Exercise 12

1. Lengths of sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse:
(i) 3 cm, 8 cm, 6 cm
(ii) 13 cm, 12 cm, 5 cm
(iii) 1.4 cm, 4.8 cm, 5 cm

Solution:
We use the Pythagoras theorem to check whether the triangles are right triangles.
We have $h^2 = b^2+a^2$ [Pythagoras theorem]
Where h is the hypotenuse, b is the base and a is the altitude.

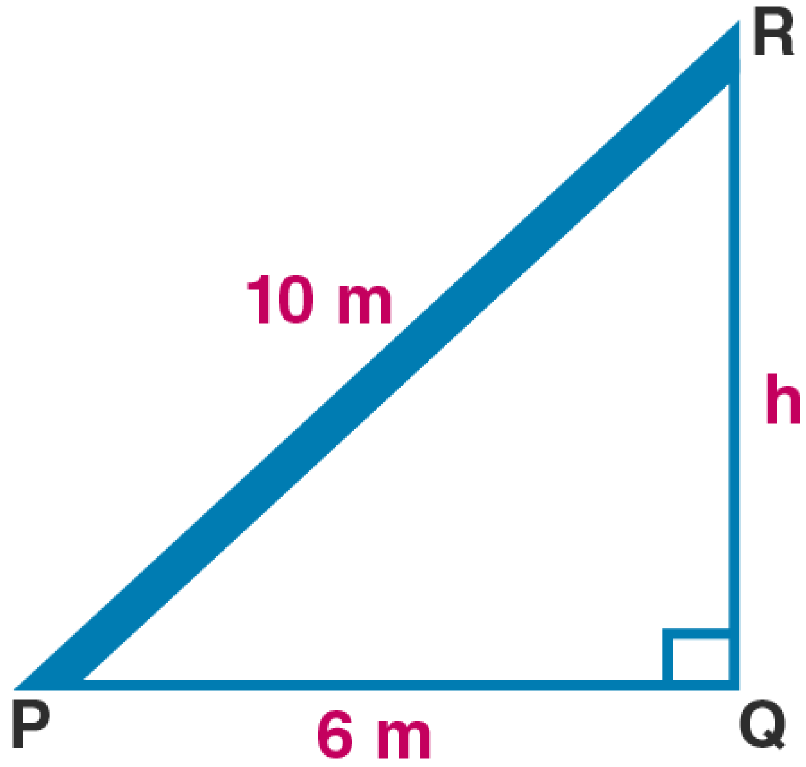
(i)Given sides are 3 cm, 8 cm and 6 cm
$b^2+a^2 = 3^2+ 6^2 = 9+36 = 45$
$h^2 = 8^2 = 64$
here $45 \neq 64$
Hence the given triangle is not a right triangle.

(ii) Given sides are 13 cm, 12 cm and 5 cm
$b^2+a^2 = 12^2+ 5^2 = 144+25 = 169$
$h^2 = 13^2 = 169$
here $b^2+a^2 = h^2$
Hence the given triangle is a right triangle.
Length of the hypotenuse is 13 cm.

(iii) Given sides are 1.4 cm, 4.8 cm and 5 cm
$b^2+a^2 = 1.4^2+ 4.8^2 = 1.96+23.04 = 25$
$h^2 = 5^2 = 25$
here $b^2+a^2 = h^2$
Hence the given triangle is a right triangle.
Length of the hypotenuse is 5 cm.

2. Foot of a 10 m long ladder leaning against a vertical well is 6 m away from the base of the wall. Find the height of the point on the wall where the top of the ladder reaches.

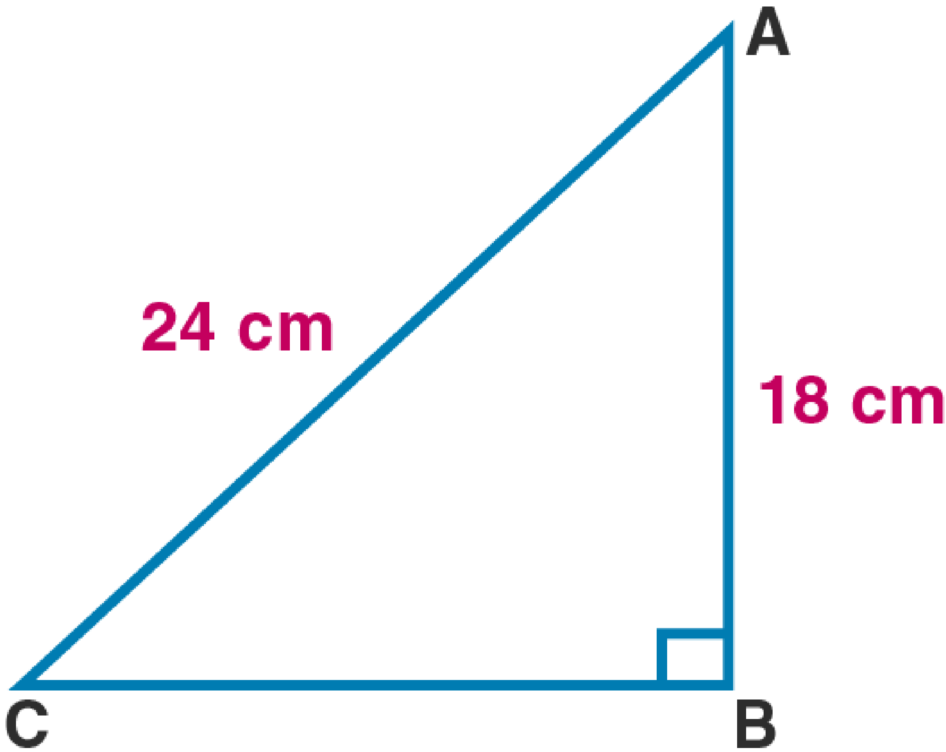
Solution:



Let PR be the ladder and QR be the vertical wall.
Length of the ladder PR = 10 m
PQ = 6 m
Let height of the wall, QR = h
According to Pythagoras theorem,
$PR^2 = PQ^2+QR^2$
$10^2 = 6^2+QR^2$
$100 = 36+QR^2$
$\therefore$ $QR^2 = 100-36$
$\therefore$ $QR^2 = 64$
Taking square root on both sides,
$\therefore$ QR = 8
Hence the height of the wall where the top of the ladder reaches is 8 m.

3. A guy attached a wire 24 m long to a vertical pole of height 18 m and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be tight?

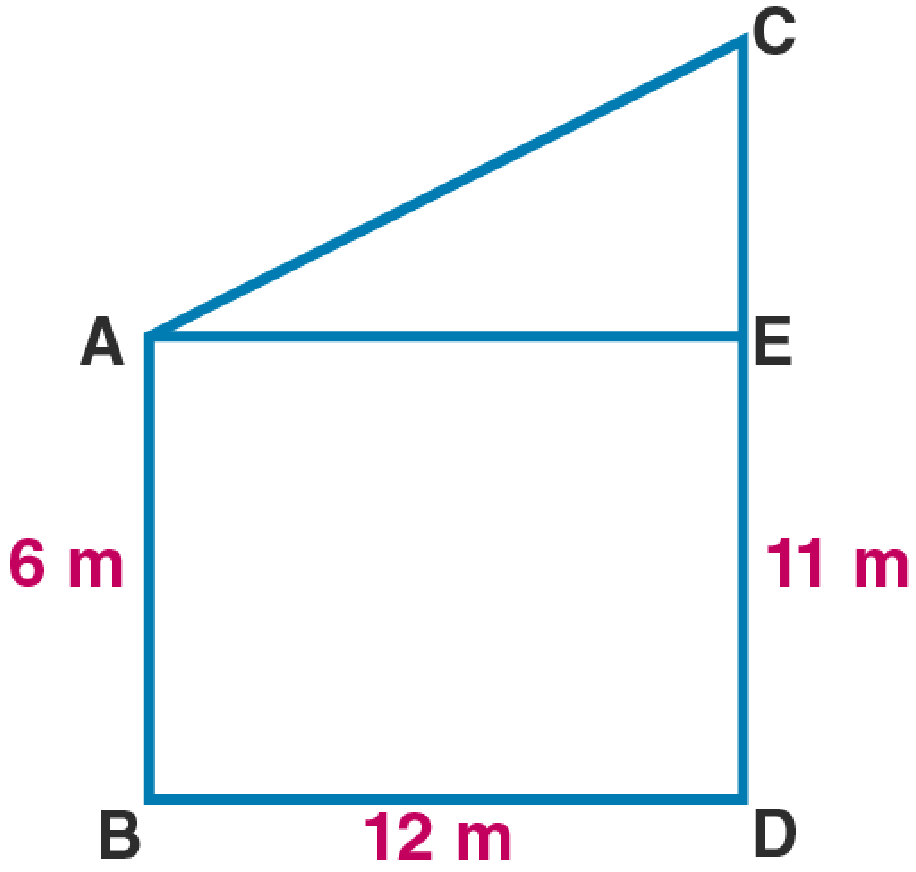
Solution:



Let AC be the wire and AB be the height of the pole.
AC = 24 cm
AB = 18 cm
According to Pythagoras theorem,
$AC^2 = AB^2+BC^2$
$24^2 = 18^2+BC^2$
$576 = 324+BC^2$
$\Rightarrow BC^2 = 576-324$
$\Rightarrow BC^2 = 252$
Taking square root on both sides,
$BC = \sqrt{252}$
$= \sqrt{(4\times9\times7)}$
$= 2\times3\sqrt{7}$
$= 6\sqrt{7}$ cm
Hence the distance is $6\sqrt{7}$ cm.

4. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

Solution:



Let AB and CD be the poles which are 12 m apart.
AB = 6 m
CD = 11 m
BD = 12 m
Draw AE ∥ BD
CE = 11-6 = 5 m
AE = 12 m
According to Pythagoras theorem,
$AC^2 = AE^2+CE^2$
$AC^2 = 12^2+5^2$
$AC^2 = 144+25$
$AC^2 = 169$
Taking square root on both sides
AC = 13
Hence the distance between their tops is 13 m.

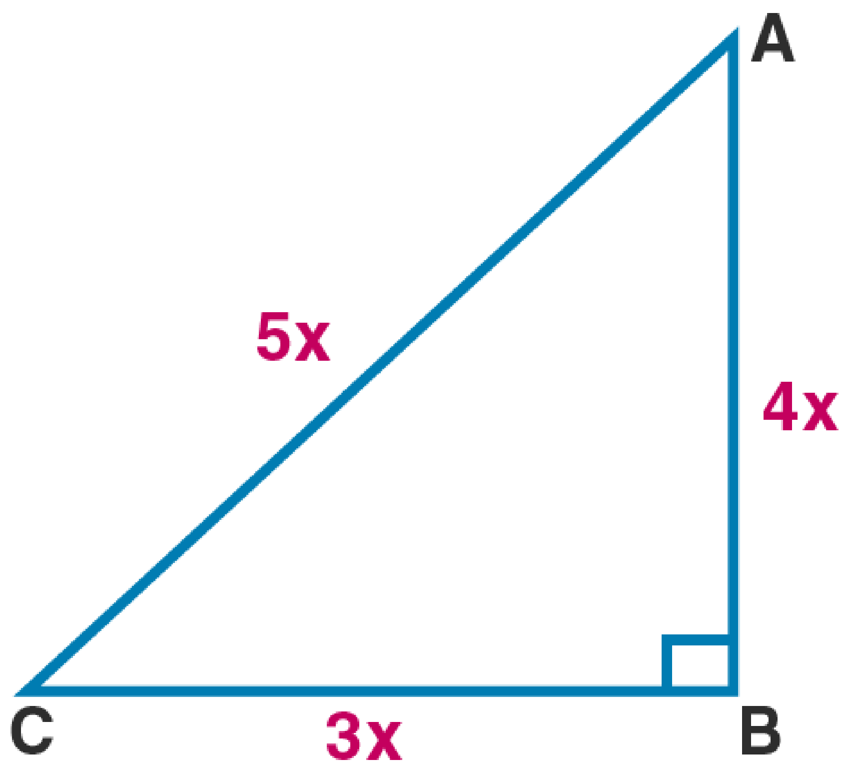
5. In a right-angled triangle, if hypotenuse is 20 cm and the ratio of the other two sides is 4:3, find the sides.

Solution:

Given hypotenuse, h = 20 cm
Ratio of other two sides, a:b = 4:3
Let altitude of the triangle be 4x and base be 3x.
According to Pythagoras theorem,
$h^2 = b^2+a^2$
$\therefore 20^2 = (3x)^2+(4x)^2$
$\therefore 400 = 9x^2+16x^2$
$\Rightarrow 25x^2 = 400$
$\Rightarrow x^2 = 400/25$
$\Rightarrow x^2 = 16$
Taking square root on both sides
x = 4
so base, b = 3x = 3×4 = 12
altitude, a = 4x = 4×4 = 16
Hence the other sides are 12 cm and 16 cm.

6. If the sides of a triangle are in the ratio 3:4:5, prove that it is right-angled triangle.

Solution:

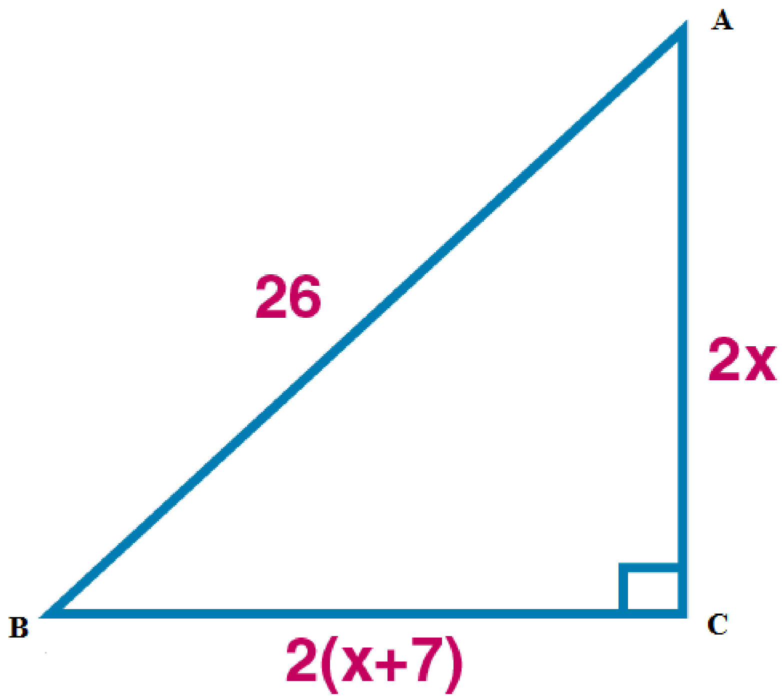


Given the sides are in the ratio 3:4:5.
Let ABC be the given triangle.
Let the sides be 3x, 4x and hypotenuse be 5x.
According to Pythagoras theorem,
$AC^2 = BC^2+AB^2$

$BC^2+AB^2 = (3x)^2+(4x)^2$
$= 9x^2+16x^2$
$= 25x^2$
$AC^2 = (5x)^2 = 25x^2$
$\therefore AC^2 = BC^2+AB^2$
Hence $\triangle ABC$ is a right angled triangle.

