Andhra Pradesh SSC Class 10th Maths Question Paper 1 With Solution 2015

QUESTION PAPER CODE 15E(A)

SECTION - I
GROUP - A

(5 * 2 = 10)

Answer ANY 5 Questions choosing two from each of the following groups.

Question 1: Find L.C.M and H.C.F of 72 and 108 by the prime factorisation method.

Solution:
H.C.F of 72 and 108

2|72, 108
 |__________
2| 36, 54
 |__________
3| 18, 27
 |__________
3| 6. 9
 |__________
 | 2, 3
 |__________

H.C.F. of 72 and 108 = 2*2*3*3
= 36

L.C.M. of 72 and 108

2|72, 108

|________
2|36, 54
|________
2|18, 27
|________
3| 9, 27
|________
3| 3,   9
|________
3| 1,   3
|________
| 1,   1

L.C.M. of 72 and 108 = 2*2*2*3*3*3
= 216

So, H.C.F. of 72 and 108 is 36 and L.C.M of 72 and 108 is 216

Question 2: If V = {a, e, i, o, u} and B = {a, i, k, u}, then find V – B and B – V. Are they equal?

Solution:
V – B = {e, o}
B – V = {k}
No; they are not equal.

Question 3: Find a quadratic polynomial, the sum and product of whose zeroes are 1 / 4, – 1 respectively.

Solution:
Let the roots be a,b.
Then the equation of polynomial is given by
$x^2 - (a + b)x + ab$ --------------(1)
The sum of roots = 1/4
a + b = 1 / 4

product of roots = -1

ab = -1

Substituting above values in (1)

∴ Equation of the required polynomial is $x^2 - (1/4)x - 1 = 0$ or $4x^2 - x - 4 = 0$.

Question 4: Find two numbers, whose sum is 27 and product is 182.

Solution:

Let the first number be x and the second number is 27 - x.

Therefore, their product = x (27 - x)

It is given that the product of these numbers is 182.

Therefore, x(27 - x) = 182

$\Rightarrow x^2 - 27x + 182 = 0$

$\Rightarrow x^2 - 13x - 14x + 182 = 0$

$\Rightarrow x(x - 13) - 14(x - 13) = 0$

$\Rightarrow (x - 13)(x - 14) = 0$

Either x = -13 = 0 or x - 14 = 0

$\Rightarrow x = 13$ or $x = 14$

If first number = 13, then the other number = 27 - 13 = 14

If first number = 14, then the other number = 27 - 14 = 13

Therefore, the numbers are 13 and 14.

GROUP - B

Question 5: For what value of ‘k’, the pair equations 3x + 4y + 2 = 0 and 9x + 12y + k = 0 represent coincident lines.

Solution:

The coincident lines are 3x + 4y + 2 = 0 and 9x + 12y + k = 0.

Comparing the equations with the general equations

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Then,

$a_1 = 3$, $b_1 = 4$, $c_1 = 2$, $a_2 = 9$, $b_2 = 12$, $c_2 = k$

Since, the lines are coincident, then,

$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$

$3 / 9 = 4 / 12 = 2 / k$

$1 / 3 = 1 / 3 = 2 / k$

$1 / 3 = 2 / k$

$k = 6$

Question 6: In a flower bed, there are 23 rose plants in the first row, 21 in the second row, 19 in the third row, and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?

Solution:

$a_1 = 23$, $a_2 = 21$, $a_3 = 19$ and $a_n = 5 = l$; $S_n = ?$

$a_n = a + (n - 1)d$

=> $5 = 23 + (n - 1) (-2)$

=> $5 = 23 - 2n + 2$

=> $5 = 25 - 2n$

=> $2n = 25 - 5$

=> $n = 20 / 2$

=> $n = 10$

$S_n = [n / 2] (2a + [n - 1] d)$

$S_{10} = [10 / 2] (2 * 23 + 9 * -2)$

$S_{10} = 5 \times 28$

$= 140$

So, there are 140 rose plants in the flower bed.

Question 7: Find the coordinates of a point A, where AB is the diameter of a circle, whose centre is (2, -3) and B is (1, 4).

Solution:

$2 = [x + 1] / 2$

$4 - x = 1$

x = 3
-3 = [y + 4] / 2
-6 - y = 4
y = -10
The coordinates of A are (3, -10).

Question 8: Find the area of the triangle, whose vertices are (-5, -1), (3, -5), (5, 2).

