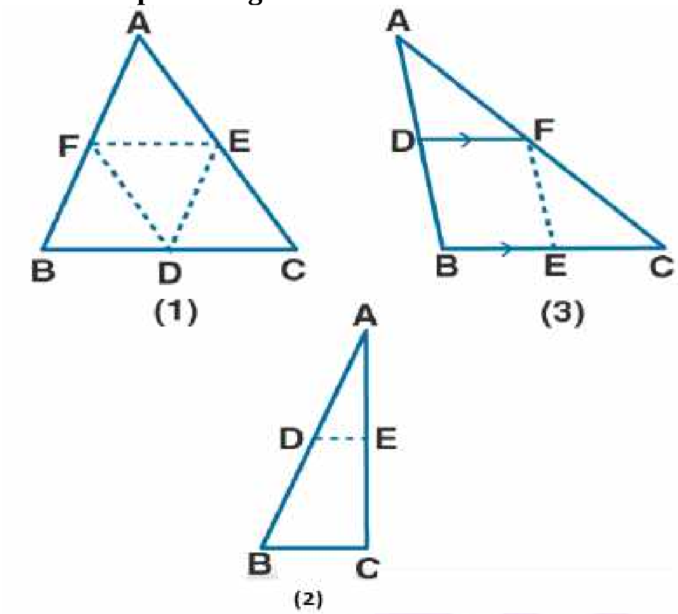
EXERCISE 11

1. (a) In the figure (1) given below, D, E and F are mid-points of the sides BC, CA and AB respectively of Δ ABC. If AB = 6 cm, BC = 4.8 cm and CA = 5.6 cm, find the perimeter of (i) the trapezium FBCE (ii) the triangle DEF.

(b) In the figure (2) given below, D and E are mid-points of the sides AB and AC respectively. If BC = 5.6 cm and ∠B = 72^0, compute (i) DE (ii) ∠ADE.

(c) In the figure (3) given below, D and E are mid-points of AB, BC respectively and DF || BC. Prove that DBEF is a parallelogram. Calculate AC if AF = 2.6 cm.

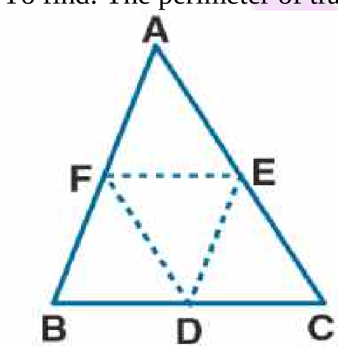


Solution:

(a) (i) It is given that

AB = 6 cm, BC = 4.8 cm and CA = 5.6 cm

To find: The perimeter of trapezium FBCA



It is given that

F is the mid-point of AB

We know that

BF = ½ AB = ½ × 6 = 3 cm ……. (1)

It is given that

E is the mid-point of AC

We know that

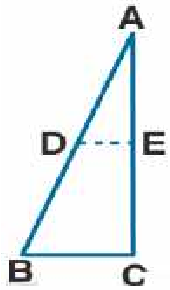
CE = ½ AC = ½ × 5.6 = 2.8 cm ……. (2)

Here F and E are the mid-point of AB and CA
FE || BC
We know that
FE = ½ BC = ½ × 4.8 = 2.4 cm …… (3)

Here
Perimeter of trapezium FBCE = BF + BC + CE + EF
Now substituting the value from all the equations
= 3 + 4.8 + 2.8 + 2.4
= 13 cm

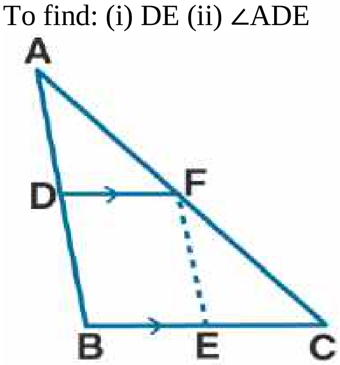
Therefore, the perimeter of trapezium FBCE is 13 cm.

(ii) D, E and F are the midpoints of sides BC, CA and AB of Δ ABC
Here EF || BC
EF = ½ BC = ½ × 4.8 = 2.4 cm
DE = ½ AB = ½ × 6 = 3 cm
FD = ½ AC = ½ × 5.6 = 2.8 cm



We know that
Perimeter of Δ DEF = DE + EF + FD
Substituting the values
= 3 + 2.4 + 2.8
= 8.2 cm

(b) It is given that
D and E are the mid-point of sides AB and AC
BC = 5.6 cm and ∠B = 72^0
To find: (i) DE (ii) ∠ADE



We know that
In Δ ABC

D and E is the mid-point of the sides AB and AC
Using mid-point theorem
DE || BC

(i) DE = ½ BC = ½ × 5.6 = 2.8 cm

(ii) ∠ADE = ∠B are corresponding angles
It is given that
∠B = 72^0 and BC || DE
∠ADE = 72^0