7. For going to a city B from city A, there is route via city C such that AC ⊥ CB, AC = 2x km and CB=2(x+ 7) km. It is proposed to construct a 26 km highway which directly connects the two cities A and B. Find how much distance will be saved in reaching city B from city A after the construction of highway.

Solution:

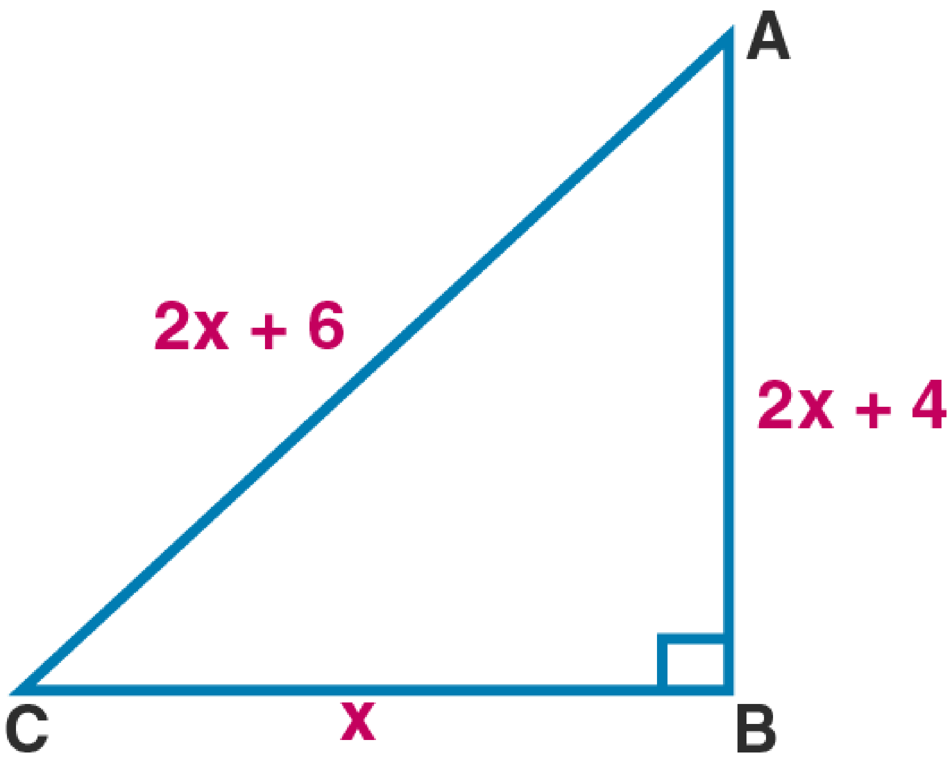


Given AC = 2x km
CB = 2(x+7)km
AB = 26
Given AC⊥ CB.
According to Pythagoras theorem,
$AB^2 = CB^2+AC^2$
$\therefore 26^2 = (2(x+7))^2+(2x)^2$
$676 = 4(x^2+14x+49) + 4x^2$
$\Rightarrow 4x^2+56x+196+4x^2 = 676$
$\Rightarrow 8x^2+56x+196 = 676$
$\Rightarrow 8x^2+56x +196-676 = 0$
$\Rightarrow 8x^2+56x -480 = 0$
$\Rightarrow x^2+7x -60 = 0$
$\Rightarrow (x-5)(x+12) = 0$

$\Rightarrow$(x-5) = 0 or (x+12) = 0
$\Rightarrow$x = 5 or x = -12
Length cannot be negative. So x = 5
$\therefore$ BC = 2(x+7) = 2(5+7) = 2×12 = 24 km
AC = 2x = 2×5 = 10 km
Total distance = AC + BC = 10+24 = 34 km
Distance saved = 34-26 = 8 km
Hence the distance saved is 8 km.

8. The hypotenuse of right triangle is 6m more than twice the shortest side. If the third side is 2m less than the hypotenuse, find the sides of the triangle.

Solution:

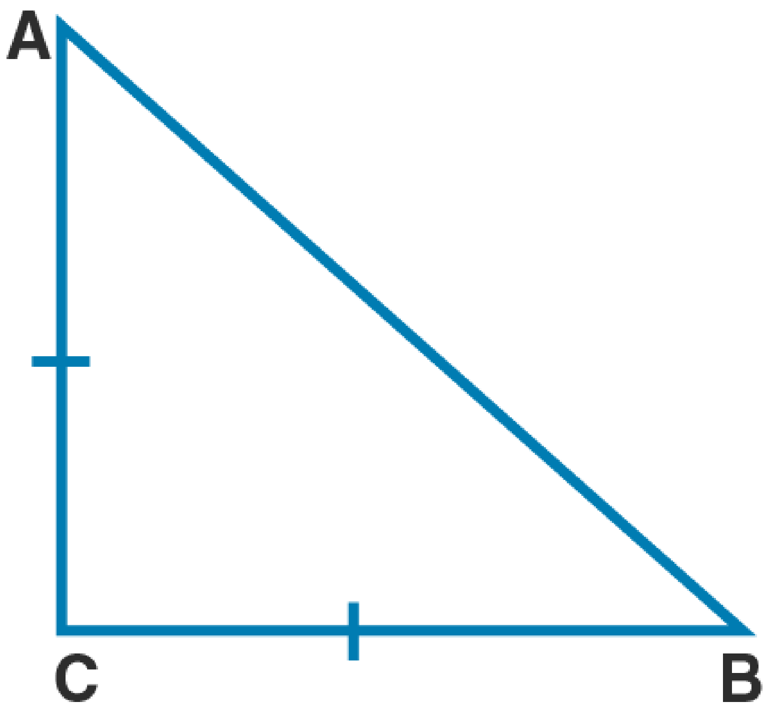


Let the shortest side be x.
Then hypotenuse = 2x+6
Third side = 2x+6-2 = 2x+4
According to Pythagoras theorem,
$AB^2 = CB^2+AC^2$
$(2x+6)^2 = x^2+(2x+4)^2$
$4x^2+24x+36 = x^2+4x^2+16x+16$
$\Rightarrow x^2-8x-20 = 0$
$\Rightarrow$(x-10)(x+2) = 0
$\Rightarrow$ x-10 = 0 or x+2 = 0
x = 10 or x = -2
x cannot be negative.

So shortest side is 10 m.
Hypotenuse = 2x+6
= 2×10+6
= 20+6
= 26 m
Third side = 2x+4
= = 2×10+4
= 20+4
= 24 m
Hence the shortest side, hypotenuse and third side of the triangle are 10 m, 26 m and 24 m respectively.

9. ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.

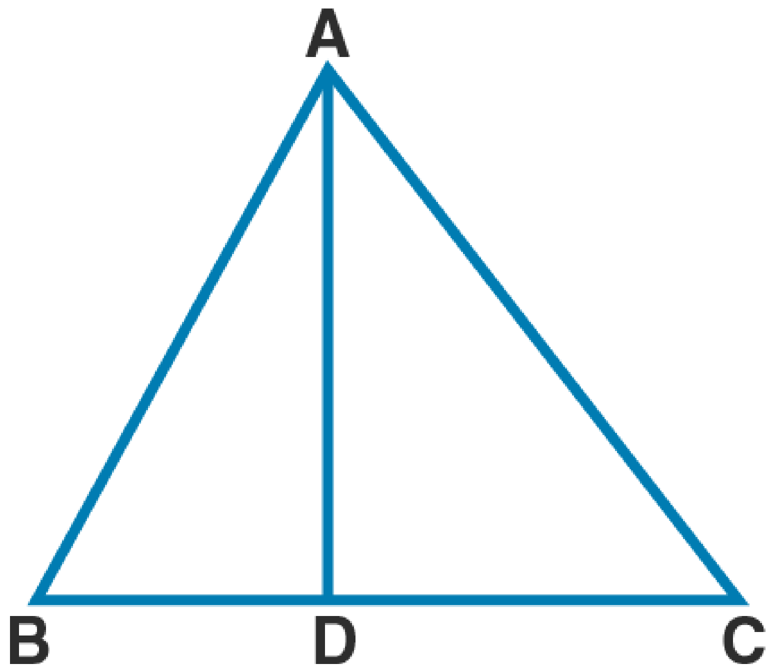
Solution:



Let ABC be the isosceles right angled triangle .
$\angle C = 90^\circ$
AC = BC [isosceles triangle]
According to Pythagoras theorem,
$AB^2 = BC^2+AC^2$
$AB^2 = AC^2+AC^2$ [$\because$ AC = BC]
$\therefore\ AB^2 = 2AC^2$
Hence proved.

10. In a triangle ABC, AD is perpendicular to BC. Prove that $AB^2 + CD^2 = AC^2 + BD^2$.

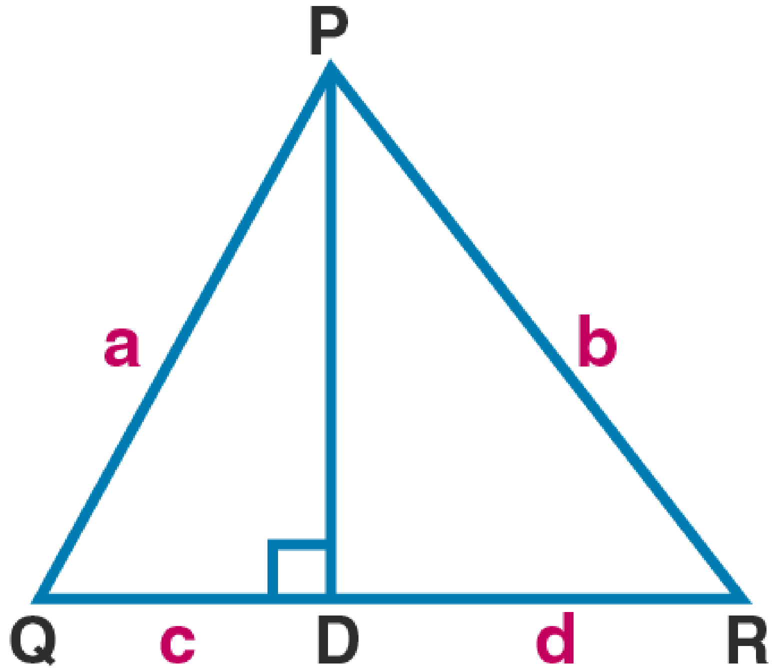
Solution:



Given AD ⊥ BC.
So △ADB and △ADC are right triangles.
In △ADB,
$AB^2 = AD^2+BD^2$ [Pythagoras theorem]
$AD^2 = AB^2- BD^2$ …(i)
In △ADC,
$AC^2 = AD^2+CD^2$ [Pythagoras theorem]
$AD^2 = AC^2- CD^2$ …(ii)
Comparing (i) and (ii)
$AB^2- BD^2 = AC^2- CD^2$
$\therefore\ AB^2+ CD^2 = AC^2+ BD^2$
Hence proved.

11. In ΔPQR, PD ⊥ QR, such that D lies on QR. If PQ = a, PR = b, QD = c and DR = d, prove that (a + b) (a – b) = (c + d) (c – d).

Solution:



Given PQ = a, PR = b, QD = c and DR = d.

PD $\perp$ QR.

So $\triangle$PDQ and $\triangle$PDR are right triangles.

In $\triangle$PDQ,

$PQ^2 = PD^2+QD^2$ [Pythagoras theorem]

$\therefore PD^2 = PQ^2- QD^2$

$\therefore PD^2 = a^2- c^2$ …(i) [$\because$ PQ = a and QD = c]

In $\triangle$PDR,

$PR^2 = PD^2+DR^2$ [Pythagoras theorem]

$\therefore PD^2 = PR^2- DR^2$

$\therefore PD^2 = b^2- d^2$ …(ii) [$\because$ PR = b and DR = d]

Comparing (i) and (ii)

$a^2- c^2= b^2- d^2$

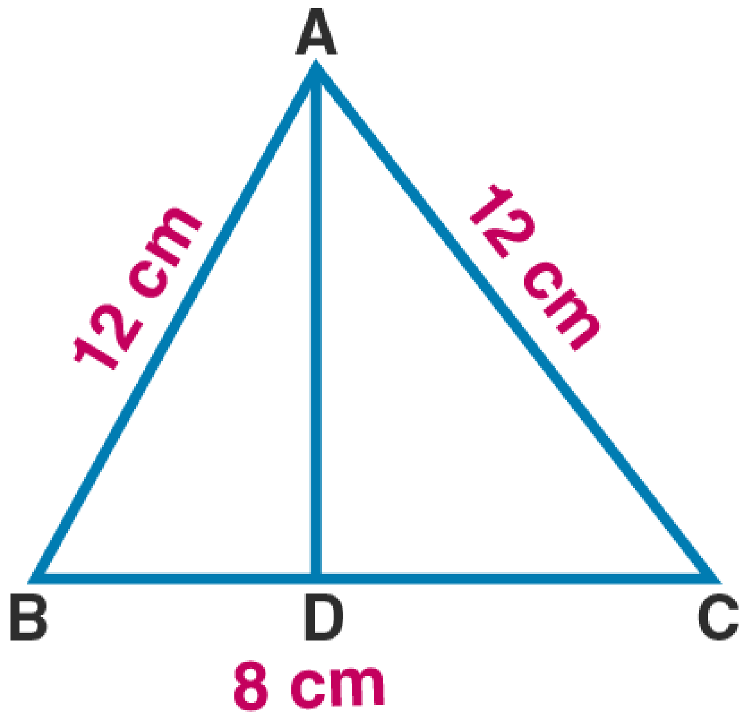
$a^2- b^2= c^2- d^2$

$\therefore (a+b)(a-b) = (c+d)(c-d)$

Hence proved.

12. ABC is an isosceles triangle with AB = AC = 12 cm and BC = 8 cm. Find the altitude on BC and Hence, calculate its area.

