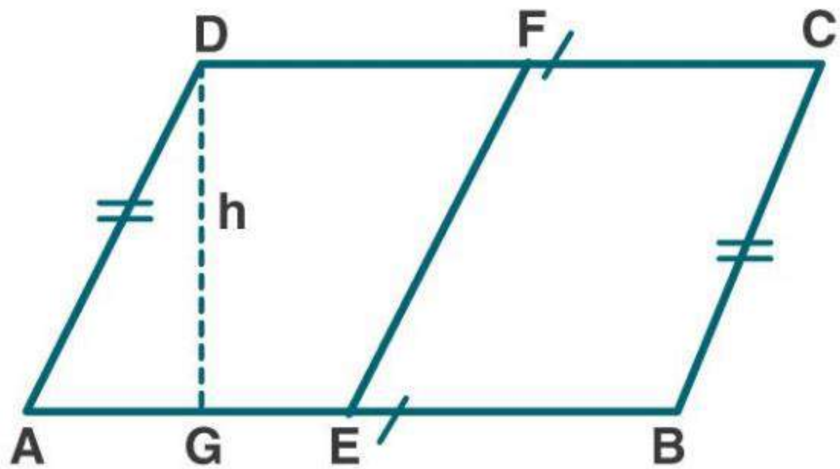
EXERCISE 14

1. Prove that the line segment joining the mid-points of a pair of opposite sides of a parallelogram divides it into two equal parallelograms.

Solution:

Let us consider ABCD be a parallelogram in which E and F are mid-points of AB and CD. Join EF.

<u>To prove:</u> ar (|| AEFD) = ar (|| EBCF)



Let us construct DG ⊥ AG and let DG = h where, h is the altitude on side AB.

<u>Proof:</u>

ar (|| ABCD) = AB × h

ar (|| AEFD) = AE × h

= ½ AB × h ….. (1) [Since, E is the mid-point of AB]

ar (|| EBCF) = EF × h

= ½ AB × h …… (2) [Since, E is the mid-point of AB]

From (1) and (2)

ar (|| ABFD) = ar (|| EBCF)

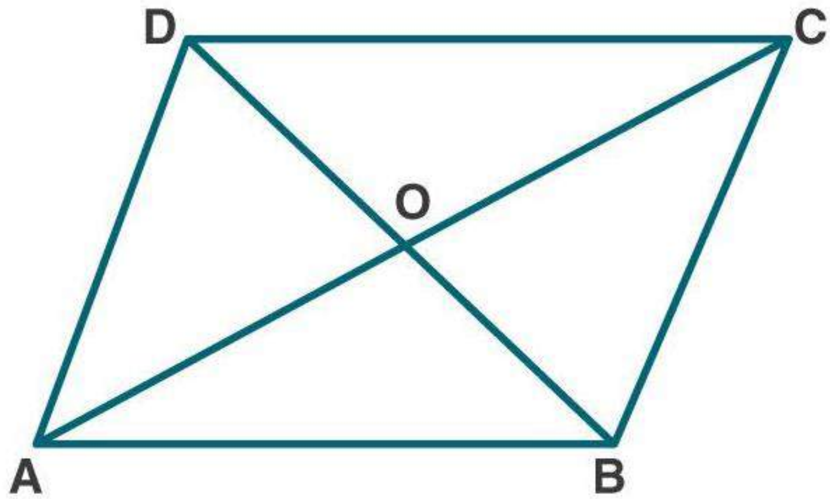
Hence proved.

2. Prove that the diagonals of a parallelogram divide it into four triangles of equal area.

Solution:

Let us consider in a parallelogram ABCD the diagonals AC and BD are cut at point O.

<u>To prove:</u> ar (ΔAOB) = ar (ΔBOC) = ar (ΔCOD) = ar (ΔAOD)



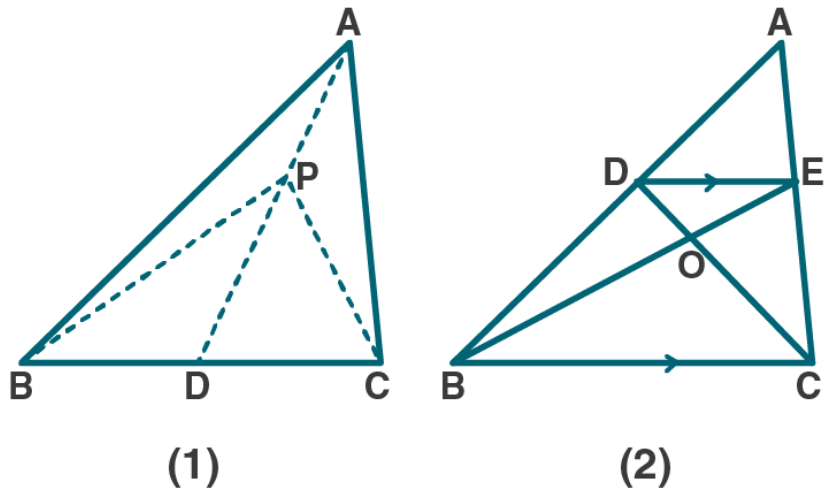
Proof:
In parallelogram ABCD the diagonals bisect each other.
AO = OC
In ΔACD, O is the mid-point of AC. DO is the median.
ar (ΔAOD) = ar (COD) ….. (1) [Median of Δ divides it into two triangles of equal arreas]

Similarly, in Δ ABC
ar (ΔAOB) = ar (ΔCOB) ….. (2)

In ΔADB
ar (ΔAOD) = ar (ΔAOB) …. (3)

In ΔCDB
ar (ΔCOD) = ar (ΔCOB) …. (4)
From (1), (2), (3) and (4)
ar (ΔAOB) = ar (ΔBOC) = ar (ΔCOD) = ar (ΔAOD)
Hence proved.

3. (a) In the figure (1) given below, AD is median of ΔABC and P is any point on AD. Prove that
(i) Area of ΔPBD = area of ΔPDC.
(ii) Area of ΔABP = area of ΔACP.
(b) In the figure (2) given below, DE || BC. Prove that
(i) area of ΔACD = area of Δ ABE.
(ii) Area of ΔOBD = area of ΔOCE.



