

# KSEEB Class 10 Maths Question Paper Solution 2017

## QUESTION PAPER CODE 81-E

### SECTION - I

1. If  $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ ,  $A = \{1, 2, 3\}$  and  $B = \{2, 3, 4, 5\}$ , then  $(A \cup B)'$  is

- (A)  $\{5, 6, 7\}$
- (B)  $\{6, 7, 8\}$
- (C)  $\{3, 4, 5\}$
- (D)  $\{1, 2, 3\}$

**Solution:**

Correct answer: (B)

Given,

$$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$A = \{1, 2, 3\}$$

$$B = \{2, 3, 4, 5\}$$

$$A \cup B = \{1, 2, 3\} \cup \{2, 3, 4, 5\}$$

$$= \{1, 2, 3, 4, 5\}$$

$$(A \cup B)' = U - (A \cup B)$$

$$= \{1, 2, 3, 4, 5, 6, 7, 8\} - \{1, 2, 3, 4, 5\}$$

$$= \{6, 7, 8\}$$

2. LCM of 18 and 45 is

- (A) 9
- (B) 45
- (C) 90
- (D) 81

**Solution:**

Correct answer: (C)

Prime factorization of 18:

$$18 = 2 \times 3 \times 3$$

Prime factorization of 45:

$$45 = 3 \times 3 \times 5$$

$$\text{LCM}(18, 45) = 2 \times 3 \times 3 \times 5 = 90$$

3. The mean ( $\bar{x}$ ) and the standard deviation ( $\sigma$ ) of certain scores are 60 and 3 respectively. Then the coefficient of variation is

- (A) 5
- (B) 6
- (C) 7
- (D) 8

**Solution:**

Correct answer: (A)

Given,

$$\text{Mean} = 60$$

$$\text{Standard deviation} = 3$$

$$\text{Coefficient of variation} = (\text{standard deviation}/\text{mean}) \times 100$$

$$= (3/60) \times 100$$

$$= 5$$

4. Rationalising factor of  $\sqrt{x} - y$  is

(A)  $x - y$

(B)  $\sqrt{x}$

(C)  $\sqrt{x} + y$

(D)  $\sqrt{x} - y$

**Solution:**

Correct answer: (D)

Rationalising factor of  $\sqrt{x} - y$  is  $\sqrt{x} - y$ .

$$\text{Since } (\sqrt{x} - y)(\sqrt{x} - y)$$

$$= [\sqrt{(x - y)}]^2$$

$$= x - y$$

5. If  $f(x) = x^2 - 2x + 15$  then  $f(-1)$  is

(A) 14

(B) 18

(C) 15

(D) 13

**Solution:**

Correct answer: (B)

Given,

$$f(x) = x^2 - 2x + 15$$

$$f(-1) = (-1)^2 - 2(-1) + 15$$

$$= 1 + 2 + 15$$

$$= 18$$

6. In a circle, the angle subtended by a chord in the major segment is

(A) a straight angle

(B) a right angle

(C) an acute angle

(D) an obtuse angle

**Solution:**

Correct answer: (C)

In a circle, the angle subtended by a chord in the major segment is an acute angle.

7. The length of the diagonal of a square of side 12 cm is  
(A)  $5\sqrt{2}$  cm  
(B) 144 cm  
(C) 24 cm  
(D)  $12\sqrt{2}$  cm

**Solution:**

Correct answer: (D)

Given,

Side of a square =  $a = 12$  cm

Diagonal of square =  $a\sqrt{2} = 12\sqrt{2}$  cm

8. The distance between the origin and the point (-12, 5) is  
(A) 13 units  
(B) -12 units  
(C) 10 units  
(D) 5 units

**Solution:**

Correct answer: (A)

The distance of a point (x, y) from the origin is  $\sqrt{(x^2 + y^2)}$ .

The distance between the origin and the point (-12, 5) =  $\sqrt{[(-12)^2 + 5^2]}$

$$= \sqrt{(144 + 25)}$$

$$= \sqrt{169}$$

$$= 13 \text{ units}$$

## SECTION - II

9. Write the value of  ${}^{100}P_0$ .

**Solution:**

We know that,

$${}^nP_0 = 1$$

$$\text{Therefore, } {}^{100}P_0 = 1$$

(or)

$${}^{100}P_0 = 100!/(100 - 0)!$$

$$= 100!/100!$$

$$= 1$$

10. What is the probability of a certain event?

**Solution:**

Probability of a certain event or sure event is 1.

11. Find the midpoint of the class-interval 5 – 15.

**Solution:**

Given,

Class interval is 5 - 15.

Upper limit = 15

Lower limit = 5

Midpoint of the class interval =  $(\text{lower limit} + \text{upper limit})/2$

$$= (5 + 15)/2$$

$$= 20/2$$

$$= 10$$

12. Find the value of  $\cos 48^\circ - \sin 42^\circ$ .

**Solution:**

$$\cos 48^\circ - \sin 42^\circ$$

$$= \cos(90^\circ - 42^\circ) - \sin 42^\circ$$

$$= \sin 42^\circ - \sin 42^\circ$$

$$= 0$$

(or)

$$\cos 48^\circ - \sin 42^\circ$$

$$= \cos 48^\circ - \sin(90^\circ - 48^\circ)$$

$$= \cos 48^\circ - \sin 48^\circ$$

$$= 0$$

13. Write the slope and y-intercept of the line  $y = 3x$ .

**Solution:**

Given,

Equation of line is  $y = 3x$

Comparing with  $y = mx + c$

Here,

$$\text{Slope} = m = 3$$

$$\text{y-intercept} = c = 0$$

14. Write the formula used to find the total surface area of a solid hemisphere.

**Solution:**

Total surface area of a solid hemi-sphere =  $3\pi r^2$  sq.units

Here,

r = Radius of the solid hemisphere

### SECTION - III

15. If A and B are the sets such that  $n(A) = 37$ ,  $n(B) = 26$  and  $n(A \cup B) = 51$ , then find  $n(A \cap B)$ .

**Solution:**

Given,

$$n(A) = 37$$

$$n(B) = 26$$

$$n(A \cup B) = 51$$

We know that,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$51 = 37 + 26 - n(A \cap B)$$

$$n(A \cap B) = 63 - 51$$

$$n(A \cap B) = 12$$

16. Write the formula used to find

a) arithmetic mean between a and b ( $a > b$ )

b) harmonic mean between a and b ( $a > b$ ).