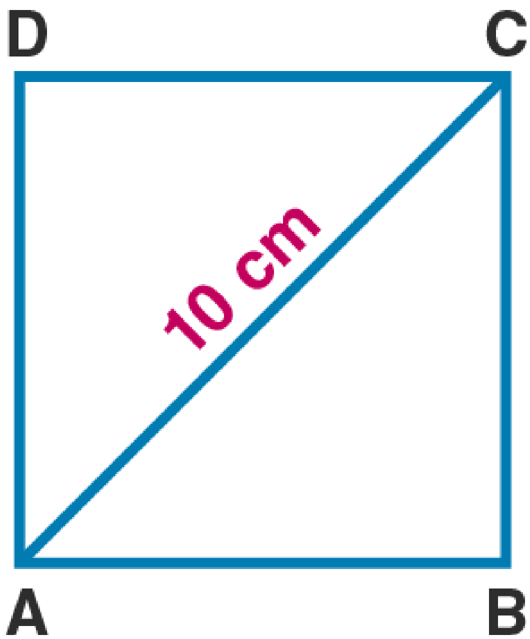
Solution:



Let AD be the altitude of △ABC.
Given AB = AC = 12 cm
BC = 8 cm
The altitude to the base of an isosceles triangle bisects the base.
So BD = DC
∴ BD = 8/2 = 4 cm
DC = 4 cm
△ADC is a right triangle.
$\therefore AB^2 = BD^2 + AD^2$ [Pythagoras theorem]
$\therefore AD^2 = AB^2 - BD^2$
$\therefore AD^2 = 12^2 - 4^2$
$\therefore AD^2 = 144-16$
$\therefore AD^2 = 128$
Taking square root on both sides,
$AD = \sqrt{128} = \sqrt{(2\times 64)} = 8\sqrt{2}$ cm
Area of △ABC = ½ ×base ×height
$= \frac{1}{2} \times 8 \times 8\sqrt{2}$
$= 4 \times 8\sqrt{2}$
$= 32\sqrt{2}$ cm^2
Hence the area of triangle is $32\sqrt{2}$ cm^2.

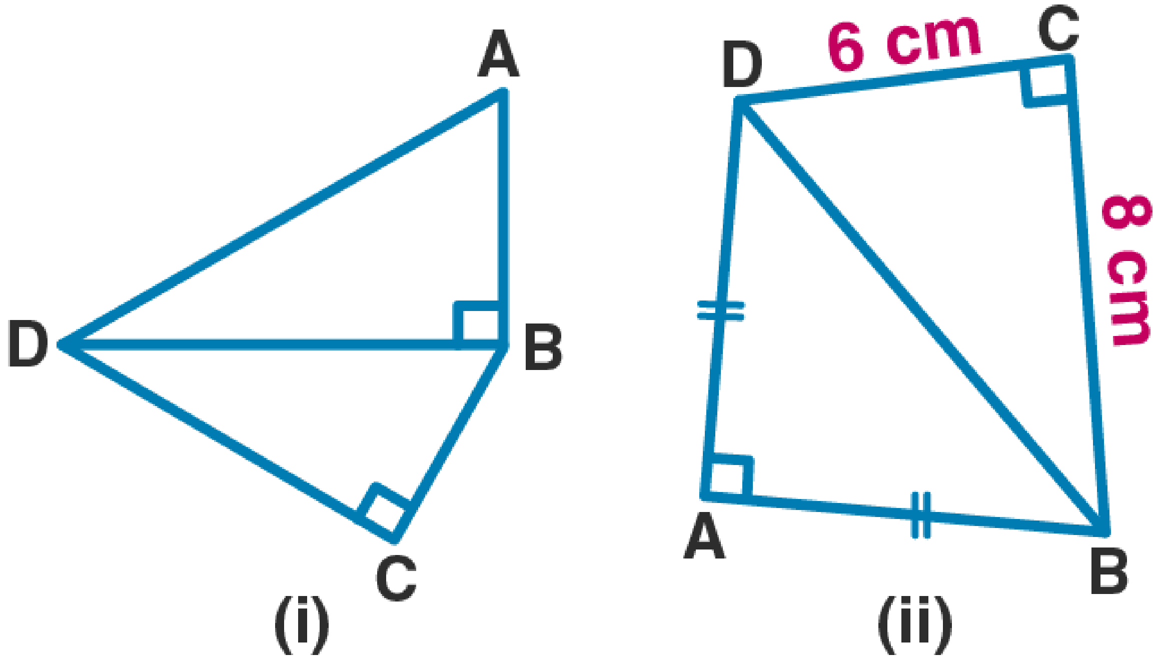
13. Find the area and the perimeter of a square whose diagonal is 10 cm long.

Solution:



Given length of the diagonal of the square is 10 cm.
AC = 10
Let AB = BC = x [Sides of square are equal in measure]
$\angle B = 90^\circ$ [All angles of a square are 90°]
$\triangle ABC$ is a right triangle.
$\therefore AC^2 = AB^2+BC^2$
$\therefore 10^2 = x^2+x^2$
$100 = 2x^2$
$x^2 = 50$
$x = \sqrt{50} = \sqrt{(25\times2)}$
$\therefore x = 5\sqrt{2}$
So area of square = x^2
$= (5\sqrt{2})^2 = 50$ cm^2
Perimeter = 4x
$= 4\times5\sqrt{2}$
$= 20\sqrt{2}$ cm
Hence area and perimeter of the square are 50 cm^2 and $20\sqrt{2}$ cm.

14. (a) In fig. (i) given below, ABCD is a quadrilateral in which AD = 13 cm, DC = 12 cm, BC = 3 cm, $\angle$ ABD = $\angle$BCD = 90°. Calculate the length of AB.
(b) In fig. (ii) given below, ABCD is a quadrilateral in which AB = AD, $\angle$A = 90° =$\angle$C, BC = 8 cm and CD = 6 cm. Find AB and calculate the area of $\triangle$ ABD.



Solution:

(i)Given AD = 13 cm, DC = 12 m

BC = 3 cm

$\angle ABD = \angle BCD = 90^\circ$

$\triangle BCD$ is a right triangle.

$\therefore BD^2 = BC^2+DC^2$ [Pythagoras theorem]

$\therefore BD^2 = 3^2+12^2$

$\therefore BD^2 = 9+144$

$\therefore BD^2 = 153$

$\triangle ABD$ is a right triangle.

$\therefore AD^2 = AB^2+BD^2$ [Pythagoras theorem]

$\therefore 13^2 = AB^2+153$

$\therefore 169 = AB^2+153$

$\therefore AB^2 = 169-153$

$\therefore AB^2 = 16$

Taking square root on both sides,

AB = 4 cm

Hence the length of AB is 4 cm.

(ii)Given AB = AD, $\angle A = 90^\circ = \angle C$, BC = 8 cm and CD = 6 cm

$\triangle BCD$ is a right triangle.

$\therefore BD^2 = BC^2+DC^2$ [Pythagoras theorem]

$\therefore BD^2 = 8^2+6^2$

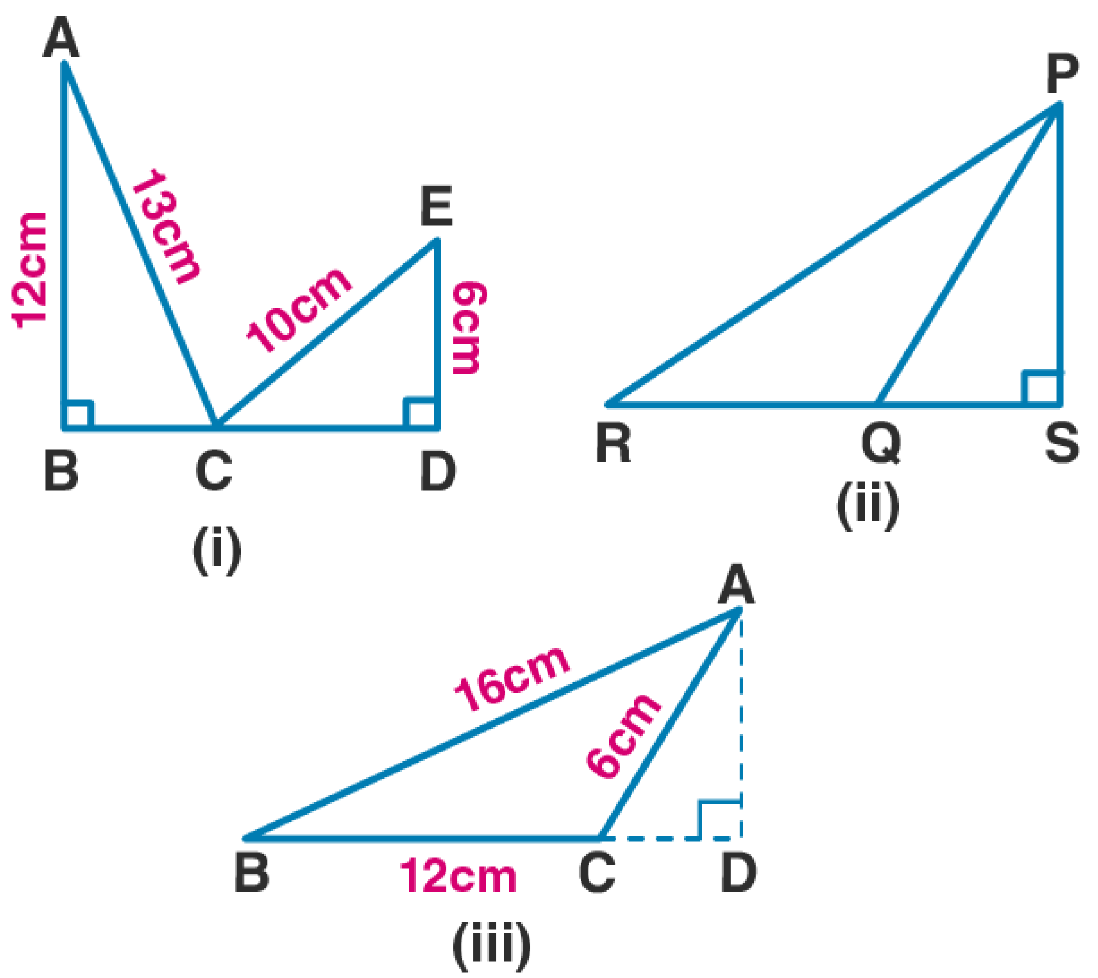
$\therefore BD^2 = 64+36$

$\therefore BD^2 = 100$

Taking square root on both sides,

BD = 10 cm
△ABD is a right triangle.
$\therefore BD^2 = AB^2+AD^2$ [Pythagoras theorem]
$10^2 = 2AB^2$ [$\because$ AB = AD]
$100 = 2AB^2$
$\therefore AB^2 = 100/2$
$\therefore AB^2 = 50$
Taking square root on both sides,
$AB = \sqrt{50}$
$AB = \sqrt{(2\times25)}$
$AB = 5\sqrt{2}$ cm
Hence the length of AB is $5\sqrt{2}$ cm.

15. (a) In figure (i) given below, AB = 12 cm, AC = 13 cm, CE = 10 cm and DE = 6 cm. Calculate the length of BD.
(b) In figure (ii) given below, ∠PSR = 90°, PQ = 10 cm, QS = 6 cm and RQ = 9 cm. Calculate the length of PR.
(c) In figure (iii) given below, ∠ D = 90°, AB = 16 cm, BC = 12 cm and CA = 6 cm. Find CD.



Solution:

(a)Given AB = 12 cm, AC = 13 cm, CE = 10 cm and DE = 6 cm

△ABC is a right triangle.

$\therefore AC^2 = AB^2+BC^2$ [Pythagoras theorem]

$\therefore 13^2 = 12^2+BC^2$

$\therefore BC^2 = 13^2-12^2$

$\therefore BC^2 = 169-144$

$\therefore BC^2 = 25$

Taking square root on both sides,

BC = 5 cm

△CDE is a right triangle.

$\therefore CE^2 = CD^2+DE^2$ [Pythagoras theorem]

$\therefore 10^2 = CD^2+6^2$

$\therefore 100 = CD^2+36$

$\therefore CD^2 = 100-36$

$\therefore CD^2 = 64$

Taking square root on both sides,

CD = 8 cm

$\therefore$ BD = BC +CD

$\therefore$ BD = 5+8

$\therefore$ BD = 13 cm

Hence the length of BD is 13 cm.

(b) Given ∠PSR = 90°, PQ = 10 cm, QS = 6 cm and RQ = 9 cm

△PSQ is a right triangle.

$\therefore PQ^2 = PS^2+QS^2$ [Pythagoras theorem]

$10^2 = PS^2+6^2$

$100 = PS^2+36$

$\therefore PS^2 = 100-36$

$\therefore PS^2 = 64$

Taking square root on both sides,

PS = 8 cm

△PSR is a right triangle.

RS = RQ+QS

RS = 9+6

RS = 15 cm

$\therefore PR^2 = PS^2+RS^2$ [Pythagoras theorem]

$PR^2 = 8^2+15^2$

$PR^2 = 64+225$

$PR^2 = 289$

Taking square root on both sides,

PR = 17 cm

Hence the length of PR is 17 cm.

(c) ∠D = 90°, AB = 16 cm, BC = 12 cm and CA = 6 cm

△ADC is a right triangle.

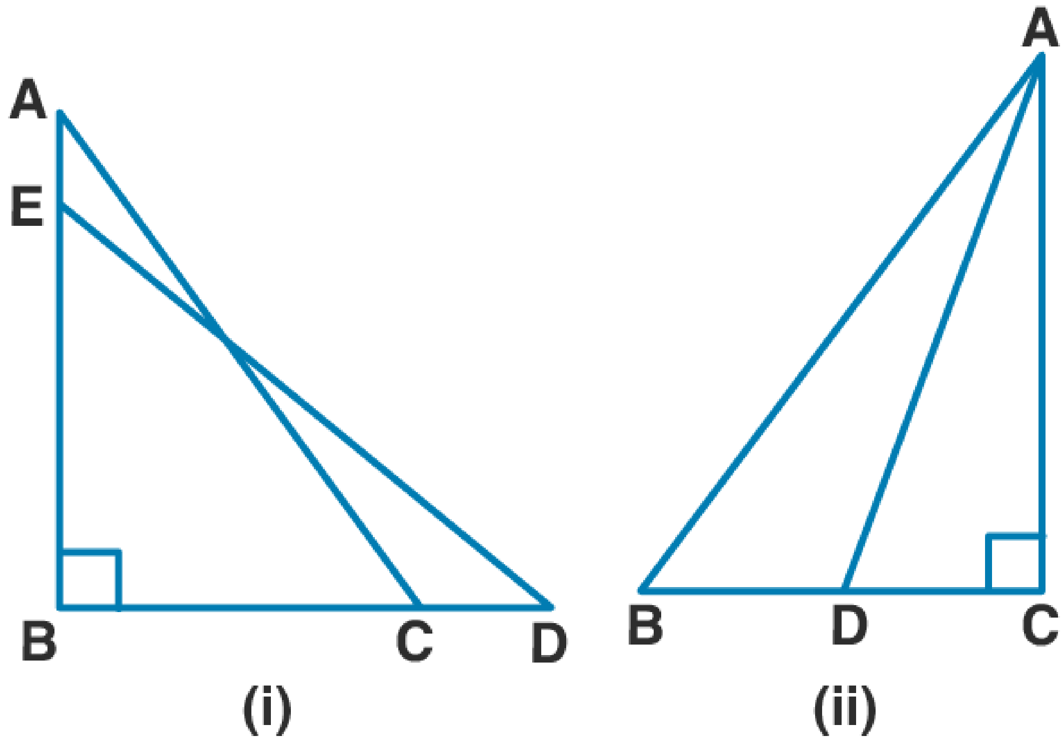
$\therefore AC^2 = AD^2+CD^2$ [Pythagoras theorem]

$6^2 = AD^2+CD^2$ …..(i)
$\triangle ABD$ is a right triangle.
$\therefore AB^2 = AD^2+BD^2$ [Pythagoras theorem]
$16^2 = AD^2+(BC+CD)^2$
$16^2 = AD^2+(12+CD)^2$
$256 = AD^2+144+24CD+CD^2$
$256-144 = AD^2+CD^2+24CD$
$AD^2+CD^2 = 112-24CD$
$6^2 = 112-24CD$ [from (i)]
$36 = 112-24CD$
$24CD = 112-36$
$24CD = 76$
$\therefore CD = 76/24 = 19/6$

$$\therefore CD = 3\ \frac{1}{6}$$

Hence the length of CD is $3\ \frac{1}{6}$ cm

16. (a) In figure (i) given below, BC = 5 cm,
∠B =90°, AB = 5AE, CD = 2AE and AC = ED. Calculate the lengths of EA, CD, AB and AC.
(b) In the figure (ii) given below, ABC is a right triangle right angled at C. If D is mid-point of BC, prove that $AB^2 = 4AD^2 – 3AC^2$.



