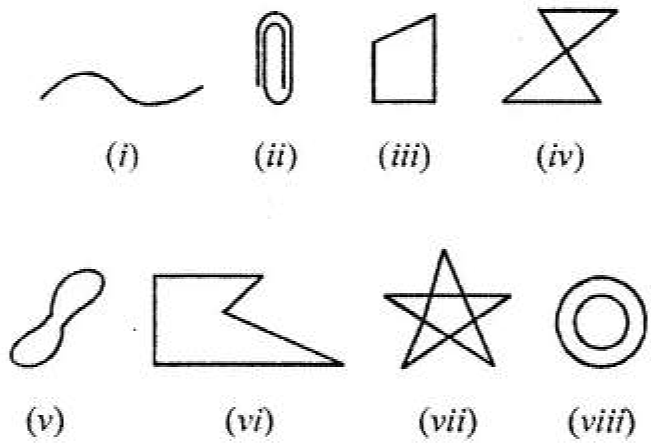
EXERCISE 13.1

1. Some figures are given below.

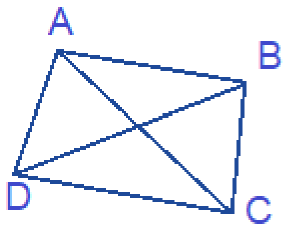


Classify each of them on the basis of the following:
(a) Simple curve
(b) Simple closed curve
(c) Polygon
(d) Convex polygon
(e) Concave polygon
Solution:-
The given figure are classified as,
(a) Figure (i), Figure (ii), Figure (iii), Figure (v) and Figure (vi) are Simple curves.
Simple curve is a curve that does not cross itself.
(b) Figure (iii), Figure (v) and Figure (vi) are Simple closed curves.
In simple closed curves the shapes are closed by line-segments or by a curved line.
(c) Figure (iii) and Figure (vi) are Polygons.
A Polygon is any 2-dimensional shape formed with straight lines.
(d) Figure (iii) is a Convex polygon.
In a convex polygon, every diagonal of the figure passes only through interior points of the polygon.
(e) Figure (vi) is a Concave polygon.
In a concave polygon, at least one diagonal of the figure contains points that are exterior to the polygon.

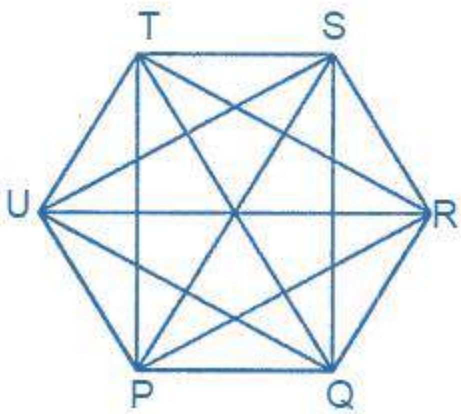
2. How many diagonals does each of the following have?
(a) A convex quadrilateral
(b) A regular hexagon

Solution:

(a) A convex quadrilateral has two diagonals.



(b) A regular hexagon has 9 diagonals as shown.



3. Find the sum of measures of all interior angles of a polygon with the number of sides:

(i) 8

(ii) 12

Solution:

From the question it is given that,

(i) 8

We know that,

Sum of measures of all interior angles of 8 sided polygons = $(2n - 4) \times 90^o$

Where, n = 8

$= ((2 \times 8) - 4) \times 90^o$

$= (16 - 4) \times 90^o$

$= 12 \times 90^o$

$= 1080^o$

(ii) 12

We know that,

Sum of measures of all interior angles of 12 sided polygons = $(2n - 4) \times 90^o$

Where, n = 12

$= ((2 \times 12) - 4) \times 90^o$
$= (24 - 4) \times 90^o$
$= 20 \times 90^o$
$= 1800^o$

4. Find the number of sides of a regular polygon whose each exterior angles has a measure of
(i) 24^o
(ii) 60^o
(iii) 72^o
Solution:-
(i) The number of sides of a regular polygon whose each exterior angles has a measure of 24^o
Let us assume the number of sides of the regular polygon be n,
Then, $n = 360^o/24^o$
$n = 15$
Therefore, the number of sides of a regular polygon is 15.
(ii) The number of sides of a regular polygon whose each exterior angles has a measure of 60^o
Let us assume the number of sides of the regular polygon be n,
Then, $n = 360^o/60^o$
$n = 6$
Therefore, the number of sides of a regular polygon is 6.
(iii) The number of sides of a regular polygon whose each exterior angles has a measure of 72^o
Let us assume the number of sides of the regular polygon be n,
Then, $n = 360^o/72^o$
$n = 5$
Therefore, the number of sides of a regular polygon is 5.

5. Find the number of sides of a regular polygon if each of its interior angles is
(i) 90^o
(ii) 108^o
(iii) 165^o
Solution:-
(i) The number of sides of a regular polygon whose each interior angles has a measure of 90^o

Let us assume the number of sides of the regular polygon be n,

Then, we know that $90^o = ((2n - 4)/n) \times 90^o$

$90^o/90^o = (2n - 4)/n$

$1 = (2n - 4)/n$

$2n - 4 = n$

By transposing we get,

$2n - n = 4$

$n = 4$

Therefore, the number of sides of a regular polygon is 4.

So, it is a Square.

(ii) The number of sides of a regular polygon whose each interior angles has a measure of 108^o

Let us assume the number of sides of the regular polygon be n,

Then, we know that $108^o = ((2n - 4)/n) \times 90^o$

$108^o/90^o = (2n - 4)/n$

$6/5 = (2n - 4)/n$

By cross multiplication,

$5(2n - 4) = 6n$

$10n - 20 = 6n$

By transposing we get,

$10n - 6n = 20$

$4n = 20$

$n = 20/4$

$n = 5$

Therefore, the number of sides of a regular polygon is 5.

So, it is a Pentagon.

(iii) The number of sides of a regular polygon whose each interior angles has a measure of 165^o

Let us assume the number of sides of the regular polygon be n,

Then, we know that $165^o = ((2n - 4)/n) \times 90^o$

$165^o/90^o = (2n - 4)/n$

$11/6 = (2n - 4)/n$

By cross multiplication,

$6(2n - 4) = 11n$

$12n - 24 = 11n$

By transposing we get,

$12n - 11n = 24$

$n = 24$

Therefore, the number of sides of a regular polygon is 24.

6. Find the number of sides in a polygon if the sum of its interior angles is:
(i) 1260^o
(ii) 1980^o
(iii) 3420^o
Solution:-

(i) We know that,

Sum of measures of all interior angles of polygons = $(2n - 4) \times 90^o$

Given, interior angle = 1260^o

$1260 = (2n - 4) \times 90^o$

$1260/90 = 2n - 4$

$14 = 2n - 4$

By transposing we get,

$2n = 14 + 4$

$2n = 18$

$n = 18/2$

$n = 9$

Therefore, the number of sides in a polygon is 9.

(ii) We know that,

Sum of measures of all interior angles of polygons = $(2n - 4) \times 90^o$

Given, interior angle = 1980^o

$1980 = (2n - 4) \times 90^o$

$1980/90 = 2n - 4$

$22 = 2n - 4$

By transposing we get,

$2n = 22 + 4$

$2n = 26$

$n = 26/2$

$n = 13$

Therefore, the number of sides in a polygon is 13.

(ii) We know that,

Sum of measures of all interior angles of polygons = $(2n - 4) \times 90^o$
Given, interior angle = 3420^o

$3420 = (2n – 4) \times 90^o$
$3420/90 = 2n – 4$
$38 = 2n – 4$

By transposing we get,

$2n = 38 + 4$
$2n = 42$
$n = 42/2$
$n = 21$

Therefore, the number of sides in a polygon is 21.