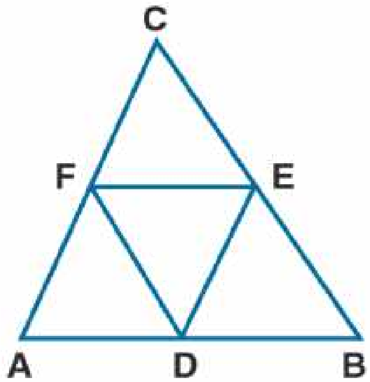
(c) It is given that
D and E are the midpoints of AB and BC respectively
DF || BC and AF = 2.6 cm
To find: (i) BEF is a parallelogram
(ii) Calculate the value of AC

Proof:
(i) In Δ ABC
D is the midpoint of AB and DF || BC
F is the midpoint of AC ….. (1)
F and E are the midpoints of AC and BC
EF || AB ….. (2)
Here DF || BC
DF || BE ….. (3)
Using equation (2)
EF || AB
EF || DB ….. (4)
Using equation (3) and (4)
DBEF is a parallelogram

(ii) F is the midpoint of AC
So we get
AC = 2 × AF = 2 × 2.6 = 5.2 cm

2. Prove that the four triangles formed by joining in pairs the mid-points of the sides C of a triangle are congruent to each other.
Solution:

It is given that
In Δ ABC
D, E and F are the mid-points of AB, BC and CA
Now join DE, EF and FD
To find:
Δ ADF ≅ Δ DBE ≅ Δ ECF ≅ Δ DEF

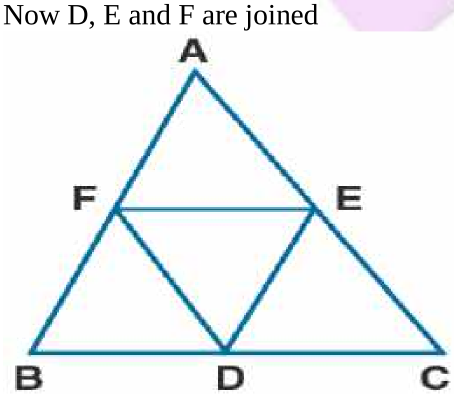


To prove:
In Δ ABC
D and E are the mid-points of AB and BC
DE || AC or FC
Similarly DF || EC
DECF is a parallelogram

We know that
Diagonal FE divides the parallelogram DECF in two congruent triangles DEF and CEF
Δ DEF ≅ Δ ECF …… (1)
Here we can prove that
Δ DBE ≅ Δ DEF …. (2)
Δ DEF ≅ Δ ADF ……. (3)
Using equation (1), (2) and (3)
Δ ADF ≅ Δ DBE ≅ Δ ECF ≅ Δ DEF

3. If D, E and F are mid-points of sides AB, BC and CA respectively of an isosceles triangle ABC, prove that Δ DEF is also isosceles.
Solution:

It is given that
ABC is an isosceles triangle in which AB = AC
D, E and F are the midpoints of the sides BC, CA and AB
Now D, E and F are joined



To find:
Δ DEF is an isosceles triangle

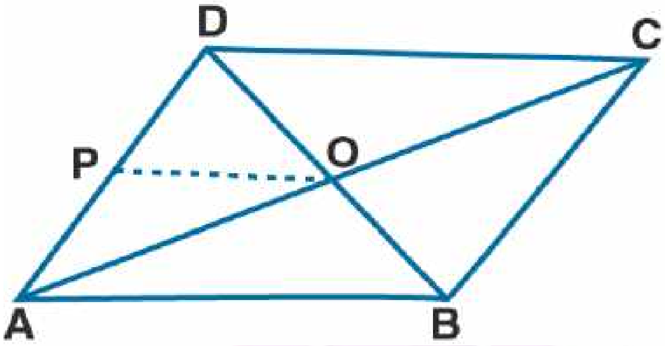
Proof:

D and E are the midpoints of BC and AC
Here DE || AB and DE = ½ AB ….. (1)
D and F are the midpoints of BC and AB
Here DF || AC and DF = ½ AC ….. (2)
It is given that
AB = BC and DE = DF

Hence, ∆ DEF is an isosceles triangle.

4. The diagonals AC and BD of a parallelogram ABCD intersect at O. If P is the midpoint of AD, prove that
(i) PQ || AB
(ii) PO = ½ CD.
Solution:

It is given that
ABCD is a parallelogram in which diagonals AC and BD intersect each other
At the point O, P is the midpoint of AD
Join OP
To find: (i) PQ || AB (ii) PQ = ½ CD

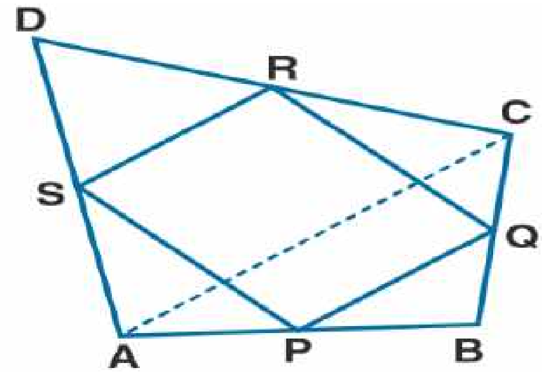


Proof:
(i) In parallelogram diagonals bisect each other
BO = OD
Here O is the mid-point of BD

In ∆ ABD
P and O is the midpoint of AD and BD
PO || AB and PO = ½ AB ….. (1)
Hence, it is proved that PO || AB.