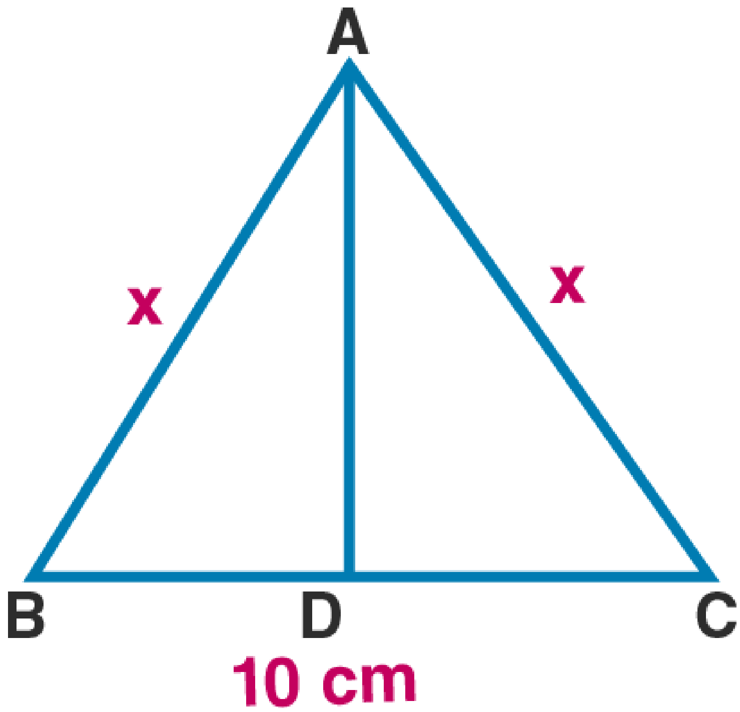
Solution:

(a)Given BC = 5 cm,
$\angle B = 90°$, AB = 5AE,
CD = 2AE and AC = ED
$\triangle ABC$ is a right triangle.
$\therefore AC^2 = AB^2+BC^2$ …(i) [Pythagoras theorem]
$\triangle BED$ is a right triangle.
$\therefore ED^2 = BE^2+BD^2$ [Pythagoras theorem]
$\therefore AC^2 = BE^2+BD^2$ …(ii) [$\because$ AC = ED]
Comparing (i) and (ii)
$AB^2+BC^2 = BE^2+BD^2$
$(5AE)^2+5^2 = (4AE)^2+(BC+CD)^2$ [$\because$ BE = AB-AE = 5AE-AE = 4AE]
$(5AE)^2+25 = (4AE)^2+(5+2AE)^2$ …(iii) [$\because$ BC = 5, CD = 2AE]
Let AE = x. So (iii) becomes,
$(5x)^2+25 = (4x)^2+(5+2x)^2$
$25x^2+25 = 16x^2+25+20x+4x^2$
$25x^2 = 20x^2+20x$
$5x^2 = 20x$
$\therefore x = 20/5 = 4$
$\therefore$ AE = 4 cm
$\therefore$ CD = 2AE = 2×4 = 8 cm
$\therefore$ AB = 5AE
$\therefore$ AB = 5×4 = 20 cm
$\triangle ABC$ is a right triangle.
$\therefore AC^2 = AB^2+BC^2$ [Pythagoras theorem]
$\therefore AC^2 = 20^2+5^2$
$\therefore AC^2 = 400+25$
$\therefore AC^2 = 425$
Taking square root on both sides,
$AC = \sqrt{425} = \sqrt{(25\times17)}$
$AC = 5\sqrt{17}$ cm
Hence EA = 4 cm, CD = 8 cm, AB = 20 cm and AC = $5\sqrt{17}$ cm.

(b)Given D is the midpoint of BC.
$\therefore DC = \frac{1}{2} BC$
$\triangle ABC$ is a right triangle.
$\therefore AB^2 = AC^2+BC^2$ …(i) [Pythagoras theorem]
$\triangle ADC$ is a right triangle.
$\therefore AD^2 = AC^2+DC^2$ …(ii) [Pythagoras theorem]
$AC^2 = AD^2-DC^2$
$AC^2 = AD^2-(\frac{1}{2} BC)^2$ [$\because DC = \frac{1}{2} BC$]
$AC^2 = AD^2-\frac{1}{4} BC^2$
$4AC^2 = 4AD^2-BC^2$
$AC^2+3AC^2 = 4AD^2-BC^2$
$AC^2+BC^2 = 4AD^2-3AC^2$
$\therefore AB^2 = 4AD^2-3AC^2$ [from (i)]
Hence proved.

17. In △ ABC, AB = AC = x, BC = 10 cm and the area of △ ABC is 60 cm². Find x.

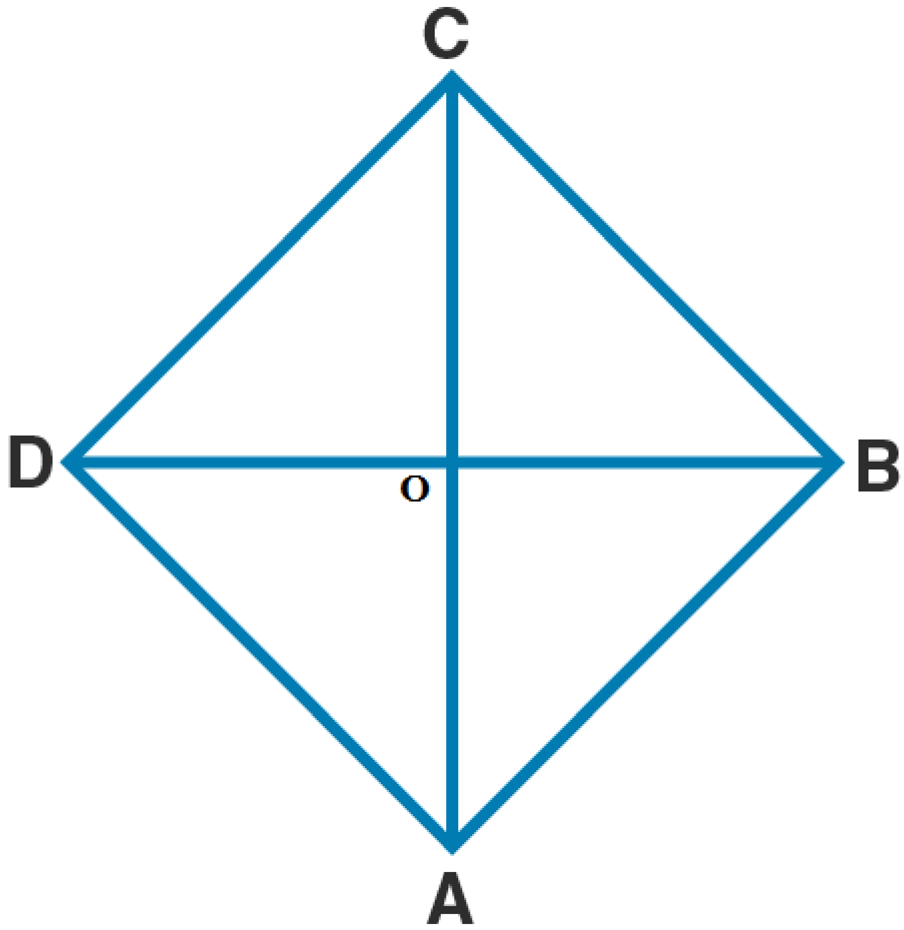
Solution:



Given AB = AC = x
So ABC is an isosceles triangle.
AD ⊥ BC
The altitude to the base of an isosceles triangle bisects the base.
∴ BD = DC = 10/2 = 5 cm
Given area = 60 cm^2
∴ ½ ×base ×height = ½ ×10×AD = 60
∴ AD = 60×2/10
∴ AD = 60/5
∴ AD = 12cm
△ADC is a right triangle.
∴ $AC^2 = AD^2+DC^2$
∴ $x^2 = 12^2+5^2$
∴ $x^2 = 144+25$
∴ $x^2 = 169$
Taking square root on both sides
x = 13 cm
Hence the value of x is 13 cm.

18. In a rhombus, If diagonals are 30 cm and 40 cm, find its perimeter.

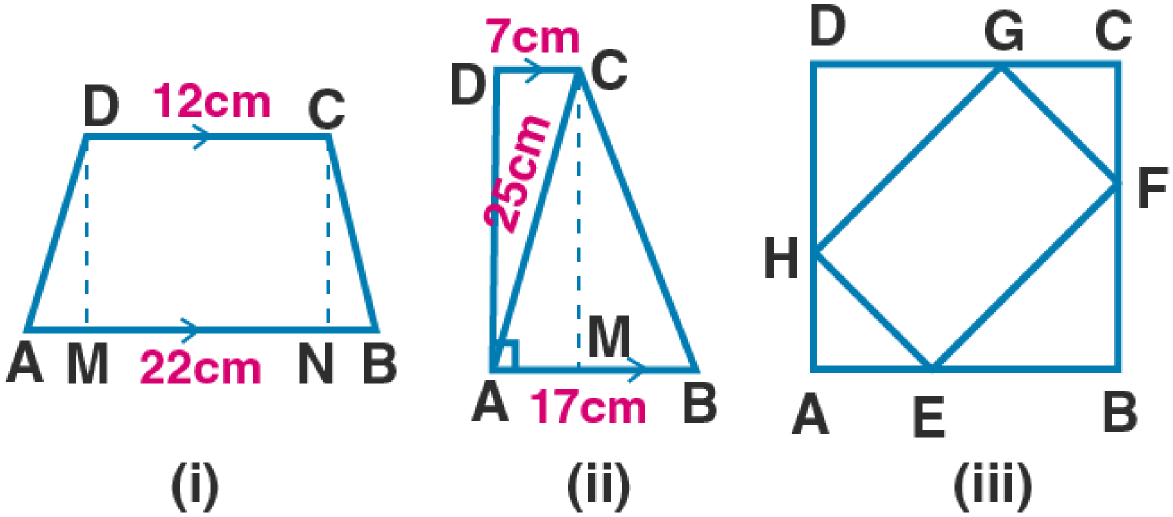
Solution:



Let ABCD be the rhombus.
Given AC = 30cm
BD = 40 cm
Diagonals of a rhombus are perpendicular bisectors of each other.
$\therefore$ OB = ½ BD = ½ ×40 = 20 cm
OC = ½ AC = ½ ×30 = 15 cm
△OCB is a right triangle.
$\therefore BC^2 = OC^2+OB^2$ [Pythagoras theorem]
$\therefore BC^2 = 15^2+20^2$
$\therefore BC^2 = 225+400$
$\therefore BC^2 = 625$
Taking square root on both sides
BC = 25 cm
So side of a rhombus, a = 25 cm.
Perimeter = 4a = 4×25 = 100 cm
Hence the perimeter of the rhombus is 100 cm.

19. (a) In figure (i) given below, AB || DC, BC = AD = 13 cm. AB = 22 cm and DC = 12cm. Calculate the height of the trapezium ABCD.

(b) In figure (ii) given below, AB || DC, ∠ A = 90°, DC = 7 cm, AB = 17 cm and AC = 25 cm. Calculate BC.
(c) In figure (iii) given below, ABCD is a square of side 7 cm. if
AE = FC = CG = HA = 3 cm,
(i) prove that EFGH is a rectangle.
(ii) find the area and perimeter of EFGH.

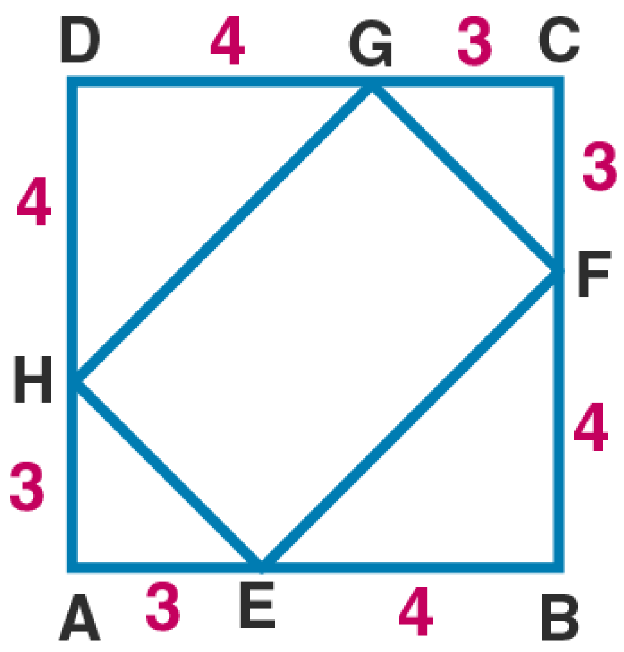


Solution:
(i) Given AB || DC, BC = AD = 13 cm.
AB = 22 cm and DC = 12cm
Here DC = 12
∴ MN = 12 cm
AM = BN
AB = AM+MN+BN
22 = AM+12+AM [∵ AM = BN]
2AM = 22-12 = 10
∴ AM = 10/2
∴ AM = 5 cm
△AMD is a right triangle.
$AD^2 = AM^2+DM^2$ [Pythagoras theorem]
$13^2 = 5^2+DM^2$
∴ $DM^2 = 13^2-5^2$
∴ $DM^2 = 169-25$
∴ $DM^2 = 144$
Taking square root on both sides,
DM = 12 cm
Hence the height of the trapezium is 12 cm.