7. If the angles of a pentagon are in the ratio 7 : 8 : 11 : 13 : 15, find the angles.

Solution:-

From the question it is given that,
The angles of a pentagon are in the ratio 7 : 8 : 11 : 13 : 15
We know that, Sum of measures of all interior angles of polygons = $(2n - 4) \times 90^o$
Given, n = 5

$= ((2 \times 5) – 4) \times 90^o$
$= (10 – 4) \times 90^o$
$= 6 \times 90^o$
$= 540^o$

Let us assume the angles of the pentagon be 7a, 8a, 11a, 13a and 15a.
Then, $7a + 8a + 11a + 13a + 15a = 540^o$

$54a = 540^o$
$a = 540/54$
$a = 10^o$

Therefore, the angles are $7a = 7 \times 10 = 70^o$

$8a = 8 \times 10 = 80^o$
$11a = 11 \times 10 = 110^o$
$13a = 13 \times 10 = 130^o$
$15a = 15 \times 10 = 150^o$

8. The angles of a pentagon are x^o, $(x – 10)^o$, $(x + 20)^o$, $(2x – 44)^o$ and $(2x – 70)^o$ Calculate x.

Solution:-

From the question it is given that, angles of a pentagon are x^o, $(x – 10)^o$, $(x + 20)^o$, (2x –

44)° and (2x – 70)°

We know that, Sum of measures of all interior angles of polygons = (2n - 4) × 90°

Where, n = 5

= ((2 × 5) – 4) × 90°

= (10 – 4) × 90°

= 6 × 90°

= 540°

Then, x°+ (x – 10)° + (x + 20)° + (2x – 44)° + (2x – 70)° = 540

x + x – 10° + x + 20° + 2x – 44° + 2x – 70° = 540°

7x + 20° – 124° = 540°

7x – 104° = 540°

By transposing we get,

7x = 540° + 104°

7x = 644°

x = 644°/7

x = 92°

Therefore, the value of x is 92°.

9. The exterior angles of a pentagon are in ratio 1 : 2 : 3 : 4 : 5. Find all the interior angles of the pentagon.

Solution:-

From the question it is given that, the exterior angles of a pentagon are in ratio 1 : 2 : 3 : 4 : 5.

We know that, sum of exterior angles of pentagon is equal to 360°.

So, let us assume the angles of the pentagon be 1a, 2a, 3a, 4a and 5a.

1a + 2a + 3a + 4a + 5a = 360°

15a = 360°

a = 360°/15

a = 24°

Therefore, the angles of pentagon are, 1a = 1 × 24 = 24°

2a = 2 × 24 = 48°

3a = 3 × 24 = 72°

4a = 4 × 24 = 96°

5a = 5 × 24 = 120°

Then, interior angles of the pentagon are, 180° – 24° = 156°

180° – 48° = 132°

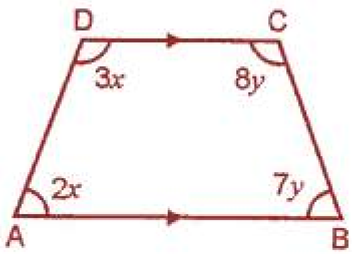
180° – 72° = 108°

$180^\circ - 96^\circ = 84^\circ$
$180^\circ - 120^\circ = 60^\circ$

10. In a quadrilateral ABCD, AB || DC. If ∠A : ∠D = 2:3 and ∠B : ∠C = ∠7 : 8, find the measure of each angle.

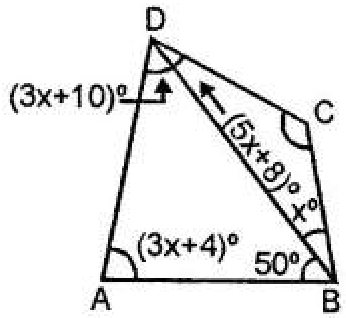
Solution:-

From the question it is given that,
In a quadrilateral ABCD, AB || DC. If ∠A : ∠D = 2:3 and ∠B : ∠C = ∠7 : 8,



Then, ∠A + ∠D = 180°
Let us assume the angle ∠A = 2a and ∠D = 3a
$2a + 3a = 180^\circ$
$5a = 180^\circ$
$a = 180^\circ/5$
$a = 36^\circ$
Therefore, ∠A = 2a = $2 \times 36^\circ = 72^\circ$
∠D = 3a = $3 \times 36^\circ = 108^\circ$
Now, ∠B + ∠C = 180°
Let us assume the angle ∠B = 7b and ∠C = 8b
$7b + 8b = 180^\circ$
$15b = 180^\circ$
$b = 180^\circ/15$
$b = 12^\circ$
Therefore, ∠B = 7b = $7 \times 12^\circ = 84^\circ$
∠C = 8b = $8 \times 12^\circ = 96^\circ$

11. From the adjoining figure, find
(i) x
(ii) ∠DAB
(iii) ∠ADB



Solution:-

(i) From the given figure,

ABCD is a quadrilateral

$\angle A + \angle B + \angle C + \angle D = 360^o$

$(3x + 4) + (50 + x) + (5x + 8) + (3x + 10) = 360^o$

$3x + 4 + 50 + x + 5x + 8 + 3x + 20 = 360^o$

$12x + 72 = 360^o$

By transposing we get,

$12x = 360^o – 72$

$12x = 288$

$x = 288/12$

$x = 24$

(ii) $\angle DAB = (3x + 4)$

$= ((3 \times 24) + 4)$

$= 72 + 4$

$= 76^o$

Therefore, $\angle DAB = 76^o$

(iii) Consider the triangle ABD,

We know that, sum of interior angles of triangle is equal to 180^o,

$\angle DAB + \angle ABD + \angle ADB = 180^o$

$76^o + 50^o + \angle ADB = 180^o$

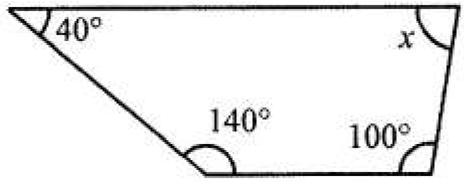
$\angle ADB + 126^o = 180^o$

$\angle ADB = 180^o – 126^o$

Therefore, $\angle ADB = 54^o$

12. Find the angle measure x in the following figures:

(i)



Solution:-

From the given quadrilateral three angles are 40°, 100° and 140°

We have to find the value of x,

We know that, sum of four angles of quadrilateral is equal to 360°.

So, 40° + 100° + 140° + x = 360°

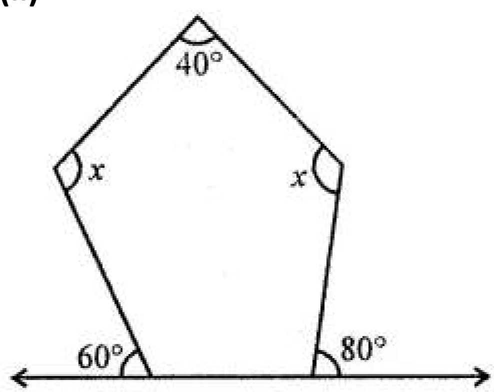
280° + x = 360°

x = 360° – 280°

x = 80°

Therefore, the value of x is 80°.

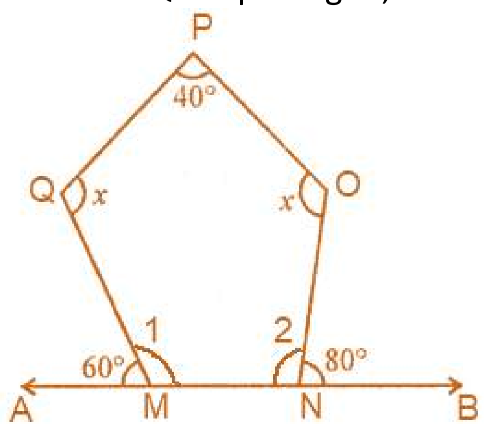
(ii)



Solution:-

From the given figure,

Let MNOPQ is a pentagon,



We know that, sum of angles linear pair is equal to 180^o

So, $\angle 1 + 60^o = 180^o$

$\angle 1 = 180^o - 60^o$

$\angle 1 = 120^o$

And $\angle 2 + 80^o = 180^o$

$\angle 2 = 180^o - 80^o$

$\angle 2 = 100^o$

Also We know that, Sum of measures of all interior angles of polygons = $(2n - 4) \times 90^o$

Where, n = 5

$= ((2 \times 5) - 4) \times 90^o$

$= (10 - 4) \times 90^o$

$= 6 \times 90^o$

$= 540^o$

Then, $\angle M + \angle N + \angle O + \angle Q + \angle P = 540^o$

$120^o + 100^o + x + x + 40^o = 540^o$

$260^o + 2x = 540^o$

By transposing we get,

$2x = 540^o - 260^o$

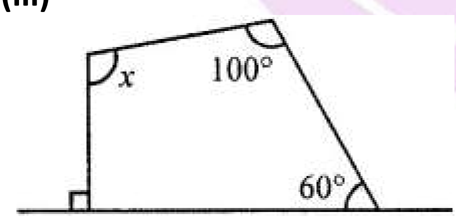
$2x = 280^o$

$x = 280^o/2$

$x = 140^o$

Therefore, the value of x is 140^o.

