64^{10}

Unit digit of base 64 is 4 and power is 10 (even power).

Therefore, unit digit of 64^{10} is 6.

Try these

Find the unit digit of the following exponential numbers:

(i) 64^{11} (ii) 29^{18}

(iii) 79^{19} (iv) 104^{32}

Example 3.13 Find the unit digit of the large numbers: (i) 9^{12} (ii) 49^{17}

Solution

(i) 9^{12}

Unit digit of base 9 is 9 and power is 12 (even power).

Therefore, unit digit of 9^{12} is 1 .

(ii) 49^{17}

Unit digit of base 49 is 9 and power is 17 (odd power).

Therefore, unit digit of 49^{17} is 9.

The following activity will help you to find the unit digits of exponent numbers whose base is ending with 2,3,7 and 8.

Activity

Observe the table given below. The numbers in first column, that is 2,3,7 and 8 denotes the unit digit of base of the given exponent number and the numbers in the first row, that is 1,2,3 and 0 stands for the remainder when power is divided by 4.

Unit digit of base	The remainder when power is divided by 4			
	1	2	3	0
2	2	4	8	6
3	3	9	7	1
7	7	9	3	1
8	8	4	2	6

For example, consider 2^6

Unit digit of base 2 is 2 and power is 6. When the power 6 is divided by 4, we get the remainder as 2.

From the table, we see that 2 and 2 corresponds to 4. Therefore, the unit digit of 2^6 is 4. To verify, $2^6 = 2\times2\times2\times2\times2\times2 = 64$.

Similarly, consider 117^{20}

Unit digit of base 117 is 7 and power is 20. When the power 20 is divided by 4, we get the remainder as 0.

From the table, we see that 7 and 0 corresponds to 1. Therefore, the unit digit of 117^{20} is 1.

Now, you can extend this activity for finding the unit digits of any exponent number whose base is ending with any of 2,3,7 or 8.

Exercise 3.2

1. Fill in the blanks.

   (i) Unit digit of $124\times36\times980$ is ____________

   (ii) When the unit digit of the base and its expanded form of that number is 9, then the exponent must be _____________ power.

2. Match the following:

Group A Exponential form		Group B Unit digit of the number	
(i)	20^{10}	(a)	6
(ii)	121^{11}	(b)	4
(iii)	444^{41}	(c)	0
(iv)	25^{100}	(d)	1
(v)	716^{83}	(e)	9
(vi)	729^{725}	(f)	5

3. Find the unit digit of expanded form.

(i) 25^{23} (ii) 11^{10} (iii) 46^{15} (iv) 100^{12}

(v) 29^{21} (vi) 19^{12} (vii) 24^{25} (viii) 34^{16}

4. Find the unit digit of the following numeric expressions.

(i) $114^{20}+115^{21}+116^{22}$ (ii) $10000^{10000}+11111^{11111}$

Objective type questions

5. Observe the equation $(10+y)^4=50625$ and find the value of y.

(i) 1 (ii) 5 (iii) 4 (iv) 0

6. The unit digit of $(32\times 65)^0$ is

(i) 2 (ii) 5 (iii) 0 (iv) 1

7. The unit digit of the numeric expression $10^{71}+10^{72}+10^{73}$ is

(i) 0 (ii) 3 (iii) 1 (iv) 2

3.5 Degree of Expression

Let us recall about algebraic expression which we have studied earlier.

3.5.1 Recap of Algebraic expression

We have learnt that while constructing an algebraic expression, we use mathematical operators like addition, subtraction, multiplication and division to combine variables and constants.

Now, we have learnt about exponents. Note that exponential notations also can be used in the construction of Algebraic expressions.

Let us recall some basic concepts about expressions.

Consider the expression $2x+3$, which is obtained by multiplying the variable x with the constant 2 and then adding the constant 3 to the product.

This expression is binomial as it contains two terms. The term $2x$ is a variable term and 3 is a constant term. 2 is the co-efficient of x.

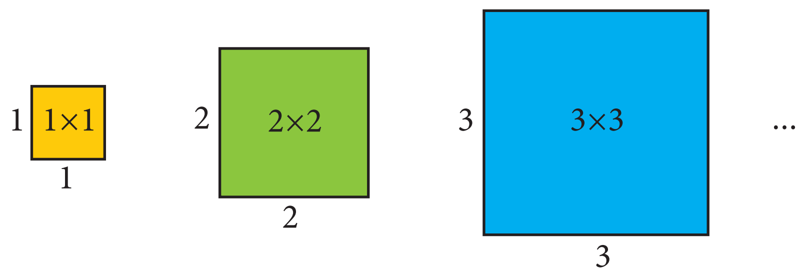
The terms with same variables are called like terms. For example, $-7x$, $2x$ and $5x$ are like terms. But, term with different variables are called unlike terms. For example, $-2x, 7y$ are unlike terms.

We can add or subtract like terms only. We know that $2x+5x=7x$. But, when we add unlike terms, it results in new expression. For example, $2x$ and $5y$ are unlike terms, thus resulting in new expression $2x+5y$.

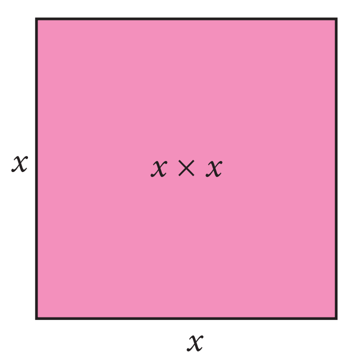
3.5.2 Degree of Expressions

To know the degree of an expression, first let us try to understand the degree of a variable by relating it with the exponents of numbers. Let us consider the square numbers. They have different base and same exponents.

The geometrical representation of square numbers are given below.



In general, if we consider the side of a square as a variable 'x' then its area will be $x \times x$ *sq. units*. This can be denoted as 'x^2'. Thus we have an algebraic expression with exponent notation.



If we consider the term x^2 as a monomial expression, the highest power of the expression is its exponent, that is 2.

Similarly, when length 'l' units and breadth 'b' units are variables of a rectangle, then its area is $l \times b = lb$ sq. units. We can consider lb as a term in an algebraic expression, where l and b are factors of lb.

The highest power of the expression lb is also 2, as we have to add up the powers of the variable factors.

Note

(i) When no exponent is explicitly shown in a variable of a term, it is understood to be 1. For example, $11p = 11p^1$.

(ii) For an expression in x, if the terms of the expressions are in descending powers of x and the like terms are added, then we say that it is in the standard form.
For example, $x^4 - 3x^3 + 5x^2 - 7x + 9$ is in the standard form. It is easier to find the highest power term when the expression is in standard form. Highest power of this expression is 4.

(iii) The highest degree term of an algebraic expression is called as leading term.

Let us consider an algebraic expression: $x^3 - 3x^2 + 4$.

The terms of the above expression are x^3, $-3x^2$ and 4. Exponent of the term x^3 is 3 and $-3x^2$ is 2.

Thus, the term x^3 has the highest exponent, that is 3.

Now, consider the expression, $3x^4 - 4x^3y^2 + 8xy + 7$.

Take each term and check its power. In $3x^4$, exponent is 4, hence its degree is 4. In $-4x^3y^2$, the sum of powers of x and y is 5, hence its degree is 5. In $8xy$, the sum of powers is 2.

Therefore, the term with highest power in the above expression is $-4x^3y^2$ and its power is 5, which is called as degree of this expression.

The term(s) containing the highest power of the variables in an expression is called the degree of expression.

The degree of any term in an expression can only be a positive integer. Also, degree of expression doesn't depend on the number of terms, but on the power of variables in the individual terms. The degree of constant term is 0.

Try these

1. Complete the following table:

S. No.	Algebraic expression	Degree of the terms				Degree of the expression
		Term-I	Term-II	Term-III	Term-IV	
1.	$7x^3 - 11x^2 + 2x - 5$	3	2	1	0	3
2.	$9x^5 - 4x^2 + 2x - 11$	5	2	1	0	5
3.	$6b^2 - 3a^2 + 5a^2b^2$					
4.	$p^4+p^3+p^2+1$					
5.	$6x^2y^3 - 7x^3y + 5xy$					
6.	$9 + 2x^2 + 5xy - 5x^3$					

2. Identify the like terms from the following:

(i) $2x^2y,\ 2xy^2,\ 3xy^2,\ 14x^2y,\ 7yx$

(ii) $3x^3y^2,\ y^3x,\ y^3x^2,\ -y^3x,\ 3y^3x$

(iii) $11pq,\ -pq,\ 11pqr,\ -11pq,\ pq$

Example 3.14 Find the degree of the following expressions.

N1S4Q4

(i) x^5 (ii) $-3p^3q^2$ (iii) $-4xy^2z^3$

(iv) $12xyz-3x^3y^2z+z^8$ (v) $3a^3b^4-16c^6+9b^2c^5+7$

Solution

(i) In x^5, the exponent is 5. Thus, the degree of the expression is 5.

(ii) In $-3p^3q^2$, the sum of powers of p and q is 5 (that is, 3+2). Thus, the degree of the expression is 5.

(iii) In $-4xy^2z^3$, the sum of powers of x, y and z is 6 (that is, 1+2+3). Thus, the degree of the expression is 6.

(iv) The terms of the given expression are $12xyz,\ 3x^3y^2z,\ z^8$

Degree of each of the terms: 3, 6, 8

Terms with highest degree: z^8.

Therefore, degree of the expression is 8.

(v) The terms of the given expression are $3a^3b^4, -16c^6,\ 9b^2c^5,\ 7$

Degree of each of the terms: 7, 6, 7, 0

Terms with highest degree: $3a^3b^4, 9b^2c^5$

Therefore, degree of the expression is 7.

Example 3.15 Add the expressions $4x^2+3xy+9y^2$ and $2x^2-9xy+6y^2$ and find the degree.

Solution

This can be written as $(4x^2+3xy+9y^2)+(2x^2-9xy+6y^2)$

Let us group the like terms, thus we have

$$\left(4x^2+2x^2\right)+\left(3xy-9xy\right)+\left(9y^2+6y^2\right) = x^2\left(4+2\right)+xy\left(3-9\right)+y^2\left(9+6\right)$$

$$= 6x^2-6xy+15y^2$$

Thus, the degree of the expression is 2.

Example 3.16 Subtract x^3-x^2+x+3 from $3x^3-2x^2-7x+6$ and find the degree.

Solution

This can be written as $\left(3x^3-2x^2-7x+6\right)-\left(x^3-x^2+x+3\right)$

When there is a –ve sign before the brackets, it can be removed by changing the sign of every term inside the bracket.

$$\begin{aligned}\left(3x^3-2x^2-7x+6\right)-\left(x^3-x^2+x+3\right) &= 3x^3-2x^2-7x+6-x^3+x^2-x-3\\ &=\left(3x^3-x^3\right)+\left(-2x^2+x^2\right)+\left(-7x-x\right)+\left(6-3\right)\\ &=x^3\left(3-1\right)+x^2\left(-2+1\right)+x\left(-7-1\right)+\left(6-3\right)\\ &=2x^3-x^2-8x+3\end{aligned}$$

Hence, the degree of the expression is 3.

Example 3.17 Simplify and find the degree of the expression

$$\left(4m^2+3n\right)-\left(3m+9n^2\right)-\left(3m^2-6n^2\right)+\left(5m-n\right)$$

Solution

$$\begin{aligned}&\left(4m^2+3n\right)-\left(3m+9n^2\right)-\left(3m^2-6n^2\right)+\left(5m-n\right)\\ &= 4m^2+3n-3m-9n^2-3m^2+6n^2+5m-n\\ &=\left(4m^2-3m^2\right)+\left(3n-n\right)+\left(-3m+5m\right)+\left(-9n^2+6n^2\right)\\ &=m^2+2n+2m-3n^2\end{aligned}$$

Hence, the degree of the expression is 2.

Exercise 3.3

1. Fill in the blanks.
   (i) The degree of the term $a^3b^2c^4d^2$ is ___________.
   (ii) Degree of the constant term is ______________.
   (iii) The coefficient of leading term of the expression $3z^2y+2x-3$ is _________.

2. Say True or False.
   (i) The degree of m^2n and mn^2 are equal.
   (ii) $7a^2b$ and $-7ab^2$ are like terms.
   (iii) The degree of the expression $-4x^2yz$ is -4.
   (iv) Any integer can be the degree of the expression.

3. Find the degree of the following terms.
   (i) $5x^2$ (ii) $-7ab$ (iii) $12pq^2r^2$ (iv) -125 (v) $3z$

4. Find the degree of the following expressions.
   (i) x^3-1 (ii) $3x^2+2x+1$ (iii) $3t^4-5st^2+7s^3t^2$
   (iv) $5-9y+15y^2-6y^3$ (v) $u^5+u^4v+u^3v^2+u^2v^3+uv^4$

5. Identify the like terms : $12x^3y^2z,\ -y^3x^2z,\ 4z^3y^2x,\ 6x^3z^2y,\ -5y^3x^2z$

6. Add and find the degree of the following expressions.

 (i) $(9x+3y)$ and $(10x-9y)$ (ii) $(k^2-25k+46)$ and $(23-2k^2+21k)$

 (iii) $(3m^2n+4pq^2)$ and $(5nm^2-2q^2p)$

7. Simplify and find the degree of the following expressions.

 (i) $10x^2-3xy+9y^2-(3x^2-6xy-3y^2)$ (ii) $9a^4-6a^3-6a^4-3a^2+7a^3+5a^2$

 (iii) $4x^2-3x-[8x-(5x^2-8)]$

Objective type questions

8. $3p^2-5pq+2q^2+6pq-q^2+pq$ is a

 (i) Monomial (ii) Binomial (iii) Trinomial (iv) Quadrinomial

9. The degree of $6x^7-7x^3+4$ is

 (i) 7 (ii) 3 (iii) 6 (iv) 4

10. If $p(x)$ and $q(x)$ are two expressions of degree 3, then the degree of $p(x)+q(x)$ is

 (i) 6 (ii) 0 (iii) 3 (iv) Undefined

Exercise 3.4

Miscellaneous Practice problems

J7U3I5

1. $6^2 \times 6^m = 6^5$, find the value of '*m*'.

2. Find the unit digit of $124^{128} \times 126^{124}$.

3. Find the unit digit of the numeric expression: $16^{23}+71^{48}+59^{61}$

4. Find the value of $\dfrac{(-1)^6 \times (-1)^7 \times (-1)^8}{(-1)^3 \times (-1)^5}$.

5. Identify the degree of the expression, $2a^3bc+3a^3b+3a^3c-2a^2b^2c^2$

6. If $p=-2,\ q=1$ and $r=3$, find the value of $3p^2q^2r$.

Challenge Problems

7. **LEADERS** is a WhatsApp group with 256 members. Every one of its member is an admin for their own WhatsApp group with 256 distinct members. When a message is posted in LEADERS and everybody forwards the same to their own group, then how many members in total will receive that message?

8. Find x such that $3^{x+2}=3^x+216$.

9. If $X=5x^2+7x+8$ and $Y=4x^2-7x+3$, then find the degree of $X+Y$.

10. Find the degree of $\left(2a^2+3ab-b^2\right)-\left(3a^2-ab-3b^2\right)$

11. Find the value of w, given that $x=3$, $y=4$, $z=-2$ and $w=x^2-y^2+z^2-xyz$.

12. Simplify and find the degree of $6x^2+1-\left[8x-\left\{3x^2-7-\left(4x^2-2x+5x+9\right)\right\}\right]$

13. The two adjacent sides of a rectangle are $2x^2-5xy+3z^2$ and $4xy-x^2-z^2$. Find the perimeter and the degree of the expression.

DO YOU KNOW?

Srinivasa Ramanujan, the great Indian mathematician created so many beautiful equations in his childhood using exponents. Here is an interesting exponential equation from his most popular "**Notebooks**".

$$2^2\times6^6\times1^1\times1^1=3^3\times3^3\times4^4$$

The base and power are equal in each factor. Moreover the sum of base (or power) in both the sides are also equal. (that is, $2+6+1+1=3+3+4=10$). It can be easily proved with the help of laws of exponents.

LHS $=2^2\times6^6\times1^1\times1^1=2^2\times6^6\times1=2^2\times(2\times3)^6$

$= 2^2\times 2^6\times 3^6$ [since, $(a\times b)^m=a^m\times b^m$]

$=2^{2+6}\times3^{3+3}$ [since, $a^m\times a^n=a^{m+n}$]

$=2^8\times3^3\times3^3$

$=2^{2\times4}\times3^3\times3^3$

$=\left(2^2\right)^4\times3^3\times3^3$ [since, $a^{m\times n}=\left(a^m\right)^n$]

$=4^4\times3^3\times3^3 \ =3^3\times3^3\times4^4$

= RHS.

Try to prove some of his other equations given below:

$8^8\times9^9\times1^1=3^3\times3^3\times12^{12}$ (Base total is 18)

$4^4\times20^{20}\times30^{30}\times1^1=6^6\times24^{24}\times25^{25}$ (Base total is 55)

Summary

- If 'a' is any integer, then $a\times a\times a\times\ldots\times a$ (n times) $=a^n$. Here 'a' is the base and n is the exponent or power or index.
- $(-1)^n=\begin{cases}1, & \text{if } n \text{ is even}\\ -1, & \text{if } n \text{ is odd}\end{cases}$
- When a number 'a' is multiplied by itself, the product is called the square of that number and denoted by a^2. Similarly, the square of a number (a^2) is multiplied by 'a', then the product is called the cube of that number and is denoted by a^3.