(b) Given AB || DC, ∠A = 90°, DC = 7 cm,
AB = 17 cm and AC = 25 cm
△ADC is a right triangle.

$\therefore AC^2 = AD^2+DC^2$ [Pythagoras theorem]
$25^2 = AD^2+7^2$
$\therefore AD^2 = 25^2-7^2$
$\therefore AD^2 = 625-49$
$\therefore AD^2 = 576$
Taking square root on both sides
AD = 24 cm
$\therefore$ CM = 24 cm [$\because$ AB ॥ CD]
DC = 7 cm
$\therefore$ AM = 7 cm
BM = AB-AM
$\therefore$ BM = 17-7 = 10 cm
$\triangle$BMC is a right triangle.
$\therefore BC^2 = BM^2+CM^2$
$BC^2 = 10^2+24^2$
$BC^2 = 100+576$
$BC^2 = 676$
Taking square root on both sides
BC = 26 cm
Hence length of BC is 26 cm.

(c) (i)Proof:



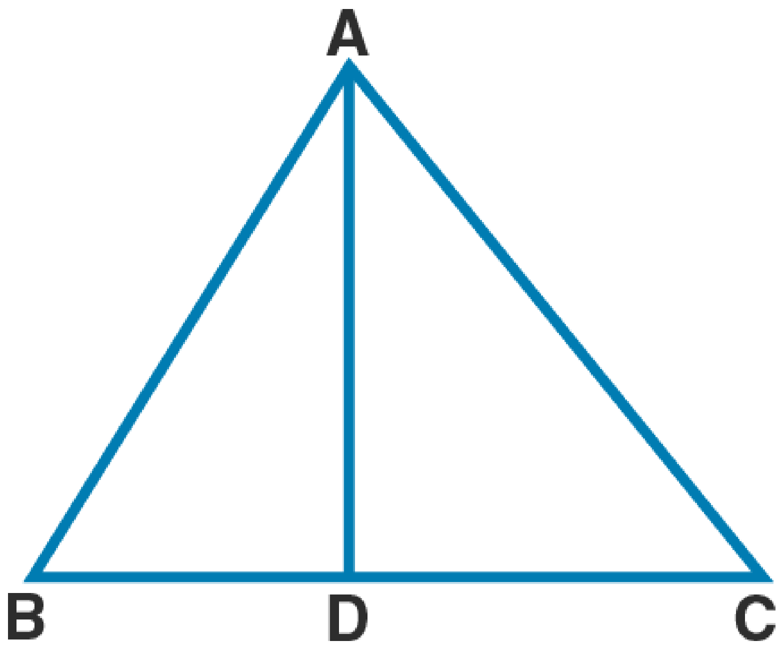
Given ABCD is a square of side 7 cm.
So AB = BC = CD = AD = 7 cm
Also given AE = FC = CG = HA = 3 cm

BE = AB-AE = 7-3 = 4 cm
BF = BC-FC = 7-3 = 4 cm
GD = CD-CG = 7-3 = 4 cm
DH = AD-HA = 7-3 = 4 cm
$\angle A = 90^\circ$ [Each angle of a square equals 90°]
$\triangle$AHE is a right triangle.
$\therefore HE^2 = AE^2+AH^2$ [Pythagoras theorem]
$\therefore HE^2 = 3^2+3^2$
$\therefore HE^2 = 9+9 = 18$
HE = $\sqrt{(9\times2)} = 3\sqrt{2}$ cm
Similarly GF = $3\sqrt{2}$ cm
$\triangle$EBF is a right triangle.
$\therefore EF^2 = BE^2+BF^2$ [Pythagoras theorem]
$\therefore EF^2 = 4^2+4^2$
$\therefore EF^2 = 16+16 = 32$
Taking square root on both sides
EF = $\sqrt{(16\times2)} = 4\sqrt{2}$ cm
Similarly HG = $4\sqrt{2}$ cm
Now join EG
In $\triangle$EFG
$EG^2 = EF^2+GF^2$
$EG^2 = (4\sqrt{2})^2+(3\sqrt{2})^2$
$EG^2 = 32+18 = 50$
$\therefore EG = \sqrt{50} = 5\sqrt{2}$ cm …(i)
Join HF.
Also $HF^2 = EH^2+HG^2$
$= (3\sqrt{2})^2+(4\sqrt{2})^2$
= 18+32 = 50
HF = $\sqrt{50} = 5\sqrt{2}$ cm …(ii)
From (i) and (ii)
EG = HF
Diagonals of the quadrilateral are congruent. So EFGH is a rectangle.
Hence proved.

(ii)Area of rectangle EFGH = length × breadth
= HE ×EF
= $3\sqrt{2}\times4\sqrt{2}$
= 24 cm^2
Perimeter of rectangle EFGH = 2(length+breadth)
= $2\times(4\sqrt{2}+3\sqrt{2})$
= $2\times7\sqrt{2}$
= $14\sqrt{2}$ cm
Hence area of the rectangle is 24 cm^2 and perimeter is $14\sqrt{2}$ cm.

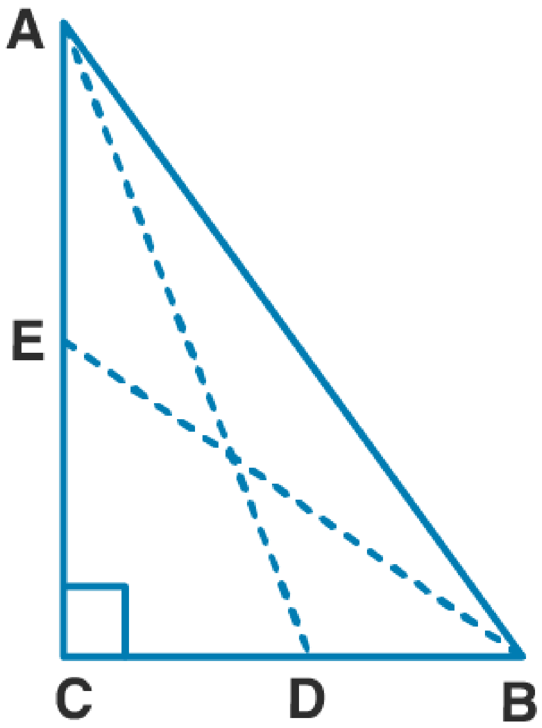
20. AD is perpendicular to the side BC of an equilateral Δ ABC. Prove that $4AD^2 = 3AB^2$.

Solution:



Given AD ⊥ BC
$\angle D = 90^\circ$
Proof:
Since ABC is an equilateral triangle,
AB = AC = BC
△ABD is a right triangle.
According to Pythagoras theorem,
$AB^2 = AD^2+BD^2$
BD = ½ BC
$\therefore\ AB^2 = AD^2+(\frac{1}{2}BC)^2$
$AB^2 = AD^2+(\frac{1}{2}AB)^2$ [$\because$ BC = AB]
$AB^2 = AD^2+\frac{1}{4}AB^2$
$AB^2 = (4AD^2+AB^2)/4$
$\therefore 4AB^2 = 4AD^2+AB^2$
$\therefore 4AD^2 = 4AB^2-AB^2$
$\therefore 4AD^2 = 3AB^2$
Hence proved.

21. In figure (i) given below, D and E are mid-points of the sides BC and CA respectively of a ΔABC, right angled at C.
Prove that :
(i)$4AD^2 = 4AC^2+BC^2$
(ii)$4BE^2 = 4BC^2+AC^2$
(iii)$4(AD^2+BE^2) = 5AB^2$



Solution:

Proof:

(i)$\angle C = 90^\circ$

So $\triangle ACD$ is a right triangle.

$AD^2 = AC^2+CD^2$ [Pythagoras theorem]

Multiply both sides by 4, we get

$4AD^2 = 4AC^2+4CD^2$

$4AD^2 = 4AC^2+4BD^2$ [$\because$ D is the midpoint of BC, CD = BD = ½ BC]

$4AD^2 = 4AC^2+(2BD)^2$

$4AD^2 = 4AC^2+BC^2$….(i) [$\because$ BC = 2BD]

Hence proved.

(ii)$\triangle BCE$ is a right triangle.

$\therefore BE^2 = BC^2+CE^2$ [Pythagoras theorem]

Multply both sides by 4 , we get

$4BE^2 = 4BC^2+4CE^2$

$4BE^2 = 4BC^2+(2CE)^2$

$4BE^2 = 4BC^2+AC^2$ ….(ii) [$\because$ E is the midpoint of AC, AE = CE = ½ AC]

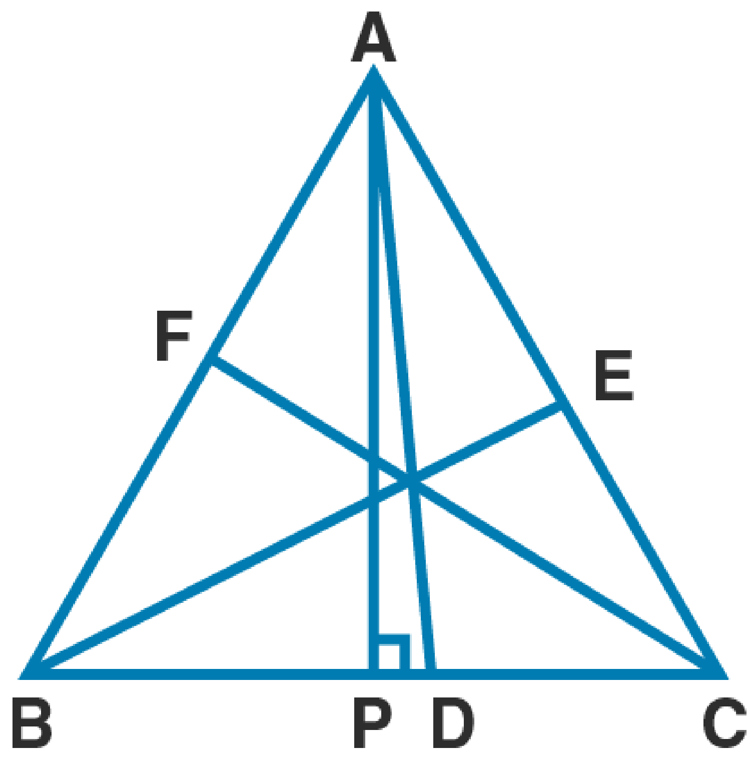
Hence proved.

(iii)Adding (i) and (ii)

$4AD^2+4BE^2 = 4AC^2+BC^2+4BC^2+AC^2$
$4AD^2+4BE^2 = 5AC^2+5BC^2$
$4(AD^2+BE^2) = 5(AC^2+BC^2)$
$4(AD^2+BE^2) = 5(AB^2)$ [$\because$ ABC is a right triangle, $AB^2 = AC^2+BC^2$]
Hence proved.

22. If AD, BE and CF are medians of △ABC, prove that 3(AB² + BC² + CA²) = 4(AD² + BE² + CF²).

Solution:



Construction:
Draw AP⊥BC
Proof:
△APB is a right triangle.
$\therefore AB^2 = AP^2+BP^2$ [Pythagoras theorem]
$\therefore AB^2 = AP^2+(BD-PD)^2$
$\therefore AB^2 = AP^2+BD^2+PD^2-2BD\times PD$
$\therefore AB^2 = (AP^2+PD^2)+BD^2-2BD\times PD$
$\therefore AB^2 = AD^2+ (\frac{1}{2} BC)^2-2\times(\frac{1}{2} BC)\times PD$ [$\because AP^2+PD^2 = AD^2$ and $BD = \frac{1}{2} BC$]

$\therefore AB^2 = AD^2 + \frac{1}{4} BC^2 - BC \times PD$ ....(i)

$\triangle APC$ is a right triangle.

$AC^2 = AP^2+PC^2$ [Pythagoras theorem]

$AC^2 = AP^2+(PD^2+DC^2)$

$AC^2 = AP^2+PD^2+DC^2+2\times PD\times DC$

$AC^2 = (AP^2+PD^2)+ (\frac{1}{2} BC)^2+2\times PD\times(\frac{1}{2} BC)$ [$DC = \frac{1}{2} BC$]

$AC^2 = (AD)^2+ \frac{1}{4} BC^2+PD\times BC$ ...(ii) [In $\triangle APD$, $AP^2+PD^2 = AD^2$]

Adding (i) and (ii), we get

$AB^2+AC^2 = 2AD^2+ \frac{1}{2} BC^2$ .....(iii)

Draw perpendicular from B and C to AC and AB respectively.

Similarly we get,

$BC^2+CA^2 = 2CF^2+ \frac{1}{2} AB^2$ ....(iv)

$AB^2+BC^2 = 2BE^2+ \frac{1}{2} AC^2$ ....(v)

Adding (iii), (iv) and (v), we get

$2(AB^2+BC^2+CA^2) = 2(AD^2+BE^2+CF^2)+ \frac{1}{2} (BC^2+AB^2+AC^2)$

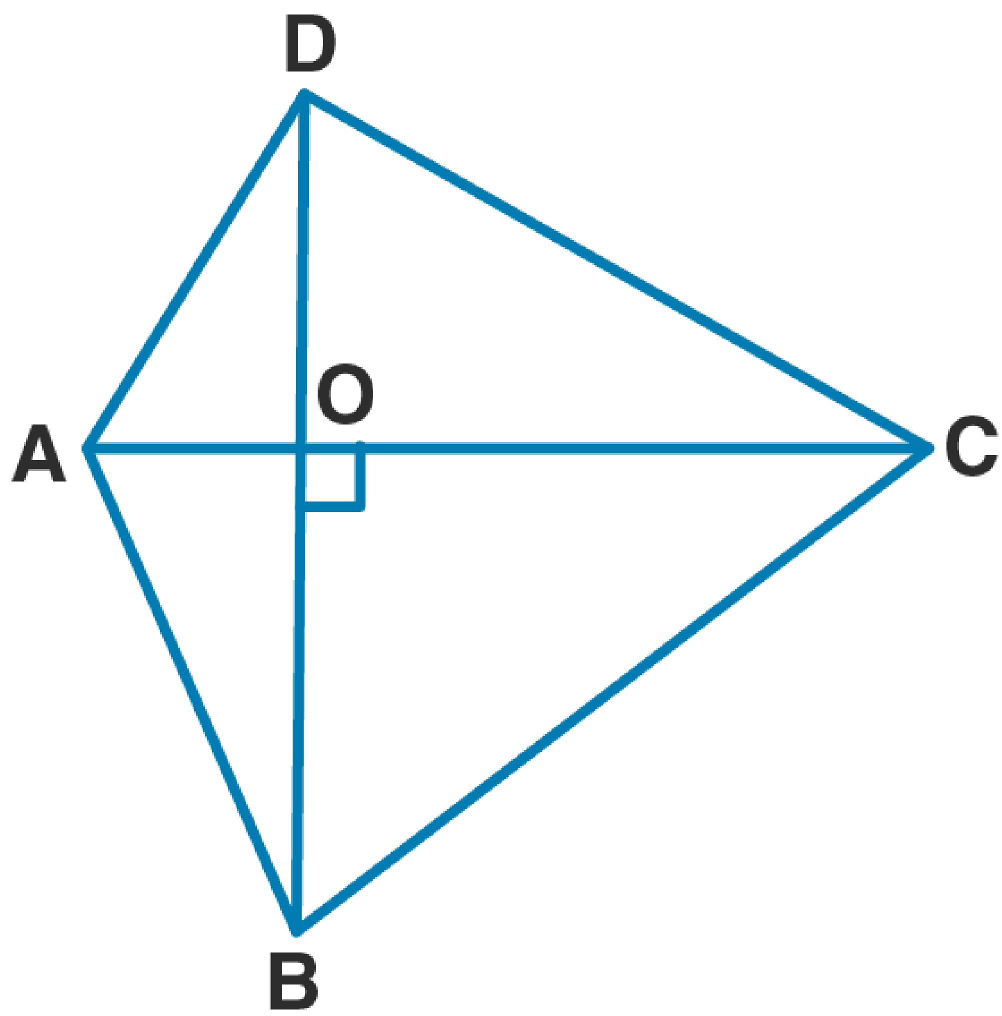
$2(AB^2+BC^2+CA^2) = 2(AB^2+BC^2+CA^2) - \frac{1}{2} (AB^2+BC^2+CA^2)$

$2(AD^2+BE^2+CF^2) = (3/2)\times (AB^2+BC^2+CA^2)$

$\therefore 4(AD^2+BE^2+CF^2) = 3(AB^2+BC^2+CA^2)$

Hence proved.