(iii)



Solution:-

From the given quadrilateral angles are 60^o and 100^o,

We know that, sum of angles linear pair is equal to 180^o

So, another angle is $180^o - 90^o = 90^o$

We have to find the value of x,

We know that, sum of four angles of quadrilateral is equal to 360^o.

So, $60^o + 100^o + 90^o + x = 360^o$

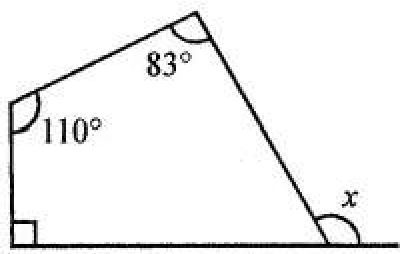
$250^o + x = 360^o$

$x = 360^o - 250^o$

$x = 110^o$

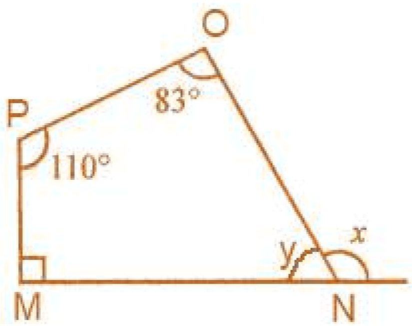
Therefore, the value of x is 110^o.

(iv)



Solution:-

We know that, sum of four angles of quadrilateral is equal to 360^o.



Consider Quadrilateral MNOP,

$\angle M + \angle N + \angle O + \angle P = 360^o$

$90^o + y + 83^o + 110^o = 360^o$

$283^o + y = 360^o$

$y = 360^o – 283^o$

$y = 77^o$

Therefore, the value of y is 110^o

We know that, sum of angles linear pair is equal to 180^o

So, $y + x = 180^o$

$77^o + x = 180^o$

By transposing we get,

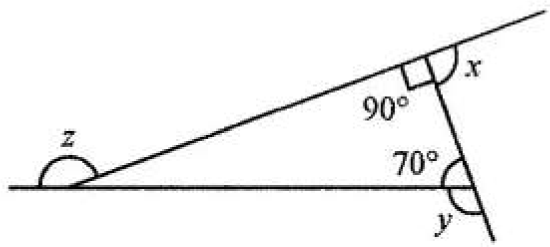
$x = 180^o – 77^o$

$x = 103^o$

Therefore, the value of x 103^o.

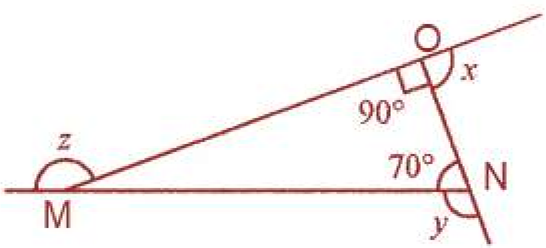
13.

(i) In the given figure, find x + y + z.



Solution:-

From the figure,



Consider the triangle MNO,

We know that, sum of measures of interior angles of triangle is equal to 180^o.

$\angle M + \angle N + \angle O = 180^o$

$\angle M + 70^o + 90^o = 180^o$

$160^o + \angle M = 180^o$

$\angle M = 180^o - 160$

$\angle M = 20^o$

We know that, sum of angles linear pair is equal to 180^o

So, $x + 90 = 180^o$

By transposing we get,

$x = 180^o - 90^o$

$x = 90^o$

Therefore, the value of x is 90^o.

Then, $y + 70^o = 180^o$

By transposing we get,

$y = 180^o - 70^o$

$y = 110^o$

Therefore, the value of y is 110^o.

Similarly, $z + 20 = 180^o$

By transposing we get,

$z = 180^o - 20^o$

$z = 160^o$

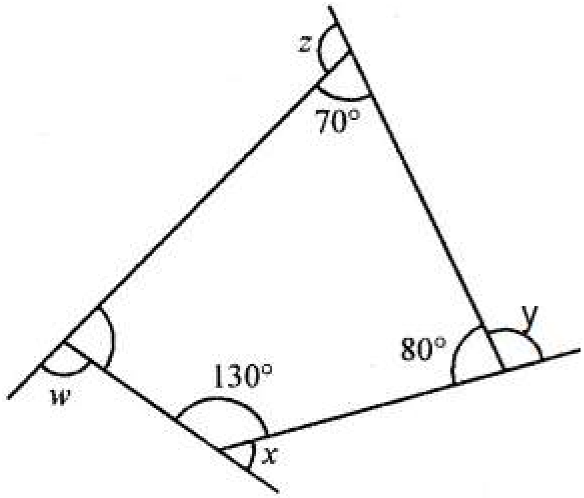
Therefore, the value of z is 160^o.

Hence, x + y + z

= $90^\circ + 110^\circ + 160^\circ$

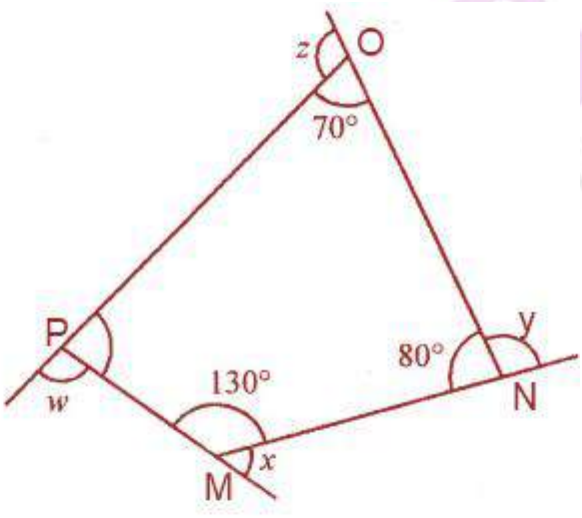
= 360°

(ii) In the given figure, find x + y + z + w



Solution:-

Let MNOP is a quadrilateral,



We know that, sum of four angles of quadrilateral is equal to 360°.

$\angle M + \angle N + \angle O + \angle P = 360^\circ$

$130^\circ + 80^\circ + 70^\circ + \angle P = 360^\circ$

$280^\circ + \angle P = 360^\circ$

$\angle P = 360^\circ - 280^\circ$

$\angle P = 80^\circ$

We know that, sum of angles linear pair is equal to 180°

So, $x + 130^\circ = 180^\circ$

By transposing we get,

$x = 180^o - 130^o$

$x = 50^o$

Therefore, the value of x is 50^o.

Then, $y + 80^o = 180^o$

By transposing we get,

$y = 180^o - 80^o$

$y = 100^o$

Therefore, the value of y is 100^o.

Similarly, $z + 70 = 180^o$

By transposing we get,

$z = 180^o - 70^o$

$z = 110^o$

Therefore, the value of z is 110^o.

Similarly, $w + 80 = 180^o$

By transposing we get,

$z = 180^o - 80^o$

$z = 100^o$

Therefore, the value of z is 110^o.

Hence, $x + y + z + w$

$= 50^o + 100^o + 110^o + 100$

$= 360^o$

14. A heptagon has three equal angles each of 120^o and four equal angles. Find the size of equal angles.

Solution:-

From the question it is given that,

A heptagon has three equal angles each of 120^o

Four equal angles = ?

We know that, Sum of measures of all interior angles of polygons = $(2n - 4) \times 90^o$

Where, n = 7

$= ((2 \times 7) - 4) \times 90^o$

$= (14 - 4) \times 90^o$

$= 10 \times 90^o$

$= 900^o$

Sum of 3 equal angles = $120^o + 120^o + 120^o = 360^o$

Let us assume the sum of four equal angle be 4x,

So, sum of 7 angles of heptagon = 900^o
Sum of 3 equal angles + Sum of 4 equal angles = 900^o
$360^o + 4x = 900^o$
By transposing we get,
$4x = 900^o – 360^o$
$4x = 540^o$
$x = 540^o/4$
$x = 134^o$
Therefore, remaining four equal angle measures 135^o each.