**Solution:**

a) Arithmetic mean A.M. =  $(a + b)/2$  (where  $a > b$ )

b) Harmonic mean H.M. =  $2ab/(a + b)$  (where  $a > b$ )

17. Find the sum to infinity of the geometric series  $2 + (2/3) + (2/9) + \dots$

**Solution:**

Given,

$$2 + (2/3) + (2/9) + \dots$$

Here,

$$a = 2$$

$$r = 2/(3/2) = 1/3$$

$$S_{\infty} = a/(1 - r)$$

$$= 2/[1 - (1/3)]$$

$$= 2/[(3 - 1)/3]$$

$$= 2/(2/3)$$

$$= 3$$

Therefore, the sum to infinity of the given geometric series is 3.

18. Prove that  $3 + \sqrt{5}$  is an irrational number.

**Solution:**

Let  $3 + \sqrt{5}$  be a rational number.

Thus,  $3 + \sqrt{5} = p/q$ , where p, q are coprime integers and  $q \neq 0$ .

$$\Rightarrow \sqrt{5} = (p/q) - 3$$

$$\Rightarrow \sqrt{5} = (p - 3q)/q$$

Since p and q are integers,  $(p - 3q)/q$  is a rational number.

$\Rightarrow \sqrt{5}$  is also a rational number.

This is the contradiction to the fact that  $\sqrt{5}$  is an irrational number.

Hence, our assumption that  $3 + \sqrt{5}$  is a rational number is wrong.

Therefore,  $3 + \sqrt{5}$  is an irrational number.

Hence proved.

19. Find how many triangles can be drawn through 8 points on a circle.

**Solution:**

We know that a triangle is formed by joining 3 non-collinear points.

$\therefore$  Total number of triangles that can be drawn out of 8 non-collinear

points =  ${}^8C_3$

$${}^nC_r = n!/(n - r)!r!$$

Here,  $n = 8$  and  $r = 3$

$${}^8C_3 = 8!/(8 - 3)!3!$$

$$= (8 \times 7 \times 6 \times 5!)/5! (3 \times 2)$$

$$= (8 \times 7 \times 6)/6$$

$$= 56$$

Hence, the required number of triangles is 56.

20. If  $(1/8!) + (1/9!) = x/10!$ , then find the value of  $x$ .

**Solution:**

$$(1/8!) + (1/9!) = x/10!$$

$$(1/8!) + (1/9 \times 8!) = x/(10 \times 9 \times 8!)$$

$$(1/8!) [1 + (1/9)] = x/(10 \times 9 \times 8!)$$

$$(9 + 1)/9 = x/90$$

$$10/9 = x/90$$

$$\Rightarrow x = 900/9$$

$$\Rightarrow x = 100$$

21. A box has 4 red and 3 black marbles. Four marbles are picked up randomly. Find the probability that two marbles are red.

**Solution:**

Given,

A box has 4 red and 3 black marbles.

Out of 7 marbles, 4 marbles can be drawn in  ${}^7C_4 = 35$  ways

Thus,  $n(S) = 35$

Two red marbles can be drawn in  ${}^4C_2 = 6$  ways

The remaining 2 marbles must be black and they can be drawn

in  ${}^3C_2 = 3$  ways

$$n(A) = 6 \times 3 = 18$$

$$P(A) = n(A)/n(S)$$

$$= 18/35$$

22. Calculate standard deviation for the following scores:

5, 6, 7, 8, 9.

**Solution:**

Using direct method:

x	$x^2$
5	25
6	36
7	49
8	64
9	81
$\Sigma x = 35$	$\Sigma x^2 = 255$

$$N = 5$$

Standard deviation

$$\begin{aligned}\sigma &= \sqrt{\frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2} \\ &= \sqrt{\frac{255}{5} - \left(\frac{35}{5}\right)^2} \\ &= \sqrt{51 - 49} \\ &= \sqrt{2} \\ \sigma &= 1.414\end{aligned}$$

23. Solve  $x^2 - 2x - 4 = 0$  by using formula.

**Solution:**

Given quadratic equation is:

$$x^2 - 2x - 4 = 0$$

Comparing with the standard form  $ax^2 + bx + c = 0$ ,

$$a = 1, b = -2, c = -4$$

Using the quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4 + 16}}{2}$$

$$= \frac{2 \pm 2\sqrt{5}}{2}$$

$$= 2(1 \pm \sqrt{5})/2$$

$$= 1 \pm \sqrt{5}$$

Hence, the roots of the given equation are  $(1 + \sqrt{5})$  and  $(1 - \sqrt{5})$ .

**OR**

Determine the nature of the roots of the equation  $x^2 - 2x - 3 = 0$ .

**Solution:**

Given quadratic equation is:

$$x^2 - 2x - 3 = 0$$

Comparing with the standard form  $ax^2 + bx + c = 0$ ,

$$a = 1, b = -2, c = -3$$

$$\Delta = b^2 - 4ac$$

$$= (-2)^2 - 4(1)(-3)$$

$$= 4 + 12$$

$$= 16$$

$$\Delta > 0$$

Discriminant is greater than 0.

Therefore, the roots of the equation are real and distinct.