23.(a) In fig. (i) given below, the diagonals AC and BD of a quadrilateral ABCD intersect at O, at right angles. Prove that $AB^2 + CD^2 = AD^2 + BC^2$.



Solution:

Given diagonals of quadrilateral ABCD, AC and BD intersect at O at right angles.

Proof:

$\triangle$AOB is a right triangle.

$\therefore AB^2 = OB^2+OA^2$ …(i) [Pythagoras theorem]

$\triangle$COD is a right triangle.

$\therefore CD^2 = OC^2+OD^2$ …(ii) [Pythagoras theorem]

Adding (i) and (ii), we get

$AB^2+ CD^2 = OB^2+OA^2+ OC^2+OD^2$

$AB^2+ CD^2 = (OA^2+OD^2)+ (OC^2+OB^2)$ …(iii)

$\triangle$AOD is a right triangle.

$\therefore AD^2 = OA^2+OD^2$ …(iv) [Pythagoras theorem]

$\triangle$BOC is a right triangle.

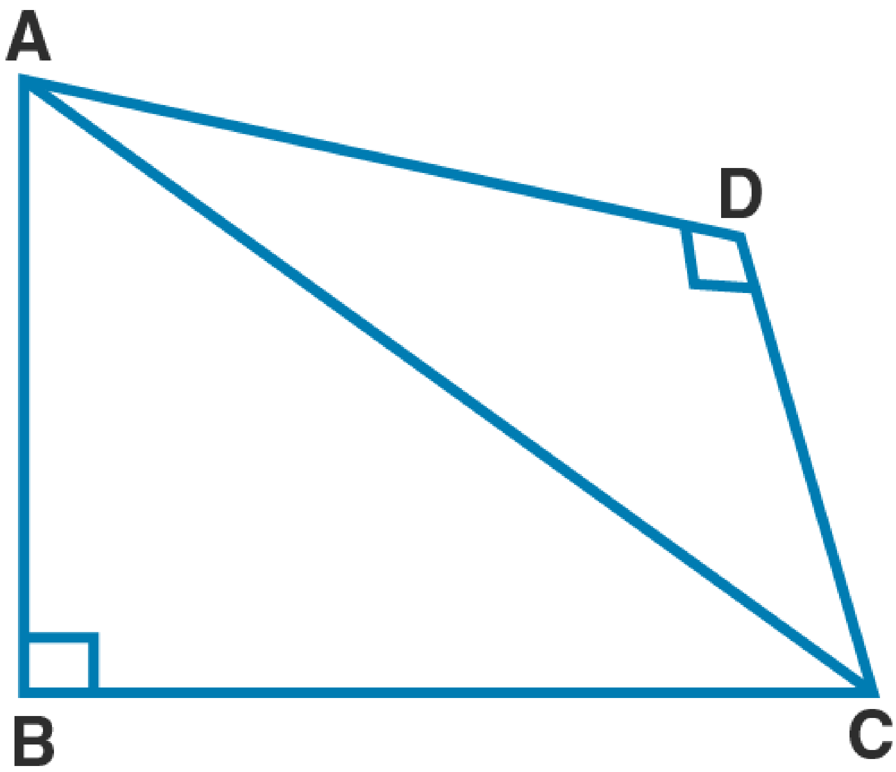
$\therefore BC^2 = OC^2+OB^2$ …(v) [Pythagoras theorem]

Substitute (iv) and (v) in (iii), we get

$AB^2 + CD^2 = AD^2 + BC^2$
Hence proved.

24. In a quadrilateral ABCD, ∠B = 90° = ∠D. Prove that 2 AC² – BC² = AB² + AD² + DC².

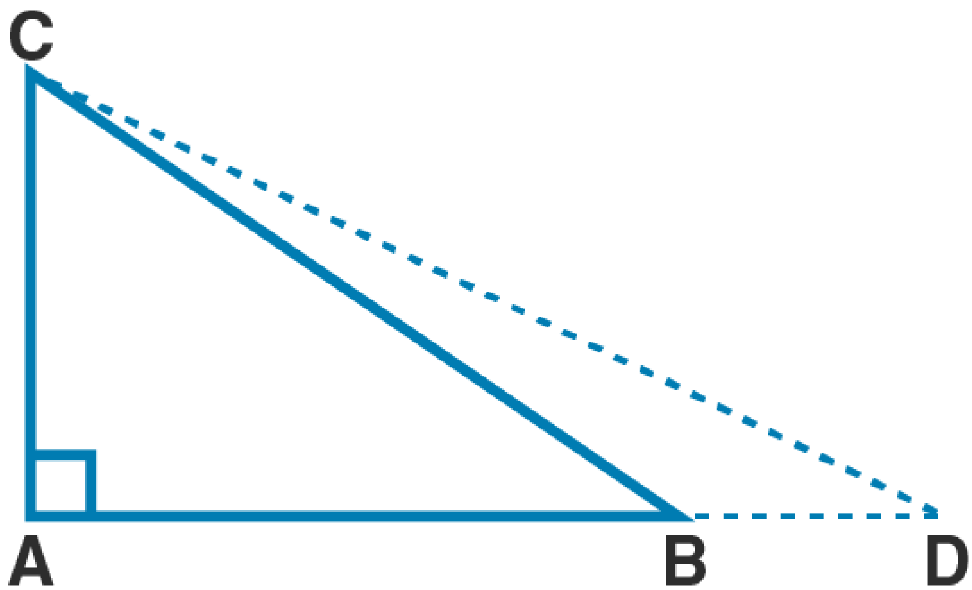
Solution:



Given $\angle B = \angle D = 90^\circ$
So $\triangle ABC$ and $\triangle ADC$ are right triangles.
In $\triangle ABC$,
$AC^2 = AB^2 + BC^2$ …(i) [Pythagoras theorem]
In $\triangle ADC$,
$AC^2 = AD^2 + DC^2$ …(ii) [Pythagoras theorem]
Adding (i) and (ii)
$2AC^2 = AB^2 + BC^2 + AD^2 + DC^2$
$\therefore 2AC^2 - BC^2 = AB^2 + AD^2 + DC^2$
Hence proved.

25. In a △ ABC, ∠ A = 90°, CA = AB and D is a point on AB produced. Prove that : DC² – BD² = 2AB×AD.

Solution:



Given $\angle A = 90^\circ$

CA = AB

Proof:

In $\triangle ACD$,

$DC^2 = CA^2+AD^2$ [Pythagoras theorem]

$DC^2 = CA^2+(AB+BD)^2$

$DC^2 = CA^2+AB^2+BD^2+2AB\times BD$

$DC^2 - BD^2 = CA^2+AB^2+2AB\times BD$

$DC^2 - BD^2 = AB^2+AB^2+2AB\times BD$ [$\because$ CA = AB]

$DC^2 - BD^2 = 2AB^2+2AB\times BD$

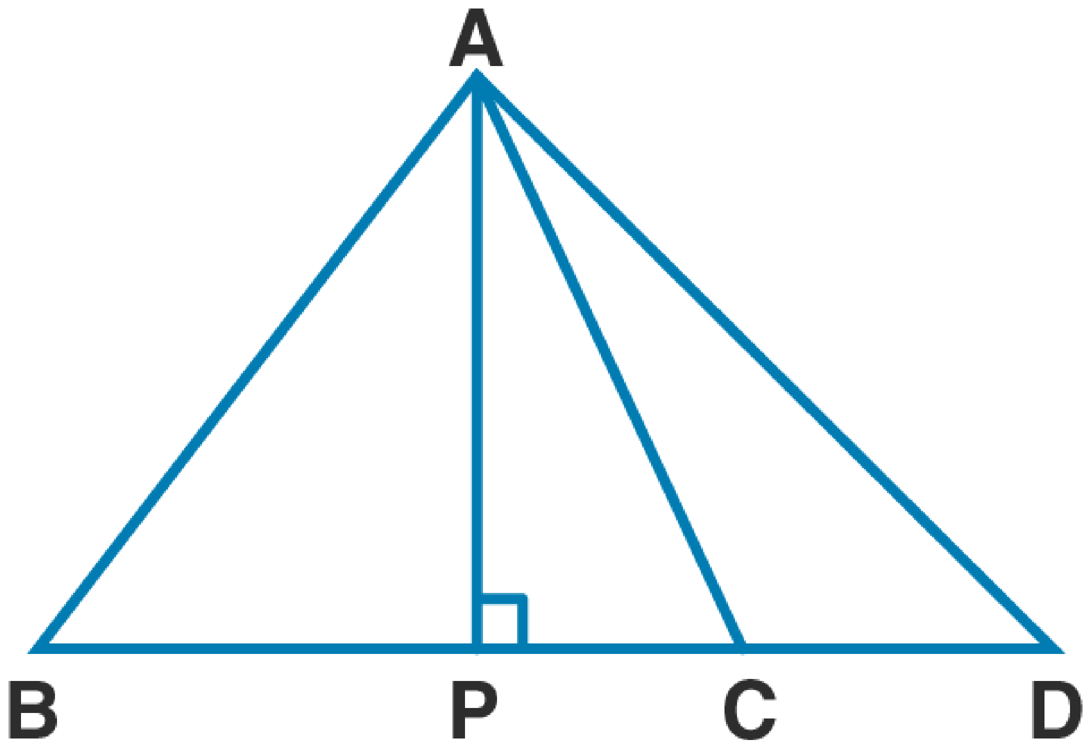
$DC^2 - BD^2 = 2AB(AB+BD)$

$DC^2 - BD^2 = 2AB\times AD$ [A-B-D]

Hence proved.

26. In an isosceles triangle ABC, AB = AC and D is a point on BC produced. Prove that $AD^2 = AC^2+BD.CD$.

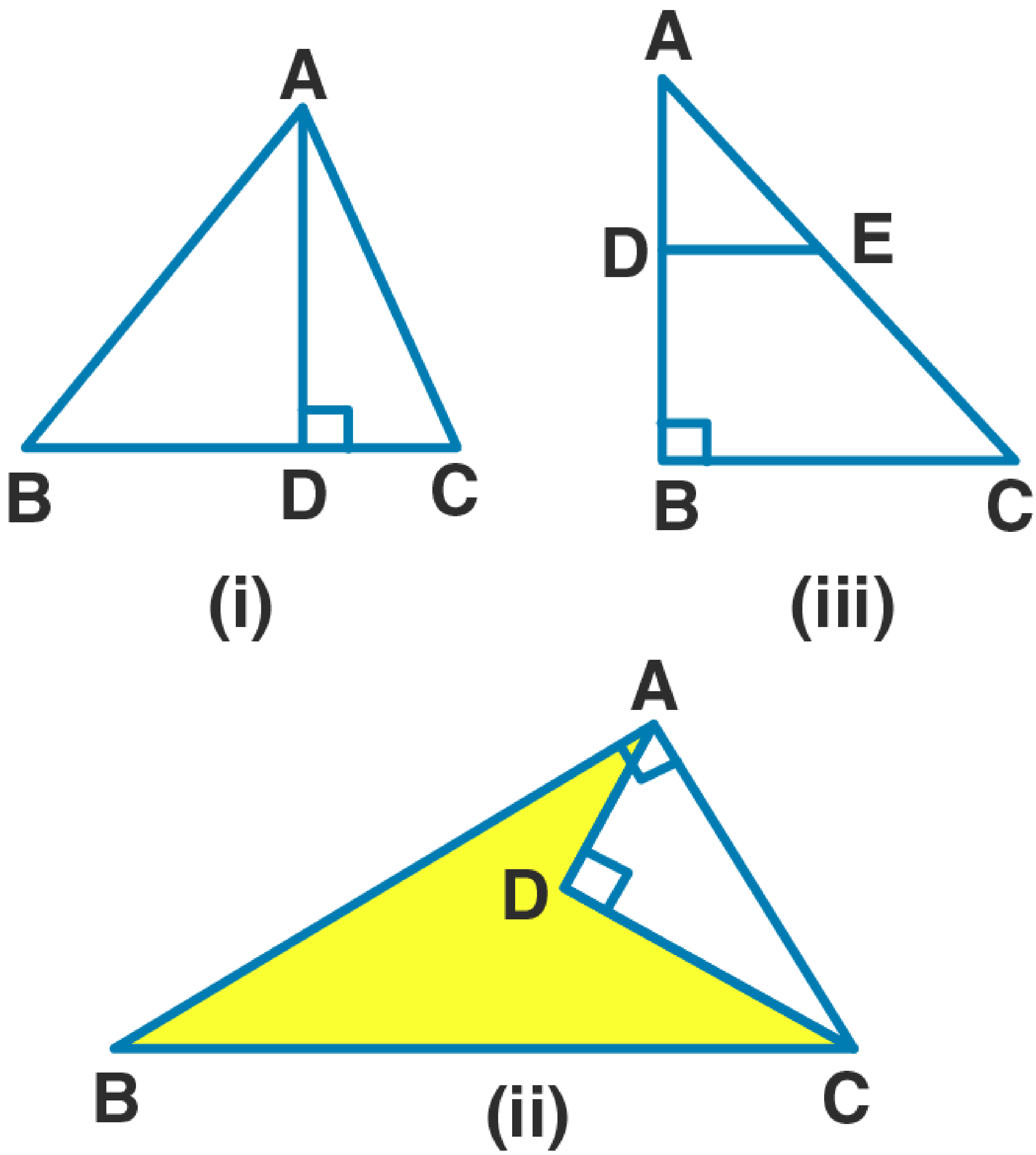
Solution:



Given $\triangle ABC$ is an isosceles triangle.
AB = AC
Construction: Draw $AP \perp BC$
Proof:
$\triangle APD$ is a right triangle.
$\therefore AD^2 = AP^2+PD^2$ [Pythagoras theorem]
$\therefore AD^2 = AP^2+(PC+CD)^2$ [PD = PC+CD]
$\therefore AD^2 = AP^2+PC^2+CD^2+2PC\times CD$ ….(i)
$\triangle APC$ is a right triangle.
$\therefore AC^2 = AP^2+PC^2$ …(ii) [Pythagoras theorem]
Substitute (ii) in (i)
$\therefore AD^2 = AC^2+CD^2+2PC\times CD$ ….(iii)
Since $\triangle ABC$ is an isosceles triangle,
PC = ½ BC [The altitude to the base of an isosceles triangle bisects the base]
$\therefore AD^2 = AC^2+CD^2+2\times \frac{1}{2} BC\times CD$
$\therefore AD^2 = AC^2+CD^2+BC\times CD$
$\therefore AD^2 = AC^2+CD(CD+BC)$
$\therefore AD^2 = AC^2+CD\times BD$ [CD+BC = BD]
$\therefore AD^2 = AC^2+BD\times CD$
Hence proved.