- If 'a' and 'b' are any non-zero numbers and 'm' and 'n' are natural numbers, then

  (i) $a^m \times a^n = a^{m+n}$ (Product rule)

  (ii) $a^m \div a^n = a^{m-n}$, $m>n$ (Quotient rule)

  (iii) $\left(a^m\right)^n = a^{m\times n}$ (Power rule)

  (iv) $\left(a\times b\right)^m = a^m \times b^m$

  (v) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

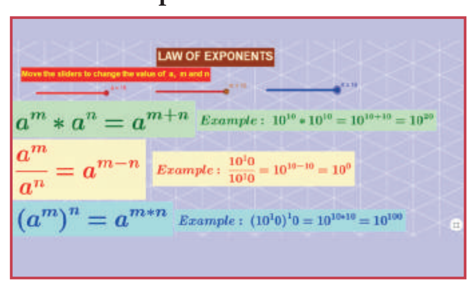
- For the base number whose unit digits are 0,1,5 and 6, the unit digit of a number corresponding to any positive exponent remains unchanged.
- For base ending with 4, the unit digit is 4 for odd power and is 6 for even power. Similarly, for base ending with 9, the unit digit is 9 for odd power and is 1 for even power.
- The largest power of a variable in an expression is called its degree. If it has more than one variable, then one has to take the sum of the powers of variables in each term and take the maximum of all these sums.

ICT Corner

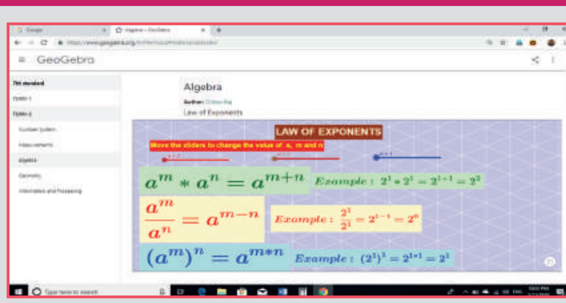
Step-1 : Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "Algebra" will open. There is a work sheet named "Law of Exponents"

Step-2 : Move the sliders a, m and n, observe the results and practice the laws.

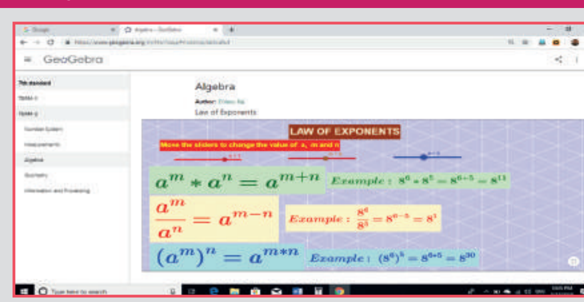
Expected outcome



Step 1



Step 2



Browse in the link

Algebra: https://www.geogebra.org/m/f4w7csup#material/ab5ra9uf
or Scan the QR Code.

B350_7_MATHS_EM

Chapter
4

GEOMETRY

R3T2F7

Learning Objectives

- To apply angle sum property of triangles.
- To understand the concept of congruency of triangles.
- To know the criteria for congruence of triangles.

Recap

Triangles

In first term we studied about different types of angles made by intersecting lines and parallel lines with transversals. Further we learnt about triangles, types of triangles and properties of triangles. In this term we are going to apply the properties of triangles.

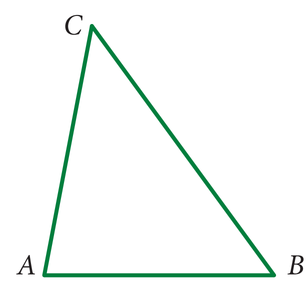


Fig. 4.1

A triangle is a closed figure formed by three line segments. It has three vertices, three sides and three angles.

In the triangle ABC (Fig 4.1) vertices are $A,B,C,$ the sides are $\overline{AB}$, $\overline{BC}$, $\overline{CA}$ and the angles are $\angle CAB$, $\angle ABC$, $\angle BCA$. We have already learnt to classify triangles based on sides and angles.

The classification of triangles are shown below.

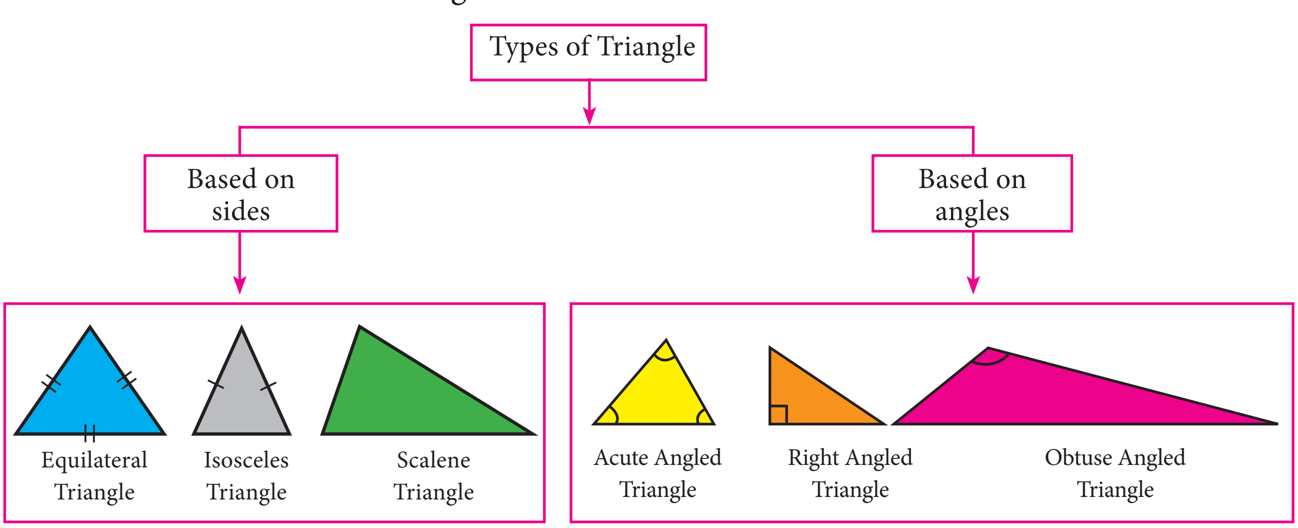
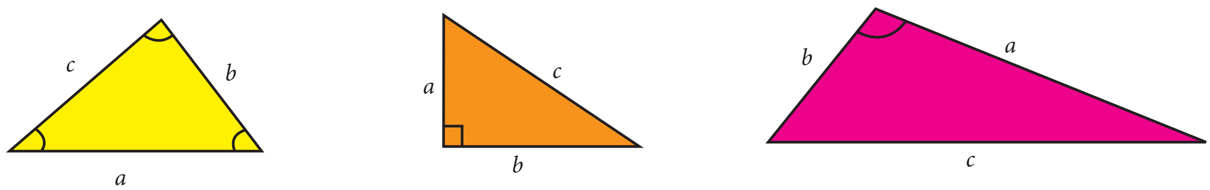


Fig. 4.2

In any triangle drawn by joining three non-collinear points, the sum of the lengths of any two sides is greater than the length of the third side. This property is called as Triangle inequality.

To verify this property, let us consider three types of triangles based on angles.



Acute angled triangle | Right angled triangle | Obtuse angled triangle

Fig. 4.3

For each of these triangles the following statements are true.

1. $a+b>c$ 2. $b+c>a$ 3. $c+a>b$

This property is also true for three types of triangles based on sides.

Try these

Anwser the following questions.

1. Triangle is formed by joining three _______________ points.
2. A triangle has _______________ vertices and _______________ sides.
3. A point where two sides of a triangle meet is known as ________ of a triangle.
4. Each angle of an equilateral triangle is of _______________ measure.
5. A triangle has angle measurements of 29°, 65° and 86°. Then it is _______ triangle.
   (i) an acute angled (ii) a right angled (iii) an obtuse angled (iv) a scalene
6. A triangle has angle measurements of 30°, 30° and 120°. Then it is _______ triangle.
   (i) an acute angled (ii) scalene (iii) obtuse angled (iv) right angled
7. Which of the following can be the sides of a triangle?
   (i) 5,9,14 (ii) 7,7,15 (iii) 1, 2, 4 (iv) 3, 6, 8
8. Ezhil wants to fence his triangular garden. If two of the sides measure 8 feet and 14 feet then the length of the third side is __________
   (i) 11 ft (ii) 6 ft (iii) 5 ft (iv) 22 ft
9. Can we have more than one right angle in a triangle?
10. How many obtuse angles are possible in a triangle?
11. In a right triangle, what will be the sum of other two angles?
12. Is it possible to form an isosceles right angled triangle? Explain.

4.1 Introduction

Triangles are an effective tool for construction and architecture. They are used in the design of buildings and other structures for more strength and stablility. The knowledge of the properties of triangles is essential to understand its use in architecture. The triangle's use in architecture dates back more years than other common shapes like dome, arch, cylinder. Usage of triangles even predates the wheel. The most sturdy of the triangles are equilateral and isosceles; their symmetry aids in distributing weight.

This chapter is a continuation of properties of triangles that we studied in class VI.

MATHEMATICS ALIVE - Geometry in Real Life

Electric Transformer

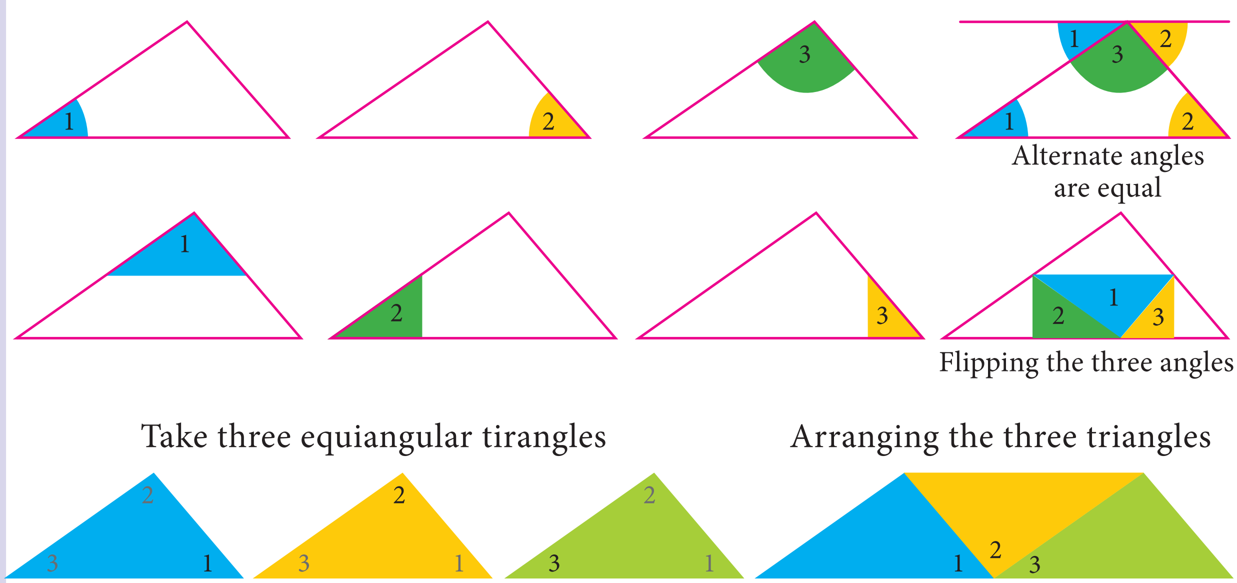
Howrah Bridge

4.2 Application of Angle Sum Property of Triangle

We are familiar with the angle sum property of a triangle which can be stated as the sum of all angles in a triangle is 180°. We can verify this by doing the following activity.

Activity

Draw any triangle and colour the angles. Check the property as shown below:



From the above we have verified that the sum of three angles of any triangle is 180°

From the above activity, we get the result as, the sum of three angles of any triangle is 180°.

Now we prove the result in a formal method.

Given: A triangle *ABC*, where $\angle A = x$, $\angle B = y$ and $\angle C = z$.

Now let us prove, $x + y + z = 180^\circ$.

To show this we need to extend *BC* to *D* and draw a line *CE*, parallel to *AB*.

Now *CE* makes two angles, $\angle ACE$ and $\angle ECD$.

Let it be u and v respectively.

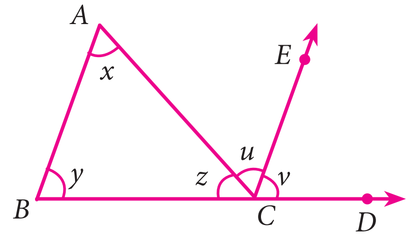


Fig. 4.4

Now z, u, v are the angles formed at a point on a straight line. Therefore $z + u + v = 180^\circ$. ... (1)

Since *AB* and *CE* are parallel and *DB* is a transversal,

$v = y$ (corresponding angles).

Again *AB* and *CE* are parallel lines and *AC* is a transversal,

$u = x$ (alternate angles). Also $z + u + v = 180^\circ$ [by (1)]

Hence, by replacing u as x and v as y, we get $x + y + z = 180^\circ$. Hence the sum of all three angles in a triangle is 180°.

F9G1H6

Example 4.1 Can the following angles form a triangle?

(i) 80°, 70°, 50° (ii) 56°, 64°, 60°

Solution

(i) Given angles 80°, 70°, 50°

Sum of the angles = 80°+70°+ 50° = 200° ≠ 180°

The given angles cannot form a triangle.

(ii) Given angles 56°, 64°, 60°

Sum of the angles = 56°+ 64°+ 60° = 180°

The given angles can form a triangle.

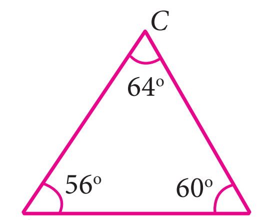


Fig. 4.5

Example 4.2 Find the measure of the missing angle in the given triangle *ABC*.

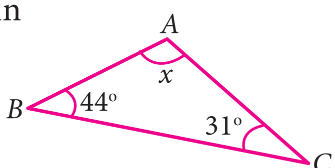


Fig. 4.6

Solution

Let $\angle A = x$

We know that, $\angle A + \angle B + \angle C = 180^\circ$ (angle sum property)

$x + 44^\circ + 31^\circ = 180^\circ$

$$x + 75° = 180°$$
$$x = 180° - 75°$$
$$x = 105°$$

Example 4.3 In ΔSTU, if $SU = UT$, $\angle SUT = 70°$, $\angle STU = x$, find the value of x.

Solution

Given, $\angle SUT = 70°$

$\angle UST = \angle STU = x$ [Angles opposite to equal sides]

$\angle SUT + \angle UST + \angle STU = 180°$

$$70° + x + x = 180°$$
$$70° + 2x = 180°$$
$$2x = 180° - 70°$$
$$2x = 110°$$
$$x = \frac{110°}{2} = 55°$$

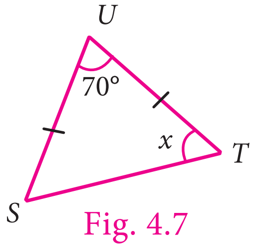


Fig. 4.7

Example 4.4 If two angles of a triangle having measures 65° and 35°, find the measure of the third angle.

Solution

Given angles are 65° and 35°.

Let the third angle be x

$$65° + 35° + x = 180°$$
$$100° + x = 180°$$
$$x = 180° - 100°$$
$$x = 80°$$

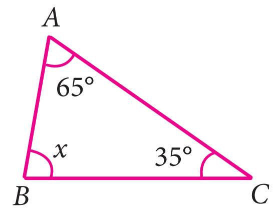


Fig. 4.8

4.3 Exterior Angles

We know that any triangle is made up of three vertices, three sides and three angles. Now observe the triangle given in Fig. 4.9

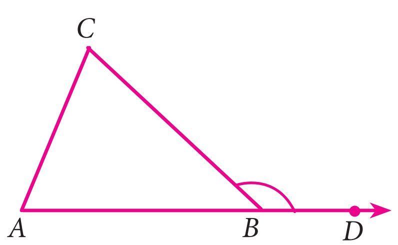


Fig. 4.9

In ΔABC, the side AB is extended to D. Observe $\angle CBD$ which is formed by the side BC and BD. $\angle CBD$ is known as the exterior angle of ΔABC at B.

We can observe that, $\angle ABC$ and $\angle CBD$ are adjacent and they form a linear pair.

Think

Can CB be extended to F to get another exterior angle of ΔABC at B?

Besides, the other two angles namely, $\angle CAB$ and $\angle ACB$ are non-adjacent angles to $\angle CBD$. They are called interior opposite angles of $\angle CBD$.

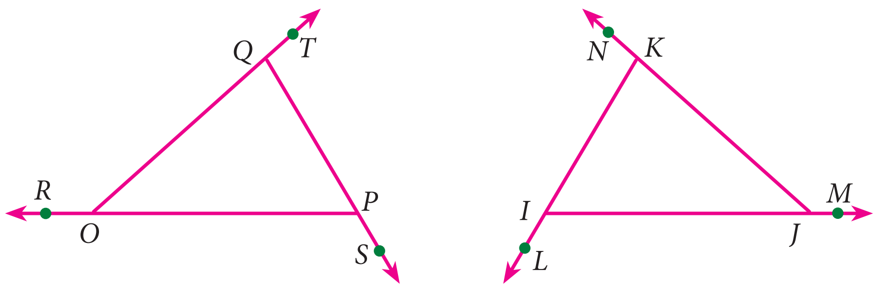
Note

In ΔABC, we can also extend the sides BC to E and CA to F to form exterior angles at C and A.