Solution:
Area = [1 / 2] [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]
= [1 / 2] [-5 * (-5 -2) + 3 (2 + 1) + 5 (-1 + 5)]
= [1 / 2] [35 + 9 + 20]
= [1 / 2] * 64
= 32 cm^2

SECTION - II

[4 * 1 = 4]

Answer ANY 4 of the following six questions.

Question 9: Expand log 15.

Solution:
log 15 = log5 3
= log5 log3

Question 10: Write roster and set builder form of *"The set of all-natural numbers, which divide 42"*.

Solution:
(i) x = {1, 2, 3, 6, 7}
(ii) x = {x : x = 1, 2, 3, 6, 7}

Question 11: If p(t) = t^3 – 1, find the values of p (1), p (–2).

Solution:

p(1) = 1^3 - 1 = 0

p(-2) = -2^3 - 1 = -8 - 1 = -9

Question 12: Formulate a pair of linear equations in two variables "5 pencils and 7 pens together cost Rs. 50, whereas 7 pencils and 5 pens together cost Rs. 46".

Solution:

Let pencil = x

pen = y

5x + 7y = 50 * 5

7x + 5y = 46 * 7

25x + 35y = 250

49x + 35y = 322

24x = 72

x = 42 / 24

x = 3

Pencil = 3

y = [46 - 7x] / 5

= [46 - 7 * 3] / 5

= 25 / 5

= 5

y = 5

Pen = 5

Question 13: If $b^2 - 4ac \geq 0$, then write the roots of a quadratic equation $ax^2 + bx + c = 0$.

Solution:

Real, equal and distinct.

Question 14: Write the formula for the sum of first 'n' positive integers.

Solution:

n [n + 1] / 2

SECTION - III

[4 * 4 = 16]

Answer ANY 4 questions, choosing atleast two from each of the following groups.

GROUP - A

Question 15: Prove that √5 is irrational by the method of Contradiction.

Solution:

Say, $\sqrt{5}$ is a rational number.

∴ It can be expressed in the form p / q where p,q are co-prime integers.

$\Rightarrow \sqrt{5} = p / q$

$\Rightarrow 5 = p^2 / q^2$ {Squaring both the sides}

$\Rightarrow 5q^2 = p^2$ --- (1)

$\Rightarrow p^2$ is a multiple of 5. {Euclid's Division Lemma}

$\Rightarrow$ p is also a multiple of 5. {Fundamental Theorem of arithmetic}

$\Rightarrow p = 5m$

$\Rightarrow p^2 = 25m^2$ --- (2)

From equations (1) and (2), we get,

$5q^2 = 25m^2$

$\Rightarrow q^2 = 5m^2$

$\Rightarrow q^2$ is a multiple of 5. {Euclid's Division Lemma}

$\Rightarrow$ q is a multiple of 5.{Fundamental Theorem of Arithmetic}

Hence, p,q have a common factor 5.

This contradicts that they are co-primes.

Therefore, p / q is not a rational number.

So, $\sqrt{5}$ is an irrational number.

Question 16: If A = {3, 6, 9, 12, 15, 18, 21}; B = {4, 8, 12, 16, 20}; C = {2, 4, 6, 8, 10, 12, 14, 16}; D = {5, 10, 15, 20}; find
(i) A – B
(ii) B – A
(iii) C – A
(iv) D – A
(v) B – C
(vi) B – D
(vii) C – B
(viii) D – B

Solution:
(i) A – B = {3, 6, 9, 15, 18, 21}
(ii) B – A = {4, 8, 16, 20}
(iii) C – A = {2, 4, 8, 10, 14, 16}
(iv) D – A = {5, 10, 20}
(v) B – C = {20}
(vi) B – D = {4, 8, 12, 16}
(vii) C – B = {2, 6, 10, 14}
(viii) D – B = {5, 10, 15}

Question 17: Verify that 1, -1 and -3 are the zeroes of the cubic polynomial $x^3 + 3x^2 - x - 3$ and check the relationship between zeroes and the coefficient.