(ii) ABCD is a parallelogram
AB = CD ……. (2)
Using both (1) and (2)
PO = ½ CD

5. In the adjoining figure, ABCD is a quadrilateral in which P, Q, R and S are mid-points of AB, BC, CD and DA respectively. AC is its diagonal. Show that
(i) SR || AC and SR = ½ AC
(ii) PQ = SR
(iii) PQRS is a parallelogram.



Solution:

It is given that
In quadrilateral ABCD
P, Q, R and S are the mid-points of sides AB, BC, CD and DA
AC is the diagonal

To find:
(i) SR || AC and SR = ½ AC
(ii) PQ = SR
(iii) PQRS is a parallelogram

Proof:
(i) In Δ ADC
S and R are the mid-points of AD and DC
SR || AC and SR = ½ AC….. (1) Using the mid-point theorem

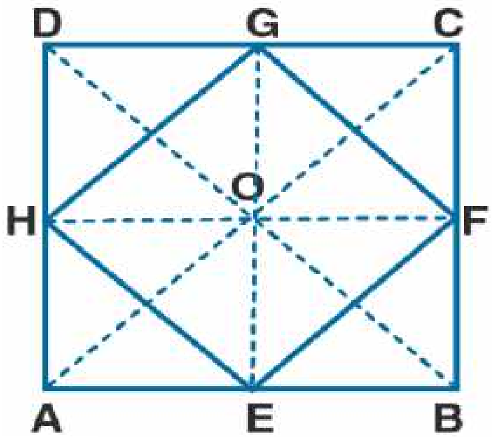
(ii) In Δ ABC
P and Q are the midpoints of AB and BC
PQ || AC and PQ = ½ AC ….. (2)

Using equation (1) and (2)
PQ = SR and PQ || SR

(iii) PQ = SR and PQ || SR
Hence, PQRS is a parallelogram.

6. Show that the quadrilateral formed by joining the mid-points of the adjacent sides of a square, is also a square.
Solution:

It is given that
A square ABCD in which E, F, G and H are mid-points of AB, BC, CD and DA
Join EF, FG, GH and HE.



To find:
EFGH is a square
Construct AC and BD

Proof:
In Δ ACD
G and H are the mid-points of CD and AC
GH || AC and GH = ½ AC ….. (1)

In Δ ABC, E and F are the mid-points of AB and BC
EF || AC and EF = ½ AC …… (2)

Using both the equations
EF || GH and EF = GH = ½ AC ….. (3)

In the same way we can prove that
EF || GH and EH = GF = ½ BD
We know that the diagonals of square are equal
AC = BD
By dividing both sides by 2
½ AC = ½ BD ….. (4)
Using equation (3) and (4)
EF = GH = EH = GF …. (5)
Therefore, EFGH is a parallelogram

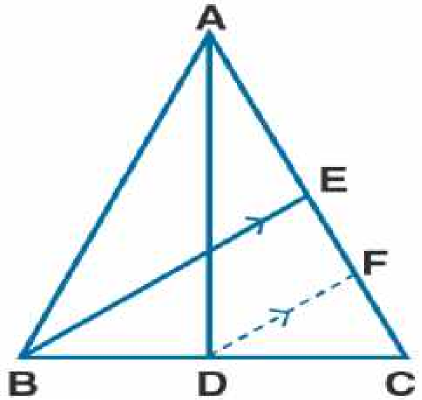
In Δ GOH and Δ GOF
OH = OF as the diagonals of parallelogram bisect each other
OG = OG is common
Using equation (5)
GH = GF
Δ GOH ≅ Δ GOF (SSS axiom of congruency)
∠GOH = ∠GOF (c.p.c.t)

We know that
∠GOH + ∠GOF = 180^0 as it is a linear pair
∠GOH + ∠GOH = 180^0
So we get

$2\angle GOH = 180^0$
$\angle GOH = 180^0/2 = 90^0$
So the diagonals of a parallelogram ABCD bisect and perpendicular to each other

Hence, it is proved that EFGH is a square.

7. In the adjoining figure, AD and BE are medians of Δ ABC. If DF || BE, prove that CF = ¼ AC.



Solution:

It is given that
AD and BE are the medians of Δ ABC
Construct DF || BE

To find:
CF = ¼ AC

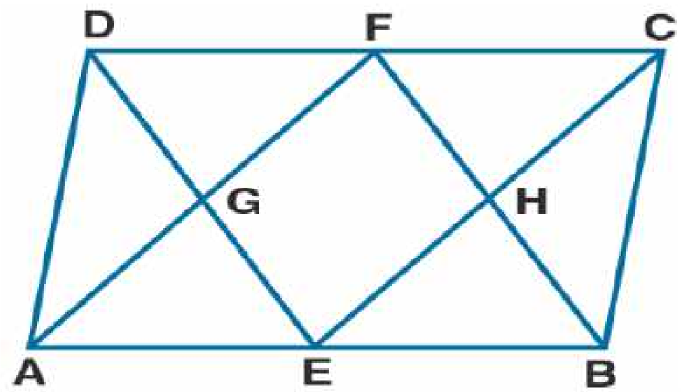
Proof:
In Δ BCE
D is the mid-point of BC and DF || BE
F is the mid-point of EC
CF = ½ EC …… (1)

E is the mid-point of AC
EC = ½ AC ….. (2)

Using both the equations
CF = ½ EC = ½ (½ AC)
So we get
CF = ¼ AC

Hence, it is proved.