Exterior Angle Properties of a Triangle

Activity

To understand exeterior angle properties of a triangle list all the exterior angles of triangles that are shown below.



Measure and express each exterior angle as a sum of its interior opposite angles and complete the table.

Exterior angle	Sum of its interior opposite angle

From the above activity we observe that an exterior angle of a triangle is equal to the sum of its interior opposite angles.

Now we prove this result in a formal method.

Proof

In ΔABC, consider, the angles at A, B and C as a, b and c respectively and take exterior angles at A, B and C as x, y and z respectively.

We are going to prove $x=b+c$, $y=a+c$ and $z=a+b$

$a + x = 180°$ [linear pair of angles are supplementary]

This gives, $x = 180° - a$ … (1)

Now, $a + b + c = 180°$ [Sum of 3 angles in a triangle is 180°]

This gives, $b + c = 180° - a$ …(2)

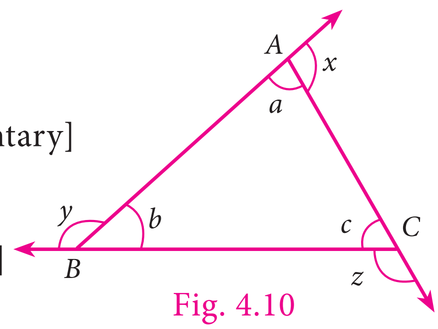


Fig. 4.10

From (1) and (2),

x and $b + c$ both are equal.

Therefore, $x = b + c$.

In the same way, we can prove the result for other exterior angles.

Now we are going to learn one more result on the exterior angle of a triangle for which we do the following activity.

Activity

Imagine a person standing in one of the vertices (corners) and walking along the boundary of the triangle until he reaches the starting point. At each of the vertex of the triangle he would turn an angle equal to the exterior angle at that vertex. Hence after the complete journey around the triangle he would have turned through an angle equal to one complete revolution, that is 360°.

We prove this result as below.

Since the angles on a straight line is $180°$,

We have $a + x = 180°$ [linear pair of angles are supplementary]

$x = 180° - a$

Similarly, $y = 180° - b$

and $z = 180° - c$

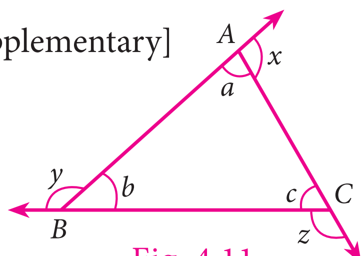


Fig. 4.11

Therefore, $x + y + z = (180° - a) + (180° - b) + (180° - c)$

$= 540 - (a + b + c)$

$= 540° - 180°$ [sum of all three angles of a triangle is 180°]

$= 360°$

Hence, the sum of all exterior angles of a triangle is 360°.

Therefore, the properties of exterior angle gives us the following two results.

(i) An exterior angle of a triangle is equal to the sum of its interior opposite angles.

(ii) The sum of exterior angles of a triangle is $360°$.

Example 4.5 In ΔPQR, find the exterior angle, $\angle SRQ$.

Solution

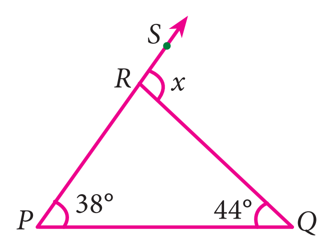


Fig. 4.12

Let, $\angle SRQ = x$

Exterior angle = sum of two interior opposite angles

$x = 38° + 44° = 82°$

Example 4.6 In ΔLMN, LM is extended to O. If $\angle L = 62°$ and $\angle N = 31°$, find $\angle NMO$.

Solution

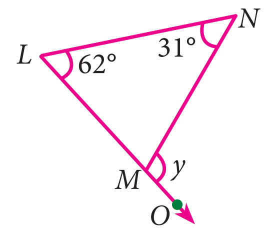


Fig. 4.13

Let, $\angle NMO = y$

Exterior angle = sum of two interior opposite angles

$y = 62° + 31°$

$= 93°$

Example 4.7 In the $\triangle ABC$ shown in the figure, find the angel z.

Solution

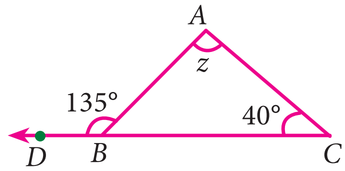


Fig. 4.14

Exterior angle = sum of two interior opposite angles

$135° = z + 40°$

Subtract 40° on both sides

$135° - 40° = z + 40° - 40°$

$z = 95°$

Example 4.8 In the given isoceles triangle IJK (Fig. 4.15), if $\angle IKL = 128°$, find the value of x.

Solution

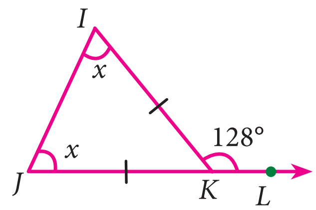


Fig. 4.15

Exterior angle = sum of two interior opposite angles

$128° = x + x$

$128 = 2x$

$\frac{128}{2} = \frac{2x}{2}$ [on both sides, divide by 2]

$x = 64°$

Example 4.9 With the given data in the Fig. 4.16, find $\angle UWY$.
What do you infer about $\angle XWV$?

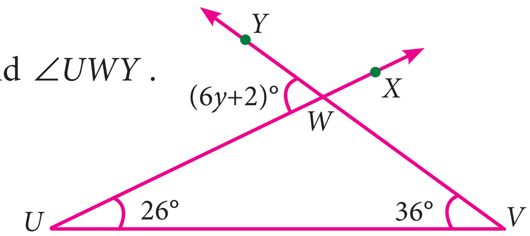


Fig. 4.16

Solution

Exterior angle = sum of two interior opposite angles

$6y + 2 = 26° + 36°$

$6y + 2 = 62°$

Subtract 2 from both sides

$6y = 62 - 2$

$6y = 60°$

$$\frac{6y}{6} = \frac{60}{6} \quad \text{[on both sides, divide by 6]}$$

$$y = 10°$$

$\angle UWY = 6y + 2 = 6(10) + 2 = 62°$.

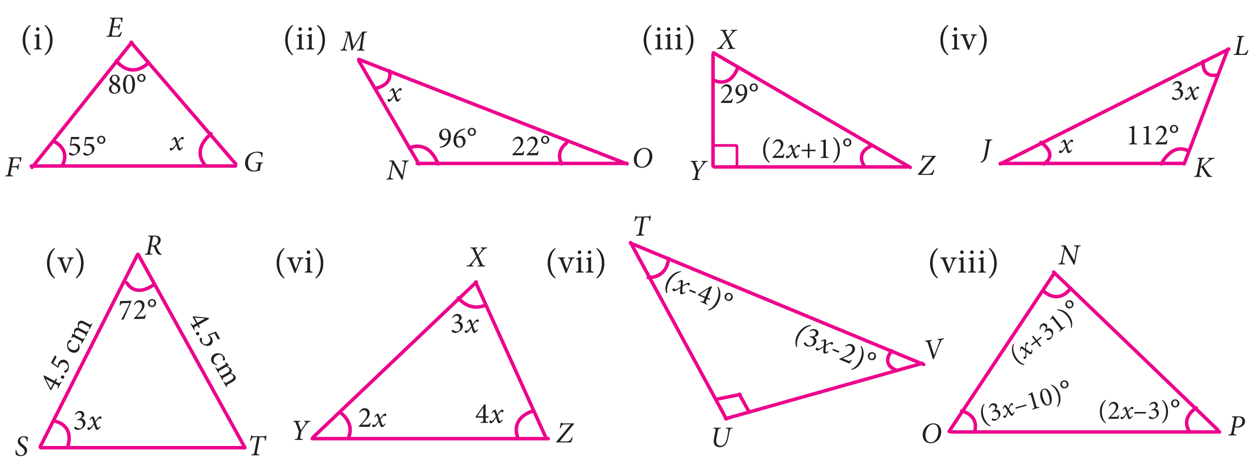
We can conclude that $\angle XWV = \angle UWY$, because they are vertically opposite angles as well as exterior angles.

Exercise 4.1

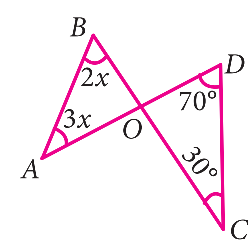
1. Can 30°, 60° and 90° be the angles of a triangle?

2. Can you draw a triangle with 25°, 65° and 80° as angles?

3. In each of the following triangles, find the value of x.

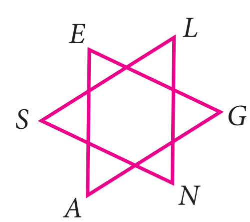


4. Two line segments $\overline{AD}$ and $\overline{BC}$ intersect at O. Joining $\overline{AB}$ and $\overline{DC}$ we get two triangles, $\triangle AOB$ and $\triangle DOC$ as shown in the figure. Find the $\angle A$ and $\angle B$.



5. Observe the figure and find the value of

$$\angle A + \angle N + \angle G + \angle L + \angle E + \angle S.$$



6. If the three angles of a triangle are in the ratio $3:5:4$, then find them.

7. In $\triangle RST$, $\angle S$ is 10° greater than $\angle R$ and $\angle T$ is 5° less than $\angle S$, find the three angles of the triangle.

8. In $\triangle ABC$, if $\angle B$ is 3 times $\angle A$ and $\angle C$ is 2 times $\angle A$, then find the angles.

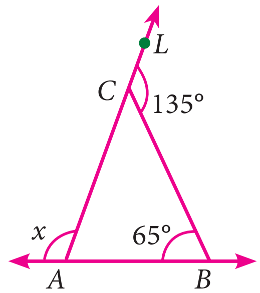
9. In $\triangle XYZ$, if $\angle X : \angle Z$ is 5 : 4 and $\angle Y = 72°$. Find $\angle X$ and $\angle Z$

10. In a right angled triangle ABC, $\angle B$ is right angle, $\angle A$ is $x+1$ and $\angle C$ is $2x+5$. Find $\angle A$ and $\angle C$.

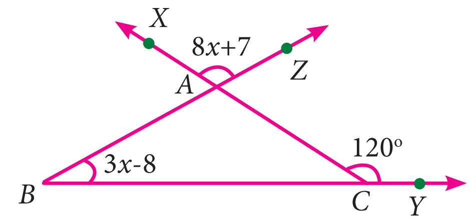
11. In a right angled triangle MNO, $\angle N = 90°$, MO is extended to P. If $\angle NOP = 128°$, find the other two angles of $\triangle MNO$.

12. Find the value of x in each of the given triangles.

(i)

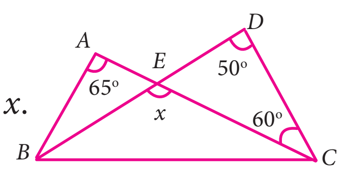


(ii)

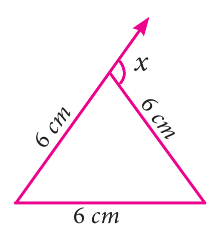


13. In $\triangle LMN$, MN is extended to O. If $\angle MLN = 100 - x$, $\angle LMN = 2x$ and $\angle LNO = 6x - 5$, find the value of x.

14. Using the given figure find the value of x.



15. Using the diagram find the value of x.



Objective type questions

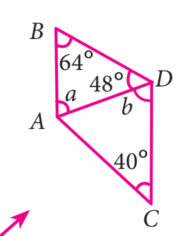
16. The angles of a triangle are in the ratio 2:3:4. Then the angles are

(i) 20, 30, 40 (ii) 40, 60, 80 (iii) 80, 20, 80 (iv) 10, 15, 20

17. One of the angles of a triangle is 65°. If the difference of the other two angles is 45°, then the two angles are

(i) 85°, 40° (ii)70°, 25° (iii) 80° , 35° (iv) 80° , 135°

18. In the given figure, AB is parallel to CD. Then the value of b is

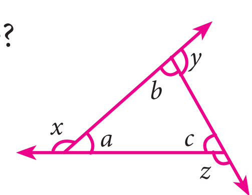


(i) 112° (ii) 68° (iii) 102° (iv) 62°

19. In the given figure, which of the following statement is true?

(i) $x + y + z = 180°$ (ii) $x + y + z = a + b + c$

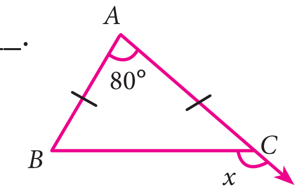
(iii) $x + y + z = 2(a + b + c)$ (iv) $x + y + z = 3(a + b + c)$



20. An exterior angle of a triangle is 70° and two interior opposite angles are equal. Then measure of each of these angle will be

(i) 110° (ii) 120° (iii) 35° (iv) 60°

21. In a ΔABC, $AB = AC$. The value of x is _____.



(i) 80° (ii) 100°

(iii) 130° (iv) 120°

22. If an exterior angle of a triangle is 115° and one of the interior opposite angles is 35°, then the other two angles of the triangle are

(i) 45°, 60° (ii) 65°, 80° (iii) 65°, 70° (iv) 115°, 60°

4.4 Congruency of Triangles

We are now going to learn a very important geometrical concept called congruence. To understand the concept of congruency of triangles, let us first look into the congruency of shapes.

4.4.1 Congruency of Shapes

Let us observe the following pictures.

Currency notes of same denomination | A pack of playing cards | Rulers of same size

Fig. 4.17

To understand congruency of shapes, let us take a pack of cards and choose any two cards and place them one over the other. We are able to place them in such a way that one card matches exactly with the other in size and shape. Hence all the cards in a pack are equal in both size and shape.

Any pair or set of objects with this property are said to be congruent.

How can we check the congruence of two objects or figures?

To check the congruence of the objects, we can use the method of superposition. In this method we have to make a trace-copy of one figure and place it over the other. If the figures match each other completely then, they are said to be congruent. In this method to match the original with the trace copy, we are not allowed to bend, twist or stretch, but we can translate or rotate.

4.4.2 Congruence of Line Segments

Measure and observe the following line segments.

Fig. 4.18

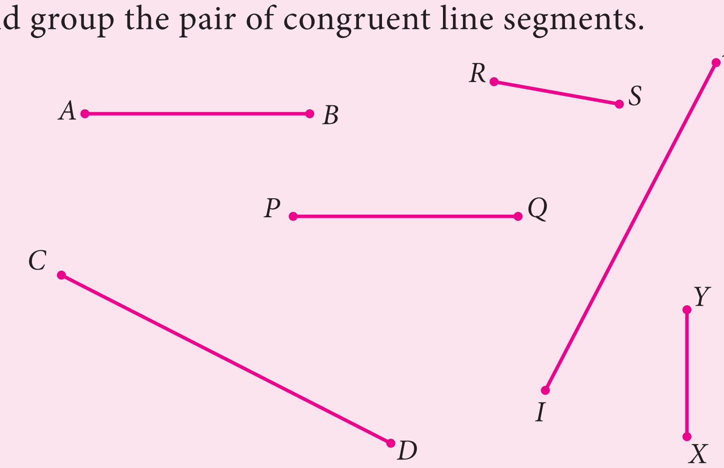
The line segments $\overline{AB}$ and $\overline{CD}$ have the same length. If we use the superposition method $\overline{AB}$ and $\overline{CD}$ will match each other. Hence the line segments are congruent and we can write it as $\overline{AB} \cong \overline{CD}$.

Since we are considering the lengths of the line segments for congruence we can also write $\overline{AB} \cong \overline{CD}$ as $AB = CD$.

From the above cases, we understand that lines are congruent if they have the same length.

Try these

Measure and group the pair of congruent line segments.



4.4.3 Congruence of Angles

Look at the following angles.

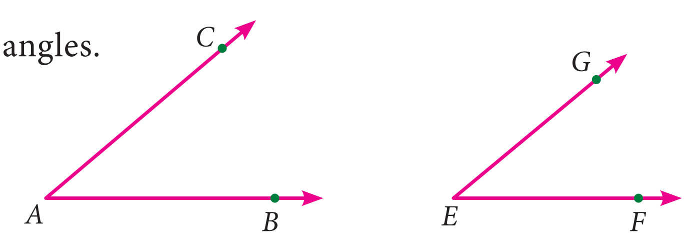


Fig. 4.19

To check the congruency of angles $\angle BAC$ and $\angle FEG$, make a trace copy of $\angle BAC$. By superposition method, if we try to match $\angle BAC$ with the $\angle FEG$ by placing the arm AB on the arm EF, then the arm AC will coincide with the arm EG. Even if length of the arms are different, the angles $\angle BAC$ matches exactly with $\angle FEG$. Hence the angles are congruent.