Solution:
(a) Given:
ΔABC in which AD is the median. P is any point on AD. Join PB and PC.
To prove:
(i) Area of ΔPBD = area of ΔPDC.
(ii) Area of ΔABP = area of ΔACP.
Proof:
From fig (1)
AD is a median of ΔABC
So, ar (ΔABD) = ar (ΔADC) …. (1)
Also, PD is the median of ΔBPD

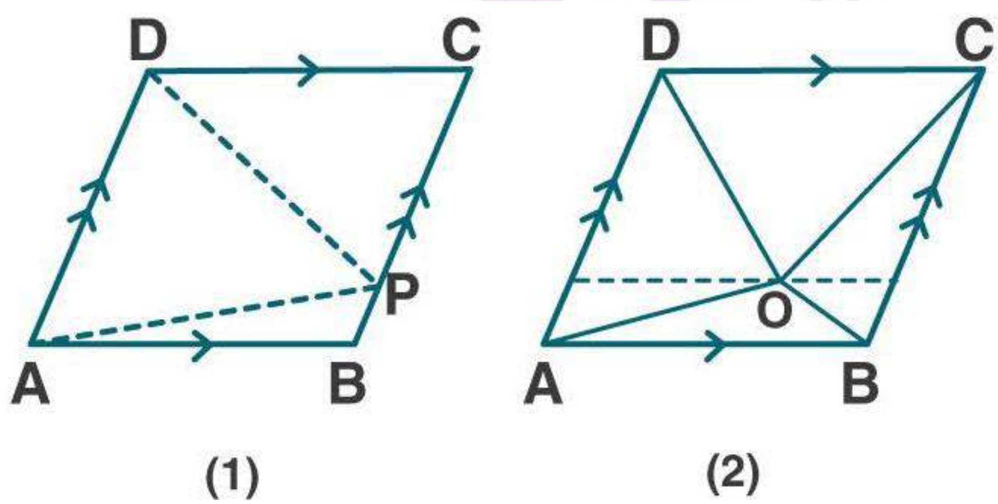
Similarly, ar (ΔPBD) = ar (ΔPDC) …. (2)
Now, let us subtract (2) from (1), we get
ar (ΔABD) - ar (ΔPBD) = ar (ΔADC) - ar (ΔPDC)
Or ar (ΔABP) = ar (ΔACP)
Hence proved.

(b) Given:
ΔABC in which DE || BC
To prove:
(i) area of ΔACD = area of Δ ABE.
(ii) Area of ΔOBD = area of ΔOCE.
Proof:
From fig (2)
ΔDEC and ΔBDE are on the same base DE and between the same || line DE and BE.

ar (ΔDEC) = ar (ΔBDE)
Now add ar (ADE) on both sides, we get
ar (ΔDEC) + ar (ΔADE) = ar (ΔBDE) + ar (ΔADE)
ar (ΔACD) = ar (ΔABE)
Hence proved.

Similarly, ar (ΔDEC) = ar (ΔBDE)
Subtract ar (ΔDOE) from both sides, we get
ar (ΔDEC) - ar (ΔDOE) = ar (ΔBDE) - ar (ΔDOE)
ar (ΔOBD) = ar (ΔOCE)
Hence proved.

4. (a) In the figure (1) given below, ABCD is a parallelogram and P is any point in BC. Prove that: Area of ΔABP + area of ΔDPC = Area of ΔAPD.
(b) In the figure (2) given below, O is any point inside a parallelogram ABCD. Prove that:
(i) area of ΔOAB + area of ΔOCD = ½ area of || gm ABCD
(ii) area of Δ OBC + area of Δ OAD = ½ area of || gm ABCD



Solution:
(a) Given:
From fig (1)
ABCD is a parallelogram and P is any point in BC.
To prove:
Area of ΔABP + area of ΔDPC = Area of ΔAPD
Proof:
ΔAPD and || gm ABCD are on the same base AD and between the same || lines AD and BC,

ar (ΔAPD) = ½ ar (|| gm ABCD) …. (1)

In parallelogram ABCD
ar(|| gm ABCD) = ar (Δ ABP) + ar (ΔAPD) + ar (ΔDPC)
Now, divide both sides by 2, we get
½ ar(|| gm ABCD) = ½ ar (Δ ABP) + ½ ar (ΔAPD) + ½ ar (ΔDPC) …. (2)
From (1) and (2)
ar (ΔAPD) = ½ ar (|| gm ABCD)
Substituting (2) in (1)
ar (ΔAPD) = ½ ar (Δ ABP) + ½ ar (ΔAPD) + ½ ar (ΔDPC)
ar (ΔAPD) - ½ ar (ΔAPD) = ½ ar (Δ ABP) + ½ ar (ΔDPC)
½ ar (ΔAPD) = ½ [ar (Δ ABP) + ar (ΔDPC)]
ar (ΔAPD) = ar (Δ ABP) + ar (ΔDPC)
Or ar (Δ ABP) + ar (ΔDPC) = ar (ΔAPD)
Hence proved.

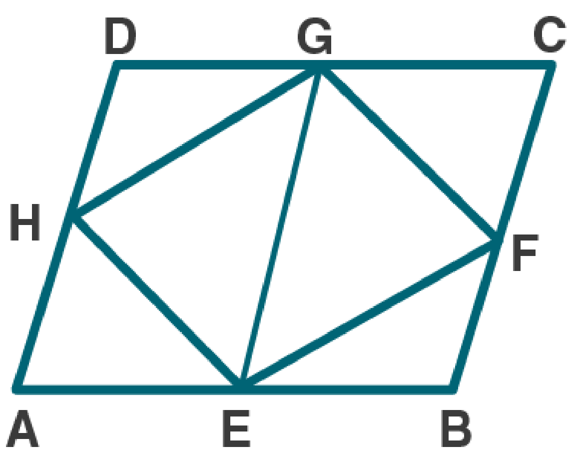
(b) Given:
From fig (2)
|| gm ABCD in which O is any point inside it.
<u>To prove:</u>
(i) area of ΔOAB + area of ΔOCD = ½ area of || gm ABCD
(ii) area of Δ OBC + area of Δ OAD = ½ area of || gm ABCD
Draw POQ || AB through O. It meets AD at P and BC at Q.
<u>Proof:</u>
(i) AB || PQ and AP || BQ
ABQP is a || gm
Similarly, PQCD is a || gm
Now, ΔOAB and || gm ABQP are on same base AB and between same || lines AB and PQ
ar (ΔOAB) = ½ ar (||gm ABQP) …. (1)

Similarly, ar (ΔOCD) = ½ ar (||gm PQCD) …. (2)
Now by adding (1) and (2)
ar (ΔOAB) + ar (ΔOCD) = ½ ar (|| gm ABQP) + ½ ar (|| gm PQCD)
= ½ [ar (|| gm ABQP) + ar (|| gm PQCD)]
= ½ ar (|| gm ABCD)
ar (ΔOAB) + ar (ΔOCD) = ½ ar (|| gm ABCD)
Hence proved.

(ii) we know that,

ar (ΔOAB) + ar (Δ OBC) + ar (ΔOCD) + ar (ΔOAD) = ar (|| gm ABCD)
[ar (ΔOAB) + ar (ΔOCD)] + [ar (Δ OBC) + ar (ΔOAD)] = ar (|| gm ABCD)
½ ar (|| gm ABCD) + ar (ΔOBC) + ar (ΔOAD) = ar (|| gm ABCD)
ar (ΔOBC) + ar (ΔOAD) = ar (|| gm ABCD) - ½ ar (|| gm ABCD)
ar (ΔOBC) + ar (ΔOAD) = ½ ar (|| gm ABCD)
Hence proved.