8. In the figure (1) given below, ABCD is a parallelogram. E and F are mid-points of the sides AB and CO respectively. The straight lines AF and BF meet the straight lines ED and EC in points G and H respectively. Prove that
(i) Δ HEB = Δ HCF
(ii) GEHF is a parallelogram.



Solution:

It is given that
ABCD is a parallelogram
E and F are the mid-points of sides AB and CD

To prove:
(i) Δ HEB = Δ HCF
(ii) GEHF is a parallelogram

Proof:
(i) We know that
ABCD is a parallelogram
FC || BE
∠CEB = ∠FCE are alternate angles
∠HEB = ∠FCH ….. (1)
∠EBF = ∠CFB are alternate angles
∠EBH = ∠CFM …… (2)
Here E and F are mid-points of AB and CD
BE = ½ AB ….. (3)
CF = ½ CD …… (4)

We know that
ABCD is a parallelogram
AB = CD
Now dividing both sides by ½
½ AB = ½ CD
Using equation (3) and (4)
BE = CF ….. (5)

In Δ HEB and Δ HCF
∠HEB = ∠FCH using equation (1)
∠EBH = ∠CFH using equation (2)
BE = CF using equation (5)
So we get
Δ HEB ≅ Δ HCF (ASA axiom of congruency)
Hence, it is proved.

(ii) It is given that
E and F are the mid-points of AB and CD
AB = CD
So we get
AE = CF
Here AE || CF
AE = CF and AE || CF
So AECF is a parallelogram.

G and H are the mid-points of AF and CE
GF || EH ……. (6)
In the same way we can prove that GFHE is a parallelogram
So G and H are the points on the line DE and BF
GE || HF …….. (7)
Using equation (6) and (7) GEHF is a parallelogram.
Hence, it is proved.

9. ABC is an isosceles triangle with AB = AC. D, E and F are mid-points of the sides BC, AB and AC respectively. Prove that the line segment AD is perpendicular to EF and is bisected by it.
Solution:

It is given that
ABC is an isosceles triangle with AB = AC
D, E and F are mid-points of the sides BC, AB and AC

To find:
AD is perpendicular to EF and is bisected by it.

Proof:
In Δ ABD and Δ ACD
ABC is an isosceles triangle
∠ABD = ∠ACD
Here D is the mid-point of BC
BD = BD
It is given that AB = AC
Δ ABD ≅ Δ ACD (SAS axiom of congruency)
∠ADB = ∠AOC (c. p. c. t)

We know that
∠ADB + ∠AOC = 180^0 is a linear pair
∠ADB + ∠ADB = 180^0
By further calculation
2 ∠ADB = 180^0
So we get
∠ADB = 180/2 = 90^0
So AD is perpendicular to BC ….. (1)

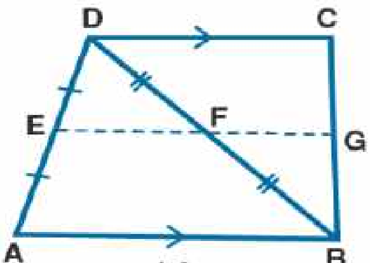
D and E are the mid-points of BC and AB
DE || AF …… (2)
D and F are the mid-points of BC and AC

EF || AD …. (3)
Using equation (2) and (3)
AEDF is a parallelogram.

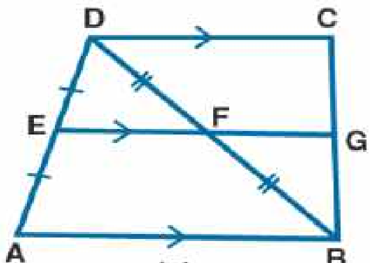
Here the diagonals of a parallelogram bisect each other
AD and EF bisect each other
Using equation (1) and (3)
EF || BC
So AD is perpendicular to EF

Hence, it is proved.

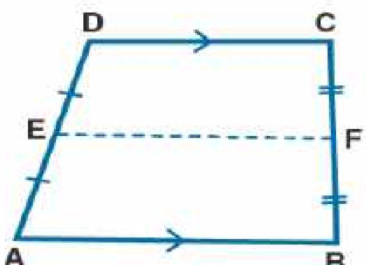
10. (a) In the quadrilateral (1) given below, AB || DC, E and F are the mid-points of AD and BD respectively. Prove that:
(i) G is mid-point of BC
(ii) EG = ½ (AB + DC)
(b) In the quadrilateral (2) given below, AB || DC || EG. If E is mid-point of AD prove that:
(i) G is the mid-point of BC
(ii) 2EG = AB + CD
(c) In the quadrilateral (3) given below, AB || DC.
E and F are mid-point of non-parallel sides AD and BC respectively. Calculate:
(i) EF if AB = 6 cm and DC = 4 cm.
(ii) AB if DC = 8 cm and EF = 9 cm.