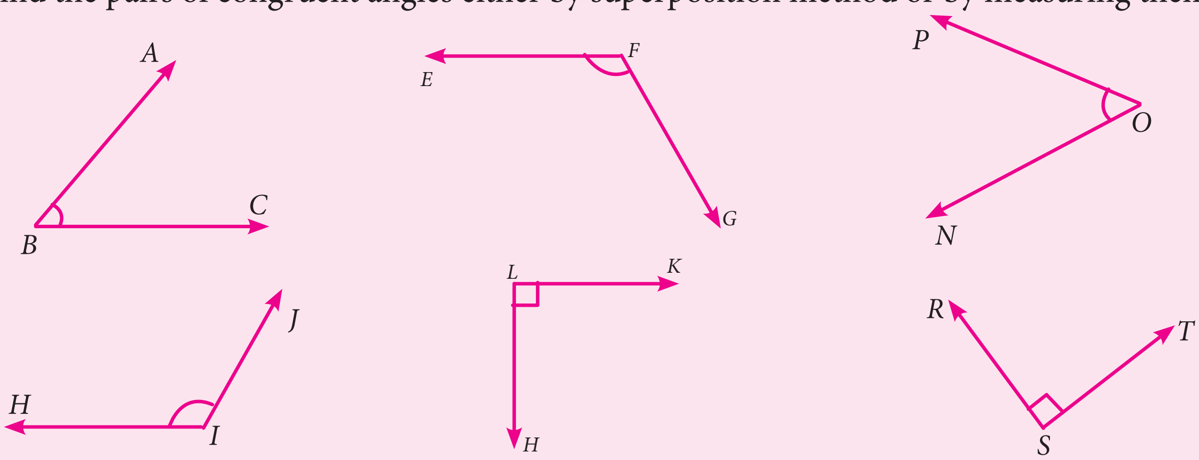
If two angles ($\angle BAC$ and $\angle FEG$) are congruent, it can be represented as $\angle BAC \cong \angle FEG$.

As in the case of line segments, congruence of angles depends only on the measures of the angles. So if we want to say two angles are congruent, the measures of the angles should be equal.

Hence we can write $\angle BAC = \angle FEG$ for $\angle BAC \cong \angle FEG$.

Try these

Find the pairs of congruent angles either by superposition method or by measuring them.



4.4.4 Congruence of Plane Figures

Examine the following pairs of plane figures.

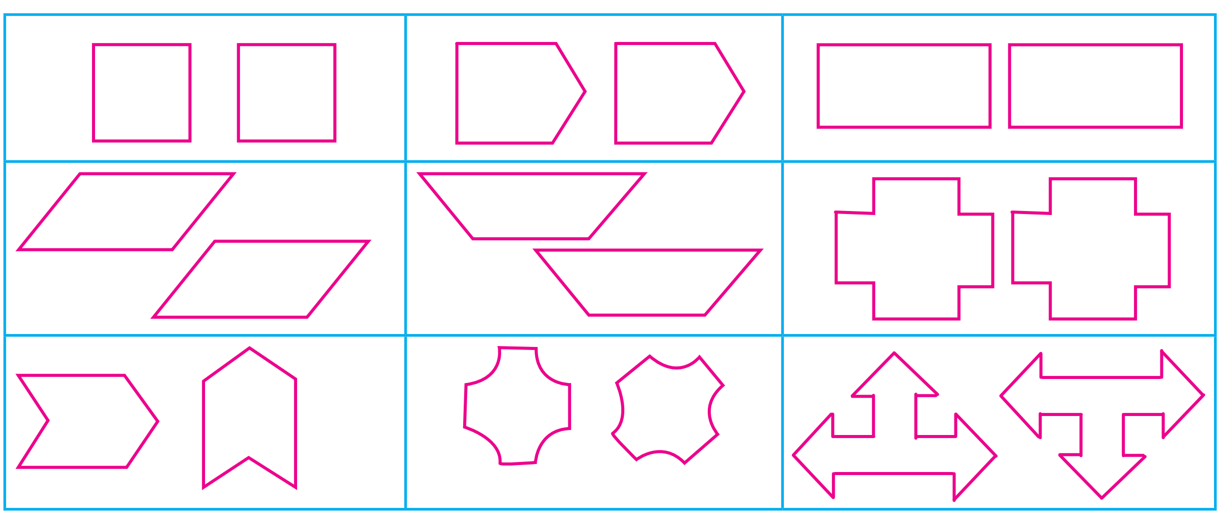


Fig. 4.20

They are identical in shape and size. Their sides (line segments) are equal and the angles are also equal.

Observe the circles given in Fig. 4.21.

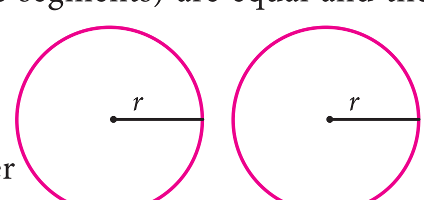


Fig. 4.21

Their radii are equal. If they are placed on each other they will coincide. These types of figures are called congruent plane figures.

Note

In congruency, the coinciding parts are called corresponding parts. The coinciding sides are called corresponding sides and the coinciding angles are called corresponding angles.

So, if the corresponding sides and corresponding angles of two plane figures are equal then they are called congruent figures. If two plane figures F_1 and F_2 are congruent, we can write $F_1 \cong F_2$

4.4.5 Congruence of Triangles

Triangle is a closed figure formed by three line segments. There are three sides and three angles. If the corresponding sides and corresponding angles of two triangles are equal, then the two triangles are said to be congruent to each other.

Observe the following two triangles $\triangle ABC$ and $\triangle XYZ$.

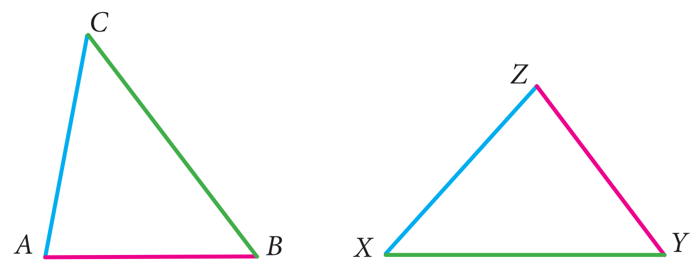


Fig. 4.22

By trace copy superposition method, it can be observed that $\triangle XYZ$ matches identically with $\triangle ABC$.

All the sides and the angles of $\triangle ABC$ are equal to the corresponding sides and angles of $\triangle XYZ$. So we can say the two triangles are congruent. We can express it as $\triangle ABC \cong \triangle XYZ$.

We can also observe that the vertex A matches with vertex Z, B matches with Y and C matches with X. They are called corresponding vertices.

The side AB matches YZ, BC matches XY and CA matches ZX. These pairs are called corresponding sides.

Also, $\angle A = \angle Z$, $\angle B = \angle Y$ and $\angle C = \angle X$. These pairs are called corresponding angles.

In the above case, the correspondance is $A \leftrightarrow Z$, $B \leftrightarrow Y$, $C \leftrightarrow X$. We write this as $ABC \leftrightarrow XYZ$.

Suppose we try to match vertex A with vertex Y or vertex X (Fig. 4.23), we can observe that the triangles do not match each other.

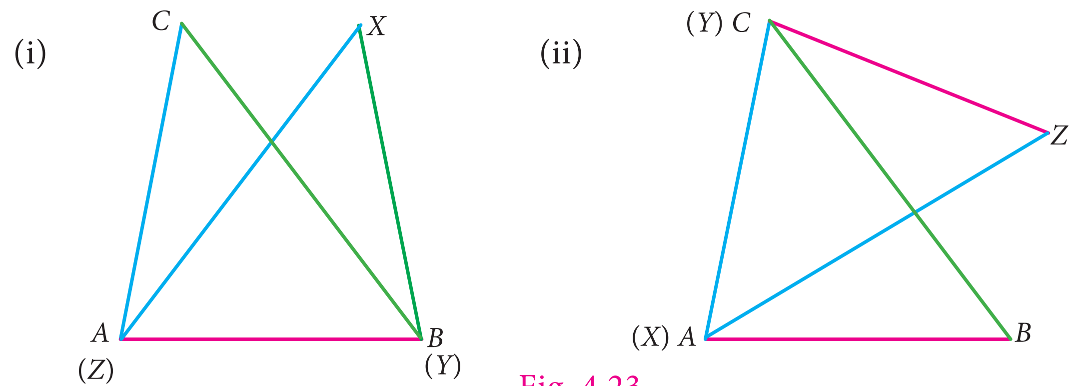


Fig. 4.23

This does not imply that the two triangles are not congruent.

So to confirm the congruency of given two triangles, we have to check the congruency of corresponding sides and corresponding angles.

Hence, the above triangles $\triangle ABC \cong \triangle XYZ$ are congruent.

So, we conclude that, if all the sides and all the angles of one triangle are equal to the corresponding sides and angles of another triangle, then the two triangles are congruent to each other.

4.4.6 Conditions for Triangles to be Congruent

We learnt to check the congruency of triangles by superposition method. Let us check the congruency of triangles using appropriate measures which will be very useful. We can study them as criterions to check the congruency of triangles.

Look at the following triangle (Fig. 4.24).

To draw a triangle congruent to the given triangle do we need to know all measures of sides and angles of the triangle?

To construct a triangle, we need only three measures. Those three measures can be any of the following.

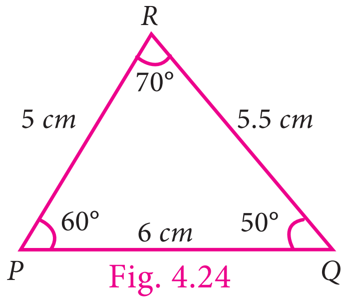


Fig. 4.24

1. The lengths of all three sides. (or)
2. The lengths of two sides and the angle included between those two sides. (or)
3. Two angles and the length of the side included by angles.

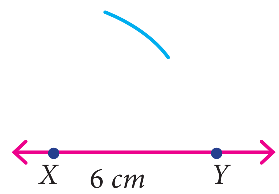
We can try the above conditions one by one.

1. **Side–Side–Side congruence criterion (SSS) - The lengths of all three sides are given**

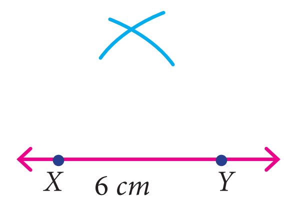
Draw a triangle XYZ given that $XY = 6\ cm$, $YZ = 5.5\ cm$ and $ZX = 5\ cm$

Step 1: Draw a line. Mark X and Y on the line such that $XY = 6\ cm$

Step 2: With X as centre, draw an arc of radius 5 *cm* above the line XY.

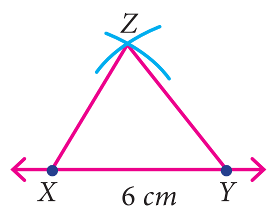


Step 3: With Y as centre, draw an arc of radius 5.5 *cm* to intersect arc drawn in step 2. Mark the point of intersection as Z.



Step 4: Join XZ and YZ.

Now XYZ is the required triangle.



Using trace copy superposition method, if we place the $\triangle XYZ$ on the $\triangle PQR$ (Fig. 4.24) in such a way that the sides PQ on XY, PR on XZ and QR on YZ, then both the triangles ($\triangle XYZ$ and $\triangle PQR$) matches exactly in size and shape.

Here we used only sides to check the congruence of the triangles, so we can say the condition as, if three sides of one triangle are equal to the corresponding sides of the other triangle then the two triangles are congruent. This criterion of congruency is known as Side – Side – Side.

Note

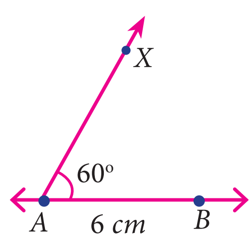
It two triangles are congruent then their corresponding parts are congruent. We can say "Corresponding Parts of Congruent Triangles are Congruent". It can be abbreviated as CPCTC.

2. **Side-Angle-Side congruence criterion (SAS) - The lengths of two sides and the angle included between the two sides are given.**

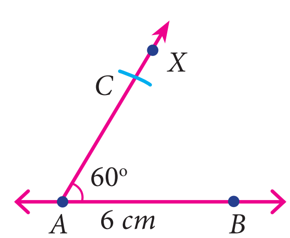
Draw a triangle ABC given that $AB = 6\ cm$, $AC = 5\ cm$ and $\angle A = 60^\circ$

Step 1: Draw a line. Mark A and B on the line such that $AB = 6\ cm$

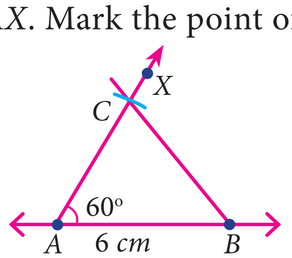
Step 2: At A, draw a ray AX making an angle of 60° with AB.



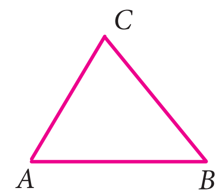
Step 3: With A as centre, draw an arc of radius 5 *cm* to cut the ray AX. Mark the point of intersection as C.



Step 4: Join BC.



ABC is the required triangle.



Using trace copy superposition method, if we place the $\triangle ABC$ on the $\triangle PQR$ (Fig. 4.24) in such a way that the sides *AB* on *PQ*, *AC* on *PR* and the $\angle A$ on $\angle P$, then both the triangles ($\triangle ABC$ and $\triangle PQR$) matches exactly in size and shape. Here, we are using only two sides and one included angle to check the congruence of triangles.

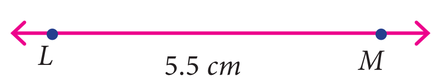
So we can say, if two sides and the included angle of a triangle are equal to the corresponding two sides and the included angle of another triangle, then the two triangles are congruent to each other.

This criterion is called Side – Angle – Side.

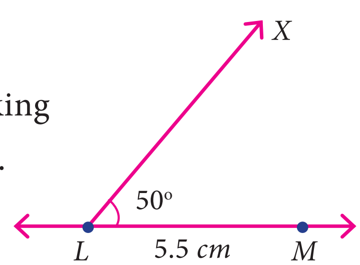
3. **Angle – Side – Angle congruence criterion (ASA) - Two angles and the length of a side included by the angles are given.**

Draw a triangle *LMN* given that $LM = 5.5\ cm$, $\angle M = 70°$ and $\angle L = 50°$.

Step 1: Draw a line. Mark *L* and *M* on the line such that $LM = 5.5\ cm$.

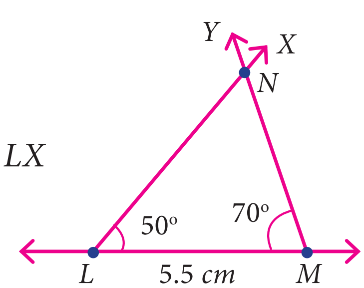


Step 2: At *L*, draw a ray *LX* making an angle of 50° with *LM*.



Step 3: At *M*, draw another ray *MY* making an angle of 70° with *LM*. Mark the point of intersection of the rays *LX* and *MY* as *N*.

LMN is the required triangle.



Using trace copy superposition method, if we place the $\triangle LMN$ on the $\triangle PQR$ (Fig. 4.24) in such a way that the angles $\angle L$ on $\angle Q$, $\angle M$ on $\angle R$ and the side *LM* on *QR*, then both the triangles ($\triangle LMN$ and $\triangle PQR$) matches exactly in size and shape. Here, we used only two angles and one included side to check the congruence of triangles.

Therefore, we can say, if two angles and the included side of one triangle are congruent to the corresponding parts of another triangle, then the two triangles are congruent. This criterion is called Angle – Side – Angle criterion.

Note

There is one more criterion to check the congruency of triangle. This criterion is called Angle– Angle – Side criterion which is a slight modification of ASA criterion. In this criterion the congruent side is not included between the congruent angles. So we can say, if two angles and a non-included side of one triangle are congruent to the corresponding parts of another triangle, then also the triangles are congruent.

Hypotenuse

In earlier class we learnt about right angled triangle. In a right angled triangle, one angle is a right angle and other angles are acute angles.

Observe the following right angled triangles.

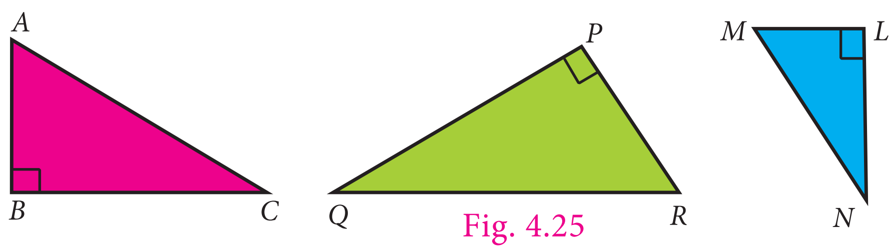


Fig. 4.25

F1G9N1

In all the given triangles the side which is opposite to the right angle is the largest side called Hypotenuse.

In the above triangles the sides *AC*, *QR* and *MN* are the hypotenuse. Remember that hypotenuse is related with right angled triangles only.

4. Right Angle – Hypotenuse – Side congruence criterion (RHS)

The name of the criterion clearly states that this criterion is used with right angled triangles only.

Observe the two given right angled triangles.

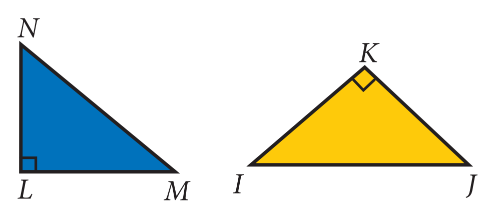


Fig. 4.26

In these two triangles right angle is common. And if we are given the sides making right angles then we can use the *SAS* criterion to check the congruency of the triangles.

Or if we are given one side containing right angle and hypotenuse, then we can have new criterion, if the hypotenuse and one side of a right angled triangle is equal to the hypotenuse and one side of another right angled triangle then the two right angled triangles are congruent.

This is called Right angle – Hypotenuse – Side criterion.