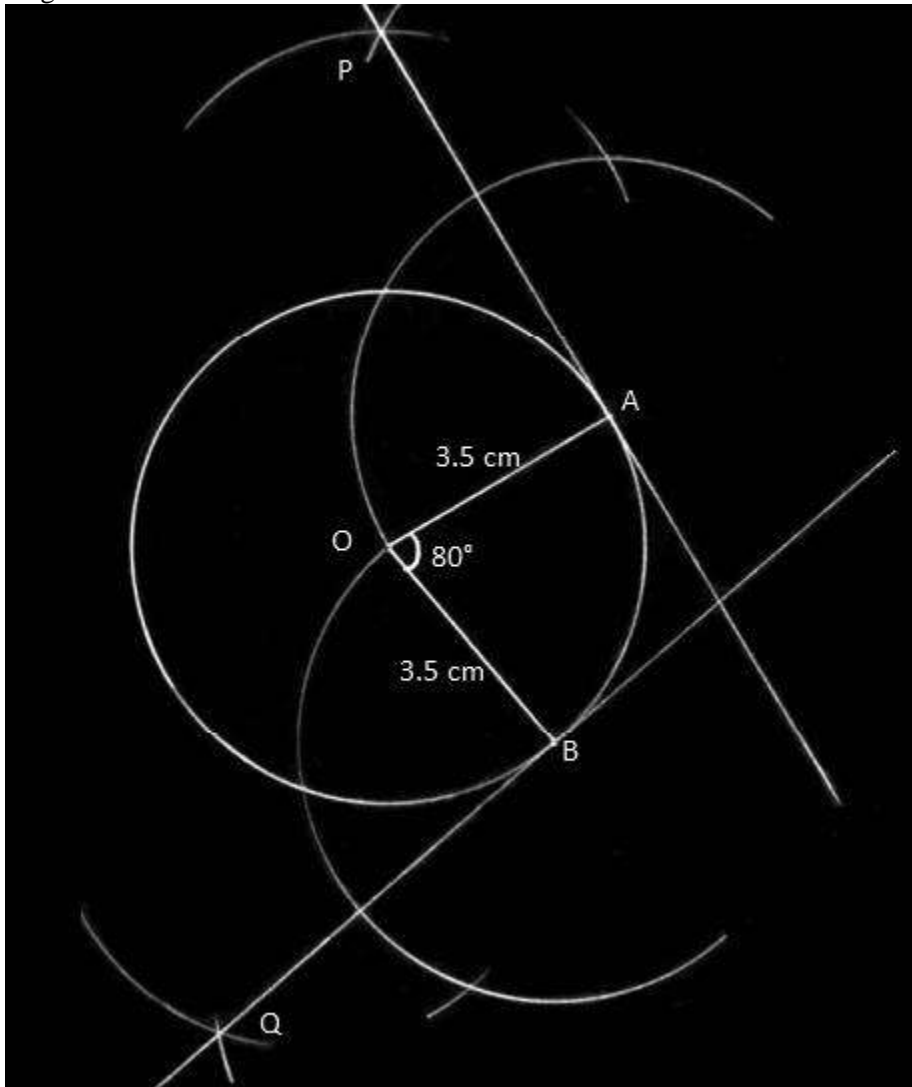
24. In a circle of radius 3.5 cm, draw two radii such that the angle between them is  $80^\circ$ . Construct tangents to the circle at the non-centre ends of the radii.

**Solution:**

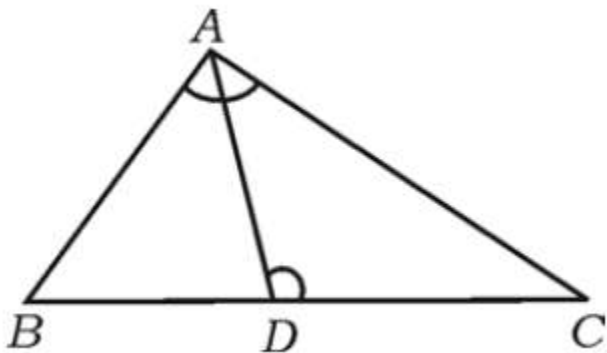
Given,

Radius = 3.5 cm

Angle between two radii =  $80^\circ$



25. In  $\triangle ABC$ , D is a point on BC such that  $\angle BAC = \angle ADC$ . Prove that  $AC^2 = BC \times DC$ .



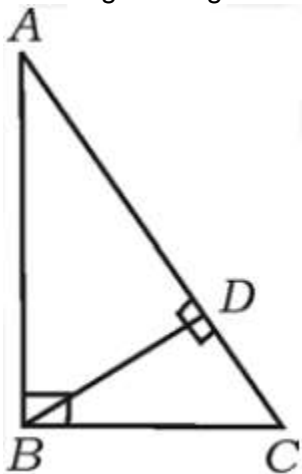


**Solution:**

In  $\triangle ABC$  and  $\triangle ADC$ ,  
 $\angle BAC = \angle ADC$  (given)  
 $\angle ACB = \angle ACD$  (common)  
 By AA similarity criterion,  
 $\triangle ACB \sim \triangle DCA$   
 $\Rightarrow AC/DC = CB/CA$   
 $\Rightarrow AC^2 = BC \times DC$   
 Hence proved.

**OR**

In the right triangle ABC,  $\angle ABC = 90^\circ$  and  $BD \perp AC$ . Prove that:  $AB^2/BC^2 = AD/CD$



**Solution:**

Given that, in right triangle ABC,  $\angle ABC = 90^\circ$  and  $BD \perp AC$ .  
 By the corollary of right angle theorem,  
 $AB^2 = AD \times AC$  ....(i)  
 And  
 $BC^2 = CD \times AC$  ....(ii)  
 Dividing (i) by (ii),  
 $AB^2/BC^2 = (AD \times AC) / (CD \times AC)$   
 $AB^2/BC^2 = AD/CD$   
 Hence proved.