(1)



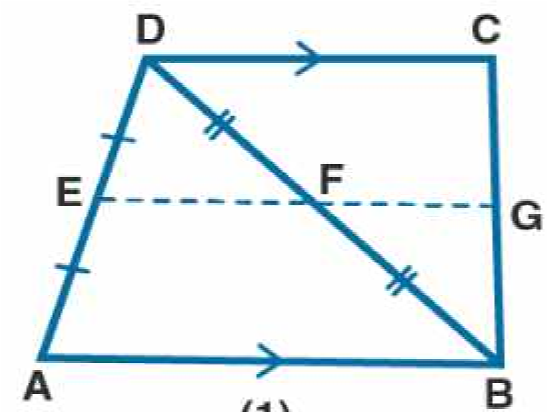
(2)



(3)

Solution:

(a) It is given that
AB || DC, E and F are mid-points of AD and BD



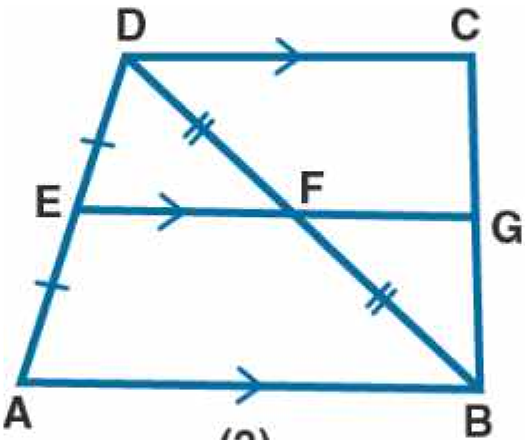
(1)

To prove:
(i) G is mid-point of BC
(ii) EG = ½ (AB + DC)
Proof:
In Δ ABD
F is the mid-point of BD
DF = BF
E is the mid-point of AD
EF || AB and EF = ½ AB ….. (1)
It is given that AB || CD
EG || CD

F is the mid-point of BD
FG || DC
G is the mid-point of BC
FG = ½ DC …….. (2)

By adding both the equations
EF + FG = ½ AB + ½ DC
Taking ½ as common
EG = ½ (AB + DC)
Therefore, it is proved.

(b) It is given that
Quadrilateral ABCD in which AB || DC || EG
E is the mid-point of AD



(2)

To prove:
(i) G is the mid-point of BC
(ii) 2EG = AB + CD

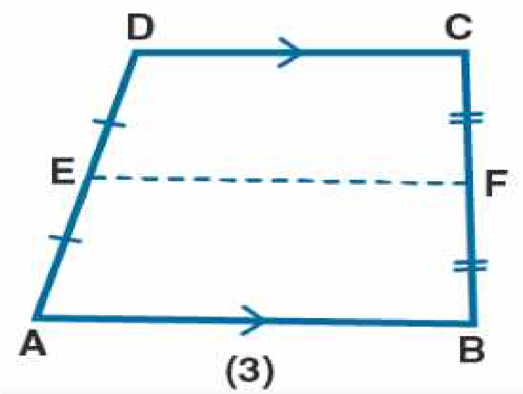
Proof:
AB || DC
EG || AB
So we get
EG || DC

In Δ DAB,
E is the mid-point of BD and EF = ½ AB ….. (1)

In Δ BCD,
F is the mid-point of BD and FG || DC
FG = ½ CD …… (2)

By adding both the equations
EF + FG = ½ AB + ½ CD
Taking out the common terms
EG = ½ (AB + CD)
Hence, it is proved.

(c) It is given that
A quadrilateral in which AB || DC
E and F are the mid-points of non-parallel sides AD and BC



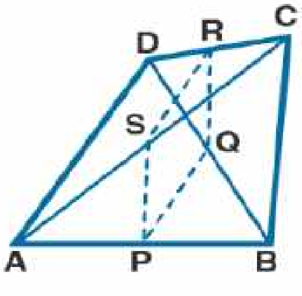
To prove:
(i) EF if AB = 6 cm and DC = 4 cm.
(ii) AB if DC = 8 cm and EF = 9 cm.

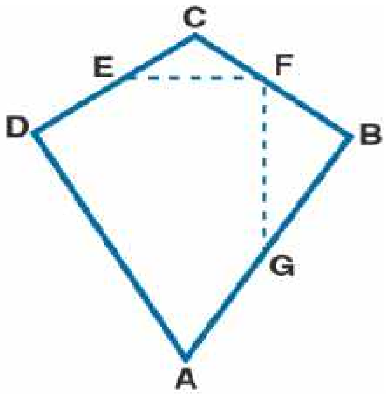
Proof:
We know that
The length of line segment joining the mid-points of two non-parallel sides is half the sum of the lengths of the parallel sides
E and F are the mid-points of AD and BC
EF = ½ (AB + CD) …… (1)

(i) AB = 6 cm and DC = 4 cm
Substituting in equation (1)
EF = ½ (6 + 4)
By further calculation
EF = ½ × 10 = 5 cm

(ii) DC = 8 cm and EF = 9 cm
Substituting in equation (1)
EF = ½ (AB + DC)
By further calculation
9 = ½ (AB + 8)
18 = AB + 8
So we get
18 – 8 = AB
AB = 10 cm

11. (a) In the quadrilateral (1) given below, AD = BC, P, Q, R and S are mid-points of AB, BD, CD and AC respectively. Prove that PQRS is a rhombus.
(b) In the figure (2) given below, ABCD is a kite in which BC = CD, AB = AD, E, F, G are mid-points of CD, BC and AB respectively. Prove that:
(i) ∠EFG = 90^0
(ii) The line drawn through G and parallel to FE bisects DA.