Note

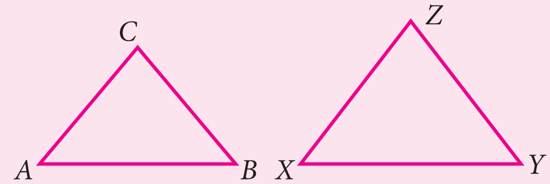
We learnt the congruency criterions of triangles. The following ordered combinations of measurement of triangles will not be sufficient to prove congruency of triangles.

Angle–Angle–Angle (AAA) combination will not always prove triangles congruent. This combination will give triangles of same shape but not of same size.

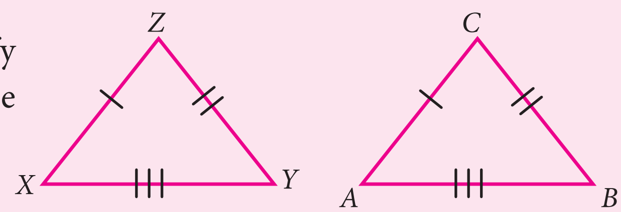
Side–Side–Angle (or) Angle–Side–Side (SSA or ASS) combination also will not prove congruency of triangles. This combination deals with two sides and a non-included angle.

Try these

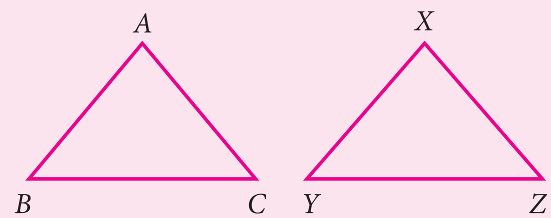
(i) If $\triangle ABC \cong \triangle XYZ$ then list the corresponding sides and corresponding angles.



(ii) Given triangles are congruent. Identify the corresponding parts and write the congruent statement.



(iii) Mention the conditions needed to conclude the congruency of the triangles with reference to the above said criterions. Give reasons for your answer.

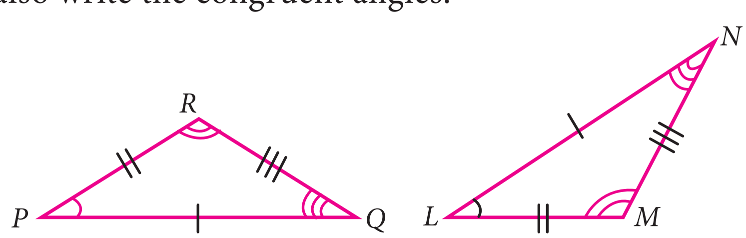


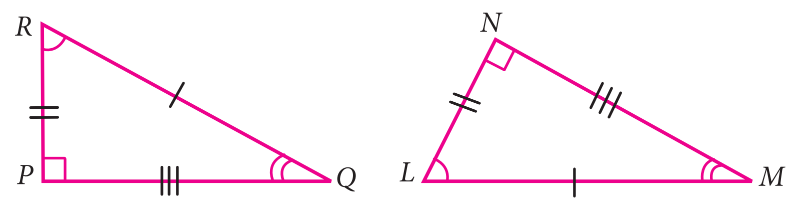
Exercise 4.2

1. Given that $\triangle ABC \cong \triangle DEF$ (i) List all the corresponding congruent sides (ii) List all the corresponding congruent angles.

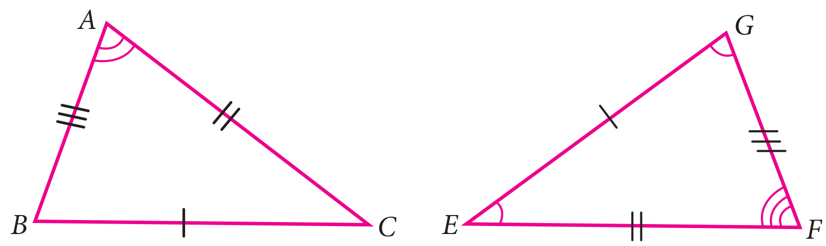
2. If the given two triangles are congruent, then identify all the corresponding sides and also write the congruent angles.

(i)



(ii) 

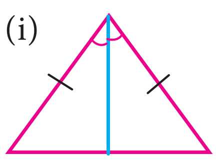
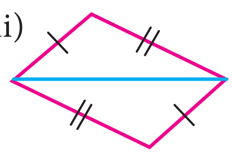
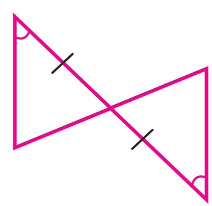
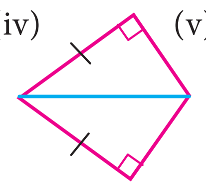
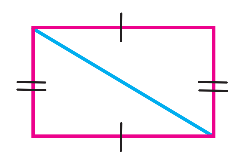
3. If the given triangles $\triangle ABC$ and $\triangle EFG$ are congruent, determine whether the given pair of sides and angles are corresponding sides or corresponding angles or not.



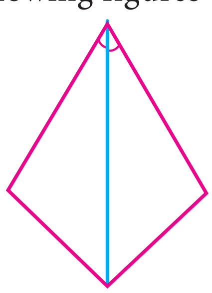
(i) $\angle A$ and $\angle G$ (ii) $\angle B$ and $\angle E$ (iii) $\angle B$ and $\angle G$

(iv) $\overline{AC}$ and $\overline{GF}$ (v) $\overline{BA}$ and $\overline{FG}$ (vi) $\overline{EF}$ and $\overline{BC}$

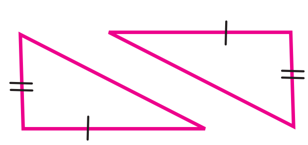
4. State whether the two triangles are congruent or not. Justify your answer.

(i)  (ii)  (iii)  (iv)  (v) 

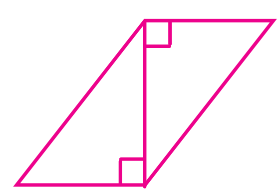
5. To conclude the congruency of triangles, mark the required information in the following figures with reference to the given congruency criterion.

(i) 

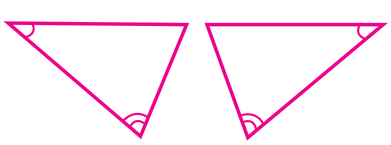
Criterion : ASA

(ii) 

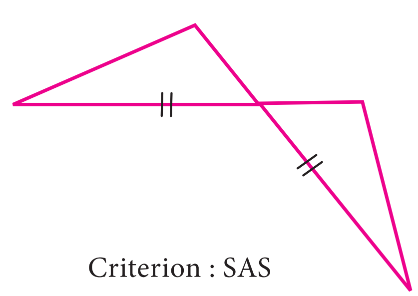
Criterion : SSS

(iii) 

Criterion : RHS

(iv) 

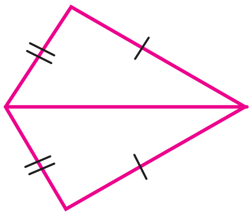
Criterion : ASA

(v) 

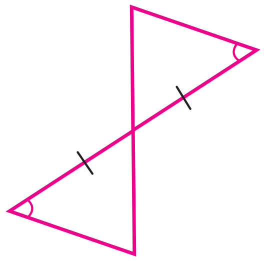
Criterion : SAS

6. For each pair of triangles state the criterion that can be used to determine the congruency?

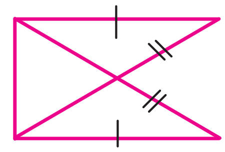
(i)



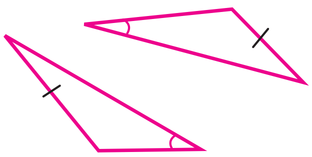
(ii)



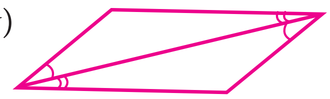
(iii)



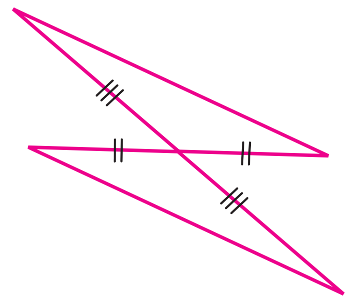
(iv)



(v)



(vi)



7. I. Construct a triangle XYZ with the given conditions.

(i) $XY = 6.4\ cm$, $ZY = 7.7\ cm$ and $XZ = 5\ cm$

(ii) An equilateral triangle of side 7.5 cm

(iii) An isosceles triangle with equal sides 4.6 cm and third side 6.5 cm

II. Construct a triangle ABC with given conditions.

(i) $AB = 7\ cm$, $AC = 6.5\ cm$ and $\angle A = 120°$.

(ii) $BC = 8\ cm$, $AC = 6\ cm$ and $\angle C = 40°$.

(iii) An isosceles obtuse triangle with equal sides 5 cm

III. Construct a triangle PQR with given conditions.

(i) $\angle P = 60°$, $\angle R = 35°$ and $PR = 7.8\ cm$

(ii) $\angle P = 115°$, $\angle Q = 40°$ and $PQ = 6\ cm$

(iii) $\angle Q = 90°$, $\angle R = 42°$ and $QR = 5.5\ cm$

B7I8G2

Objective type questions

8. If two plane figures are congruent then they have

(i) same size (ii) same shape

(iii) same angle (iv) same shape and same size

9. Which of the following methods are used to check the congruence of plane figures?

(i) translation method (ii) superposition method

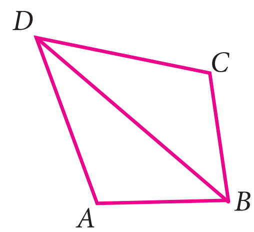
(iii) substitution method (iv) transposition method

10. Which of the following rule is not sufficient to verify the congruency of two triangles.

(i) SSS rule (ii) SAS rule (iii) SSA rule (iv) ASA rule

11. Two students drew a line segment each. What is the condition for them to be congruent?
   (i) They should be drawn with a scale.
   (ii) They should be drawn on the same sheet of paper.
   (iii) They should have different lengths.
   (iv) They should have the same length.

12. In the given figure, $AD=CD$ and $AB=CB$. Identify the other three pairs that are equal.



   (i) $\angle ADB = \angle CDB$, $\angle ABD = \angle CBD$, $BD = BD$
   (ii) $AD = AB$, $DC = CB$, $BD = BD$
   (iii) $AB =$ CD, $AD = BC$, $BD = BD$
   (iv) $\angle ADB = \angle CDB$, $\angle ABD = \angle CBD$, $\angle DAB = \angle DBC$

13. In $\triangle ABC$ and $\triangle PQR$, $\angle A=50°=\angle P$, $PQ=AB$, and $PR=AC$. By which property $\triangle ABC$ and $\triangle PQR$ are congruent?
   (i) SSS property
   (ii) SAS property
   (iii) ASA property
   (iv) RHS property

Exercise 4.3

Miscellaneous Practice problems

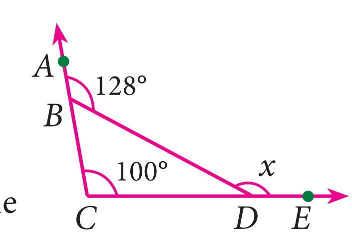
1. In an isosceles triangle one angle is 76°. If the other two angles are equal find them.

2. If two angles of a triangle are 46° each, how can you classify the triangle?

3. If one angle of a triangle is equal to the sum of the other two angles, find the type of the triangle.

4. If the exterior angle of a triangle is 140° and its interior opposite angles are equal, find all the interior angles of the triangle.

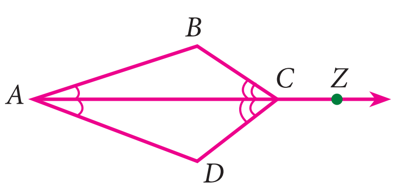
5. In $\triangle JKL$, if $\angle J = 60°$ and $\angle K = 40°$, then find the value of exterior angle formed by extending the side KL.



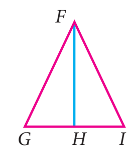
6. Find the value of 'x' in the given figure.

7. If $\triangle MNO \cong \triangle DEF$, $\angle M = 60°$ and $\angle E = 45°$ then find the value of $\angle O$.

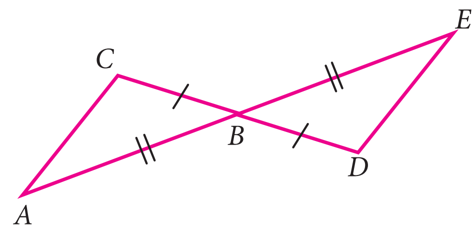
8. In the given figure ray AZ bisects $\angle BAD$ and $\angle DCB$, prove that (i) $\triangle BAC \cong \triangle DAC$ (ii) $AB = AD$.



9. In the given figure $FG = FI$ and H is midpoint of GI, prove that $\triangle FGH \cong \triangle FHI$

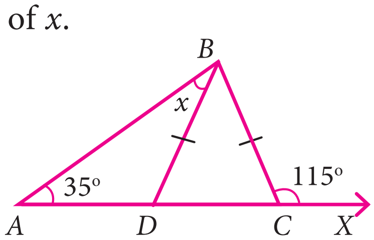


10. Using the given figure, prove that the triangles are congruent. Can you conclude that AC is parallel to DE.

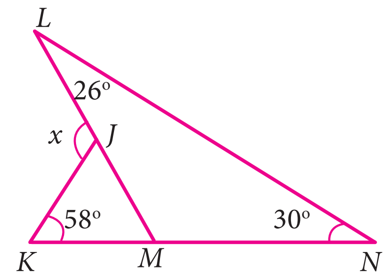


Challenge Problems

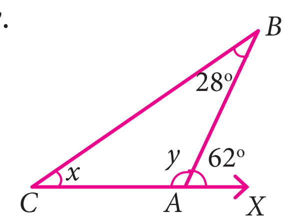
11. In given figure $BD = BC$, find the value of x.



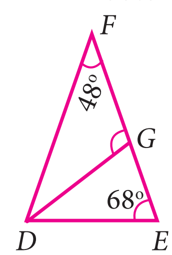
12. In the given figure find the value of x.



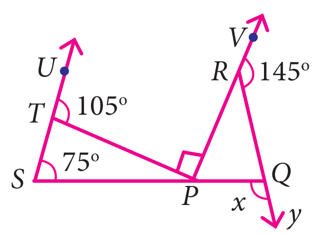
13. In the given figure find the values of x and y.



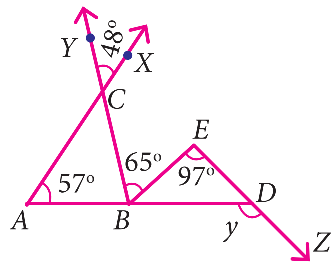
14. In ΔDEF, $\angle F = 48°$, $\angle E = 68°$ and bisector of $\angle D$ meets FE at G. Find $\angle FGD$.



15. In the figure find the value of x.



16. From the given figure find the value of y



Summary

- The sum of three angles in a triangle is $180°$.
- The exterior angle of a triangle is equal to the sum of the two interior opposite angles.
- The exterior angles of a triangle add up to $360°$.
- Two lines segments are congruent if they have the same length.
- Two angles are congruent if the measures of the angles are equal.

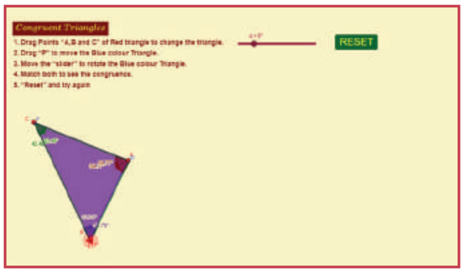
- If the corresponding sides and corresponding angles of two plane figures are equal then they are called congruent figures.
- If three sides of one triangle are equal to the corresponding sides of the other triangle then the two triangles are congruent. This is known as Side – Side – Side criterion.
- If two sides and the included angle of a triangle are equal to the corresponding two sides and the included angle of another triangle, then the two triangles are congruent to each other. This is known as Side – Angle – Side criterion.
- If two angles and the included side of one triangle are congruent to the corresponding two angles and the included side of another triangle, then the triangles are congruent This is known Angle-Side-Angle criterion.
- In right angled triangle, the side which is opposite to right angle is the largest side called Hypotenuse.
- If the hypotenuse and one side of a right angled triangle is equal to the hypotenuse and one side of another right angled triangle then the two right angled triangles are congruent. This is known as Right Angle-Hypotenuse Side criterion.

ICT Corner

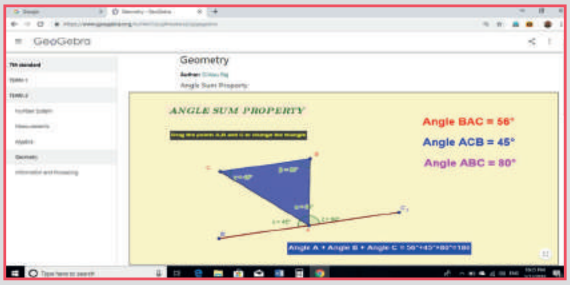
Step-1 : Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "Geometry" will open. There are three activities named "Angle sum property", "Similar triangle" and "Congruence triangle"

Step-2 : 1. In Angle sum Property Drag the vertices A,B and C to change the shape of the triangle and check the Angle sum Property. 2. In the Similar triangles change the shape and move the red triangle by holding Diamond point, over Blue triangle to match each of three angles. 3. In Congruence triangle match the triangles by moving the Blue triangle by dragging P (and rotating using the slider).