Chapter test

1. a) In fig. (i) given below, AD ⊥ BC, AB = 25 cm, AC = 17 cm and AD = 15 cm. Find the length of BC.
(b) In figure (ii) given below, ∠BAC = 90°, ∠ADC = 90°, AD = 6 cm, CD = 8 cm and BC = 26 cm.
Find :(i) AC
(ii) AB
(iii) area of the shaded region.
(c) In figure (iii) given below, triangle ABC is right angled at B. Given that AB = 9 cm, AC = 15 cm and D, E are mid-points of the sides AB and AC respectively, calculate
(i) the length of BC
(ii) the area of Δ ADE.



Solution:

(a) Given AD ⊥ BC, AB = 25 cm, AC = 17 cm and AD = 15 cm

△ADC is a right triangle.

$\therefore AC^2 = AD^2+DC^2$ [Pythagoras theorem]

$\therefore 17^2 = 15^2+DC^2$

$289 = 225+DC^2$

$\therefore DC^2 = 289-225$

$\therefore DC^2 = 64$
Taking square root on both sides,
DC = 8 cm
$\triangle$ADB is a right triangle.
$\therefore AB^2 = AD^2+BD^2$ [Pythagoras theorem]
$25^2 = 15^2+BD^2$
$625 = 225+ BD^2$
$\therefore BD^2 = 625-225 = 400$
Taking square root on both sides,
BD = 20 cm
$\therefore$ BC = BD+DC
= 20+8
= 28 cm
Hence the length of BC is 28 cm.

(b) Given $\angle BAC = 90°$, $\angle ADC = 90°$
AD = 6 cm, CD = 8 cm and BC = 26 cm.

(i) $\triangle$ADC is a right triangle.
$\therefore AC^2 = AD^2+DC^2$ [Pythagoras theorem]
$\therefore AC^2 = 6^2+8^2$
$\therefore AC^2 = 36+64$
$\therefore AC^2 = 100$
Taking square root on both sides,
AC = 10 cm
Hence length of AC is 10 cm.

(ii) $\triangle$ABC is a right triangle.
$\therefore BC^2 = AC^2+AB^2$ [Pythagoras theorem]
$\therefore 26^2 = 10^2+AB^2$
$\therefore AB^2 = 26^2-10^2$
$\therefore AB^2 = 676-100$
$\therefore AB^2 = 576$
Taking square root on both sides,
AB = 24 cm
Hence length of AB is 24 cm.

(iii)Area of $\triangle$ABC = ½ ×AB×AC
= ½ ×24×10
= 120 cm^2
Area of $\triangle$ADC = ½ ×AD×DC
= ½ ×6×8
= 24 cm^2
Area of shaded region = area of $\triangle$ABC- area of $\triangle$ADC
= 120-24
= 96 cm^2
Hence the area of shaded region is 96 cm^2.

(c) Given $\angle B = 90^\circ$.

AB = 9 cm, AC = 15 cm .

D, E are mid-points of the sides AB and AC respectively.

(i)△ABC is a right triangle.

$\therefore AC^2 = AB^2+BC^2$ [Pythagoras theorem]

$\therefore 15^2 = 9^2+BC^2$

$\therefore 225 = 81+BC^2$

$\therefore BC^2 = 225-81$

$BC^2 = 144$

Taking square root on both sides,

BC = 12 cm

Hence the length of BC is 12 cm.

(ii) AD = ½ AB [D is the midpoint of AB]

$\therefore AD = ½ \times 9 = 9/2$

AE = ½ AC [E is the midpoint of AC]

$\therefore AE = ½ \times 15 = 15/2$

△ADE is a right triangle.

$\therefore AE^2 = AD^2+DE^2$ [Pythagoras theorem]

$\therefore (15/2)^2 = (9/2)^2+DE^2$

$DE^2 = (15/2)^2 - (9/2)^2$

$DE^2 = 225/4 -81/4$

$DE^2 = 144/4$

Taking square root on both sides,

DE = 12/2 = 6 cm.

$\therefore$ Area of △ADE = ½ ×DE×AD

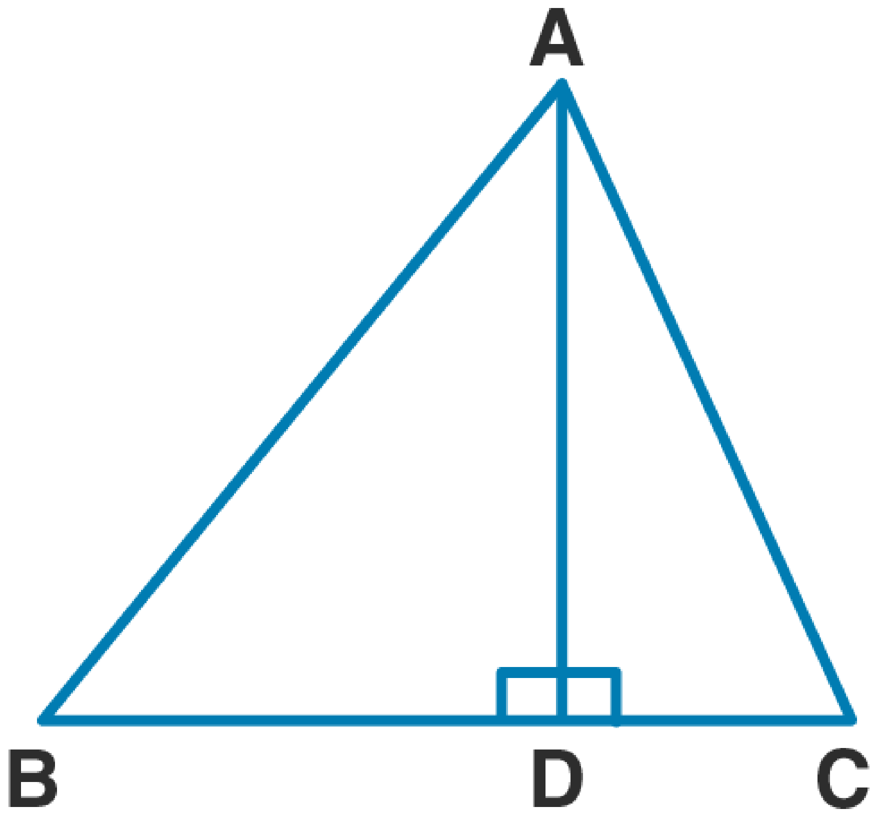
= ½ ×6×9/2

= 13.5 cm^2

Hence the area of the △ADE is 13.5 cm^2.

2. If in △ ABC, AB > AC and AD⊥ BC, prove that $AB^2 – AC^2 = BD^2 – CD^2$

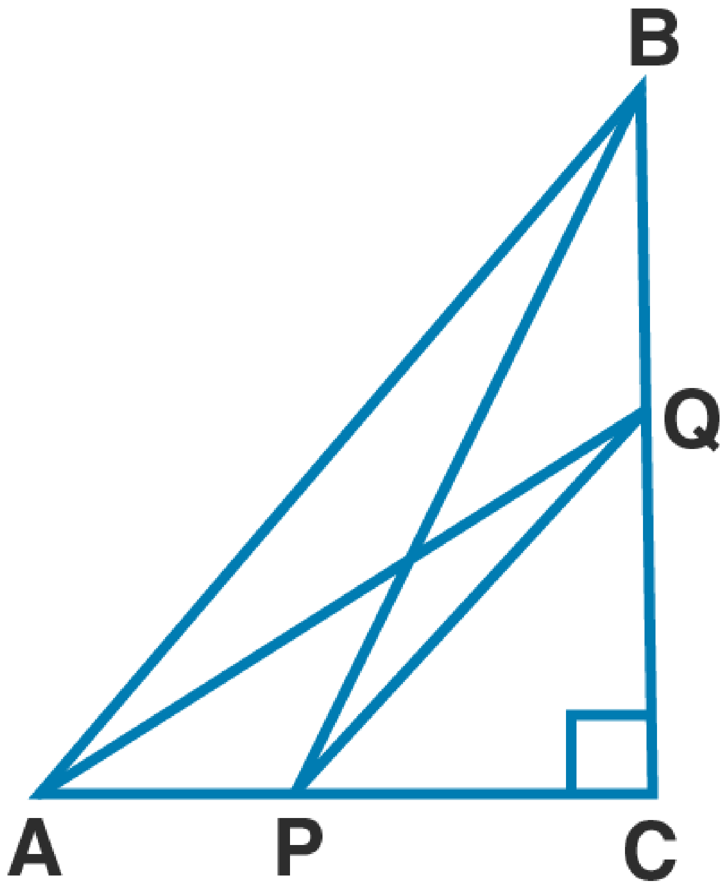
Solution:



Given AD ⊥ BC, AB>AC
So △ADB and △ADC are right triangles.
Proof:
In △ADB,
$AB^2 = AD^2+BD^2$ [Pythagoras theorem]
$\therefore AD^2 = AB^2-BD^2$ …(i)
In △ADC,
$AC^2 = AD^2+CD^2$ [Pythagoras theorem]
$\therefore AD^2 = AC^2-CD^2$ …(ii)
Equating (i) and (ii)
$AB^2-BD^2 = AC^2-CD^2$
$\therefore AB^2-AC^2 = BD^2- CD^2$
Hence proved.

3. In a right angled triangle ABC, right angled at C, P and Q are the points on the sides CA and CB respectively which divide these sides in the ratio 2:1.
Prove that
(i) $9AQ^2 = 9AC^2 + 4BC^2$
(ii) $9BP^2 = 9BC^2 + 4AC^2$
(iii) $9(AQ^2 + BP^2) = 13AB^2$.

Solution:



Construction:
Join AQ and BP.
Given $\angle C = 90^\circ$
Proof:
(i) In $\triangle ACQ$,
$AQ^2 = AC^2+CQ^2$ [Pythagoras theorem]
Multiplying both sides by 9, we get
$9AQ^2 = 9AC^2+9CQ^2$
$9AQ^2 = 9AC^2+(3CQ)^2$ …(i)
Given BQ: CQ = 1:2
$\therefore CQ/BC = CQ/(BQ+CQ)$
$\therefore CQ/BC = 2/3$
$\Rightarrow 3CQ = 2BC$ ….(ii)
Substitute (ii) in (i)
$9AQ^2 = 9AC^2+(2BC)^2$
$\Rightarrow 9AQ^2 = 9AC^2+4BC^2$ …(iii)
Hence proved.

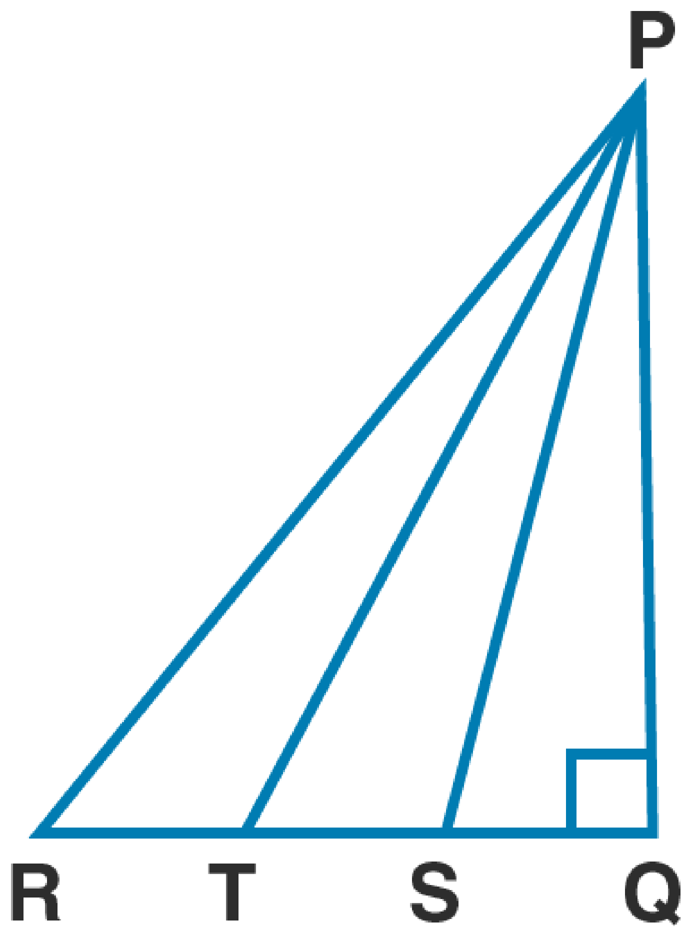
(ii)) In $\triangle BPC$,

$BP^2 = BC^2+CP^2$ [Pythagoras theorem]
Multiplying both sides by 9, we get
$9BP^2 = 9BC^2+9CP^2$
$9BP^2 = 9BC^2+(3CP)^2$ …(iv)
Given AP: PC = 1:2
$\therefore CP/AC = CP/AP+PC$
$\therefore CP/AC = 2/3$
$\Rightarrow 3CP = 2AC$ ….(v)
Substitute (v) in (iv)
$9BP^2 = 9BC^2+(2AC)^2$
$9BP^2 = 9BC^2+4AC^2$ ..(vi)
Hence proved.