Solution:
To verify roots, put root inside equations $x^3 + 3x^2 - x - 3$
Root 1 ⟶ $1^3 + 3(1)^2 – 1 – 3 = 1 + 3 – 1 – 3 = 0$
Root (-1) = $-1^3 + 3(-1)^2 + 1 - 3 = – 1 + 3 + 1 – 3 = 0$
Root (-3) = $(-3)^3 + 3(-3)^2 - (-3) - 3 = – 27 + 27 + 3 – 3 = 0$
Sum of root = -b / a = -3 / 1 = -3
From roots = (1 – 1 – 3) = –3
Product of roots = -d / a = 3 / 1 = 3
From roots = 1 * -1 * -3 = 3
Product of roots = pq + qr + rp = c / a = -1 / 1 = -1

From roots = 1 * –1 + –1 * –3 + –3 * 1 = – 1 + 3 – 3 = –1
Hence verified.

Question 18: Find the root of equation $2x^2$ - x - 4 = 0 by the method of completing the square.

Solution:

$2x^2 - 1x - 4 = 0$

$a \neq 1$ so divide through by 2

$[2 / 2]\, x^2 - [1 / 2]\, x - [4 / 2] = 0 / 2$

$x^2 - [1 / 2]\, x - 2 = 0$

Keep x terms on the left and move the constant to the right side by adding it on both sides

$x^2 - [1 / 2]\, x = 2$

Take half of the x term and square it

$\{[-1 / 2] \cdot [1 / 2]\}^2 = 1 / 16$

Add the result to both sides

$x^2 - [1 / 2]\, x + [1 / 16] = 2 + [1 / 16]$

Rewrite the perfect square on the left

$(x - (1 / 4))^2 = 2 + [1 / 16]$

Combine terms on the right

$(x - [1 / 4])^2 = 33 / 16$

Take the square root of both sides

$x - [1 / 4] = \pm \sqrt{33 / 16}$

Simplify the Radical term (0):

$x - [1 / 4] = \pm \sqrt{33} / 4$

Isolate the x on the left side and solve for x (1)

$x = [1 / 4] \pm \sqrt{33} / 4$

$x = [1 / 4] + \sqrt{33} / 4$

$x = [1 / 4] - \sqrt{33} / 4$

GROUP - B

Question 19: Solve the equations [5] / [x - 1] + [1] / [y - 2] and [6] / [x - 1] - [3] / [y - 2] = 1.

Solution:

[5] / [x - 1] + [1] / [y - 2] --- 1

[6] / [x - 1] - [3] / [y - 2] = 1 --- 2

let assume, 1 / [x - 1] = u, 1 / [y - 2] = v

15 * u = 3 * v = 6 ---) 3

Add equations 2 and 3

21* u = 7

u = (1 / 3)

x - 1 = 3

x = 4

b = 1 / 3

y – 2 = 3

y = 5

Question 20: A fraction becomes 4 / 5 if 1 is added to both numerator and denominator. If, however, 5 is subtracted from both numerator and denominator, the fraction becomes 1 / 2. What is the fraction?

Solution:

Let the fraction be p / q.

The fraction will become 4 / 5 if 1 is added to numerator and denominator

So,

4 / 5 = (p + 1) / (q + 1)

4q + 4 = 5p + 5

(p - 5) / (q - 5) = 1 / 2

2p - 10 = q - 5

q = 2p - 5

4(2p - 5) + 4 = 5p + 5

8p - 20 + 4 = 5p + 5

3p = 21

p = 7

Therefore q = 2 * 7 - 5

q = 9

So the fraction is 7/9.

Question 21: The sum of the 4th and 8th terms of an A.P is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the A.P.

Solution:

Let the first term of an AP = a and the common difference of the given AP = d.

$a_n = a + (n - 1) d$

$a_4 = a + (4 - 1) d$

$a_4 = a + 3d$

Similarly,

$a_8 = a + 7d$

$a_6 = a + 5d$

$a_{10} = a + 9d$

The sum of 4^{th} and 8^{th} term = 24

$a_4 + a_8 = 24$

a + 3d + a + 7d = 24

2a + 10d = 24

a + 5d = 12.................... (i)

The sum of 6^{th} and $10t^{h}$ term = 44

$a_6 + a_{10} = 44$

a + 5d + a + 9d = 44

2a + 14d = 44

a + 7d = 22 ....(ii)

Solving (i) and (ii),

a + 7d = 22

a + 5d = 12

2d = 10

d = 10 / 2

d = 5

From equation (i), we get,

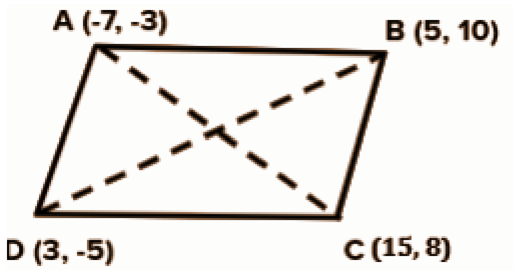
a + 5d = 12

a + 5 (5) = 12

$a + 25 = 12$
$a = -13$
$a_2 = a + d = -13 + 5 = -8$
$a_3 = a_2 + d = -8 + 5 = -3$

Question 22: Prove that the points (-7, -3), (5, 10), (15, 8) and (3, -5) taken in order are the vertices of a parallelogram.