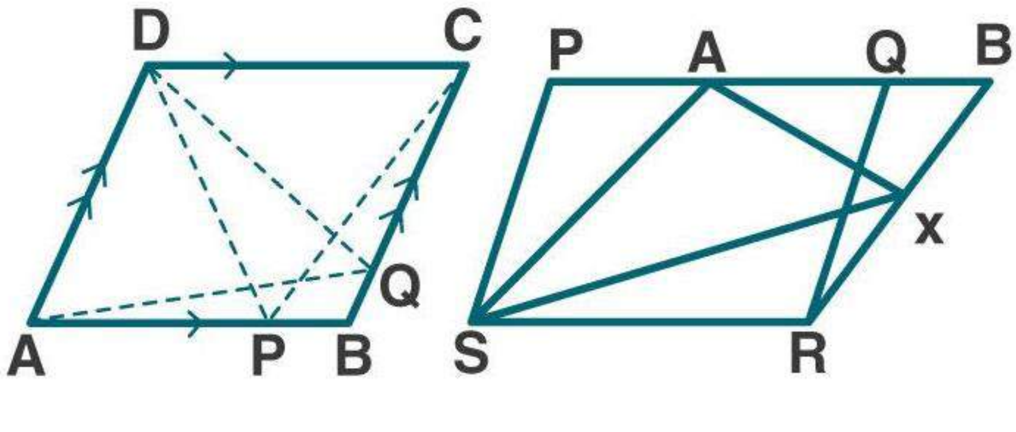
5. If E, F, G and H are mid-points of the sides AB, BC, CD and DA respectively of a parallelogram ABCD, prove that area of quad. EFGH = 1/2 area of || gm ABCD.
Solution:
Given:
In parallelogram ABCD, E, F, G, H are the mid-points of its sides AB, BC, CD and DA.
Join EF, FG, GH and HE.
To prove:
area of quad. EFGH = ½ area of || gm ABCD



Proof:
Let us join EG.
We know that, E and G are mid-points of AB and CD.
EG || AD || BC
AEGD and EBCG are parallelogram
Now, || gm AEGD and ΔEHG are on the same base and between the parallel lines.
ar ΔEHG = ½ ar || gm AEGD …. (1)
Similarly,
ar ΔEFG = ½ ar || gm EBCG …. (2)
Now by adding (1) and (2)
ar ΔEHG + ar ΔEFG = ½ ar || gm AEGD + ½ ar || gm EBCG
area quad. EFGH = ½ ar || gm ABCD

Hence proved.

6. (a) In the figure (1) given below, ABCD is a parallelogram. P, Q are any two points on the sides AB and BC respectively. Prove that, area of Δ CPD = area of Δ AQD.



(b) In the figure (2) given below, PQRS and ABRS are parallelograms and X is any point on the side BR. Show that area of Δ AXS = ½ area of ||gm PQRS.

Solution:

(a) Given:

From fig (1)

||gm ABCD in which P is a point on AB and Q is a point on BC.

To prove:

area of Δ CPD = area of Δ AQD.

Proof:

Δ CPD and ||gm ABCD are on the same base CD and between the same parallels AB and CD.

ar (Δ CPD) = ½ ar (||gm ABCD) …. (1)

Δ AQD and ||gm ABCD are on the same base AD and between the same parallels AD and BC.

ar (ΔAQD) = ½ ar (||gm ABCD) …. (2)

from (1) and (2)

ar (Δ CPD) = ar (ΔAQD)

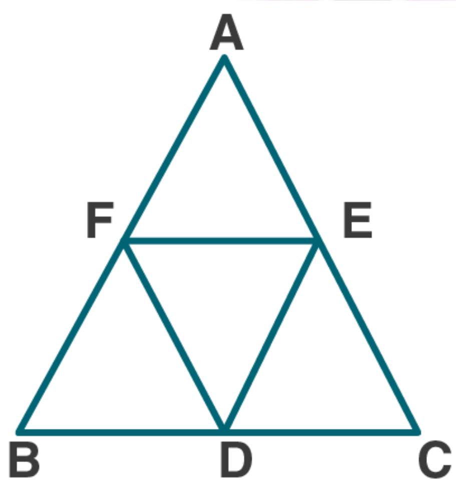
Hence proved.

(b) From fig (2)

Given:
PQRS and ABRS are parallelograms on the same base SR. X is any point on the side BR. Join AX and SX.
To prove:
area of Δ AXS = ½ area of ||gm PQRS
we know that, || gm PQRS and ABRS are on the same base SR and between the same parallels.
So, ar ||gm PQRS = ar ||gm ABRS …. (1)

we know that, Δ AXS and || gm ABRS are on the same base AS and between the same parallels.
So, ar Δ AXS = ½ ar ||gm ABRS
= ½ ar ||gm PQRS [From (1)]
Hence proved.

7. D, E and F are mid-point of the sides BC, CA and AB respectively of a Δ ABC. Prove that
(i) FDCE is a parallelogram
(ii) area of Δ DEF = ¼ area of Δ ABC
(iii) area of || gm FDCE = ½ area of Δ ABC
Solution:
Given:
D, E and F are mid-point of the sides BC, CA and AB respectively of a Δ ABC.



To prove:
(i) FDCE is a parallelogram
(ii) area of Δ DEF = ¼ area of Δ ABC
(iii) area of || gm FDCE = ½ area of Δ ABC

Proof:

(i) F and E are mid-points of AB and AC.
So, FE || BC and FE = ½ BC ….. (1)
Also, D is mid-point of BC
CD = ½ BC …… (2)
From (1) and (2)
FE || BC and FE = CD
FE || CD and FE = CD …… (3)

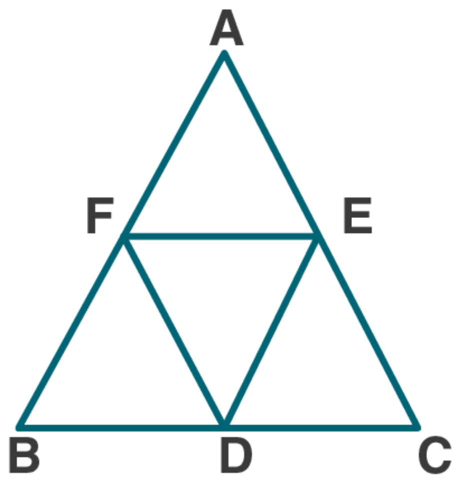
Similarly,
D and F are mid-points of BC and AB.
So, DF || EC is a parallelogram.
Hence proved.

(ii) we know that, FDCE is a parallelogram.
And DE is a diagonal of ||gm FDCE
So, ar (Δ DEF) = ar (ΔDEC) ….. (4)

Similarly, we know BDEF and DEAF are ||gm
So, ar (Δ DEF) = ar (Δ BDF) = ar (Δ AFE) ….. (5)
From (4) and (5)
ar (Δ DEF) = ar (ΔDEC) = ar (Δ BDF) = ar (Δ AFE)
Now, ar (Δ ABC) = ar (Δ DEF) + ar (Δ DEF) + ar (Δ DEF) + ar (Δ DEF)
= 4 ar (Δ DEF)
ar (Δ DEF) = ¼ ar (Δ ABC) ….. (6)
Hence proved.