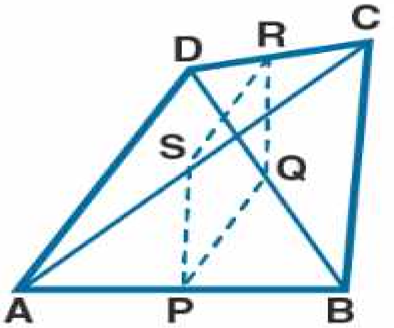




Solution:

(a) It is given that
A quadrilateral ABCD in which AD = C
P, Q, R and S are mid-points of AB, BD, CD and AC

To prove:
PQRS is a rhombus

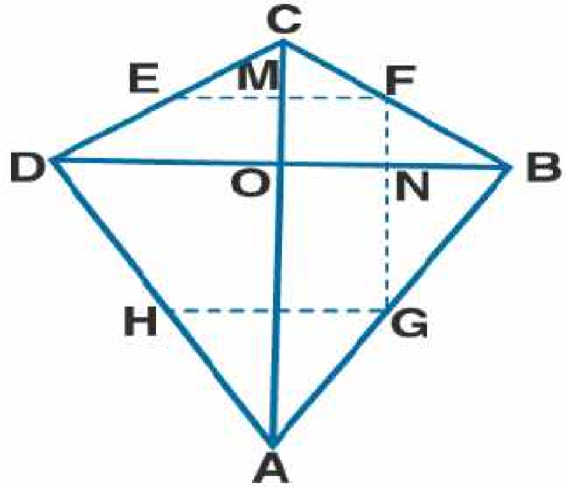


Proof:
In Δ ABD
P and Q are mid points of AB and BD
PQ || AD and PQ = ½ AB …. (1)

In Δ BCD,
R and Q are mid points of DC and BD
RQ || BC and RQ = ½ BC …… (2)
P and S are mid-points of AB and AC
PS || BC and PS = ½ BC ……. (3)
AD = BC
Using all the equations
PS || RQ and PQ = PS = RQ
Here PS || RQ and PS = RQ
PQRS is a parallelogram
PQ = RS = PS = RQ
PQRS is a rhombus

Therefore, it is proved.

(ii) It is given that
ABCD is a kite in which BC = CD, AB = AD, E, F, G are mid-points of CD, BC and AB



To prove:
(i) $\angle EFG = 90^0$
(ii) The line drawn through G and parallel to FE bisects DA

Construction:
Join AC and BD
Construct GH through G parallel to FE

Proof:
(i) We know that
Diagonals of a kite interest at right angles
$\angle MON = 90^0$ ….. (1)

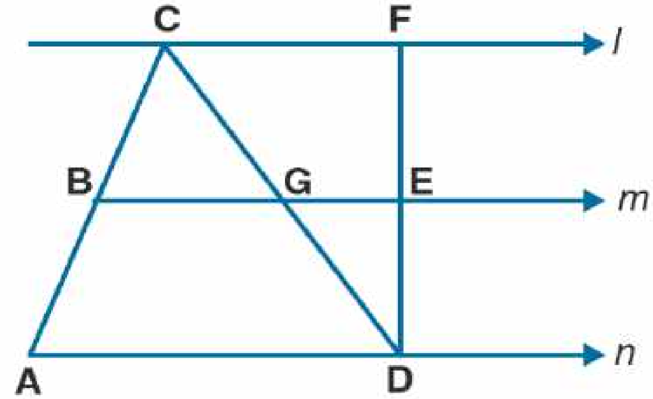
In Δ BCD
E and F are mid-points of CD and BC
EF || DB and EF = ½ DB …… (2)
EF || DB
MF || ON

Here
$\angle MON + \angle MFN = 180^0$
$90^0 + \angle MFN = 180^0$
By further calculation
$\angle MFN = 180 – 90 = 90^0$
So $\angle EFG = 90^0$
Hence, it is proved.

(ii) In Δ ABD
G is the mid-point of AB and HG || DB
Using equation (2)
EF || DB and EF || HG
HG || DB
Here H is the mid-point of DA

Therefore, the line drawn through G and parallel to FE bisects DA.