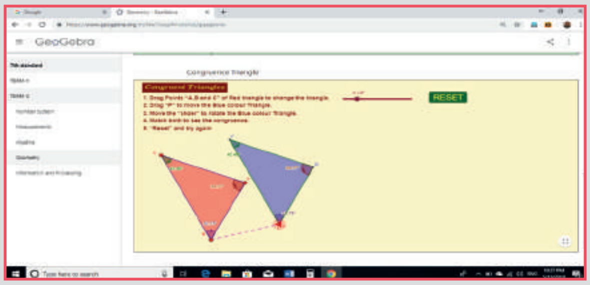
Expected outcome



Step 1



Step 2



Browse in the link

Geometry : https://www.geogebra.org/m/f4w7csup#material/gqagexmx
or Scan the QR Code.

B350_7_MATHS_EM

Chapter 5

INFORMATION PROCESSING

I3H7D4

Learning Objectives

- To identify the relationship between the two variables in a pattern.
- To generalize the pattern using tabular form of representation.
- To familiarise the type of patterns in Pascal's triangle.

5.1 Introduction

We can observe patterns in both nature and man made things. In nature, we can see patterns in leaves, trees, snowflakes, movements in celestial bodies and many others. In man made things, we see patterns in structures, buildings, fabric designs, tessellations and many others. This kind of repeated patterns brings happiness as well as interest in knowing more about them. It is really wonder to know all beautiful patterns are based on mathematics. Let us try to find the relationship among the pattern of shapes by representing it in the tabular form in the situations given below.

MATHEMATICS ALIVE - Information Processing in Real Life

Six flags pattern in snowflakes

Patterns in fabric designs

5.2 Tables and Patterns Leading to Linear Functions

Situation 1

Observe the following pattern

Consider a circular disc . Circular rings of same width are drawn around the disc . Proceed in the same way to get the shapes following a pattern as shown below.

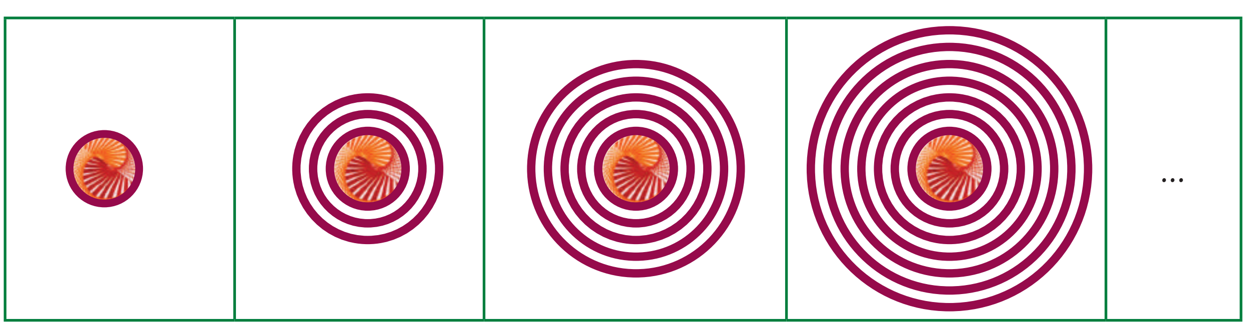


Fig. 5.1

Let us express the above pattern of shapes in the form of a table.

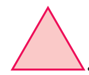
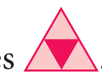
Let x denote the number of steps and y denote the total number of circular rings.

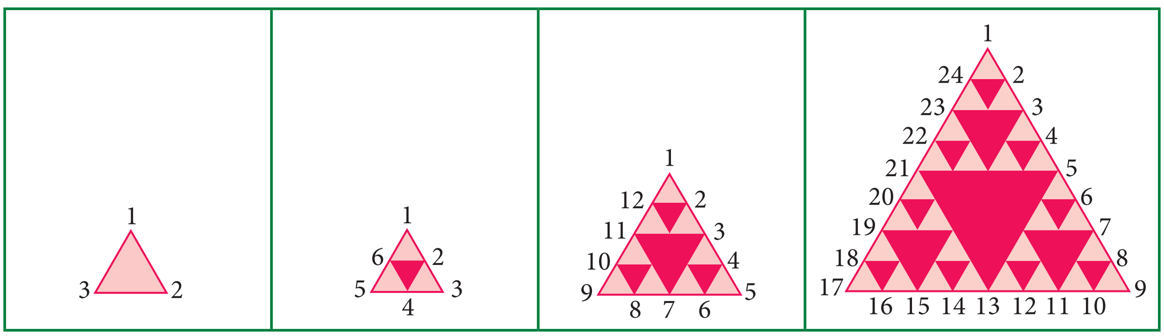
Number of steps (x)	1	2	3	4	…
Number of circular rings (y)	1	3	5	7	…

Here let us find the relationship between the number of steps and number of circular rings.

Hence the relationship between x and y can be generalised as $y = 2x - 1$

Situation 2

Consider an equilateral triangle of any size . Mark the midpoints of sides of a triangle and join them to form four equilateral triangles . Proceed in the same way to get the shapes following a pattern as shown below.



Let us express this repeated process in the form of a table.

Here let us find the relationship between the number of steps (x) and total number of vertices (y).

Number of steps (x)	1	2	3	4	…
Number of vertices (y)	$3 \times 2^0 = 3$	$3 \times 2^1 = 6$	$3 \times 2^2 = 12$	$3 \times 2^3 = 24$	…

Hence, the relationship between x and y can be generalised as $y = 3 \times \left(2^{x-1}\right)$.

From the above situations, you could have understood that even if an action is repeatedly done, the properties of the shape do not change.

Example 5.1

In the following figures, polygons are formed by increasing the number of sides using matchsticks as given below.

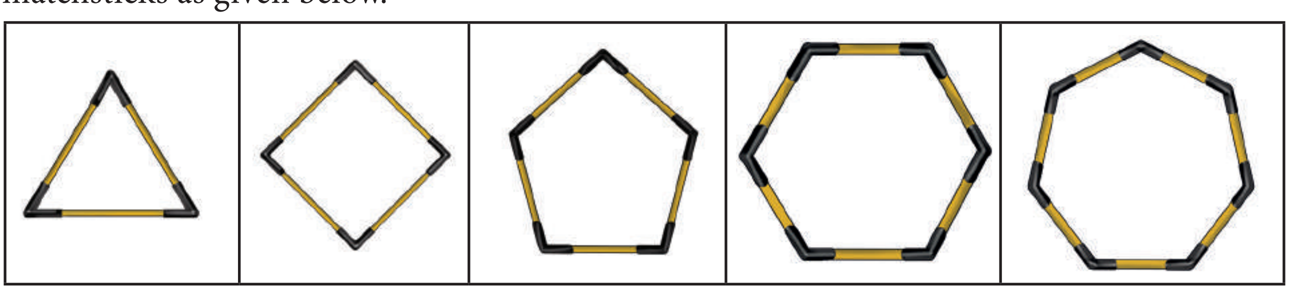


Fig. 5.2

Find the number of sticks required to form the next three shapes by tabulation and generalisation.

Solution

In the above pattern of polygons, in the first shape ($x=1$), we get a closed shape called a triangle. Similarly the second shape ($x=2$) gives a four sided polygon and the third shape ($x=3$) is a five sided polygon and continuing in the same way two more shapes are formed. If the number of match sticks required to form each of the shapes is taken as y, then the values of x and y are tabulated as given below.

x	1	2	3	4	5	...
y	3	4	5	6	7	...

Observe the table and express y in terms of x as below :

when $x=1$, $y = 3 = 1+2$

when $x=2$, $y = 4 = 2+2$

when $x=3$, $y = 5 = 3+2$

when $x=4$, $y = 6 = 4+2$

when $x=5$, $y = 7 = 5+2$

Hence, each of the values of y which we get from the table is 2 more than x. That is $y=x+2$.

Therefore,

6th shape (x=6) will have $y = 8 = 6+2$ (8 match sticks)

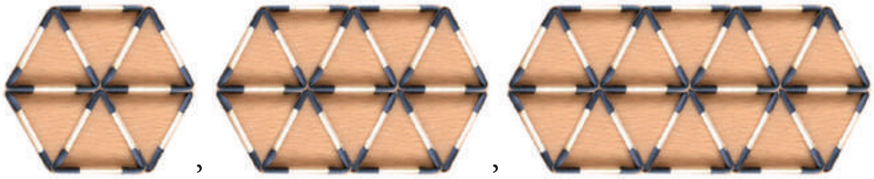
7th shape (x=7) will have $y = 9 = 7+2$ (9 match sticks)

8th shape (x=8) will have $y = 10 = 8+2$ (10 match sticks)

We clearly see that the next three shapes will require 8, 9 and 10 matchsticks.

Activity

Observe the pattern given below. Continue the pattern for three more steps.

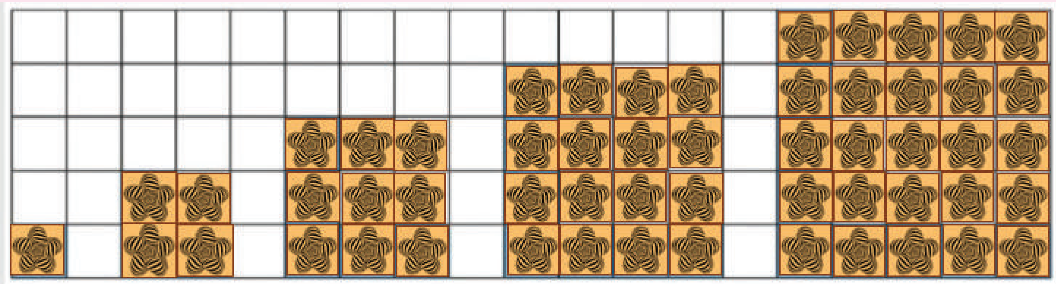


, ____ , ____ , ____ .

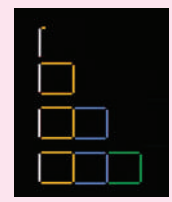
Let, 'x' be the number of steps and 'y' be the number of match sticks. Tabulate the values of 'x' and 'y' and verify the relationship $y = 7x+5$.

Try these

1. In the given figure, let x denote the number of steps and y denote its area. Find the relationship between x and y by tabulation.



2. In the figure, let x denotes the number of steps and y denotes the number of matchsticks used. Find the relationship between x and y by tabulation.



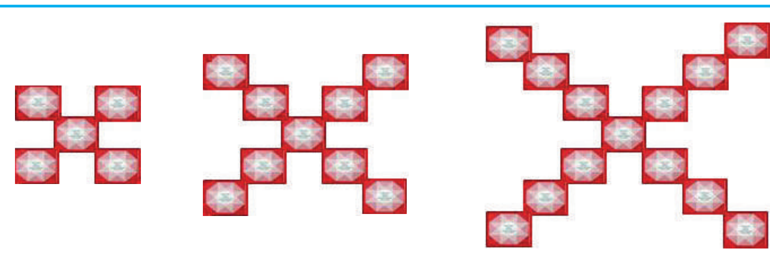
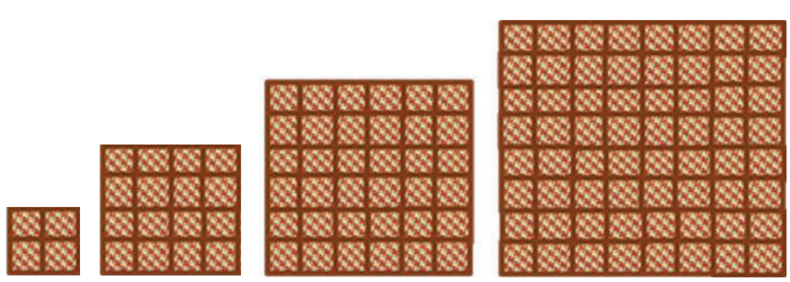
3. Observe the table given below.

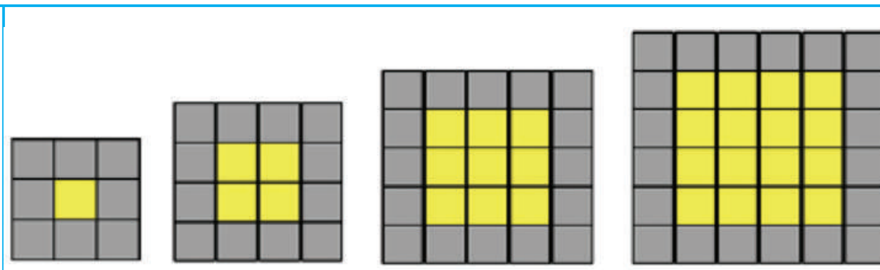
x	-2	-1	0	1	2	8	...
y	-4	-2	0	2	4	?	...

Find the relationship between x and y. What will be the value of y, when $x = 8$.

Exercise 5.1

1. Match the given patterns of shapes with the appropriate number pattern and its generalization.

(i)		a) Sequence : 5,9,13,17… General form: $y = 4n+1$
(ii)		b) Sequence : 3,4,5,6,… General form: $y = x+2$
(iii)		c) Sequence : 1,4,9,16… General form: $y = n^2$

(iv)		d) Sequence : 2,4,6,8… General from: $y = 2n$
(v)		e) Sequence : 4,16,36,64… General from: $y = 4n^2$

Objective type questions

2. Identify the correct relationship between x and y from the given table.

x	1	2	3	4	...
y	4	8	12	16	...

(i) $y = 4x$ (ii) $y = x + 4$ (iii) $y = 4$ (iv) $y = 4 \times 4$

3. Identify the correct relationship between x and y from the given table.

x	–2	–1	0	1	2	...
y	6	3	0	–3	–6	...

(i) $y = -2x$ (ii) $y = +2x$ (iii) $y = +3x$ (iv) $y = -3x$

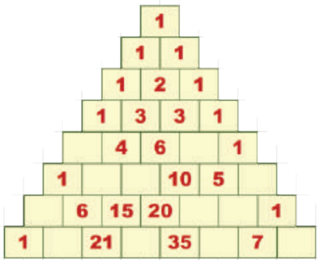
5.3 Pascal's Triangle

The triangle of numbers created by the famous French Mathematician and philosopher Blaise Pascal which is named after him as Pascal's Triangle. This Pascal's Triangle of numbers provides lot of scope to observe various types of number patterns in it.

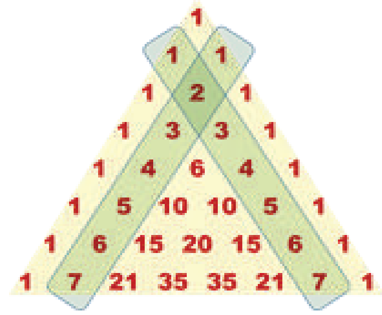
L3F2P1

Activity

1. Complete the following Pascal's Triangle by observing the number pattern.



2. Observe the above completed Pascal's Triangle and moving the slanting strips, find the sequence that you see in it and complete them. One is done for you



(i) 1, 2, 3, 4, 5, 6, 7.
(ii) 1, 3, ___, ___, ___, ___.
(iii) 1, ___, ___, ___, ___.
(iv) ___, ___, ___, ___.

3. Observe the sequence of numbers obtained in the 3rd and 4th slanting rows of Pascal's Triangle and find the difference between the consecutive numbers and complete the table given below.

(i)

3rd slanting row	1	3	6	10	15	21
Difference		2	___	4	___	6

(ii)

4^{th} slanting row	1	4	10	20	35
Difference		3	___	10	___

Example 5.2

Tabulate the 3^{rd} slanting row of the Pascal's Triangle by taking the position of the numbers in the slanting row as x and the corresponding values as y.

x	1	2	3	4	5	6	...
y	1	3	6	10	15	21	...

Verify whether the relationship, $y=\frac{x(x+1)}{2}$ between x and y for the given values is true.

Solution

Observe the table carefully. To verify the relationship between x and y, let us substitute the values of x and get the values of y.

If $x=1,\ y=\frac{1(1+1)}{2}=\frac{2}{2}=1$

If $x=2,\ y=\frac{2(2+1)}{2}=\frac{6}{2}=3$

If $x=3,\ y=\frac{3(3+1)}{2}=\frac{12}{2}=6$

Think

The values of y are obtained by half of the product of the two consecutive values of x.

If $x = 4$, $y = \frac{4(4+1)}{2} = \frac{20}{2} = 10$

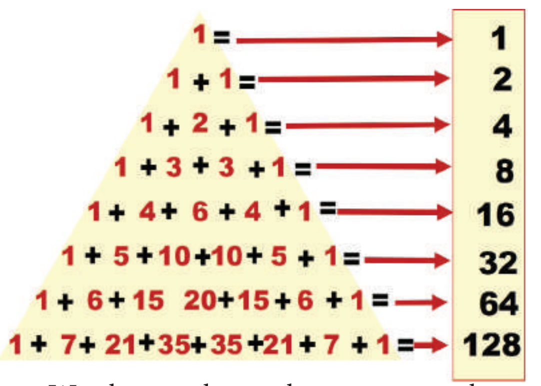
If $x = 5$, $y = \frac{5(5+1)}{2} = \frac{30}{2} = 15$

Hence, $y = \frac{x(x+1)}{2}$ is verified.

Example 5.3 Can row sum of elements in a Pascal's Triangle form a pattern?

Solution

The row sum of elements of a Pascal's Triangle are shown below:



We observe that each row sum can be expressed in powers of 2.