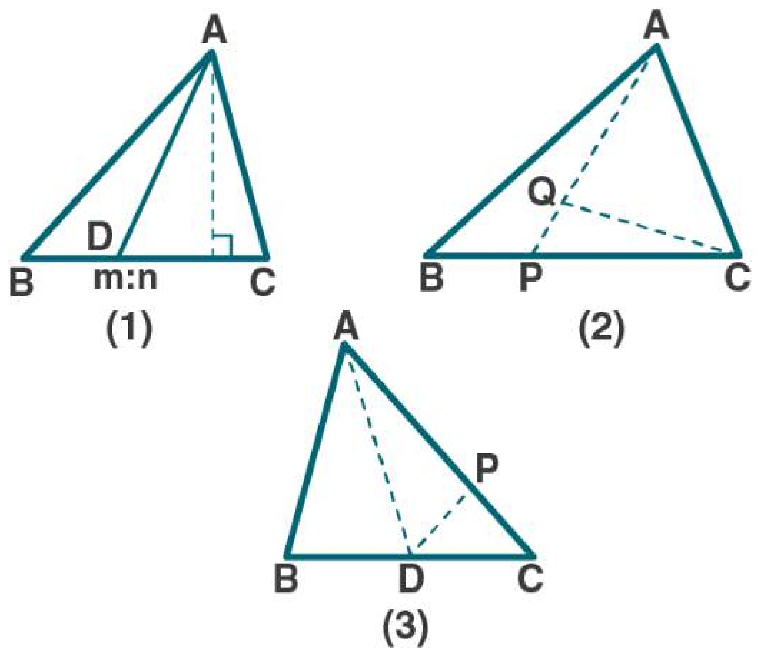
(iii) ar of || gm FDCE = ar (Δ DEF) + ar (Δ DEC)
= ar (Δ DEF) + ar (Δ DEF)
= 2 ar (Δ DEF) [From (4)]
= 2 [¼ ar (Δ ABC)] [From (6)]
ar of || gm FDCE = ½ ar of Δ ABC
Hence proved.

8. In the given figure, D, E and F are mid points of the sides BC, CA and AB respectively of Δ ABC. Prove that BCEF is a trapezium and area of trap. BCEF = ¾ area of Δ ABC.



Solution:
Given:
In ∆ABC, D, E and F are mid points of the sides BC, CA and AB.
To prove:
area of trap. BCEF = ¾ area of ∆ ABC
Proof:
We know that D and E are the mid-points of BC and CA.
So, DE || AB and ½ AB
Similarly,
EF || BC and ½ BC
And FD || AC and ½ AC
∴ BDEF, CDFE, AFDE are parallelograms which are equal in area.
ED, DF, EF are diagonals of these ||gm which divides the corresponding parallelogram into two triangles equal in area.
Hence, BCEF is a trapezium.
area of trap. BCEF = ¾ area of ∆ ABC

9. (a) In the figure (1) given below, the point D divides the side BC of ∆ABC in the ratio m: n. Prove that area of ∆ ABD: area of ∆ ADC = m: n.
(b) In the figure (2) given below, P is a point on the side BC of ∆ABC such that PC = 2BP, and Q is a point on AP such that QA = 5 PQ, find area of ∆AQC: area of ∆ABC.
(c) In the figure (3) given below, AD is a median of ∆ABC and P is a point in AC such that area of ∆ADP: area of ∆ABD = 2:3. Find
(i) AP: PC
(ii) area of ∆PDC: area of ∆ABC.



Solution:

(a) Given:

From fig (1)

In ΔABC, the point D divides the side BC in the ratio m: n.

BD: DC = m: n

To prove:

area of Δ ABD: area of Δ ADC = m: n

Proof:

area of Δ ABD = ½ × base × height

ar (Δ ABD) = ½ × BD × AE ….. (1)

ar (Δ ACD) = ½ × DC × AE ….. (2)

let us divide (1) by (2)

[ar (Δ ABD) = ½ × BD × AE] / [ar (Δ ACD) = ½ × DC × AE]

[ar (Δ ABD)] / [ar (Δ ACD)] = BD/DC

= m/n [it is given that, BD: DC = m: n]

Hence proved.

(b) Given:

From fig (2)

In ΔABC, P is a point on the side BC such that PC = 2BP, and Q is a point on AP such that QA = 5 PQ.

To Find:

area of ΔAQC: area of ΔABC

Now,
It is given that: PC = 2BP
PC/2 = BP
We know that, BC = BP + PC
Now substitute the values, we get
BC = BP + PC
= PC/2 + PC
= (PC + 2PC)/2
= 3PC/2
2BC/3 = PC
ar (ΔAPC) = 2/3 ar (ΔABC) …… (1)

It is given that, QA = 5PQ
QA/5 = PQ
We know that, QA= QA + PQ
So, QA = 5/6 AP
ar (ΔAQC) = 5/6 ar (ΔAPC)
= 5/6 (2/3 ar (ΔABC)) [From (1)]
ar (ΔAQC) = 5/9 ar (ΔABC)
ar (ΔAQC)/ ar (ΔAQC) = 5/9
Hence proved.

(c) Given:
From fig (3)
AD is a median of ΔABC and P is a point in AC such that area of ΔADP: area of ΔABD = 2:3
To Find:
(i) AP: PC
(ii) area of ΔPDC: area of ΔABC
Now,
(i) we know that AD is the median of ΔABC
ar (ΔABD) = ar (ΔADC) = ½ ar (ΔABC) ……. (1)
It is given that,
ar (ΔADP): ar (ΔABD) = 2: 3
AP: AC = 2: 3
AP/AC = 2/3
AP = 2/3 AC
Now,
PC = AC – AP

= AC – 2/3 AC
= (3AC-2AC)/3
= AC/3 …….. (2)

So,

AP/PC = (2/3 AC) / (AC/3)
= 2/1

AP: PC = 2:1

(ii) we know that from (2)

PC = AC/3

PC/AC = 1/3

So,

ar (ΔPDC)/ar (ΔADC) = PC/AC
= 1/3

ar (ΔPDC)/1/2 ar (ΔABC) = 1/3

ar (ΔPDC)/ar (ΔABC) = 1/3 × ½
= 1/6

ar (ΔPDC): ar (ΔABC) = 1: 6

Hence proved.

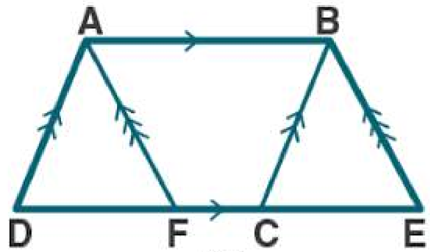
10. (a) In the figure (1) given below, area of parallelogram ABCD is 29 cm^2. Calculate the height of parallelogram ABEF if AB = 5.8 cm

(b) In the figure (2) given below, area of ΔABD is 24 sq. units. If AB = 8 units, find the height of ABC.

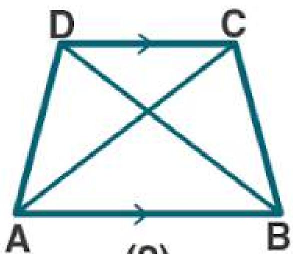
(c) In the figure (3) given below, E and F are mid points of sides AB and CD respectively of parallelogram ABCD. If the area of parallelogram ABC is 36 cm^2.

(i) State the area of Δ APD.

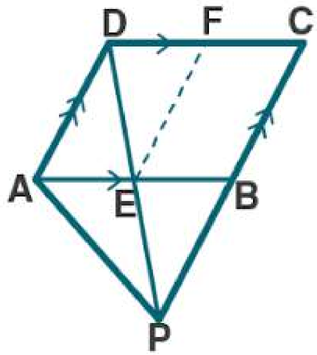
(ii) Name the parallelogram whose area is equal to the area of Δ APD.