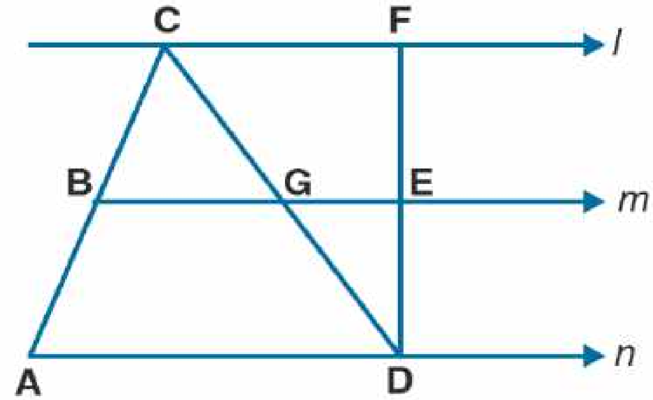
12. In the adjoining figure, the lines l, m and n are parallel to each other, and G is mid-point of CD.
Calculate:
(i) BG if AD = 6 cm
(ii) CF if GE = 2.3 cm
(iii) AB if BC = 2.4 cm
(iv) ED if FD = 4.4 cm



Solution:

It is given that
The straight line l, m and n are parallel to each other
G is the mid-point of CD

To find:
(i) BG if AD = 6 cm
(ii) CF if GE = 2.3 cm
(iii) AB if BC = 2.4 cm
(iv) ED if FD = 4.4 cm



Proof:
(i) In Δ ACD,
G is the mid-point of CD
BG || AD as m || n
Here B is the mid-point of AC and BG = ½ AD
So we get

BG = ½ × 6 = 3 cm

(ii) In Δ CDF
G is the mid-point of CD
GE || CF as m || l
Here E is the mid-point of DF and GE = ½ CF
So we get
CF = 2GE
CF = 2 × 2.3 = 4.6 cm

(iii) From (i)
B is the mid-point of AC
AB = BC
We know that
BC = 2.4 cm
So AB = 2.4 cm

(iv) From (ii)
E is the mid-point of FD
ED = ½ FD
We know that
FD = 4.4 cm
ED = ½ × 4.4 = 2.2 cm

CHAPTER TEST

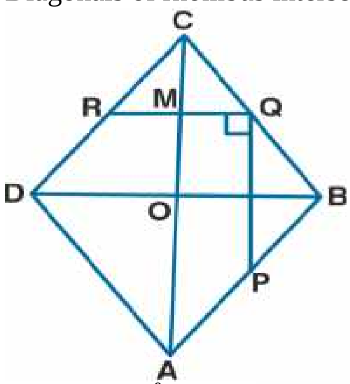
1. ABCD is a rhombus with P, Q and R as midpoints of AB, BC and CD respectively. Prove that PQ ⊥QR.
Solution:

It is given that
ABCD is a rhombus with P, Q and R as mid-points of AB, BC and CD

To prove:
PQ ⊥QR

Construction: Join AC and BD

Proof:
Diagonals of rhombus intersect at right angle

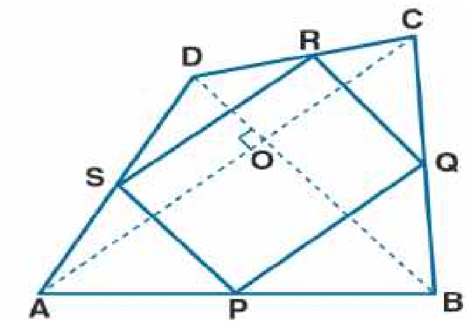


$\angle MON = 90^0$ ….. (1)
In Δ BCD
Q and R are mid-points of BC and CD.
RQ || DB and RQ = ½ DB ….. (2)
Here
RQ || DB
MQ || ON

We know that
$\angle MQN + \angle MON = 180^0$
Substituting the values
$\angle MQN + 90^0 = 180^0$
$\angle MQN = 180 – 90 = 90^0$
So NQ ⊥MQ or PQ ⊥QR

Hence, it is proved.

2. The diagonals of a quadrilateral ABCD are perpendicular. Show that the quadrilateral formed by joining the mid-points of its adjacent sides is a rectangle.
Solution:



It is given that
ABCD is a quadrilateral in which diagonals AC and BD are perpendicular to each other
P, Q, R and S are mid-points of AB, BC, CD and DA

To prove:
PQRS is a rectangle

Proof:
We know that
P and Q are the mid-points of AB and BC
PQ || AC and PQ = ½ AC ….. (1)
S and R are mid-points of AD and DC
SR || AC and SR = ½ AC ….. (2)

Using both the equations
PQ || SR and PQ = SR
So PQRS is a parallelogram

AC and BD intersect at right angles
SP || BD and BD ⊥AC
So SP ⊥ AC i.e. SP ⊥SR
$\angle RSP = 90^0$
$\angle RSP = \angle SRQ = \angle RQS = \angle SPQ = 90^0$

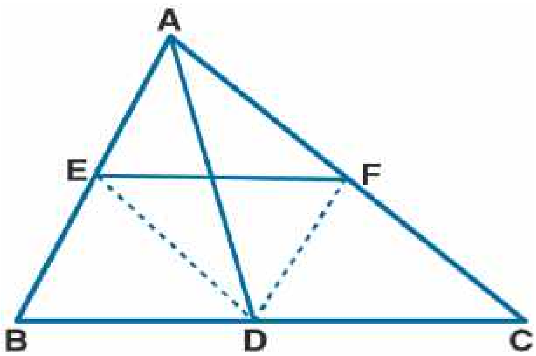
Hence, PQRS is a rectangle.