15. The ratio between an exterior angle and the interior angle of a regular polygon is 1 : 5. Find
(i) the measure of each exterior angle
(ii) the measure of each interior angle
(iii) the number of sides in the polygon.
Solution:-
From the question it is given that,
The ratio between an exterior angle and the interior angle of a regular polygon is 1: 5
Let us assume exterior angle be y
And interior angle be 5y
We know that, sum of interior and exterior angle is equal to 180^o,
$y + 5y = 180^o$
$6y = 180^o$
$y = 180^o/6$
$y = 30^o$
(i) the measure of each exterior angle = $y = 30^o$
(ii) the measure of each interior angle = $5y = 5 \times 30^o = 150^o$
(iii) the number of sides in the polygon
The number of sides of a regular polygon whose each interior angles has a measure of 150^o
Let us assume the number of sides of the regular polygon be n,
Then, we know that $150^o = ((2n – 4)/n) \times 90^o$
$150^o/90^o = (2n – 4)/n$
$5/3 = (2n – 4)/n$
By cross multiplication,
$3(2n – 4) = 5n$
$6n – 12 = 5n$

By transposing we get,

$6n - 5n = 12$

$n = 12$

Therefore, the number of sides of a regular polygon is 12.

16. Each interior angle of a regular polygon is double of its exterior angle. Find the number of sides in the polygon.

Solution:-

From the question it is given that,

Each interior angle of a regular polygon is double of its exterior angle.

So, let us assume exterior angle be y

Interior angle be 2y,

We know that, sum of interior and exterior angle is equal to 180^o,

$y + 2y = 180^o$

$3y = 180^o$

$y = 180^o/3$

$y = 60^o$

Then, interior angle = $2y = 2 \times 60^o = 120^o$

The number of sides of a regular polygon whose each interior angles has a measure of 120^o

Let us assume the number of sides of the regular polygon be n,

Then, we know that $120^o = ((2n - 4)/n) \times 90^o$

$120^o/90^o = (2n - 4)/n$

$4/3 = (2n - 4)/n$

By cross multiplication,

$3(2n - 4) = 4n$

$6n - 12 = 4n$

By transposing we get,

$6n - 4n = 12$

$2n = 12$

$n = 12/2$

$n = 6$

Therefore, the number of sides of a regular polygon is 6.

EXERCISE 13.2

1. In the given figure, ABCD is a parallelogram. Complete each statement along with the definition or property used.

(i) AD = ………..

(ii) DC = ………..

(iii) ∠DCB = ………..

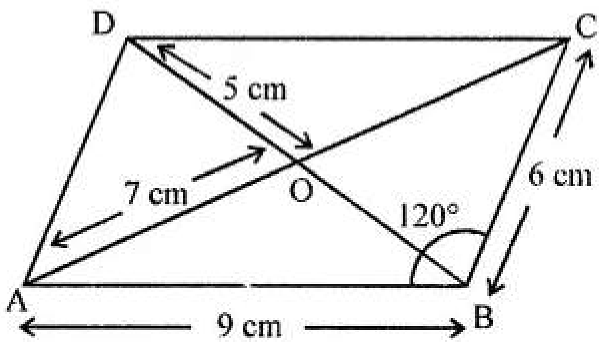
(iv) ∠ADC = ………..

(v) ∠DAB = ………..

(vi) OC = ………..

(vii) OB = ………..

(viii) m∠DAB + m∠CDA = ………..



Solution:-

From the given figure,

(i) AD = 6 cm … [because opposite sides of parallelogram are equal]

(ii) DC = 9 cm … [because opposite sides of parallelogram are equal]

(iii) ∠DCB = 60°

(iv) ∠ADC = ∠ABC = 120°

(v) ∠DAB = ∠DCB = 60°

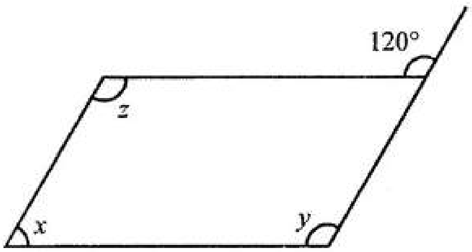
(vi) OC = AO = 7 cm

(vii) OB = OD = 5 cm

(viii) m∠DAB + m∠CDA = 180°

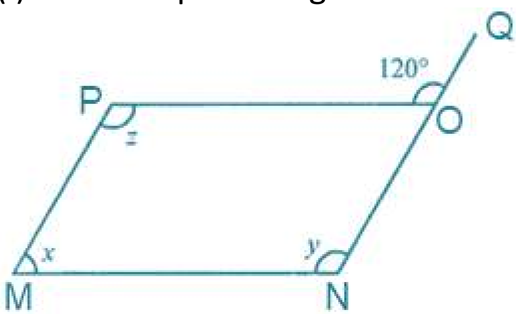
2. Consider the following parallelograms. Find the values of x, y, z in each.

(i)



Solution:-

(i) Consider parallelogram MNOP



From the figure, $\angle POQ = 120^\circ$

We know that, sum of angles linear pair is equal to 180°

So, $\angle POQ + \angle PON = 180^\circ$

$120^\circ + \angle PON = 180^\circ$

$\angle PON = 180^\circ - 120^\circ$

$\angle PON = 60^\circ$

Then, $\angle M = \angle O = 60^\circ$ … [because opposite angles of parallelogram are equal]

$\angle POQ = \angle MNO$

$120^\circ = 120^\circ$ … [because corresponding angels are equal]

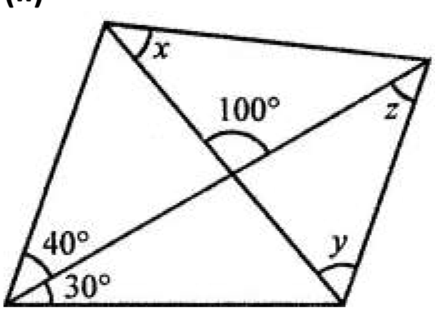
Hence, $y = 120^\circ$

Also, $z = y$

$120^\circ = 120^\circ$

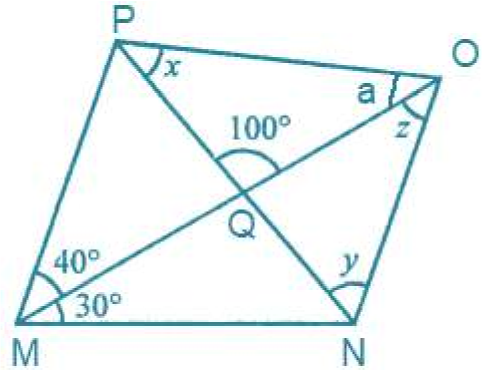
Therefore, $x = 60^\circ$, $y = 120^\circ$ and $z = 120^\circ$

(ii)



Solution:-

Solution:-



From the figure, it is given that ∠PQO = 100°, ∠OMN = 30°, ∠PMO = 40°.

Then, ∠NOM = ∠OMP … [because alternate angles are equal]

So, z = 40°

Now, ∠NMO = ∠POM … [because alternate angles are equal]

So, ∠NMO = a = 30°

Consider the triangle PQO,

We know that, sum of measures of interior angles of triangle is equal to 180°.

∠P + ∠Q + ∠O = 180°

x + 100° + 30° = 180°

x + 130° = 180°

x = 180° – 130°

x = 50°

Then, exterior angle ∠OQP = y + z

100° = y + 40°

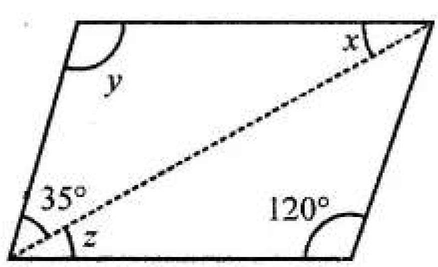
By transposing we get,

y = 100° – 40°

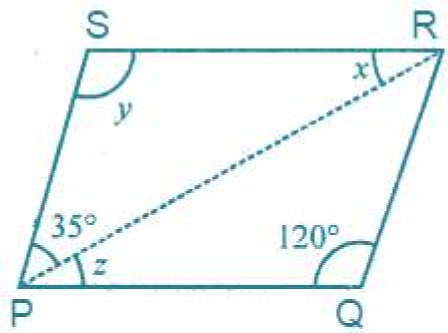
y = 60°

Therefore, the value of x = 50°, y = 60° and z = 40°.