(1)



(2)



(3)

Solution:

(a) Given:

From fig (1)

ar ||gm ABCD = 29cm^2

<u>To find:</u>

Height of parallelogram ABEF if AB = 5.8 cm

Now, let us find

We know that ||gm ABCD and ||gm ABEF with equal bases and between the same parallels so that there area are same.

ar (||gm ABEF) = ar (||gm ABCD)

ar (||gm ABEF) = 29cm^2 ….. (1) [Since, ar ||gm ABCD = 29cm^2]

also, ar (||gm ABEF = base × height)

29 = AB × height [From (1)]

29 = 5.8 × height

Height = 29/5.8

= 5

∴ Height of parallelogram ABEF is 5cm

(b) Given:

From fig (2)

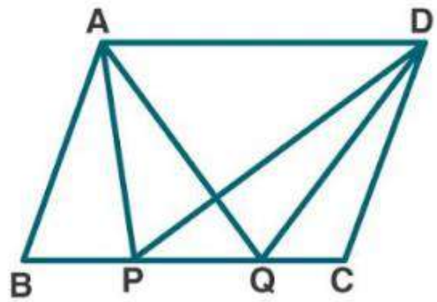
area of ΔABD is 24 sq. units. AB = 8 units

To find:
Height of ABC
Now, let us find
We know that ar ΔABD = 24 sq. units …… (1)
So, ar $\Delta ABD = \Delta ABC$ ….. (2)
From (1) and (2)
ar ΔABC = 24 sq. units
$\frac{1}{2} \times AB \times$ height = 24
$\frac{1}{2} \times 8 \times$ height = 24
$4 \times$ height = 24
Height = 24/4
= 6
∴ Height of ΔABC = 6 sq. units

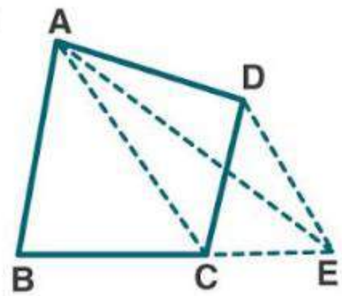
(c) Given:
From fig (3)
In ||gm ABCD, E and F are mid points of sides AB and CD respectively.
ar (||gm ABCD) = $36cm^2$
To find:
(i) State the area of Δ APD.
(ii) Name the parallelogram whose area is equal to the area of Δ APD.
Now, let us find
(i) we know that Δ APD and ||gm ABCD are on the same base AD and between the same parallel lines AD and BC.
ar (Δ APD) = ½ ar (||gm ABCD) ….. (1)
ar (||gm ABCD) = $36cm^2$ ….. (2)
From (1) and (2)
ar (Δ APD) = $\frac{1}{2} \times 36$
= $18cm^2$

(ii) we know that E and F are mid-points of AB and CD
In ΔCPD, EF || PC
Also, EF bisects the ||gm ABCD in two eual parts.
So, EF || AD and AE || DF
AEFD is a parallelogram.
ar (||gm AEFD) = ½ ar (||gm ABCD) ……. (3)
From (1) and (3)
ar (ΔAPD) = ar (||gm AEFD)
∴ AEFD is the required parallelogram which is equal to area of ΔAPD.

11. (a) In the figure (1) given below, ABCD is a parallelogram. Points P and Q on BC trisect BC into three equal parts. Prove that :
area of ∆APQ = area of ∆DPQ = 1/6 (area of ||gm ABCD)
(b) In the figure (2) given below, DE is drawn parallel to the diagonal AC of the quadrilateral ABCD to meet BC produced at the point E. Prove that area of quad. ABCD = area of ∆ABE.
(c) In the figure (3) given below, ABCD is a parallelogram. O is any point on the diagonal AC of the parallelogram. Show that the area of ∆AOB is equal to the area of ∆AOD.



(1)



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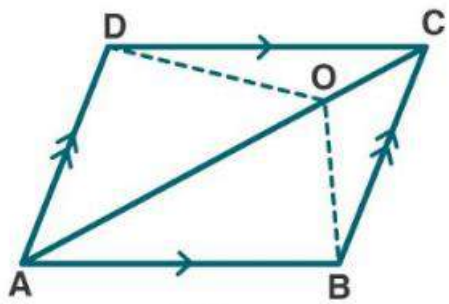
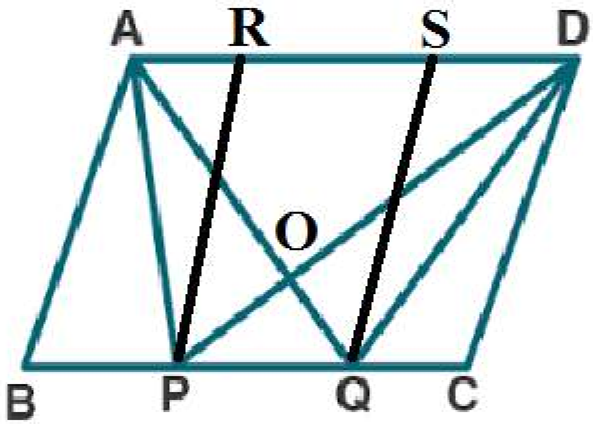


(3)

Solution:
(a) Given:
From fig (1)



In ||gm ABCD, points P and Q trisect BC into three equal parts.

To prove:

area of ΔAPQ = area of ΔDPQ = 1/6 (area of ||gm ABCD)

Firstly, let us construct: through P and Q, draw PR and QS parallel to AB and CD.

Proof:

ar (ΔAPD) = ar (ΔAQD) [Since, ΔAPD and ΔAQD lie on the same base AD and between the same parallel lines AD and BC]

ar (ΔAPD) – ar (ΔAOD) = ar (ΔAQD) – ar (ΔAOD) [On subtracting ar ΔAOD on both sides]

ar (ΔAPO) = ar (ΔOQD) ….. (1)

ar (ΔAPO) + ar (ΔOPQ) = ar (ΔOQD) + ar (ΔOPQ) [On adding ar ΔOPQ on both sides]

ar (ΔAPQ) = ar (ΔDPQ) …… (2)

We know that, ΔAPQ and ||gm PQSR are on the same base PQ and between same parallel lines PQ and AD.

ar (ΔAPQ) = ½ ar (||gm PQRS) …… (3)

Now,

[ar (||gm ABCD)/ar (||gm PQRS)] = [(BC×height)/(PQ×height)] = [(3PQ×height)/(1PQ×hight)]

ar (||gm PQRS) = 1/3 ar (||gm ABCD) …. (4)

by using (2), (3), (4), we get

ar (ΔAPQ) = ar (ΔDPQ)

= ½ ar (||gm PQRS)

= ½ × 1/3 ar (||gm ABCD)

= 1/6 ar (||gm ABCD)

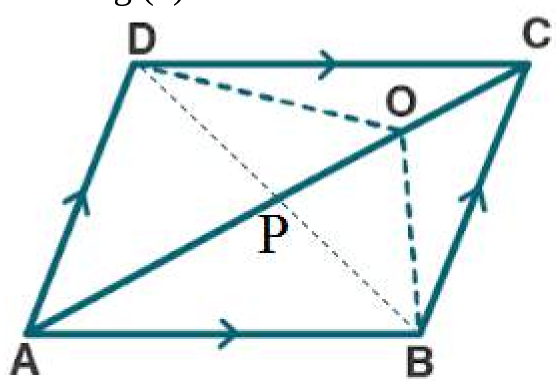
Hence proved.