26. Find the value of  $\sin 30^\circ \cdot \cos 60^\circ - \tan^2 45^\circ$ .

**Solution:**

$\sin 30^\circ = 1/2$   
 $\cos 60^\circ = 1/2$   
 $\tan 45^\circ = 1$   
 $\sin 30^\circ \cdot \cos 60^\circ - \tan^2 45^\circ$   
 $= (1/2) \times (1/2) - (1)^2$   
 $= (1/4) - 1$   
 $= (1 - 4)/4$   
 $= -3/4$

27. Find the radius of a circle whose centre is (-5, 4) and which passes through the point (-7, 1).

**Solution:**

Given,

Centre = C(-5, 4)

Circle passes through the point A = (-7, 1)

Radius of the circle = Distance between A and C

Let,  $(x_1, y_1) = (-5, 4)$

$(x_2, y_2) = (-7, 1)$

$$\begin{aligned} AC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[-7 - (-5)]^2 + (1 - 4)^2} \\ &= \sqrt{(-7 + 5)^2 + (-3)^2} \\ &= \sqrt{(-2)^2 + (-3)^2} \\ &= \sqrt{4 + 9} \\ &= \sqrt{13} \end{aligned}$$

Therefore, the radius of the circle is  $\sqrt{13}$  units.

28. The radii of two right circular cylinders are in the ratio 2 : 3 and the ratio of their curved surface areas is 5 : 6. Find the ratio of their heights.

**Solution:**

Let  $h_1$  and  $h_2$  be the heights of two right circular cylinders.

Given,

Ratio of the radii of two right circular cylinders =  $r_1 : r_2 = 2 : 3$

Ratio of the curved surface areas =  $S_1 : S_2 = 5 : 6$

$$2\pi r_1 h_1 / 2\pi r_2 h_2 = 5/6$$

$$2h_1 / 3h_2 = 5/6$$

$$h_1 / h_2 = (5 \times 3) / (6 \times 2)$$

$$h_1 / h_2 = 5/4$$

Hence, the required ratio is 5 : 4.

29. The radius of a solid metallic sphere is 10 cm. It is melted and recast into small cones of height 10 cm and base radius 5 cm. Find the number of small cones formed.

**Solution:**

Given,

Radius of solid sphere =  $R = 10$  cm

Base radius of cone =  $r = 5$  cm

Height of cone =  $h = 10$  cm

Number of small cones = Volume of sphere / Volume of one small cone

$$= (4/3)\pi R^3 / [(1/3)\pi r^2 h]$$

$$= (4 \times 10 \times 10 \times 10) / (5 \times 5 \times 10)$$

$$= 4 \times 2 \times 2$$

$$= 16$$

Hence, the number of small cones formed from the sphere = 16

30. Draw a plan by using the information given below:

[Scale: 25 metres = 1 cm]

	Metre To D	
	200	
	125	
100 to E	75	75 to C
	50	25 to B
	From A	

**Solution:**

Scale:

$$25 \text{ m} = 1 \text{ cm}$$

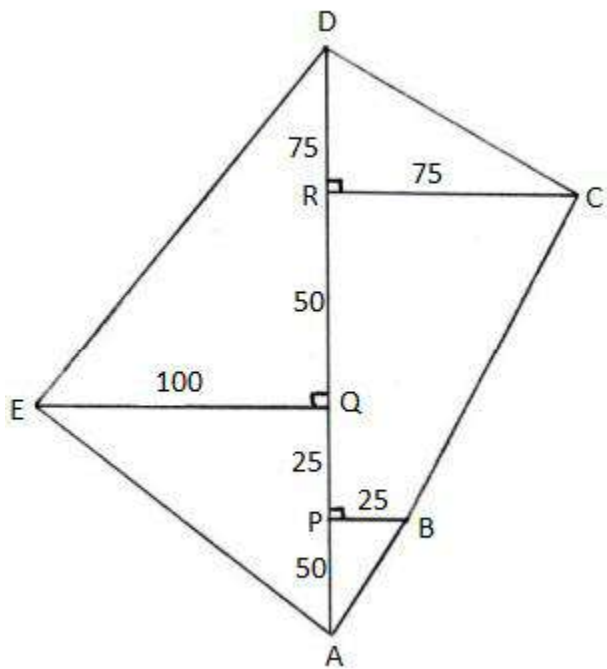
$$50 \text{ m} = 2 \text{ cm}$$

$$75 \text{ m} = 3 \text{ cm}$$

$$100 \text{ m} = 4 \text{ cm}$$

$$125 \text{ m} = 5 \text{ cm}$$

$$200 \text{ m} = 8 \text{ cm}$$



**SECTION - IV**

31. Rationalise the denominator and simplify:  
 $(\sqrt{6 + \sqrt{3}})/(\sqrt{6 - \sqrt{3}})$

**Solution:**

$$(\sqrt{6 + \sqrt{3}})/(\sqrt{6 - \sqrt{3}})$$

By rationalising the denominator,

$$= [(\sqrt{6 + \sqrt{3}})/(\sqrt{6 - \sqrt{3}})] \times [(\sqrt{6 + \sqrt{3}})/(\sqrt{6 + \sqrt{3}})]$$

$$= (\sqrt{6 + \sqrt{3}})^2 / [(\sqrt{6})^2 - (\sqrt{3})^2]$$

$$= (6 + 3 + 2\sqrt{6}\sqrt{3}) / (6 - 3)$$

$$= (9 + 2\sqrt{18})/3$$

$$= (9 + 6\sqrt{2})/3$$

$$= 3(3 + 2\sqrt{2})/3$$

$$= 3 + 2\sqrt{2}$$

32. Find the quotient  $q(x)$  and remainder  $r(x)$  on dividing  $p(x) = x^3 + 4x^2 - 5x + 6$  by  $g(x) = x + 1$  and hence verify  $p(x) = [g(x) \times q(x)] + r(x)$ .