(iii)



Solution:-



From the above figure,

∠SPR = ∠PRQ

35° = 35° … [because alternate angles are equal]

Now consider the triangle PQR,

We know that, sum of measures of interior angles of triangle is equal to 180°.

∠RPQ + ∠PQR + ∠PRQ = 180°

z + 120° + 35° = 180°

z + 155° = 180°

z = 180° – 155°

z = 25°

Then, ∠QPR = ∠PRQ

Z = x

25° = 25° … [because alternate angles are equal]

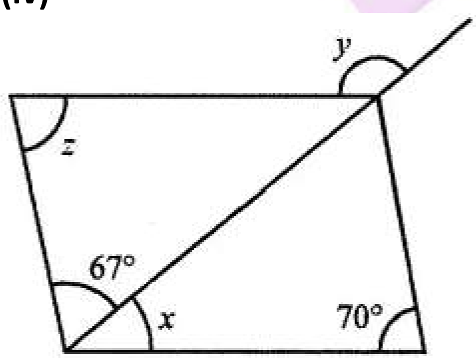
We know that, in parallelogram opposite angles are equal.

So, ∠S = ∠Q

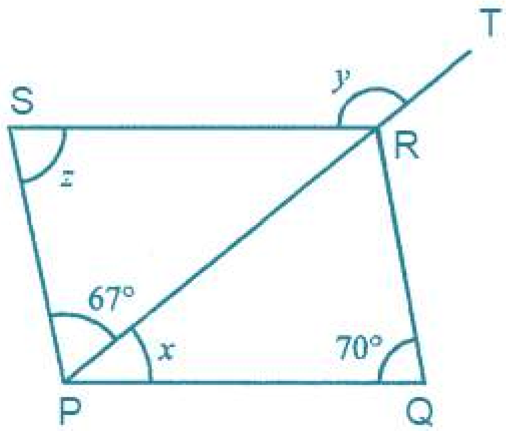
y = 120°

Therefore, value of x = 25°, y = 120° and ∠z = 25°.

(iv)



Solution:-



From the above figure, it is given that ∠SPR = 67° and ∠PQR = 70°

∠SPR = ∠PRQ

67° = 67° … [because alternate angles are equal]

Now, consider the triangle PQR

We know that, sum of measures of interior angles of triangle is equal to 180°.

∠RPQ + ∠PQR + ∠PRQ = 180°

x + 70° + 67° = 180°

x + 137° = 180°

x = 180° – 137°

x = 43°

Then, ∠PSR = ∠PQR

We know that, in parallelogram opposite angles are equal.

Z = 70°

Also we know that, exterior angle ∠SRT = ∠PSR + ∠SPR

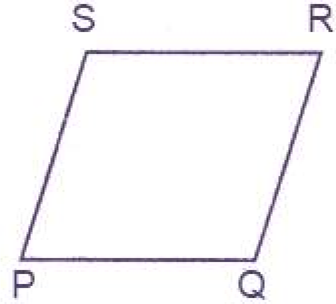
y = 70° + 67°

y = 137°

Therefore, value of x = 43°, y = 137° and z = 70°

3. Two adjacent sides of a parallelogram are in the ratio 5 : 7. If the perimeter of a parallelogram is 72 cm, find the length of its sides.

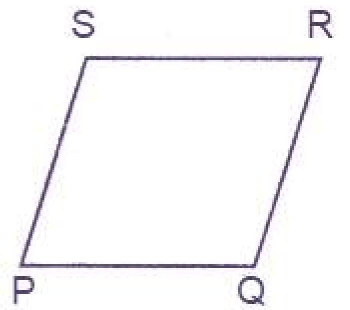
Solution:-



Consider the parallelogram PQRS,
From the question it is given that, two adjacent sides of a parallelogram are in the ratio 5 : 7.
Perimeter of parallelogram = 72 cm
2(SP + RQ) = 72 cm
SP + RQ = 72/2
SP + RQ = 36 cm
Let us assume the length of side SP = 5y and RQ = 7y,
5y + 7y = 36
12y = 36
y = 36/12
y = 3
Therefore, SP = 5y = 5 × 3 = 15 cm
RQ = 7y = 7 × 3 = 21 cm

4. The measure of two adjacent angles of a parallelogram is in the ratio 4 : 5. Find the measure of each angle of the parallelogram.

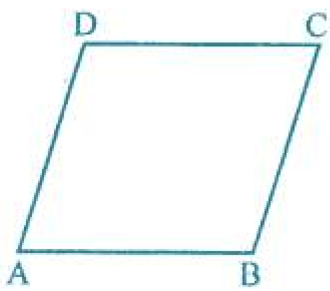
Solution:-



Consider the parallelogram PQRS,
From the question it is given that, The measure of two adjacent angles of a parallelogram is in the ratio 4 : 5.
So, ∠P: ∠Q = 4: 5
Let us assume the ∠P = 4y and ∠Q = 5y.
Then, we know that, ∠P + ∠Q = 180^o
$4y + 5y = 180^o$
$9y = 180^o$
$y = 180^o/9$
$y = 20^o$
Therefore, ∠P = 4y = 4 × 20^o = 80^o and ∠Q = 5y = (5 × 20^o) = 100^o
In parallelogram opposite angles are equal,

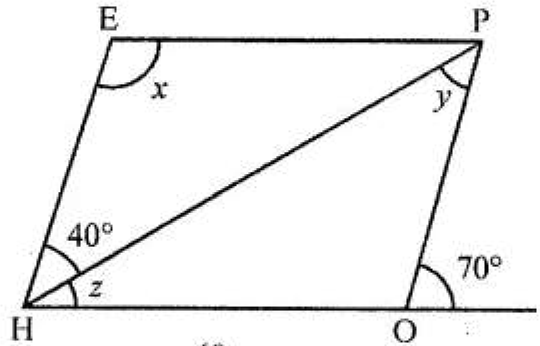
So, $\angle R = \angle P = 80^o$
$\angle S = \angle Q = 100^o$

5. Can a quadrilateral ABCD be a parallelogram, give reasons in support of your answer.
(i) $\angle A + \angle C = 180^o$?
(ii) AD = BC = 6 cm, AB = 5 cm, DC = 4.5 cm?
(iii) $\angle B = 80^o$, $\angle D = 70^o$?
(iv) $\angle B + \angle C = 180^o$?
Solution:-
From the question it is given that, quadrilateral ABCD can be a parallelogram.
We know that in parallelogram opposite sides are equal and opposites angles are equal.

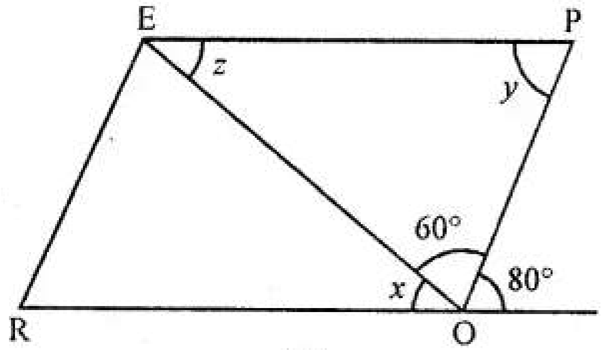


So, AB = DC and AD = BC also $\angle A = \angle C$ and $\angle B = \angle D$.
(i) $\angle A + \angle C = 180^o$
From the above condition it may be a parallelogram and may not be a parallelogram.
(ii) AD = BC = 6 cm, AB = 5 cm, DC = 4.5 cm
From the above dimension not able to form parallelogram.
Because AB ≠ DC
(iii) $\angle B = 80^o$, $\angle D = 70^o$
From the above dimension not able to form parallelogram.
Because $\angle B \neq \angle D$
(iv) $\angle B + \angle C = 180^o$
From the above condition it may be a parallelogram and may not be a parallelogram.