(b) Given:

In the figure (2) given below, DE || AC the diagonal of the quadrilateral ABCD to meet at point E on producing BC. Join AC, AE.

To prove:
area of quad. ABCD = area of ΔABE
Proof:
We know that, ΔACE and ΔADE are on the same base AC and between the same parallelogram.
ar (ΔACE) = ar (ΔADC)
Now by adding ar (ΔABC) on both sides, we get
ar (ΔACE) + ar (ΔABC) = ar (ΔADC) + ar (ΔABC)
ar (Δ ABE) = ar quad. ABCD
Hence proved.

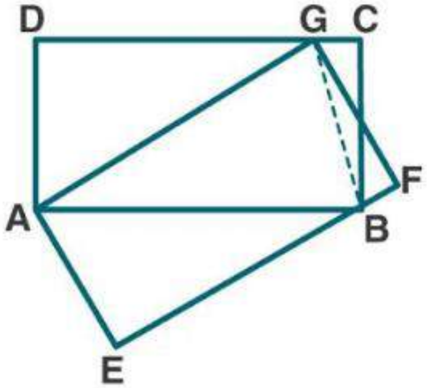
(c) Given:
From fig (3)

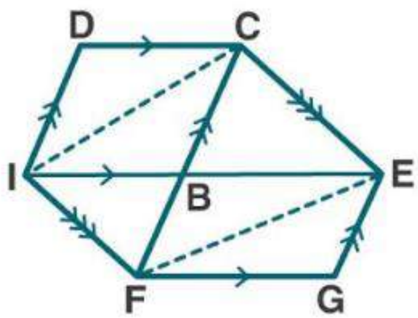


In ||gm ABCD, O is any point on diagonal AC.
To prove:
area of ΔAOB is equal to the area of ΔAOD
Proof:
Let us join BD which meets AC at P.
In ΔABD, AP is the median.
ar (ΔABP) = ar (ΔADP) ….. (1)
similarly, ar (ΔPBO) = ar (ΔPDO) …. (2)
now add (1) and (2), we get
ar (ΔABO) = ar (ΔADO) …. (3)
so,
ΔAOB = ar ΔAOD
Hence proved.

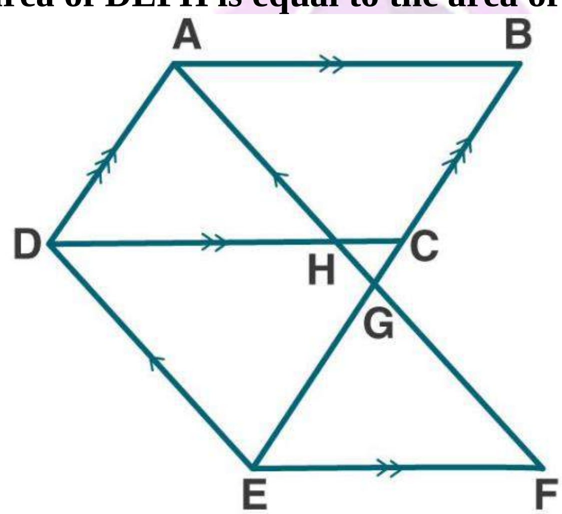
12. (a) In the figure given, ABCD and AEFG are two parallelograms.
Prove that area of || gm ABCD = area of || gm AEFG.
(b) In the fig. (2) Given below, the side AB of the parallelogram ABCD is produced

to E. A straight line through A is drawn parallel to CE to meet CB produced at F and parallelogram BFGE is Completed prove that area of || gm BFGE=Area of || gm ABCD.





(c) In the figure (3) given below AB || DC || EF, AD || BE and DE || AF. Prove the area of DEFH is equal to the area of ABCD.



Solution:

(a) Given:

From fig (1)

ABCD and AEFG are two parallelograms as shown in the figure.

To prove:
area of || gm ABCD = area of || gm AEFG
Proof:
let us join BG.
We know that,
ar (ΔABG) = ½ (ar ||gm ABCD) …… (1)
Similarly,
ar (ΔABG) = ½ (ar ||gm AEFG) …. (2)
From (1) and (2)
½ (ar ||gm ABCD) = ½ (ar ||gm AEFG)
So,
ar ||gm ABCD = ar ||gm AEFG)
Hence proved.

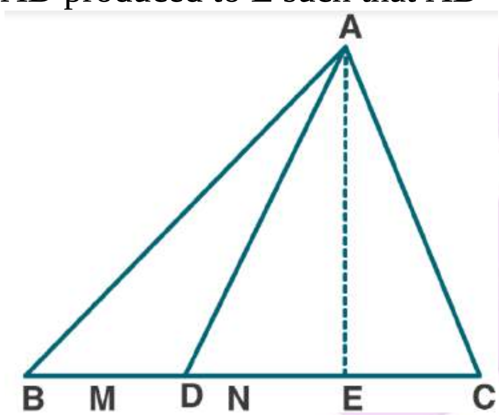
(b) Given:
From fig (2)
A parallelogram ABCD in which AB is produced to E. A straight line through A is drawn parallel to CE to meet CB produced at F and parallelogram BFGE is Completed.
To prove:
area of || gm BFGE=Area of || gm ABCD
Proof:
Let us join AC and EF.
We know that,
ar (ΔAFC) = ar (ΔAFE) ….. (1)
now subtract ar (ΔABF) on both sides, we get
ar (ΔAFC) - ar (ΔABF) = ar (ΔAFE) - ar (ΔABF)
Or ar (ΔABC) = ar (ΔBEF)

Now multiply by 2 on both sides, we get
2. ar (ΔABC) = 2. ar (ΔBEF)
Or ar (||gm ABCD) = ar (||gm BFGE)
Hence proved.

(c) Given:
From fig (3)
AB || DC || EF, AD || BE and DE || AF
To prove:
area of DEFH = area of ABCD
Proof:

We know that,
DE || AF and AD || BE
It is given that ADEG is a parallelogram.
So,
ar (||gm ABCD) = ar (||gm ADEG) ….. (1)
Again, DEFG is a parallelogram.
ar (||gm DEFH) = ar (||gm ADEG) ….. (2)
From (1) and (2)
ar (||gm ABCD) = ar (||gm DEFH)
Or ar ABCD = ar DEFH
Hence proved.