3. If D, E, F are mid-points of the sides BC, CA and AB respectively of a Δ ABC, prove that AD and FE bisect each other.
Solution:

It is given that
D, E, F are mid-points of sides BC, CA and AB of a Δ ABC

To prove:
AD and FE bisect each other

Construction:
Join ED and FD

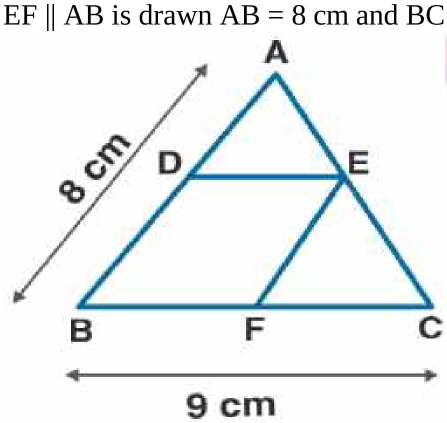


Proof:
We know that
D and E are the midpoints of BC and AB
DE || AC and DE || AF ….. (1)
D and F are the midpoints of BC and AC
DF || AB and DF || AE …. (2)
Using both the equations
ADEF is a parallelogram
Here the diagonals of a parallelogram bisect each other
AD and EF bisect each other.

Therefore, it is proved.

4. In Δ ABC, D and E are mid-points of the sides AB and AC respectively. Through E, a straight line is drawn parallel to AB to meet BC at F. Prove that BDEF is a parallelogram. If AB = 8 cm and BC = 9 cm, find the perimeter of the parallelogram BDEF.
Solution:

It is given that
In Δ ABC
D and E are the mid points of sides AB and AC
DE is joined from E
EF || AB is drawn AB = 8 cm and BC = 9 cm



To prove:
(i) BDEI is a parallelogram
(ii) Find the perimeter of BDEF

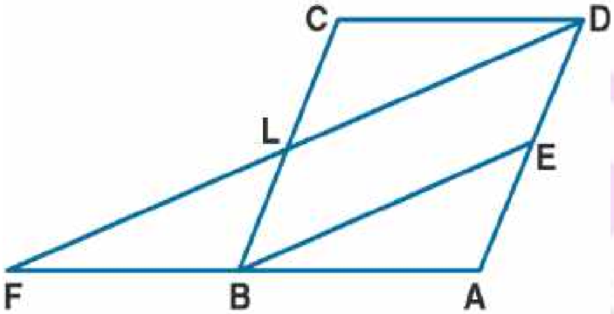
Proof:

In Δ ABC
B and E are the mid-points of AB and AC
Here DE || BC and DE = ½ BC
So EF || AB
DEFB is a parallelogram
DE = BF

So we get
DE = ½ BC = ½ × 9 = 4.5 cm
EF = ½ AB = ½ × 8 = 4 cm

We know that
Perimeter of BDEF = 2 (DE + EF)
Substituting the values
= 2 (4.5 + 4)
= 2 × 8.5
= 17 cm

5. In the given figure, ABCD is a parallelogram and E is mid-point of AD. DL || EB meets AB produced at F. Prove that B is mid-point of AF and EB = LF.



Solution:

It is given that
ABCD is a parallelogram
E is the mid-point of AD
DL || EB meets AB produced at F

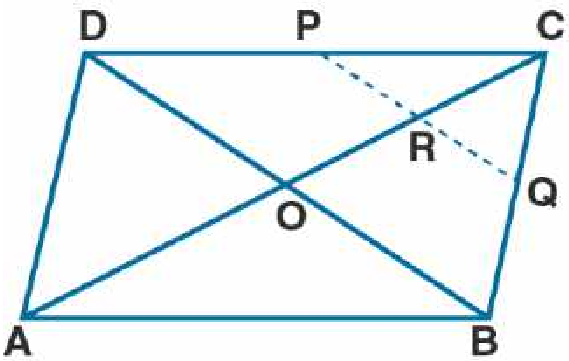
To prove:
EB = LF
B is the mid-point of AF

Proof:
We know that
BC || AD and BE || LD
BEDL is a parallelogram
BE = LD and BL = AE
Here E is the mid-point of AD
L is the mid-point of BC

In Δ FAD

E is the mid-point of AD and BE || LD at FLD
So B is the mid-point of AF
Here
EB = ½ FD = LF

6. In the given figure, ABCD is a parallelogram. If P and Q are mid-points of sides CD and BC respectively, show that CR = ½ AC.



Solution:

It is given that
ABCD is a parallelogram
P and Q are mid-points of CD and BC

To prove: CR = ¼ AC

Construction: Join AC and BD

Proof:
In parallelogram ABCD
Diagonals AC and BD bisect each other at O
AO = OC or OC = ½ AC ….. (1)

In Δ BCD
P and Q are mid points of CD and BC
PQ || BD

In Δ BCO
Q is the mid-point of BC and PQ || OB
Here is the mid-point of CO
So we get
CR = ½ OC = ½ (½ BC)
CR = ¼ BC

Hence, it is proved.