6. In the following figures, HOPE and ROPE are parallelograms. Find the measures of angles x, y and z. State the properties you use to find them.



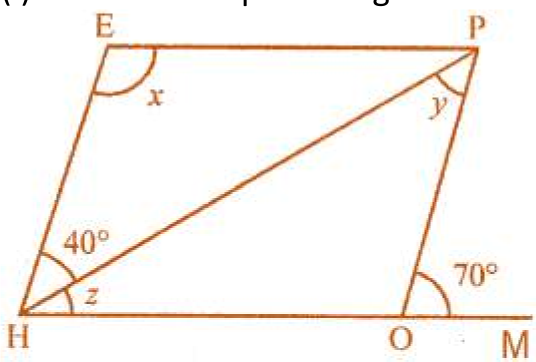
(i)



(ii)

Solution:

(i) Consider the parallelogram HOPE



We know that, sum of interior and exterior angle is equal to 180°,

$\angle HOP + \angle POM = 180^\circ$

$\angle HOP + 70^\circ = 180^\circ - 70^\circ$

$\angle HOP = 110^\circ$

Then, $\angle HEP = \angle HOP$

$x = 110^\circ$ … [because in parallelogram opposite angles are equal]

$\angle OPH = \angle PHE$

$y = 40^\circ$ … [because alternate angles are equal]

Now, consider the triangle HOP

We know that, sum of measures of interior angles of triangle is equal to 180°.

$\angle PHO + \angle HOP + \angle OPH = 180^\circ$

$z + 110^\circ + 40^\circ = 180^\circ$

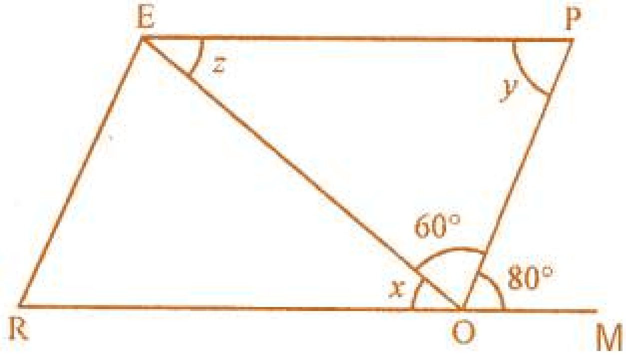
$z + 150^\circ = 180^\circ$

$z = 180^\circ - 150^\circ$

$z = 30^\circ$

Therefore, value of $x = 110^\circ$, $y = 40^\circ$ and $z = 30^\circ$.

(ii) Consider the parallelogram ROPE



From the figure, it is given that ∠POM = 80° and ∠POE = 60°.

Then, ∠OPE = ∠POM

y = 80° … [because alternate angles are equal]

We know that, angles on the same straight line are equal to 180°.

∠ROE + ∠EOP + POM = 180°

x + 60° + 80° = 180°

x + 140° = 180°

By transposing we get,

x = 180° – 140°

x = 40°

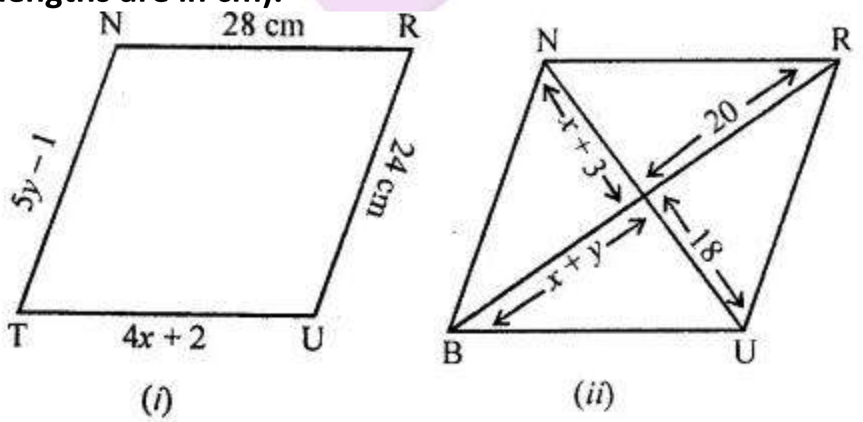
Then, ∠ROE = ∠OEP

x = z

40° = z

Therefore, value of x = 40°, y = 80° and z = 40°.

7. In the given figure TURN and BURN are parallelograms. Find the measures of x and y (lengths are in cm).

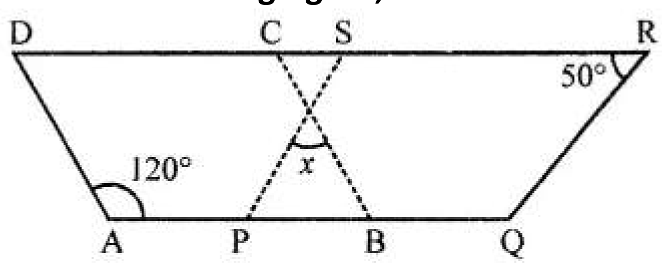


Solution:-

(i) Consider the parallelogram TURN
We know that, in parallelogram opposite sides are equal.
So, TU = RN
4x + 2 = 28
By transposing,
4x = 28 – 2
4x = 26
x = 26/4
x = 6.5 cm
and NT = RU
5y – 1 = 24
5y = 24 + 1
5y = 25
y = 25/5
y = 5
Therefore, value of x = 6.5 cm and y = 5 cm.

(ii) Consider the parallelogram BURN,
BO = OR
x + y = 20 … [equation (i)]
UO = ON
x + 3 = 18
x = 18 – 3
x = 15
substitute the value of x in equation (i),
15 + y = 20
y = 20 – 15
y = 5
Therefore, value of x = 15 and y = 5.

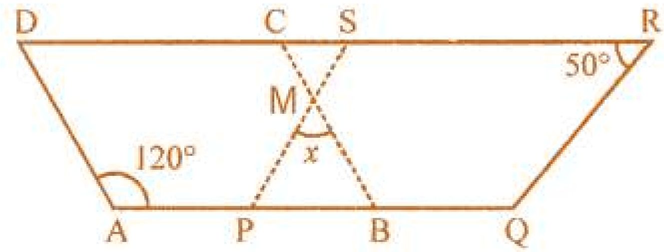
8. In the following figure, both ABCD and PQRS are parallelograms. Find the value of x.



Solution:-

From the figure it is given that, ABCD and PQRS are two parallelograms.

∠A = 120° and ∠R = 50°



We know that, ∠A + ∠B = 180°

120° + ∠B = 180°

∠B = 180° – 120°

∠B = 60°

In parallelogram opposite angles are equal,

Then, consider the triangle MPB

We know that, sum of measures of interior angles of triangle is equal to 180°.

∠PMB + ∠P + ∠B = 180°

x + 50° + 60° = 180°

x + 110° = 180°

By transposing we get,

x = 180° – 110°

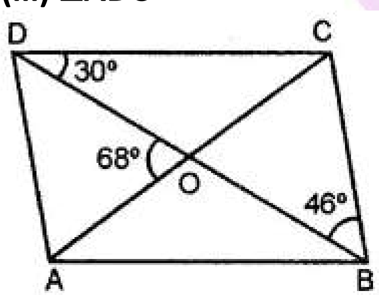
Therefore, value of x = 70°

9. In the given figure, ABCD, is a parallelogram and diagonals intersect at O. Find :

(i) ∠CAD

(ii) ∠ACD

(iii) ∠ADC



Solution:-

From the figure it is given that,

∠CBD = 46°, ∠AOD = 68° and ∠BDC = 30°

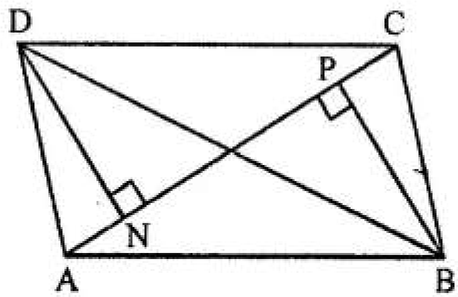
(i) ∠CBD = ∠BDA = 46° … [alternate angles are equal]