**Solution:**

Given,

$$p(x) = x^3 + 4x^2 - 5x$$

$$g(x) = x + 1$$

$$\begin{array}{r}
 x^2 + 3x - 8 \\
 x + 1 \overline{) x^3 + 4x^2 - 5x + 6} \\
 \underline{-} \\
 x^3 + x^2 \\
 \underline{-} \\
 3x^2 - 5x + 6 \\
 \underline{-} \\
 3x^2 + 3x \\
 \underline{-} \\
 -8x + 6 \\
 \underline{-} \\
 -8x - 8 \\
 \underline{-} \\
 14
 \end{array}$$

Quotient =  $q(x) = x^2 + 3x - 8$

Remainder =  $r(x) = 14$

Verification:

$[g(x) \times q(x)] + r(x)$

$= (x + 1)(x^2 + 3x - 8) + 14$

$= x^3 + 3x^2 - 8x + x^2 + 3x - 8 + 14$

$= x^3 + 4x^2 - 5x + 6$

$= p(x)$

Therefore,  $p(x) = [g(x) \times q(x)] + r(x)$

**OR**

Find the quotient and remainder by using synthetic division:

$(4x^3 - 16x^2 - 9x - 36) \div (x + 2)$

**Solution:**

$(4x^3 - 16x^2 - 9x - 36) \div (x + 2)$

Using synthetic division,

$$\begin{array}{r|rrrr}
 -2 & 4 & -16 & -9 & -36 \\
 & & -8 & 48 & (-2) \cdot 39 = -78 \\
 \hline
 & 4 & -24 & 39 & (-36) + (-78) = -114
 \end{array}$$

Therefore,

Quotient =  $q(x) = 4x^2 - 24x + 39$

Remainder =  $r(x) = -114$

**33.** Find three consecutive positive integers such that the sum of the square of the first integer and the product of the other two is 92.

**Solution:**

Let  $x$ ,  $(x + 1)$  and  $(x + 2)$  be the three consecutive positive integers.

According to the given,

$$x^2 + (x + 1)(x + 2) = 92$$

$$x^2 + x^2 + 2x + x + 2 = 92$$

$$2x^2 + 3x + 2 - 92 = 0$$

$$2x^2 + 3x - 90 = 0$$

$$2x^2 - 12x + 15x - 90 = 0$$

$$2x(x - 6) + 15(x - 6) = 0$$

$$(x - 6)(2x + 15) = 0$$

$$x - 6 = 0, 2x + 15 = 0$$

$$x = 6, x = -15/2$$

$x$  cannot be negative.

Therefore,  $x = 6$

Hence, the required three consecutive positive integers are 6, 7 and 8.

**OR**

Sum of the squares of any two numbers is 180. If the square of the smaller number is equal to 8 times the bigger number, find the two numbers.

**Solution:**

Let  $x$ ,  $y$  be the two numbers and  $x > y$ .

Sum of the squares of two numbers is 180.

$$\text{i.e. } x^2 + y^2 = 180 \dots(i)$$

Given that the square of the smaller number is equal to 8 times the bigger number.

$$\text{i.e. } y^2 = 8x \dots(ii)$$

From (i) and (ii),

$$x^2 + 8x = 180$$

$$x^2 + 8x - 180 = 0$$

$$x^2 + 18x - 10x - 180 = 0$$

$$x(x + 18) - 10(x + 18) = 0$$

$$(x + 18)(x - 10) = 0$$

$$x = -18, x = 10$$

$x = -18$  is not possible.

Thus,  $x = 10$

Substituting  $x = 10$  in (ii),

$$y^2 = 8 \times 10$$

$$= 80$$

$$y = \sqrt{80} = 4\sqrt{5}$$

Hence, the required numbers are 10 and  $4\sqrt{5}$ .

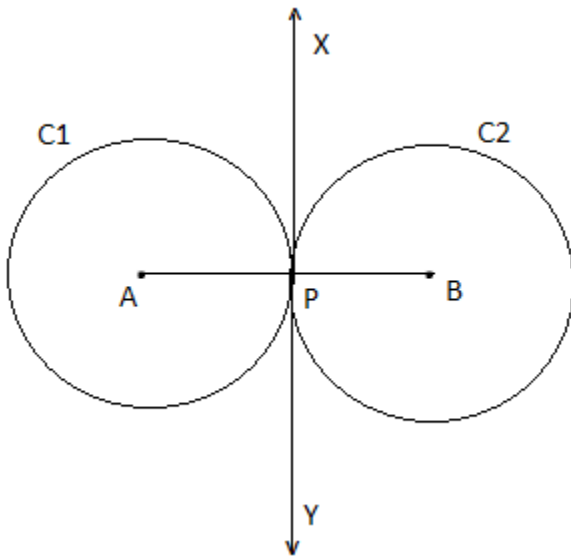
**34.** Prove that "If two circles touch each other externally, the centres and the point of contact are collinear".

**Solution:**

Let  $A$  be the centre of circle  $C_1$  and  $B$  be the centre of circle  $C_2$ .

$P$  be the point of contact.

Draw tangent  $XY$  which passes through  $P$ .