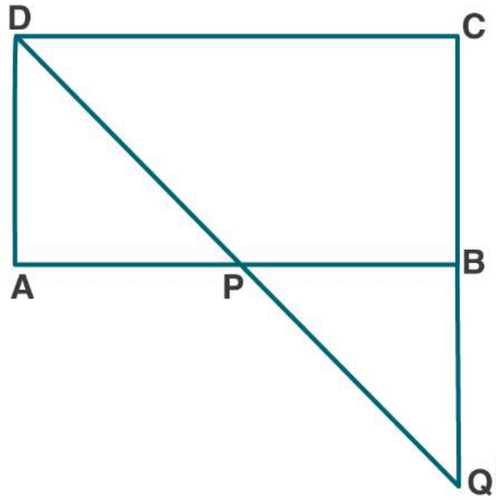
13. Any point D is taken on the side BC of, a ∆ ABC and AD is produced to E such that AD=DE, prove that area of ∆ BCE = area of ∆ ABC.
Solution:
Given:
In ∆ABC, D is taken on the side BC.
AD produced to E such that AD = DE



To prove:
area of ∆ BCE = area of ∆ ABC
Proof:
In ∆ABE, it is given that AD = DE
So, BD is the median of ∆ABE
ar (∆ABD) = ar (∆BED) ….. (1)
similarly,
In ∆ACE, CD is the median of ∆ACE
ar (∆ACD) = ar (∆CED) ….. (2)
By adding (1) and (2), we get
ar (∆ABD) + ar (∆ACD) = ar (∆BED) + ar (∆CED)

Or ar (ΔABC) = ar (ΔBCE)
Hence proved.

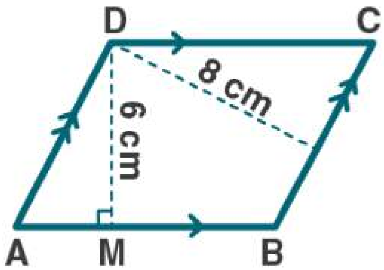
14. ABCD is a rectangle and P is mid-point of AB. DP is produced to meet CB at Q. Prove that area of rectangle ΔBCD = area of Δ DQC.
Solution:
Given:
ABCD is a rectangle and P is mid-point of AB. DP is produced to meet CB at Q.



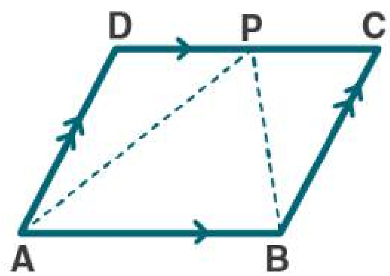
To prove:
area of rectangle ΔBCD = area of Δ DQC
Proof:
In ΔAPD and ΔBQP
AP = BP [Since, D is the mid-point of AB]
∠DAP = ∠QBP [each angle is 90^{o}]
∠APD = ∠BPQ [vertically opposite angles]
So, ΔAPD ≅ ΔBQP [By using ASA postulate]
ar (ΔAPD) = ar (ΔBQP)
Now,
ar ABCD = ar (ΔAPD) + ar PBCD
= ar (ΔBQP) + ar PBCD
= ar (ΔDQC)
Hence proved.

15. (a) In the figure (1) given below, the perimeter of parallelogram is 42 cm. Calculate the lengths of the sides of the parallelogram.
(b) In the figure (2) given below, the perimeter of Δ ABC is 37 cm. If the lengths of

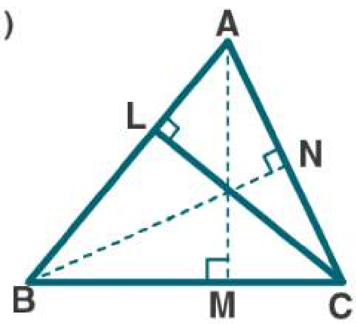
the altitudes AM, BN and CL are 5x, 6x, and 4x respectively, Calculate the lengths of the sides of △ABC.
(c) In the fig. (3) Given below, ABCD is a parallelogram. P is a point on DC such that area of △DAP = 25 cm² and area of △BCP = 15 cm². Find
(i) area of || gm ABCD
(ii) DP: PC.



(1)

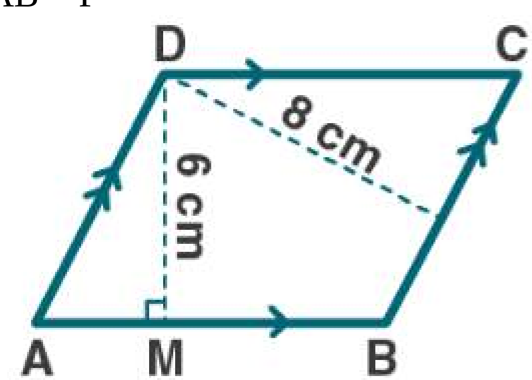


(3)



(2)

Solution:
(a) Given:
The perimeter of parallelogram ABCD = 42 cm
<u>To find:</u>
Lengths of the sides of the parallelogram ABCD.
From fig (1)
We know that,
AB = P



Then, perimeter of ||gm ABCD = 2 (AB + BC)
42 = 2(P + BC)
42/2 = P + BC
21 = P + BC
BC = 21 – P
So, ar (||gm ABCD) = AB × DM
= P × 6
= 6P ………. (1)
Again, ar (||gm ABCD) = BC × DN
= (21 - P) × 8
= 8(21 - P) ………. (2)
From (1) and (2), we get
6P = 8(21 - P)
6P = 168 – 8P
6P + 8P = 168
14P = 168
P = 168/14
= 12
Hence, sides of ||gm are
AB = 12cm and BC = (21 - 12)cm = 9cm

(b) Given:
The perimeter of Δ ABC is 37 cm. The lengths of the altitudes AM, BN and CL are 5x, 6x, and 4x respectively.
To find:
Lengths of the sides of ΔABC. i.e., BC, CA and AB.
Let us consider BC = P and CA = Q

From fig (2),
Then, perimeter of ΔABC = AB + BC + CA
37 = AB + P + Q
AB = 37 – P – Q

Area (ΔABC) = ½ × base × height
= ½ × BC × AM = ½ × CA × BN = ½ × AB × CL
= ½ × P × 5x = ½ × Q × 6x = ½ (37 – P – Q) × 4x
= 5P/2 = 3Q = 2(37 – P – Q)
Let us consider first two parts:

$5P/2 = 3Q$
$5P = 6Q$
$5P – 6Q = 0$ …… (1)
$25P – 30Q$ (multiplying by 5)….. (2)

Let us consider second and third parts:
$3Q = 2(37 – P – Q)$
$3Q = 74 – 2P – 2Q$
$3Q + 2Q + 2P = 74$
$2P + 5Q = 74$ ……. (3)
$12P + 30Q = 444$ (multiplying by 6)……. (4)

By adding (2) and (4), we get
$37P = 444$
$P = 444/37$
$= 12$
Now, substitute the value of P in equation (1), we get
$5P – 6Q = 0$
$5(12) – 6Q = 0$
$60 = 6Q$
$Q = 60/6$
$= 10$
Hence, BC = P = 12cm
CA = Q = 10cm
And AB = 37 – P – Q = 37 – 12 – 10 = 15cm