First row $= 2^{1-1} = 1$

Second row $= 2^{2-1} = 2 \times 1 = 2$

Third row $= 2^{3-1} = 2 \times 2 = 4$

Fourth row $= 2^{4-1} = 2 \times 2 \times 2 = 8$

Fifth row $= 2^{5-1} = 2 \times 2 \times 2 \times 2 = 16$

Sixth row $= 2^{6-1} = 2 \times 2 \times 2 \times 2 \times 2 = 32$

Seventh row $= 2^{7-1} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

Eighth row $= 2^{8-1} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128$

Here x denotes the row and y denotes the corresponding row sum. The values of x and y can be tabulated as follows:

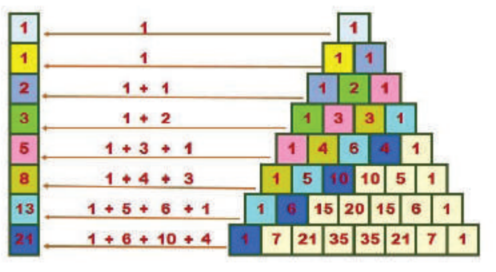
x	1	2	3	4	5	6	7	8	...
y	1	2	4	8	16	32	64	128	...

The relationship between x and y is $y = 2^{x-1}$.

DO YOU KNOW?

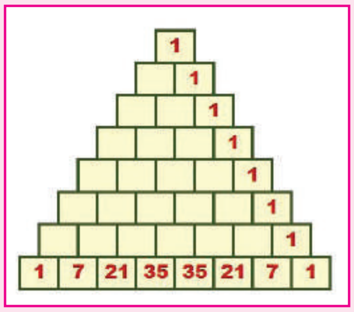
Observe the pattern obtained by adding the elements in the slanting rows of the Pascal's Triangle.

The sequence obtained is known as Fibonacci sequence.

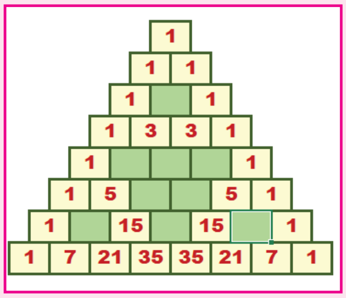


Try these

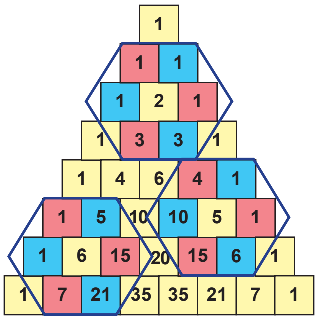
1. Observe the pattern of numbers given in the slanting rows earlier and complete the Pascal's Triangle.

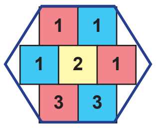


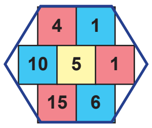
2. Complete the given Pascal's Triangle. Find the common property of the numbers filled by you. Can you relate this pattern with the pattern discussed in situation 2. Discuss.

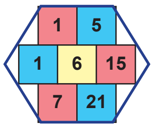


Example 5.4 Observe the numbers in the hexagonal shape given in the Pascal's Triangle. The product of the alternate three numbers in the hexagon is equal to the product of remaining three numbers. Verify this

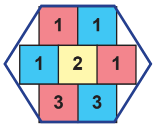
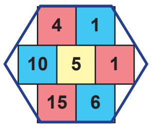
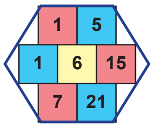


(i) 

(ii) 

(iii) 

Solution

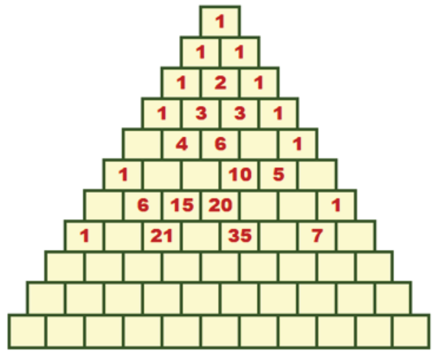
S. No.	Hexagonal Shape	Product of alternate numbers	Product of other three alternate numbers
(i)		$1\times1\times3=3$	$3\times1\times1=3$
		Both are same	
(ii)		$1\times6\times10=60$	$1\times15\times4=60$
		Both are same	
(iii)		$1\times5\times21=105$	$1\times15\times7=105$
		Both are same	

Think

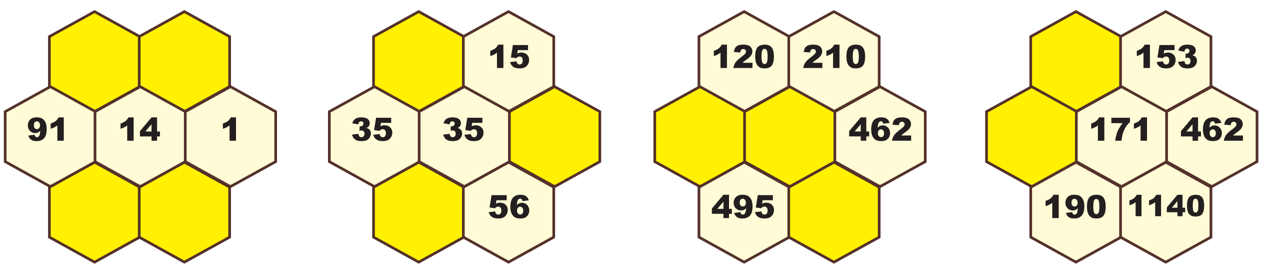
The numbers 1, 3, 6, 10, ... form triangles and are known as triangular numbers. How?

Exercise 5.2

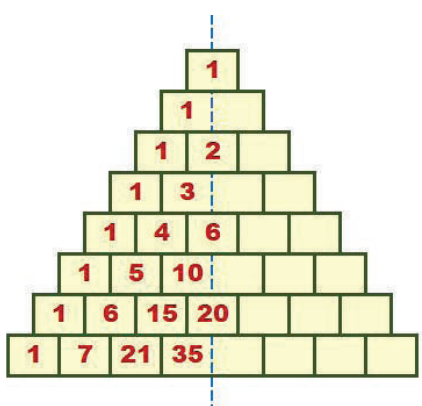
1. Complete the Pascal's Triangle.



2. The following hexagonal shapes are taken from Pascal's Triangle. Fill in the missing numbers.



3. Complete the Pascal's Triangle by taking the numbers 1,2,6,20 as line of symmetry.



A6Q3B6

Objective type questions

4. The elements along the sixth row of the Pascal's Triangle is

(i) 1,5,10,5,1 (ii) 1,5,5,1 (iii) 1,5,5,10,5,5,1 (iv) 1,5,10,10,5,1

5. The difference between the consecutive terms of the fifth slanting row containing four elements of a Pascal's Triangle is

(i) 3,6,10,… (ii) 4,10,20,… (iii) 1,4,10,… (iv) 1,3,6,…

6. What is the sum of the elements of nineth row in the Pascal's Triangle?

(i) 128 (ii) 254 (iii) 256 (iv) 126

Exercise 5.3

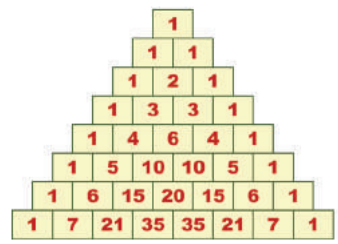
Miscellaneous Practice problems

1. Choose the correct relationship between x and y from the given table.

x	-2	-1	0	1	2	...
y	4	5	6	7	8	...

(i) $y = x+4$ (ii) $y = x+5$ (iii) $y = x+6$ (iv) $y = x+7$

2. Find the triangluar numbers from the Pascal's Triangle and colour them.

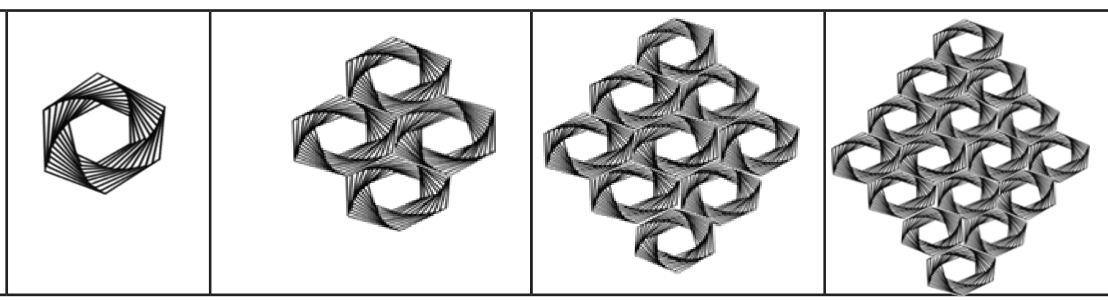


3. Write the first five numbers in the third slanting row of the Pascal's Triangle and find their squares. What do you infer?

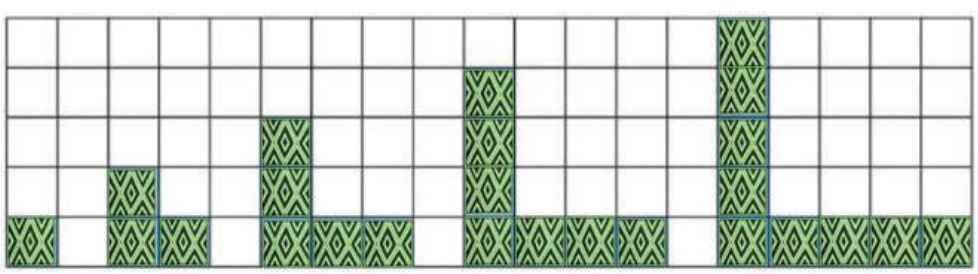
Challenge Problems

4. Tabulate and find the relationship between the variables (x and y) for the following patterns.

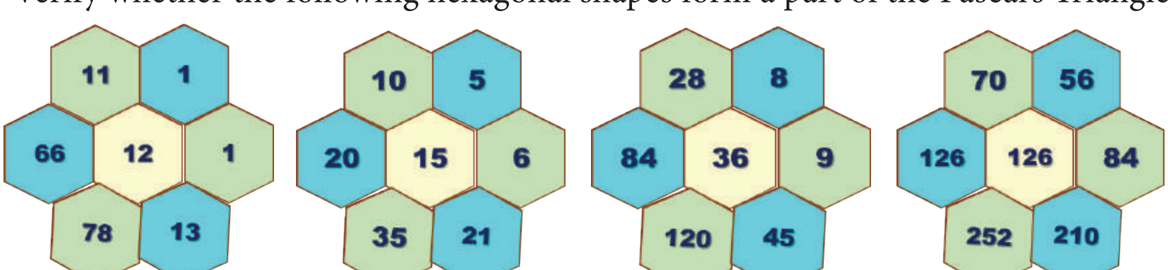
(i)



(ii)

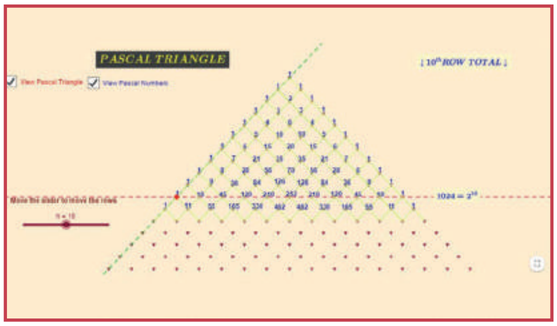


5. Verify whether the following hexagonal shapes form a part of the Pascal's Triangle.



ICT Corner

Expected Result is shown in this picture



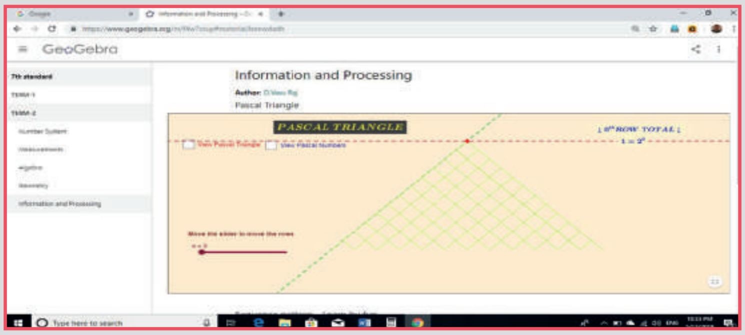
Step-1 :

Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "Information and Processing" will open. There are two activities named "Pascal Triangle" and " Sequence pattern-Learn by fun"

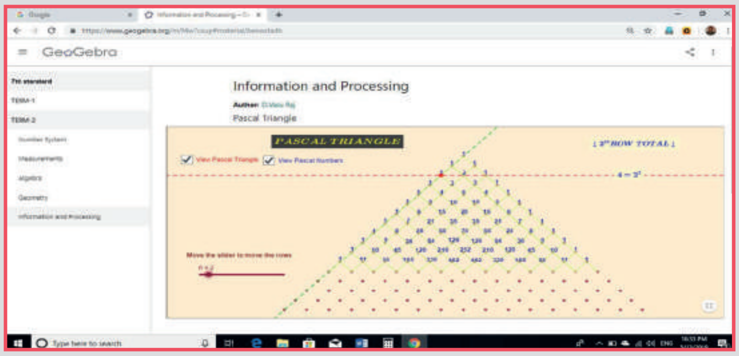
Step-2 :

1. In pascal triangle move the slider to scroll over each row and check the row total. 2. In sequence pattern move each slider *a*, *n* and *i* to generate the pattern.

Step 1



Step 2



Browse in the link

Information Processing : https://www.geogebra.org/m/f4w7csup#material/benwdadh or Scan the QR Code.

B350_7_MATHS_EM

ANSWERS

1. Number system

Exercise 1.1

1. (i) 12.2 (ii) 21.3
2. (i) 0.5 cm (ii) 0.9 cm (iii) 4.2 cm (iv) 8.9 cm (v) 37.5 cm
3. (i) 0.16 m (ii) 0.07 m (iii) 0.43 m (iv) 6.06 m (v) 2.54 m
4. (i) $30+7+\frac{3}{10}$ (ii) $600+50+8+\frac{3}{10}+\frac{7}{100}$

   (iii) $200+30+7+\frac{6}{10}$ (iv) $5000+600+70+8+\frac{3}{10}+\frac{5}{100}+\frac{8}{1000}$

5. (i)

T	O	Tenths	Hundredths
5	3	<u>6</u>	1

; $\frac{6}{10}$

(ii)

H	T	O	Tenths	Hundredths	Thousandths
2	6	3	<u>2</u>	7	1

; $\frac{2}{10}$

(iii)

T	O	Tenths	Hundredths
1	7	3	<u>9</u>

; $\frac{9}{100}$

(iv)

O	Tenths	Hundredths	Thousandths
9	6	<u>5</u>	7

; $\frac{5}{100}$

(v)

Th	H	T	O	Tenths	Hundredths	Thousandths
4	9	7	2	0	6	<u>8</u>

; $\frac{8}{1000}$

Objective type questions

6. (iv) thousandths 7. (ii) 1000 8. (iii) 30.043 9. (ii) 2.64

Exercise 1.2

1. (i) 2 (ii) 7 (iii) 0,3 (iv) 6, 0, 5
2. (i) 801.562 (ii) 932.056 (iii) 47.509 (iv) 503.007

   (v) 680.310 (vi) 109.908

3.(i)

T	O	Tenths	Hundredths	Thousandths
2	5	1	7	8

(ii)

O	Tenths	Hundredths	Thousandths
0	0	2	5

(iii)

H	T	O	Tenths	Hundredths	Thousandths
4	2	8	0	0	1

(iv)

H	T	O	Tenths	Hundredths	Thousandths
1	7	3	1	7	8

(v)

T	O	Tenths	Hundredths
1	9	5	4

4. (i) 21.237 (ii) 3.845 (iii) 6.009 (iv) 956.03 (v) 0.631

5. (i) 0.3 (ii) 3.5 (iii) 3.6 (iv) 1.5 (v) 0.8 (vi) 0.99 (vii) 3.76

6. (i) $\frac{25}{10}$ (ii) $\frac{64}{10}$ (iii) $\frac{75}{100}$ 7. (i) $\frac{117}{50}$ (ii) $\frac{9}{50}$ (iii) $\frac{89}{25}$

Objective type questions

8.(iv) 3.049 9.(iv) 0.6 10.(iii) $\frac{7}{20}$

Exercise 1.3

1. (i) 2.08 (ii) 0.99 (iii) 3.35 (iv) 5.05 (v) 12.35

2. (i) 2.35, 2.53, 3.25, 3.52, 5.32 (ii) 123.45, 123.54, 125.3, 125.34, 125.43

3. (i) 24.5 (ii) 6.95 (iii) 17.8 (iv) 235.48 (v) 0.07 (vi) 4.578

4. (i) 73.51, 71.53, 51.73, 37.51, 17.35 (ii) 745.63, 563.47, 546.37, 457.71, 456.73

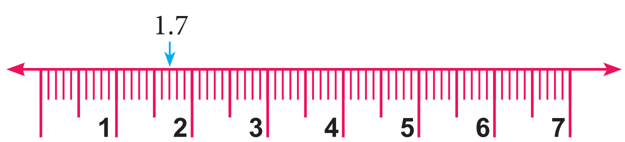
Objective type questions

5. (iii) 0.00900 6. (i) = 7.(i) <

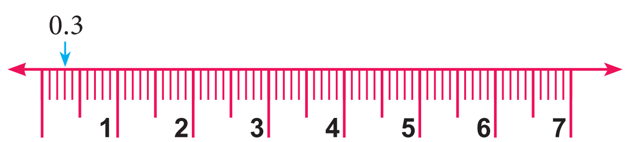
Exercise 1.4

1. P(3.6), Q(1.3), R(6.8), S(4.2)

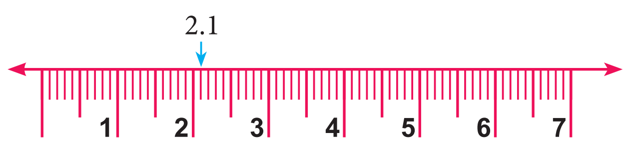
2. (i)



(ii)



(iii)



3. (i) 3 and 4 (ii) 2 and 3 (iii) 0 and 1

4. (i) 3.2 (ii) 6.5 (iii) 2.1

5. (i) 25.03 (ii) 7.01 (iii) 5.6

Objective type questions

6. (iii) 1 and 2 7. (i) 4.5

Exercise 1.5

1. (i)

H	T	O	Tenths	Hundredths
2	4	7	3	6

(ii)

H	T	O	Tenths	Hundredths	Thousandths
1	3	2	1	0	5

2. (i) 305.792 (ii) 1432.67 3. (i) 0.888 (ii) 23.915

4. C is the winner (15.6 s) 5. (i) $\frac{117}{5}$ (ii) $\frac{46301}{1000}$

6. (i) 0.256 km (ii) 4.567 km 7. Boys=0.52; Girls=0.48

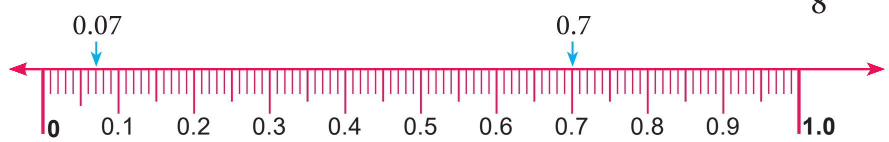
Challenge problems

8. (i) ₹809.99 (ii) ₹147.70 9. (i) 13.28 m (ii) 4.19 m

10.(i) 8.30 m (ii) 24.200 km

11. (i) 0.0023 (ii) 4.21 (iii) 3.7 12. (i) $\frac{17}{8}$ (ii) $\frac{1}{2000}$

13.