(iii)Adding (iii) and (vi), we get
$9AQ^2+9BP^2 = 9AC^2+4BC^2+9BC^2+4AC^2$
$\Rightarrow 9(AQ^2+BP)^2 = 13AC^2+13BC^2$
$\Rightarrow 9(AQ^2+BP)^2 = 13(AC^2+BC^2)$…(vii)
In $\triangle ABC$,
$AB^2 = AC^2+BC^2$ …..(viii)
Substitute (viii) in (viii), we get
$9(AQ^2+BP)^2 = 13AB^2$
Hence proved.

4. In the given figure, $\triangle PQR$ is right angled at Q and points S and T trisect side QR. Prove that $8PT^2 = 3PR^2 + 5PS^2$.

Solution:



Given $\angle Q = 90^\circ$

S and T are points on RQ such that these points trisect it.

So RT = TS = SQ

To prove : $8PT^2 = 3PR^2 + 5PS^2$.

Proof:

Let RT = TS = SQ = x

In $\triangle PRQ$,

$PR^2 = RQ^2+PQ^2$ [Pythagoras theorem]

$PR^2 = (3x)^2+PQ^2$

$PR^2 = 9x^2+PQ^2$

Multiply above equation by 3

$3PR^2 = 27x^2+3PQ^2$ ….(i)

Similarly in $\triangle PTS$,

$PT^2 = TQ^2+PQ^2$ [Pythagoras theorem]

$PT^2 = (2x)^2+PQ^2$

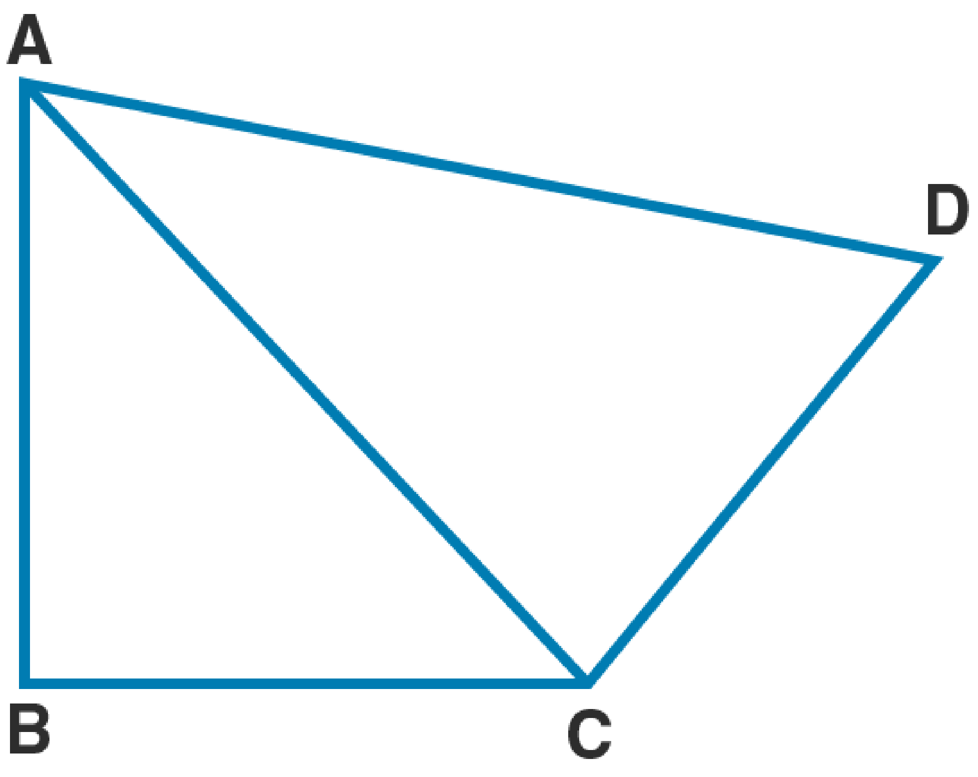
$PT^2 = 4x^2+PQ^2$

Multiply above equation by 8
$8PT^2 = 32x^2+8PQ^2$ ….(ii)

Similarly in $\triangle PSQ$,
$PS^2 = SQ^2+PQ^2$ [Pythagoras theorem]
$PS^2 = x^2+PQ^2$
Multiply above equation by 5
$5PS^2 = 5x^2+5PQ^2$ …(iii)
Add (i) and (iii), we get
$3PR^2 +5PS^2 = 27x^2+3PQ^2+5x^2+5PQ^2$
$\therefore 3PR^2 +5PS^2 = 32x^2+8PQ^2$
$\therefore 3PR^2 +5PS^2 = 8PT^2$ [From (ii)]
$\therefore$ $8PT^2 = 3PR^2 +5PS^2$
Hence proved.

5. In a quadrilateral ABCD, $\angle B = 90°$. If $AD^2 = AB^2 + BC^2 + CD^2$, prove that $\angle ACD = 90°$.

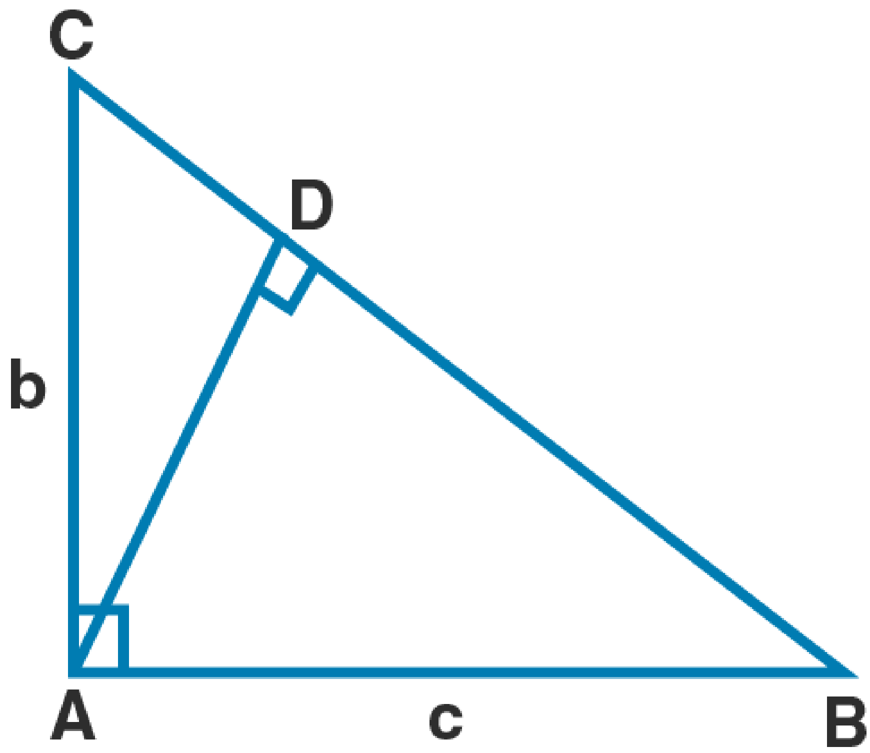
Solution:



Given : $\angle B = 90^\circ$ in quadrilateral ABCD
$AD^2 = AB^2 + BC^2 + CD^2$
To prove: $\angle ACD = 90^\circ$

Proof:
In △ABC,
$AC^2 = AB^2+BC^2$ ….(i) [Pythagoras theorem]
Given $AD^2 = AB^2 + BC^2 + CD^2$
$\therefore$ $AD^2 = AC^2+CD^2$ [from (i)]
$\therefore$ In △ACD, ∠ACD = 90° [Converse of Pythagoras theorem]
Hence proved.

6. In the given figure, find the length of AD in terms of b and c.

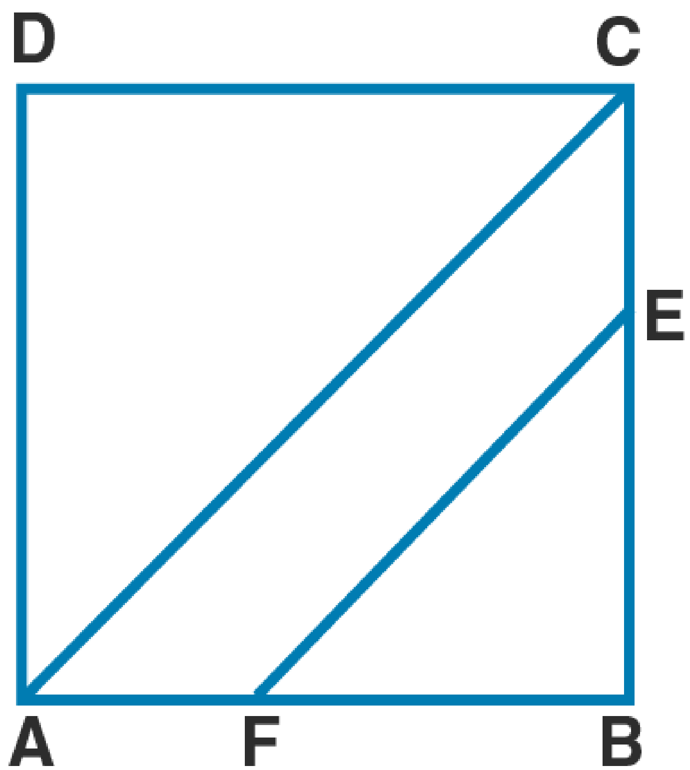


Solution:
Given : ∠A = 90°
AB = c
AC = b
∠ADB = 90°
In △ABC,
$BC^2 = AC^2+AB^2$ [Pythagoras theorem]
$BC^2 = b^2+c^2$
$BC = \sqrt{b^2+c^2}$ …(i)
Area of △ABC = ½ ×AB×AC
= ½ ×bc …(ii)
Also, Area of △ABC = ½ ×BC×AD
= ½ ×$\sqrt{b^2+c^2}$ ×AD …(iii)
Equating (ii) and (iii)

$\frac{1}{2} \times bc = \frac{1}{2} \times \sqrt{(b^2+c^2)} \times AD$
$\therefore AD = bc / (\sqrt{(b^2+c^2)})$
Hence AD is $bc / (\sqrt{(b^2+c^2)})$.

7. ABCD is a square, F is mid-point of AB and BE is one-third of BC. If area of △FBE is 108 cm², find the length of AC.

Solution:

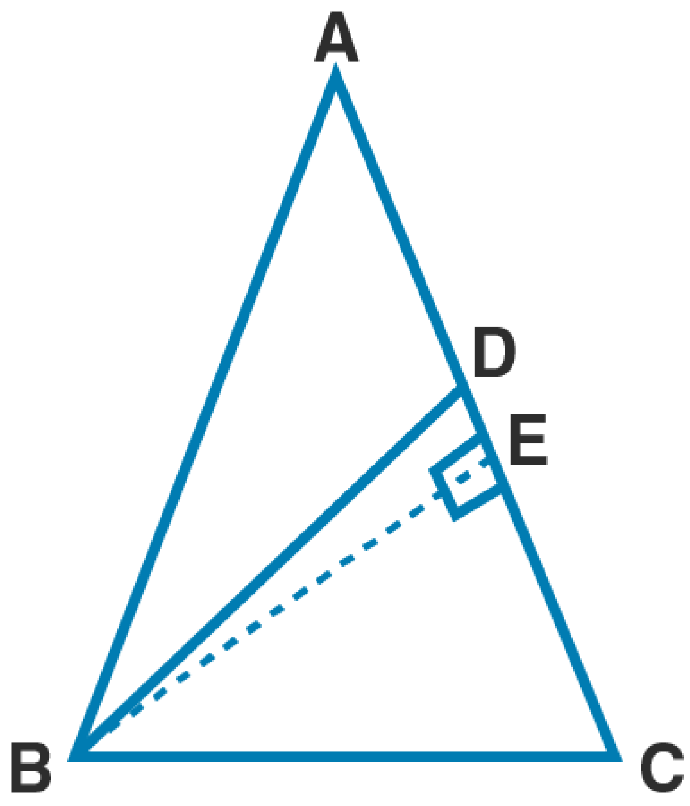


Let x be each side of the square ABCD.
FB = ½ AB [∵ F is the midpoint of AB]
$\therefore FB = \frac{1}{2} x$ …(i)
$BE = (1/3) BC$
$\therefore BE = (1/3) x$ …(ii)
$AC = \sqrt{2} \times side$ [Diagonal of a square]
$AC = \sqrt{2}x$
Area of $\triangle FBE = \frac{1}{2} FB \times BE$
$\therefore 108 = \frac{1}{2} \times \frac{1}{2} x \times (1/3)x$ [given area of $\triangle FBE = 108\ cm^2$]
$\therefore 108 = (1/12)x^2$
$\therefore x^2 = 108 \times 12$
$\therefore x^2 = 1296$
Taking square root on both sides.

$x = 36$
$\therefore AC = \sqrt{2} \times 36 = 36\sqrt{2}$
Hence length of AC is $36\sqrt{2}$ cm.

8. In a triangle ABC, AB = AC and D is a point on side AC such that $BC^2 = AC \times CD$, Prove that BD = BC.

Solution:



Given : In $\triangle ABC$, AB = AC
D is a point on side AC such that $BC^2 = AC \times CD$
To prove : BD = BC
Construction: Draw $BE \perp AC$

Proof:
In $\triangle BCE$,
$BC^2 = BE^2 + EC^2$ [Pythagoras theorem]
$BC^2 = BE^2 + (AC - AE)^2$
$BC^2 = BE^2 + AC^2 + AE^2 - 2\ AC \times AE$
$BC^2 = BE^2 + AE^2 + AC^2 - 2\ AC \times AE$ …(i)
In $\triangle ABC$,
$AB^2 = BE^2 + AE^2$ ..(ii)
Substitute (ii) in (i)
$\therefore BC^2 = AB^2 + AC^2 - 2\ AC \times AE$
$\therefore BC^2 = AC^2 + AC^2 - 2\ AC \times AE$ [$\because AB = AC$]

$\therefore BC^2 = 2AC^2 - 2\ AC \times AE$
$\therefore BC^2 = 2AC(AC-AE)$
$\therefore BC^2 = 2AC \times EC$
Given $BC^2 = AC \times CD$
$\therefore 2AC \times EC = AC \times CD$
$\Rightarrow 2EC = CD$ ..(ii)
$\therefore$ E is the midpoint of CD.
$EC = DE$ …(iii)
In $\triangle BED$ and $\triangle BEC$,
$EC = DE$ [From (iii)]
$BE = BE$ [common side]
$\angle BED = \angle BEC$
$\therefore \triangle BED \cong \triangle BEC$ [By SAS congruency rule]
$\therefore BD = BD$ [c.p.c.t]
Hence proved.