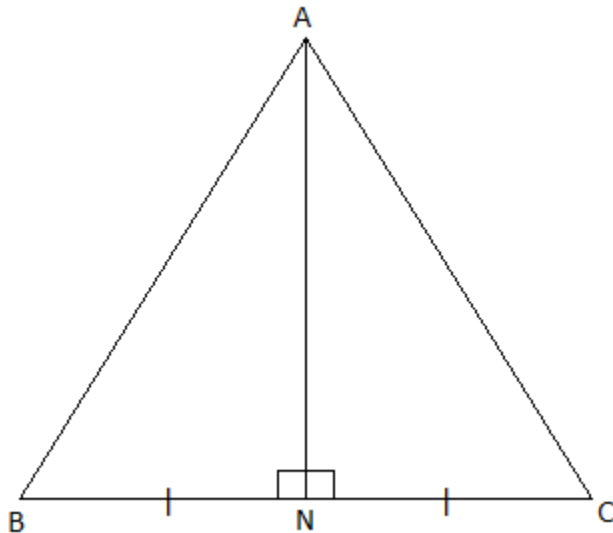


$\angle APX = \angle BPX = 90^\circ$  (radius is perpendicular to the tangent through the point of contact)  
 Now,  
 $\angle APX + \angle BPX = 90^\circ + 90^\circ = 180^\circ$   
 $180^\circ$  is the angle formed by a straight line.  
 Thus,  $\angle APX$  and  $\angle BPX$  is a linear pair.  
 Therefore, A, P and B are collinear.  
 Hence proved.

35. In an equilateral triangle ABC,  $AN \perp BC$ , prove that  $4AN^2 = 3AB^2$ .

**Solution:**

Given that in an equilateral triangle ABC,  $AN \perp BC$ .



Thus,  $BN = NC = (1/2)BC = (1/2)AB$   
 In right triangle ANB,  
 $AB^2 = AN^2 + BN^2$

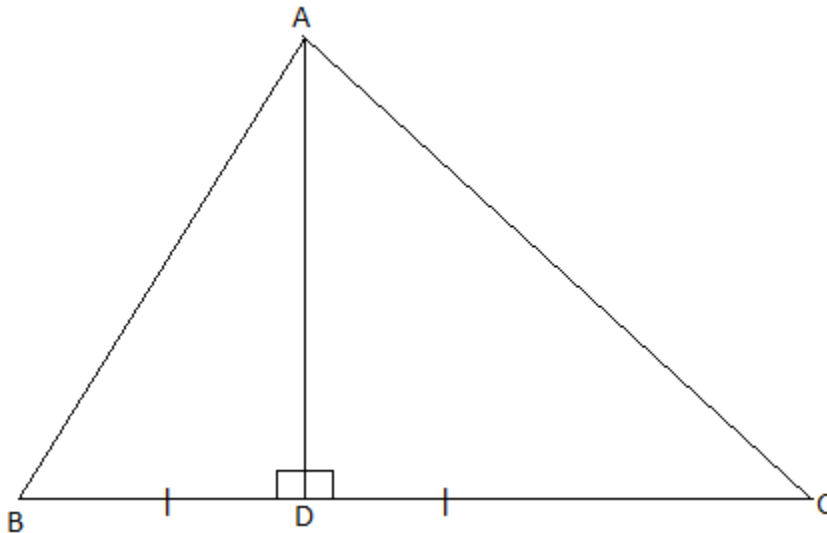
$$\begin{aligned}
 AN^2 &= AB^2 - BN^2 \\
 &= AB^2 - [(1/2)AB]^2 \\
 &= AB^2 - (AB^2/4) \\
 &= (4AB^2 - AB^2)/4 \\
 4AN^2 &= 3AB^2 \\
 \text{Hence proved.}
 \end{aligned}$$

**OR**

In  $\triangle ABC$ ,  $AD \perp BC$ , prove that  $AB^2 + CD^2 = AC^2 + BD^2$ .

**Solution:**

Given that in  $\triangle ABC$ ,  $AD \perp BC$ .



In right triangle ADB,  
 $AB^2 = AD^2 + BD^2$   
 $\Rightarrow AD^2 = AB^2 - BD^2 \dots(i)$

In right triangle ADC,  
 $AC^2 = AD^2 + DC^2$   
 $\Rightarrow AD^2 = AC^2 - DC^2 \dots(ii)$

From (i) and (ii),  
 $AB^2 - BD^2 = AC^2 - DC^2$   
 $AB^2 + DC^2 = AC^2 + BD^2$   
 Hence proved.

**36.** Prove that  $\tan^2 A - \sin^2 A = \tan^2 A \cdot \sin^2 A$ .

**Solution:**

$$\begin{aligned}
 \text{LHS} &= \tan^2 A - \sin^2 A \\
 &= (\sin^2 A / \cos^2 A) - \sin^2 A \\
 &= (\sin^2 A - \sin^2 A \cos^2 A) / \cos^2 A \\
 &= \sin^2 A (1 - \cos^2 A) / \cos^2 A \\
 \text{Using the identity } \sin^2 \theta + \cos^2 \theta &= 1, \\
 &= (\sin^2 A \sin^2 A) / \cos^2 A \\
 &= (\sin^2 A / \cos^2 A) \sin^2 A
 \end{aligned}$$



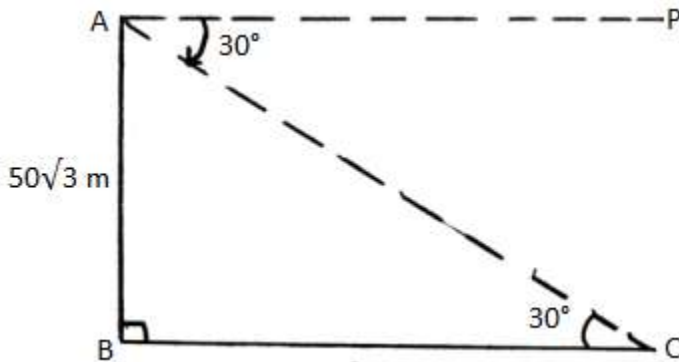
$= \tan^2 A \sin^2 A$   
 $= \text{RHS}$   
 Hence proved.