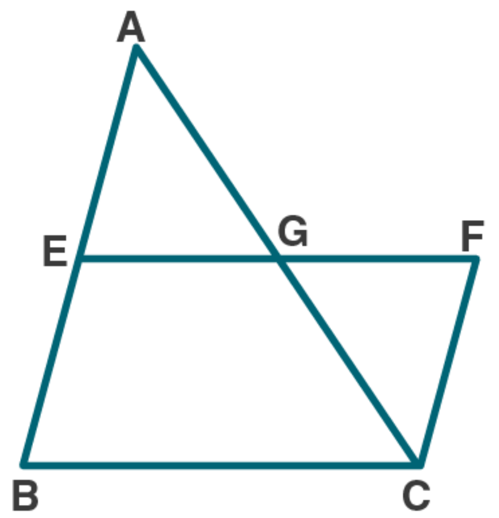
(c) Given:
ABCD is a parallelogram. P is a point on DC such that area of ΔDAP = 25 cm² and area of ΔBCP = 15 cm².
To Find:
(i) area of || gm ABCD
(ii) DP: PC
Now let us find,
From fig (3)
(i) we know that,
ar (ΔAPB) = ½ ar (||gm ABCD)
Then,
½ ar (||gm ABCD) = ar (ΔDAP) + ar (ΔBCP)
= 25 + 15

$= 40cm^2$

So, ar (||gm ABCD) = $2 \times 40 = 80cm^2$

(ii) we know that,
ΔADP and ΔBCP are on the same base CD and between same parallel lines CD and AB.
ar (ΔDAP)/ar(ΔBCP) = DP/PC
25/15 = DP/PC
5/3 = DP/PC
So, DP: PC = 5: 3

16. In the adjoining figure, E is mid-point of the side AB of a triangle ABC and EBCF is a parallelogram. If the area of Δ ABC is 25 sq. units, find the area of || gm EBCF.
Solution:



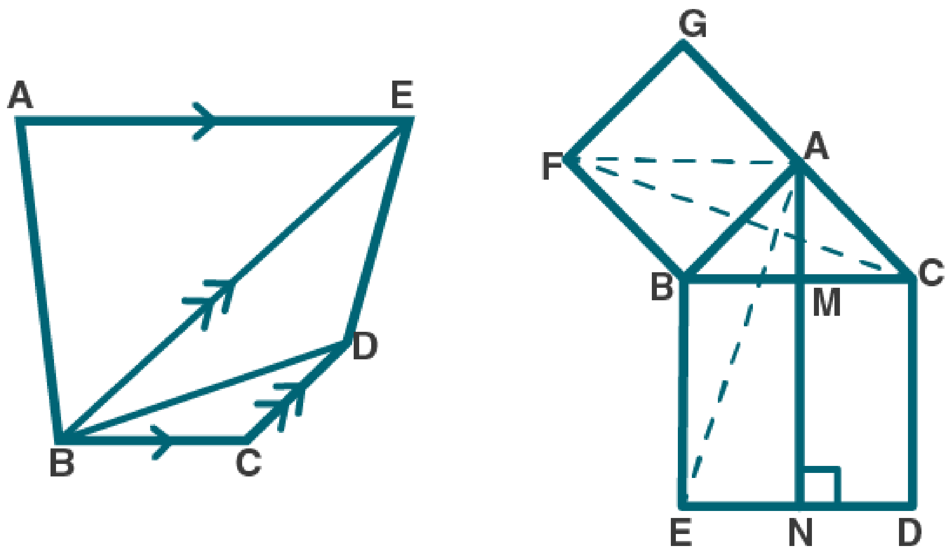
Let us consider EF, side of ||gm BCFE meets AC at G.
We know that, E is the mid-point and EF || BC
G is the mid-point of AC.
So,
AG = GC

Now, in ΔAEG and ΔCFG,
The alternate angles are: ∠EAG, ∠GCF
Vertically opposite angles are: ∠EGA = ∠CGF
So, AG = GC
Proved.

∴ ΔAEG ≅ ΔCFG
ar (ΔAEG) = ar (ΔCFG)

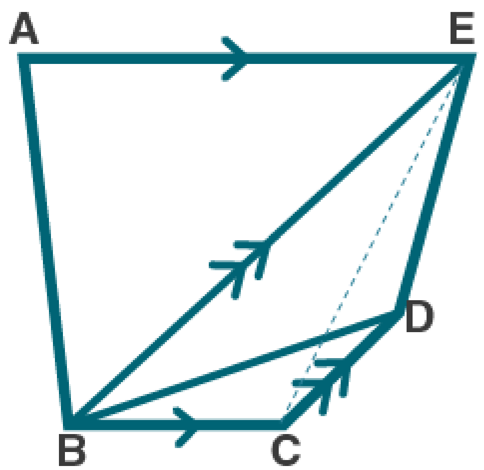
Now,
ar (||gm EBCF) = ar BCGE + ar (ΔCFG)
= ar BCGE + ar (ΔAEG)
= ar (ΔABC)
We know that, ar (ΔABC) = 25sq. units
Hence, ar (||gm EBCF) = 25sq. units

17. (a) In the figure (1) given below, BC || AE and CD || BE. Prove that: area of ΔABC= area of ΔEBD.
(b) In the figure (2) given below, ABC is right angled triangle at A. AGFB is a square on the side AB and BCDE is a square on the hypotenuse BC. If AN ⊥ ED, prove that:
(i) ΔBCF ≅ Δ ABE.
(ii) area of square ABFG = area of rectangle BENM.



(1) (2)

Solution:
(a) Given:
From fig (1)
BC || AE and CD || BE
To prove:
area of ΔABC= area of ΔEBD
Proof:



By joining CE.
We know that, from ΔABC and ΔEBC
ar (ΔABC) = ar (ΔEBC) ….. (1)
From EBC and ΔEBD
ar (ΔEBC) = ar (ΔEBD) …… (2)
From (1) and (2), we get
ar (ΔABC) = ar (ΔEBD)
Hence proved.

(b) Given:
ABC is right angled triangle at A. Squares AGFB and BCDE are drawn on the side AB and hypotenuse BC of ΔABC. AN ⊥ ED which meets BC at M.
<u>To prove:</u>
(i) ΔBCF ≅ Δ ABE.
(ii) area of square ABFG = area of rectangle BENM
From the figure (2)
(i) ∠FBC = ∠FBA + ∠ABC
So,
∠FBC = 90^o + ∠ABC ….. (1)

∠ABE = ∠EAC + ∠ABC
So,
∠ABE = 90^o + ∠ABC ….. (2)
From (1) and (2), we get
∠FBC = ∠ABE ….. (3)
So, BC = BE

Now, in ΔBCF and ΔABE

BF = AB
By using SAS axiom rule of congruency,
∴ $\Delta BCF \cong \Delta ABE$
Hence proved.

(ii) we know that,
$\Delta BCF \cong \Delta ABE$
So, ar (ΔBCF) = ar (ΔABE) ….. (4)
$\angle BAG + \angle BAC = 90^o + 90^o$
$= 180^o$
So, GAC is a straight line.

Now, from ΔBCF and square AGFB
ar (ΔBCF) = ½ ar (square AGFB) …. (5)

From ΔABE and rectangle BENM
ar (ΔABE) = ½ ar (rectangle BENM) ….. (6)
From (4), (5) and (6)
½ ar (square AGFB) = ½ ar (rectangle BENM)
ar (square AGFB) = ar (rectangle BENM)
Hence proved.