Consider the ΔAOD,
We know that, sum of measures of interior angles of triangle is equal to 180^o.
$\angle AOD + \angle ODA + \angle DAO = 180^o$
$68^o + 46^o + \angle DAO = 180^o$
$\angle DAO + 114^o + 180^o$
$\angle DAO = 180^o – 114^o$
$\angle DAO = 66^o$
Therefore, $\angle CAD = 66^o$

(ii) we know that, sum of angles on the straight line are equal to 180^o,
$\angle AOD + \angle COD = 180^o$
$68^o + \angle COD = 180^o$
$\angle COD = 180^o – 68^o$
$\angle COD = 112^o$
Now consider ΔCOD,
$\angle COD + \angle ODC + \angle DCO = 180^o$
$112^o + 30^o + \angle DCO = 180^o$
$\angle DCO + 142^o = 180^o$
By transposing we get,
$\angle DCO = 180^o – 142^o$
$\angle DCO = 38^o$
Therefore, $\angle ACD = 38^o$

(iii) $\angle ADC = \angle ADO + \angle ODC$
$\angle ADO = \angle OBC = 46^o$ … [alternate angles are equal]
Then, $\angle ADC = 46^o + 30$
$= 76^o$

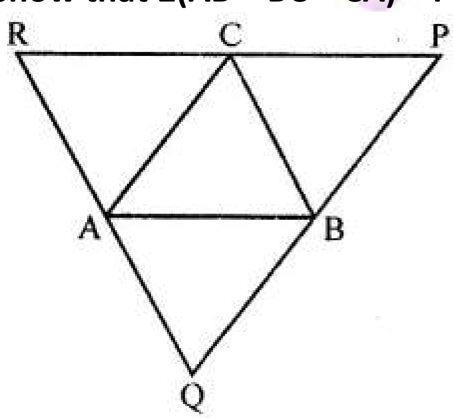
10. In the given figure, ABCD is a parallelogram. Perpendiculars DN and BP are drawn on diagonal AC. Prove that:
(i) $\Delta DCN \cong \Delta BAP$
(ii) AN = CP



Solution:-

From the figure it is given that,
ABCD is a parallelogram
Perpendiculars DN and BP are drawn on diagonal AC
We have to prove that, (i) ΔDCN ≅ ΔBAP, (ii) AN = CP
So, consider the ΔDCN and ΔBAP
AB = DC … [opposite sides of parallelogram are equal]
∠N = ∠P … [both angles are equal to 90^o]
∠BAP = ∠DCN … [alternate angles are equal]
Therefore, ΔDCN ≅ ΔBAP … [AAS axiom]
Then, NC = AP
Because, corresponding parts of congruent triangle.
So, subtracting NP from both sides we get,
NC – NP = AP – NP
AN = CP
Hence it is proved that, ΔDCN ≅ ΔBAP and AN = CP.

11. In the given figure, ABC is a triangle. Through A, B and C lines are drawn parallel to BC, CA and AB respectively, which forms a ΔPQR.
Show that 2(AB + BC + CA) = PQ + QR + RP.



Solution:-

From the figure it is given that,

Through A, B and C lines are drawn parallel to BC, CA and AB respectively.

We have to show that 2(AB + BC + CA) = PQ + QR + RP

Then, AB ∥ RC and AR ∥ CB

Therefore, ABCR is a parallelogram.

So, AB = CR … [equation (i)]

CB = AR … [equation (ii)]

Similarly, ABPC is a parallelogram.

AB ∥ CP and PB ∥ CA

AB = PC … [equation (iii)]

AC = PB … [equation (iv)]

Similarly, ACBQ is a parallelogram

AC = BQ … [equation (v)]

AQ = BC … [equation (vi)]

By adding all the equation, we get,

AB + AB + BC + BC + AC + AC = PB + PC + CR + AR + BQ +BC

2AB + 2BC + 2AC = PQ + QR + RP

By taking common we get,

2(AB + BC + AC) = PQ + QR + RP

EXERCISE 13.3

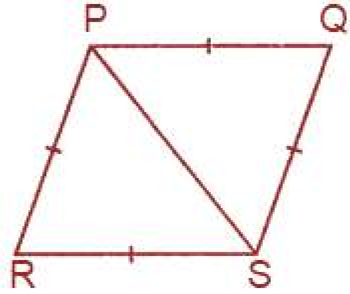
1. Identify all the quadrilaterals that have
(i) four sides of equal length
(ii) four right angles.
Solution:-
(i) The quadrilaterals that have four sides of equal length are square and rhombus.
(ii) The quadrilaterals that have four right angles are square and rectangle.

2. Explain how a square is
(i) a quadrilateral
(ii) a parallelogram
(iii) a rhombus
(iv) a rectangle.
Solution:-
(i) A square is a quadrilateral because it has four equal sides and four angles whose sum is equal to 360^o.
(ii) A square is a parallelogram because it has opposite sides equal and opposite are parallel.
(iii) A square is a rhombus because it's all four sides have equal length.
(iv) A square is a rectangle because its opposite sides are equal and parallel and each angle are equal to 90^o.

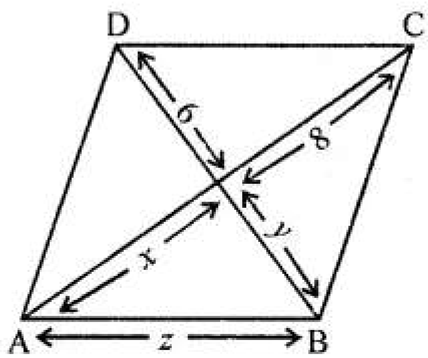
3. Name the quadrilaterals whose diagonals
(i) bisect each other
(ii) are perpendicular bisectors of each other
(iii) are equal.
Solution:-
(i) The quadrilaterals whose diagonals are bisect each other are rectangle, square, rhombus and parallelogram.
(ii) The quadrilaterals whose diagonals are perpendicular bisectors of each other are square and rhombus.
(iii) The quadrilaterals whose diagonals equal are square and rectangle.

4. One of the diagonals of a rhombus and its sides are equal. Find the angles of the rhombus.
Solution:-

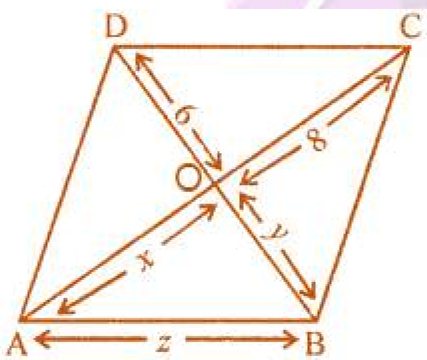
From the question it is given that, one of the diagonals of a rhombus and its sides are equal.
Therefore, the angles of the rhombus are 60^o and 120^o.



5. In the given figure, ABCD is a rhombus, find the values of x, y and z



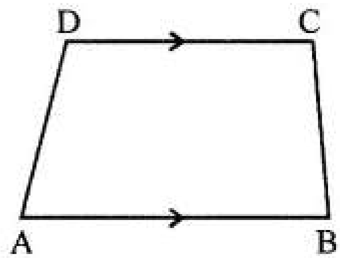
Solution:-
From the figure,
ABCD is a rhombus



Then, the diagonals of rhombus bisect each other at right angles.
So, AO = OC
x = 8 cm
Therefore, AO = 8 cm
And BO = OD
y = 6 cm
Therefore, BO = 6 cm

Consider the ΔAOB, it is a right angled triangle.
By Pythagoras theorem,
$AB^2 = AO^2 + BO^2$
$AB^2 = 8^2 + 6^2$
$AB^2 = 64 + 36$
$AB^2 = 100$
$AB = \sqrt{100}$
AB = 10 cm

6. In the given figure, ABCD is a trapezium. If ∠A : ∠D = 5 : 7, ∠B = (3x + 11)° and ∠C = (5x – 31)°, then find all the angles of the trapezium.