**OR**

From the top of a building  $50\sqrt{3}$  m high the angle of depression of an object on the ground is observed to be  $30^\circ$ . Find the distance of the object from the foot of the building.

**Solution:**

Let AB be the building and C be the object.



$AB = 50\sqrt{3}$  m  
 $\angle PAC = \angle ACB = 30^\circ$   
 Angle of depression  $= \theta = 30^\circ$   
 In right triangle ABC,  
 $\tan 30^\circ = AB/BC$   
 $1/\sqrt{3} = (50\sqrt{3})/BC$   
 $BC = (50\sqrt{3}) \times \sqrt{3}$   
 $= 50 \times 3$   
 $= 150$

Hence, the distance between the building and the object =  $BC = 150$  m

**SECTION - V**

**37.** The sum of 3rd and 5th terms of an arithmetic progression is 30 and the sum of 4th and 8th terms of it is 46. Find the arithmetic progression.

**Solution:**

Let  $a$  be the first term and  $d$  be the common difference of an AP.

According to the given,

$$a_3 + a_5 = 30$$

$$a + 2d + a + 4d = 30$$

$$2a + 6d = 30$$

$$2(a + 3d) = 30$$

$$a + 3d = 15 \dots(i)$$

And

$$a_4 + a_8 = 46$$

$$a + 3d + a + 7d = 46$$

$$2a + 10d = 46$$

$$\begin{aligned}2(a + 5d) &= 46 \\ a + 5d &= 23 \dots(ii) \\ \text{Subtracting (i) from (ii),} \\ a + 5d - (a + 3d) &= 23 - 15 \\ 2d &= 8 \\ d &= 8/2 = 4 \\ \text{Substituting } d = 4 \text{ in (i),} \\ a + 3(4) &= 15 \\ a + 12 &= 15 \\ a &= 15 - 12 \\ a &= 3 \\ a + d &= 3 + 4 = 7 \\ a + 2d &= 3 + 2(4) = 11 \\ a + 3d &= 3 + 3(4) = 15 \\ \text{Hence, the required AP is } &3, 7, 11, 15, \dots\end{aligned}$$

OR

If the fourth term of a geometric progression is 8 and its eighth term is 128, find the sum of the first ten terms of the progression.

**Solution:**

Given that in GP,

$$a_4 = 8$$

$$ar^3 = 8 \dots(i)$$

And

$$a_8 = 128$$

$$ar^7 = 128 \dots(ii)$$

Dividing (ii) by (i),

$$ar^7/ar^3 = 128/8$$

$$r^4 = 16$$

$$r^4 = (2)^4$$

$$r = 2$$

Substituting  $r = 2$  in (i),

$$a(2)^3 = 8$$

$$8a = 8$$

$$a = 8/8 = 1$$

Sum of the first n term

$$S_n = a(r^n - 1)/(r - 1)$$

$$S_{10} = 1(2^{10} - 1)/(2 - 1)$$

$$= 1024 - 1$$

$$= 1023$$

Hence, the sum of the first 10 terms of the GP is 1023.