14. (i) four and nine tenths (ii) Two hundred and twenty
(iii) seven tenths (iv) Eighty six and three tenths

15.(i) 0 and 1 (ii) 3 and 4 (iii) 3 and 4 (iv) 2 and 3
(v) 1 and 2 (vi) 1 and 2 16. 0.1 km

2. Measurements

Exercise 2.1

1. (i) $d = 30\ cm;\ c = 94.28\ cm$ (ii) $r = 280\ cm;\ d = 560\ cm$ (iii) $r = 12\ m;\ c = 75.42\ m$

2. (i) 220 cm (ii) 176 m (iii) 88 m

3. (i) 308 cm (ii) 572 mm 4. 13.2 m 5. 660 m

6. 4400 m 7. 30 frames 8. ₹59,400

Objective type questions

9. (i) $2\pi r$ units 10. (iv) radius 11. (i) 41 cm

12. (ii) more than three times of its diameter

Exercise 2.2

1. 8662.5 cm^2 2. 3.581 m^2 3. $r = 21\ cm;\ d = 42\ cm$ 4. 9856 m^2

5. 75.46 m^2 6. $r = 28\ m$ 7. 12474 cm^2 8. ₹282975 9. ₹2772

Objective type questions

10. (ii) πr^2 11. (i) 2:1 12. (iv) πn^2

Exercise 2.3

1. 2200 cm^2 2. 1386 m^2 3. 9944 cm^2 4. ₹95700 5. 1200 m^2 6. 60 m^2

7. 4.96 m^2 8. (i) 74 m^2 (ii) ₹888

Objective type questions

9. (i) $\pi\left(R^2 - r^2\right)$ $sq.units$ 10. (ii) $(L \times B) - (l \times b)$ $sq.units$ 11. (iii) $R - r$

Exercise 2.4

1. 28 cm 2. 84 m 3. 668 m^2

4. 49 m 5. (i) 196 cm^2 (ii) 38.5 cm^2 (iii) 42 cm^2

Challenge problems

6. 7 cm; 462 cm^2 7. 710 m^2 8. 30 cm; 10 cm

9. 7 m; 770 m^2 | 10. 2134 m^2 | 11. 264 cm^2 ; 336 cm^2

12. (i) 53 m^2 | (ii) 247 m^2 | (iii) ₹530

3. Algebra

Exercise 3.1

1. (i) 14 power 9 | (ii) $p \times p \times p \times q \times q$ | (iii) 12^{17} | (iv) 1
2. (i) false | (ii) false | (iii) true | (iv) true | (v) true
3. (i) 64 | (ii) 121 | (iii) 625 | (iv) 729
4. (i) 6^4 | (ii) t^2 | (iii) $5^2 \times 7^3$ | (iv) $2^2 \times a^2$
5. (i) 2^9 | (ii) 7^3 | (iii) 3^6 | (iv) 5^5
6. (i) 6^3 | (ii) 3^5 | (iii) 2^8
7. (i) 3969 | (ii) 144 | (iii) 250000
8. (i) 16 | (ii) 24 | (iii) 8000
9. (i) 3^{13} | (ii) a^{14} | (iii) 7^{x+2} | (iv) 2^2
   (v) 18^4 | (vi) 6^{12} | (vii) 1 | (viii) 27^5 | (ix) 36^y | (x) 125^6
10. (i) 17 | (ii) 23 | (iii) 25 | (iv) 1
11. (i) 4^{11} | (ii) 3^{35} | (iii) 5^5 | (iv) 1 | (v) $16a^3b$

Objective type questions

12. (i) a^5 | 13. (iv) $2^3 \times 3^2$ | 14. (i) a | 15. (iv) 20 | 16. (iii) 2^{41}

Exercise 3.2

1. (i) 0 | (ii) odd

2.

Group A	Group B
(i)	(c)
(ii)	(d)
(iii)	(b)
(iv)	(*f*)
(v)	(*a*)
(vi)	(e)

3. (i) 5 | (ii) 1 | (iii) 6 | (iv) 0
   (v) 9 | (vi) 1 | (vii) 4 | (viii) 6
4. (i) 7 | (ii) 1

Objective type questions

5. (ii) 5 | 6. (iv) 1 | 7. (i) 0

Exercise 3.3

1. (i) 11 | (ii) 0 | (iii) 3
2. (i) true | (ii) false | (iii) false | (iv) true
3. (i) 2 | (ii) 2 | (iii) 5 | (iv) 0 | (v) 1
4. (i) 3 | (ii) 2 | (iii) 5 | (iv) 3 | (v) 5
5. $-y^3x^2z$ and $-5y^3x^2z$
6. (i) $19x - 6y$; 1 | (ii) $-k^2 - 4k + 69$; 2 | (iii) $8m^2n + 2pq^2$; 3
7. (i) $7x^2 + 3xy + 12y^2$; 2 | (ii) $3a^4 + a^3 + 2a^2$; 4 | (iii) $9x^2 - 11x - 8$; 2

Objective type questions

8. (iii) trinomial 9. (i) 7 10. (iii) 3

Exercise 3.4

1. $m = 3$ 2. 6 3. 6 4. -1 5. 6 6. 36

Challenge problems

7. 65536 8. $x = 3$ 9. 2 10. 2
11. 21 12. $5x^2 - 11x - 5$; 2 13. $2x^2 - 2xy + 4z^2$; 2

4. Geometry

Exercise 4.1

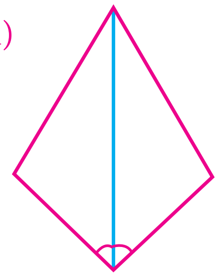
1. yes 2. cannot draw a triangle
3. (i) $45°$ (ii) $62°$ (iii) $30°$ (iv) $17°$ (v) $18°$ (vi) $20°$ (vii) $24°$ (viii) $27°$
4. $\angle A = 60°$; $\angle B = 40°$ 5. $360°$ 6. $45°, 60°, 75°$
7. $55°, 60°, 65°$ 8. $30°, 60°, 90°$ 9. $\angle X = 60°; \angle Z = 48°$
10. $\angle A = 29°; \angle C = 61°$ 11. $\angle M = 38°; \angle O = 52°$ 12.(i) $110°$ (ii) $11°$
13. $21°$ 14. $110°$ 15. $120°$

Objective type questions

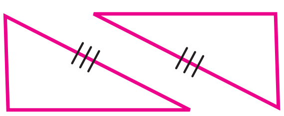
16.(ii) $40°, 60°, 80°$ 17. (iii) $80°, 35°$ 18. (ii) $68°$
19. (iii) $x + y + z = 2(a + b + c)$ 20.(iii) $35°$ 21.(iii) $130°$ 22.(ii) $65°, 80°$

Exercise 4.2

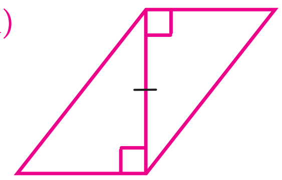
1. Corresponding sides : $AB, DE; BC, EF; AC, DF$
   Corresponding angles : $\angle ABC, \angle DEF; \angle BCA, \angle EFD; \angle CAB, \angle FDE$
2. (i) $\overline{PQ} = \overline{LN}; \overline{PR} = \overline{LM}; \overline{RQ} = \overline{MN}$
   $\angle RPQ = \angle NLM; \angle PQR = \angle LNM; \angle PRQ = \angle LMN$
   (ii) $\overline{QR} = \overline{LM}; \overline{RP} = \overline{LN}; \overline{PQ} = \overline{MN}$
   $\angle PQR = \angle LMN; \angle QRP = \angle MLN; \angle RPQ = \angle LNM$
3. (i) Not corresponding angles (ii) Not corresponding angles
   (iii) corresponding angles (iv) Not corresponding sides
   (v) corresponding sides (vi) Not corresponding sides
4. (i) congruent triangles by SAS (ii) congruent triangles by SSS
   (iii) congruent triangles by AAA (iv) congruent triangles by RHS
   (v) congruent triangles by SSS (or) RHS (or) SAS
5. (i)

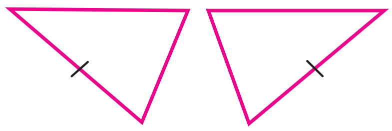


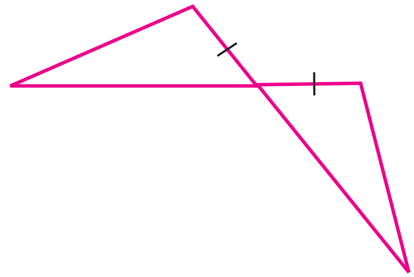
(ii)



(iii)



(iv) 

(v) 

6. (i) SSS (ii) ASA (iii) RHS
(iv) ASA (v) ASA (vi) SAS

Objective type questions

8. (iv) same shape and same size
9. (ii) superposition method
10. (iii) SSA rule
11. (iv) They should have the same length
12. (i) $\angle ADB = \angle CDB; \angle ABD = \angle CBD; BD = BD$
13. (ii) SAS property

Exercise 4.3

1. 52°, 52°
2. Isoceles triangle
3. Right angled triangle
4. 40°, 70°, 70°
5. 100°
6. 152°
7. $\angle O = 75°$
10. (SAS), $\Delta CAB \cong \Delta EBD$; $AC \parallel DE$

Challenge problems

11. $x = 30°$
12. $x = 114°$
13. $x = 34°; y = 118°$
14. 100°
15. 95°
16. $y = 137°$

5. Information Processing

Exercise 5.1

1.

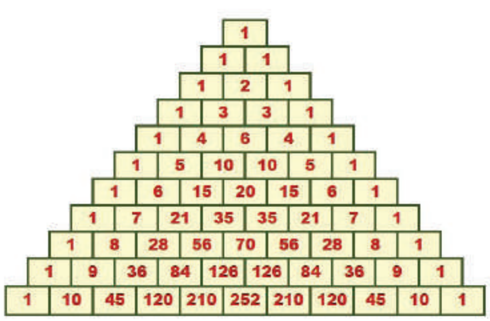
(i)	(d)
(ii)	(*a*)
(iii)	(e)
(iv)	(c)
(v)	(b)

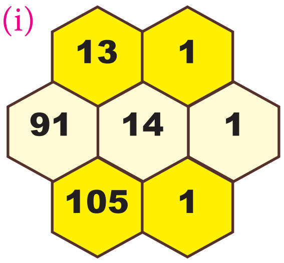
Objective type questions

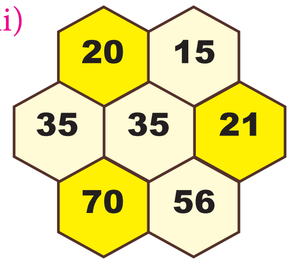
2.(i) $y = 4x$
3. (iv) $y = -3x$

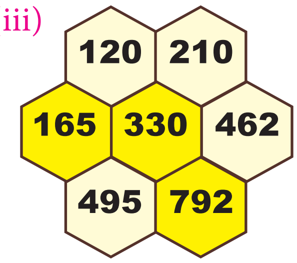
Exercise 5.2

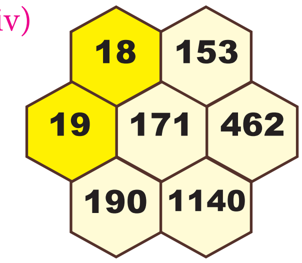
1.

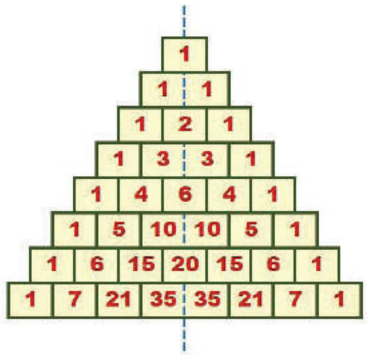


2.(i) 

(ii) 

(iii) 

(iv) 

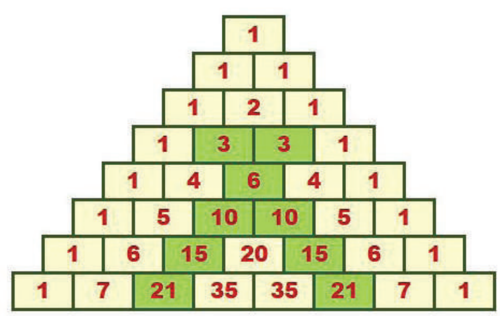
3. 

Objective type questions

4. (iv) 1,5,10,10,5,1   5. (ii) 4,10,20,...   6. (iii) 256

Exercise 5.3

1. (iii) $y = x + 6$

2. 

3. Five numbers in third slanting rows are 1, 3, 6, 10, 15 ; Their squares are 1, 9, 36, 100, 225.

Challenge Problems

4. (i)

steps (x)	1	2	3	4
shapes (y)	1	4	9	16

(ii)

steps (x)	1	2	3	4	5
shapes (y)	1	3	5	7	9

5. (i) $1 \times 13 \times 66 = 11 \times 1 \times 78$ (ii) $5 \times 21 \times 20 = 10 \times 6 \times 35$

(iii) $8 \times 45 \times 84 = 28 \times 9 \times 120$ (iv) $56 \times 210 \times 126 = 70 \times 84 \times 252$

GLOSSARY

Area	பரப்பளவு
Base	அடிமானம்
Celestial bodies	வான் பொருட்கள்
Circle	வட்டம்
Circular ring	வட்டவளையம்
Circumference	பரிதி
Congruence	சர்வசமம்
Congruency	சர்வசம தன்மை
Congruent triangle	சர்வசம முக்கோணம்
Consecutive terms	அடுத்தடுத்த உறுப்புகள்
Corresponding side	ஒத்த பக்கம்
Criterion	கொள்கை
Cube of a variable	மாறியின் கனம்
Decimal number	தசம எண்
Decimal part	தசமப் பகுதி
Decimal point	தசமப் புள்ளி
Degree	படி
Diameter	விட்டம்
Exponent	அடுக்குக் குறி, படிக் குறி
Exponent number	அடுக்கு எண்
Exponential form	அடுக்கு வடிவம்
Exterior angle	வெளிக்கோணம்
Fencing	வேலியிடுதல்

Hundredth	நூறில் ஒன்று
Hypotenuse	கர்ணம்
Integral part	முழு எண் பகுதி
Interior opposite angle	உள்ளெதிர் கோணம்
Leading Coefficient	தலையாயக் கெழு
Notation	குறியீடு
Pathway	பாதை
Polygon	பலகோணம்
Power	அடுக்கு
Quadrant	கால்வட்டம்
Radius	ஆரம்
Semi-circle	அரைவட்டம்
Slanting row	சாய்வு வரிசை
Snow flakes	பனித்திவலைகள்
Square of a variable	மாறியின் வர்க்கம்
Standard form	திட்ட வடிவம்
Superposition method	மேற்பொருந்துதல் முறை
Tenth	பத்தில் ஒன்று
Tessellations	பல்வண்ணக் கட்டமைப்பு
Thousandth	ஆயிரத்தில் ஒன்று
Traingular number	முக்கோண எண்கள்
Unit digit	ஒன்றின் இலக்கம்
Width	அகலம்

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