Solution:-
From the given figure,
ABCD is a trapezium
∠A : ∠D = 5 : 7, ∠B = (3x + 11)° and ZC = (5x – 31)°
Then, ∠B + ∠C = 180° … [because co – interior angle]
(3x + 11)° + (5x – 31)° = 180°
3x + 11 + 5x – 31 = 180°
8x – 20 = 180°
By transposing we get,
8x = 180° + 20
8x = 200°
x = 200°/8
x = 25°
Then, ∠B = 3x + 11
= (3 × 25) + 11
= 75 + 11
= 86°
∠C = 5x – 31
= (5 × 25) - 31
= 125 - 31

$= 94^o$

let us assume the angles $\angle A = 5y$ and $\angle D = 7y$

We know that, sum of co – interior angles are equal to 180^o.

$\angle A + \angle D = 180^o$

$5y + 7y = 180^o$

$12y = 180^o$

$y = 180^o/12$

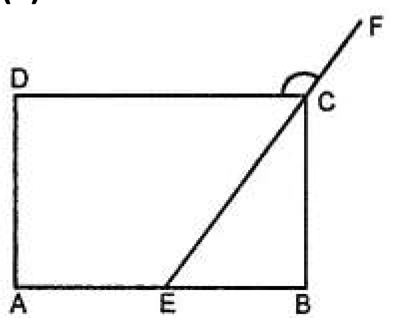
$y = 15^o$

Then, $\angle A = 5y = (5 \times 15) = 75^o$

$\angle D = 7y = (7 \times 15) = 105^o$

Therefore, the angles are $\angle A = 75^o$, $\angle B = 86^o$, $\angle C = 94^o$ and $\angle D = 105^o$.

7. In the given figure, ABCD is a rectangle. If ∠CEB : ∠ECB = 3 : 2 find
(i) ∠CEB,
(ii) ∠DCF



Solution:-

From the question it is given that,

ABCD is a rectangle

$\angle CEB : \angle ECB = 3 : 2$

We have to find, (i) ∠CEB and (ii) ∠DCF

Consider the ΔBCE,

$\angle B = 90^o$

Therefore, $\angle CEB + \angle ECB = 90^o$

Let us assume the angles be 3y and 2y

$3y + 2y = 90^o$

$5y = 90^o$

$y = 90^o/5$

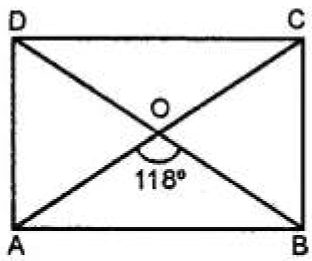
$y = 18^o$

Then, $\angle CEB = 3y = 3 \times 18 = 54^o$

$\angle CEB = \angle ECD$

$54^o = 54^o$ … [alternate angles are equal]
We know that, sum of linear pair angles equal to 180^o
$\angle ECD + DCF = 180^o$
$54^o + \angle DCF = 180^o$
By transposing we get,
$\angle DCF = 180^o - 54^o$
$\angle DCF = 126^o$

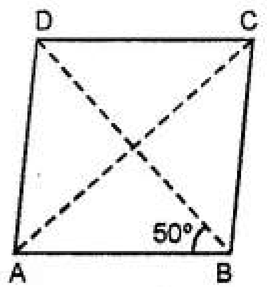
8. In the given figure, ABCD is a rectangle and diagonals intersect at O. If $\angle AOB = 118^o$, find



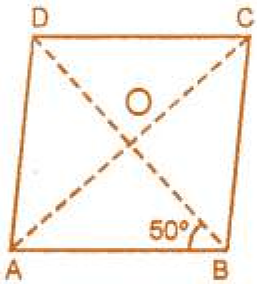
(i) $\angle ABO$
(ii) $\angle ADO$
(iii) $\angle OCB$
Solution:-
From the figure it is given that,
ABCD is a rectangle and diagonals intersect at O.
$\angle AOB = 118^o$
(i) Consider the ΔAOB,
$\angle OAB = \angle OBA$
Let us assume $\angle OAB = \angle OBA = y^o$
We know that, sum of measures of interior angles of triangle is equal to 180^o.
$\angle OAB + \angle OBA + \angle AOB = 180^o$
$y + y + 118^o = 180^o$
$2x + 118^o = 180^o$
By transposing we get,
$2y = 180^o - 118^o$
$2y = 62^o$
$y = 62/2$
$y = 31^o$
So, $\angle OAB = \angle OBA = 31^o$
Therefore, $\angle ABO = 31^o$

(ii) We know that sum of liner pair angles is equal to 180°.
∠AOB + ∠AOD = 180°
118° + ∠AOD = 180°
∠AOD = 180° – 118°
∠AOD = 62°
Now consider the ΔAOD,
Let us assume the ∠ADO = ∠DAO = x
∠AOD + ∠ADO + ∠DAO = 180°
62° + x + x = 180°
62° + 2x = 180°
By transposing we get,
2x = 180° - 62
2x = 118°
x = 118°/2
x = 59°
Therefore, ∠ADO = 59°
(iii) ∠OCB = ∠OAD = 59° … [because alternate angles are equal]

9. In the given figure, ABCD is a rhombus and ∠ABD = 50°. Find :
(i) ∠CAB
(ii) ∠BCD
(iii) ∠ADC



Solution:-
From the figure it is given that,
ABCD is a rhombus
∠ABD = 50°



(i) Consider the ΔAOB,

We know that, sum of measures of interior angles of triangle is equal to 180^o.

$\angle OAB + \angle BOA + \angle ABO = 180^o$

$\angle OAB + 90^o + 50^o = 180^o$

By transposing we get,

$\angle OAB + 140^o = 180^o$

$\angle OAB = 180^o - 140^o$

$\angle OAB = 40^o$

Therefore, $\angle CAB = 40^o$

(ii) $\angle BCD = 2\ \angle ACD$

$= 2 \times 40^o$

$= 80^o$

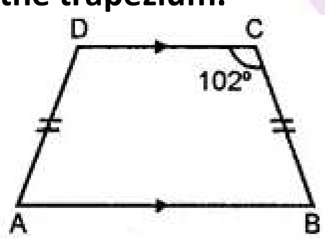
(iii) Then, $\angle ADC = 2\ \angle BDC$

$\angle ABD = \angle BDC$ because alternate angles are equal

$= 2 \times 50^o$

$= 100^o$

10. In the given isosceles trapezium ABCD, ∠C = 102°. Find all the remaining angles of the trapezium.



Solution:-

From the figure, it is given that,

Isosceles trapezium ABCD,

$\angle C = 102^o$

AB || CD

We know that sum of adjacent angles is equal to 180^o.

So, $\angle B + \angle C = 180^o$

$\angle B + 102^o = 180^o$

$\angle B = 180^o - 102^o$

$\angle B = 78^o$

Then, AD = BC

So, $\angle A = \angle B$

$78^o = 78^o$

Sum of all interior angles of trapezium is equal to 360^o.

$\angle A + \angle B + \angle C + \angle D = 360^o$

$78^o + 78^o + 102^o + \angle D = 360^o$

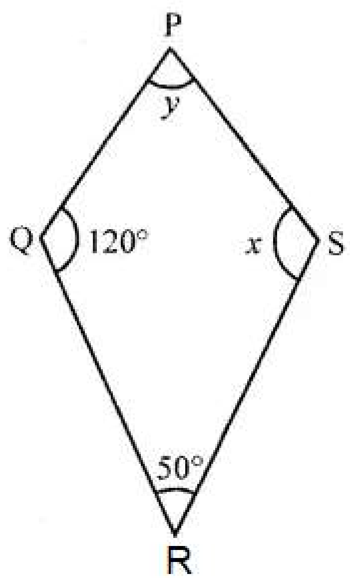
$258 + \angle D = 360^o$

By transposing we get,

$\angle D = 360^o - 258^o$

$\angle D = 102^o$

11. In the given figure, PQRS is a kite. Find the values of x and y.



Solution:-

From the figure it is given that,

PQRS is a kite.

$\angle Q = 120^o$

$\angle R = 50^o$

Then, $\angle Q = \angle S$

So, $x = 120^o$

We know that sum of all angles of Rhombus is equal to 360^o.

$\angle P + \angle Q + \angle R + \angle S = 360^o$

$y + 120^\circ + 50^\circ + 120^\circ = 360^\circ$

$y + 290^\circ = 360^\circ$

By transposing we get,

$y = 360^\circ – 290^\circ$

$y = 70^\circ$

Therefore, the value of $x = 120^\circ$ and $y = 70^\circ$