38. Solve  $x^2 - 2x - 3 = 0$  graphically.

**Solution:**

Given,

$$x^2 - 2x - 3 = 0$$

$$x^2 - (2x + 3) = 0$$

Thus, the solution will be the intersection of  $y = x^2$  and  $y = 2x + 3$

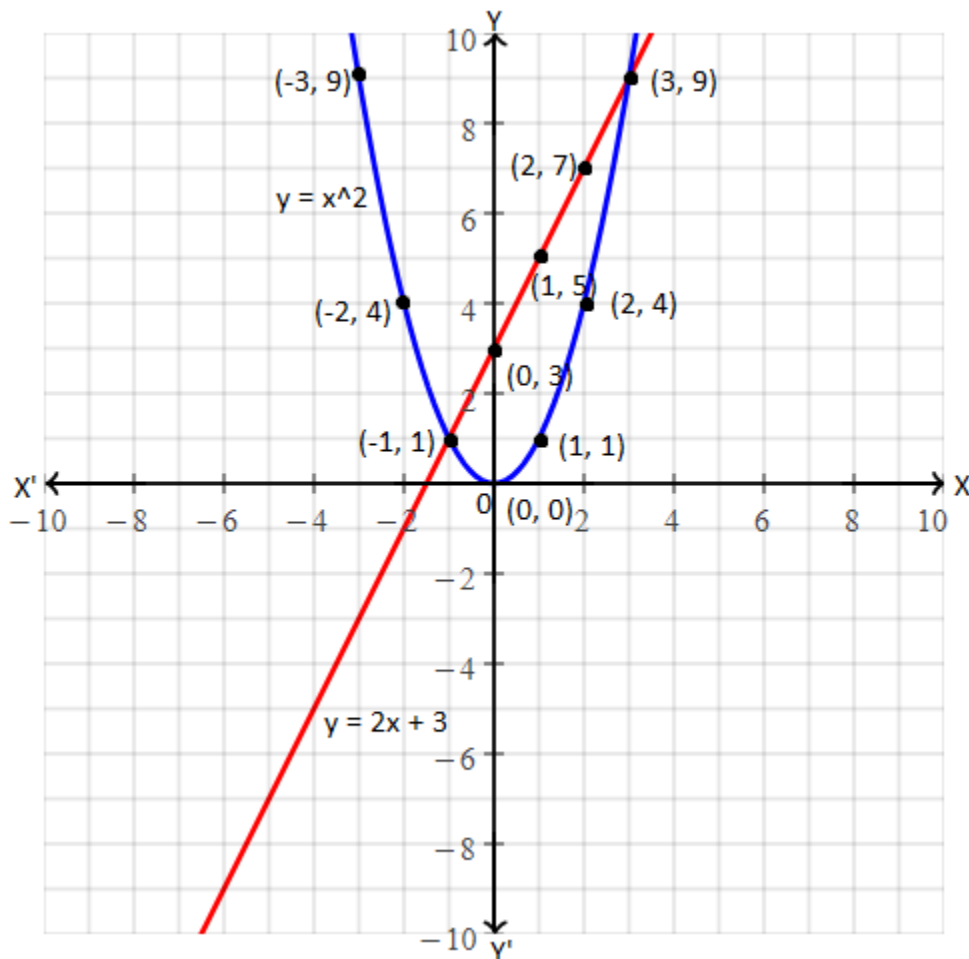
Consider,  $y = x^2$

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

$y = 2x + 3$

x	-1	0	1	2	3
y	1	3	5	7	9

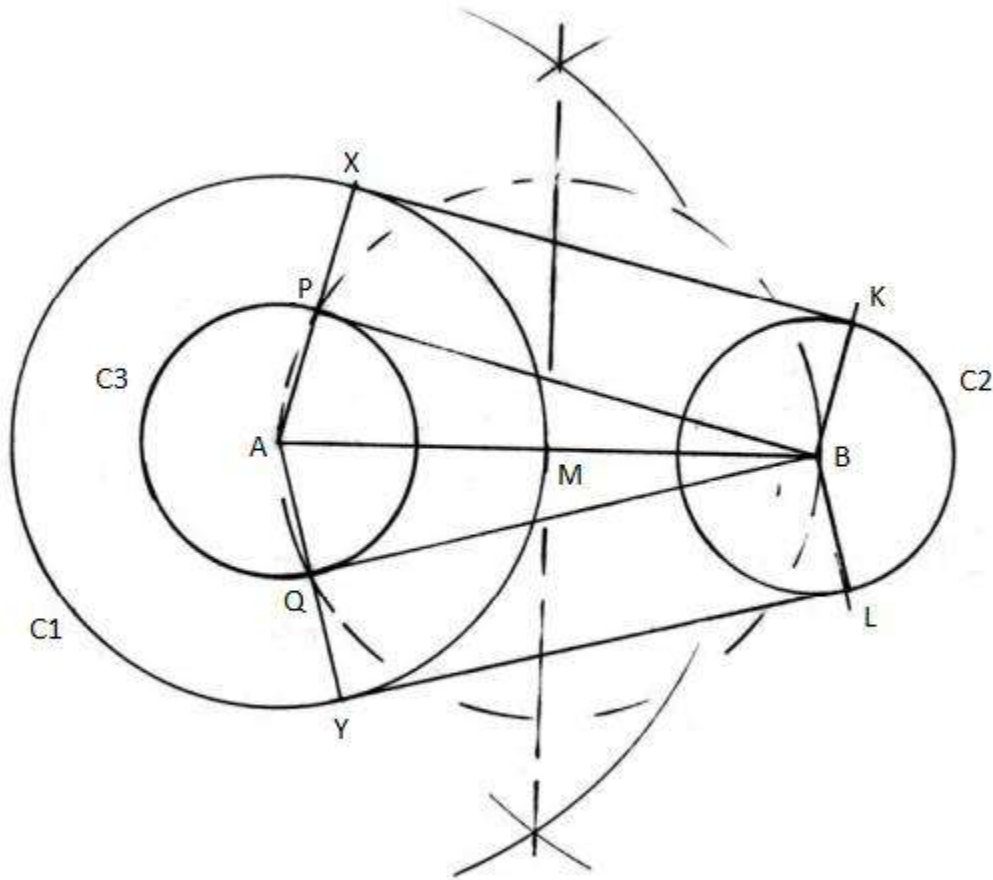
Graph:



These intersect each other at  $x = -1$  and  $x = 3$ .  
Hence, the required solution is  $x = -1$  and  $x = 3$

**39.** Construct a pair of direct common tangents to two circles of radii 4 cm and 2 cm whose centres are 8 cm apart. Measure and write the length of the direct common tangent.

**Solution:**



Therefore, KX and LY are the required tangents of length 7.8 cm.

**40.** Prove that “If two triangles are equiangular, then their corresponding sides are in proportion”.

**Solution:**

Given,

In  $\triangle ABC$  and  $\triangle DEF$ ,

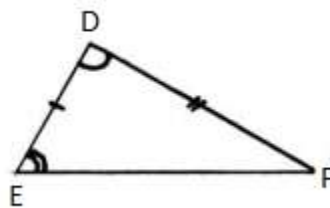
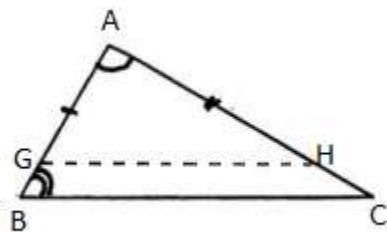
$\angle BAC = \angle EDF$

$\angle ABC = \angle DEF$

To prove:  $AB/DE = BC/EF = AC/DF$

Construction:

Points G and H are marked on AB and AC such that  $AG = DE$  and  $AH = DF$ . Join GH.



In  $\triangle AGH$  and  $\triangle DEF$ ,  
 $AG = DE$  (by construction)  
 $\angle GAH = \angle EDF$  (given)  
 $AH = DF$  (by construction)  
BY SAS congruence criterion,  
 $\triangle AGH \cong \triangle DEF$   
Thus, by CPCT,  
 $GH = EF$   
 $\angle AGH = \angle DEF$   
Given that,  $\angle DEF = \angle ABC$   
 $\angle AGH = \angle ABC$  (alternate angles)  
Therefore,  $GH \parallel BC$   
By the corollary of Thales(BPT) theorem,  
 $AB \cdot AG = BC \cdot GH = AC \cdot AH$   
And,  $AG = DE$ ,  $GH = EF$ ,  $AH = DF$   
Therefore,  
 $AB/DE = BC/EF = AC/DF$   
Hence proved.