Solution:



Let say ABCD be the points of the parallelogram.
To prove: Correct order of vertices, diagonals bisect each other in the parallelogram. Hence point O must be some from A and C.
C = [-7 + 25] / 2, [-3 + 8] / 2
= [4, (5 / 2)]
From B and D
C = [-5 + 3] / 2, [10 - 5] / 2
= [4, (5 / 2)]
Hence, both are correct.

SECTION - IV

(1 * 5 = 5)

Answer ANY ONE question from the following.

Question 23: Draw the graph of $y = x^2 - x - 6$ and find zeroes. Justify the answer.

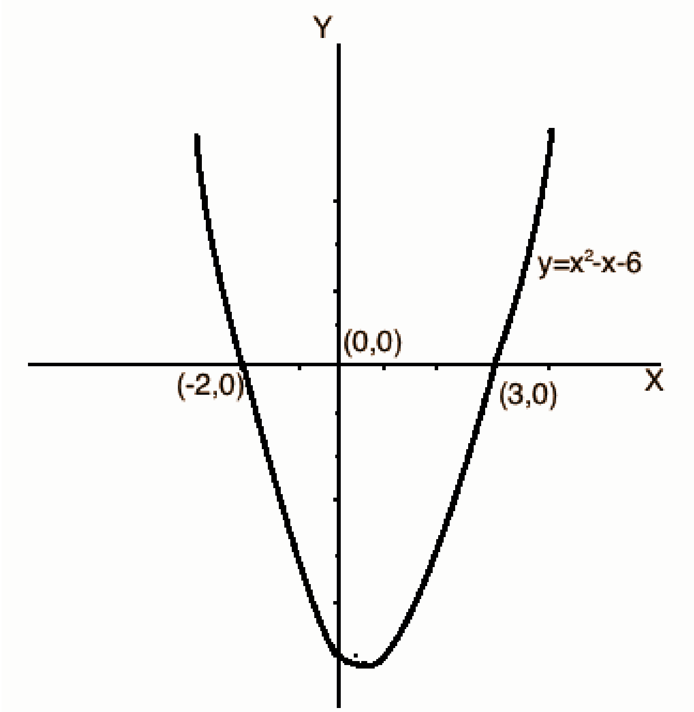
Solution:
Clearly the graph of $y = x^2 - x - 6$ cut the x−axis at $x = -2, 3$.

⇒x = −2, 3 are zeros of $y = x^2 - x - 6$.

Justification:

when $x = -2$; $y = 4 + 2 - 6 = 0$

when $x = 3$; $y = 9 - 3 - 6 = 0$

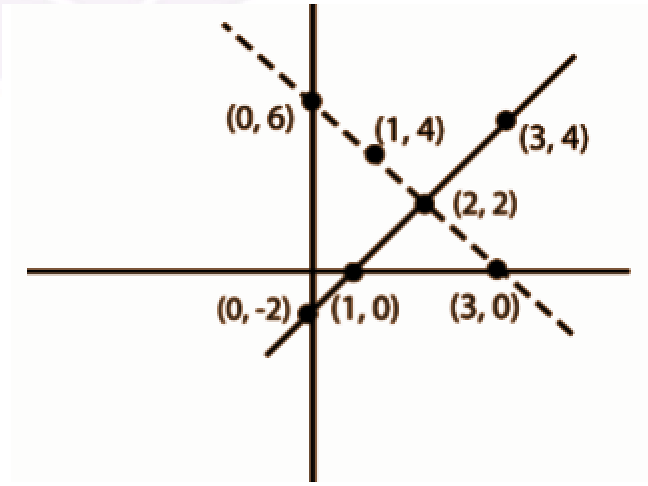


Question 24: Solve the pair linear equations graphically.

2x + y - 6 = 0

4x - 2y - 4 = 0

Solution:



Hence, the graph solution is (2,2).