EXERCISE 17

1. (a) From the figure (1) given below, find the values of:

(i) sin θ

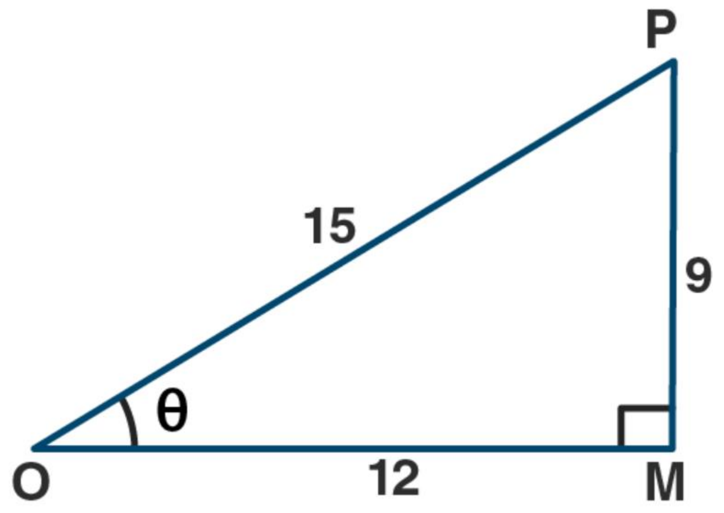
(ii) cos θ

(iii) tan θ

(iv) cot θ

(v) sec θ

(vi) cosec θ



(b) From the figure (2) given below, find the values of:

(i) sin θ

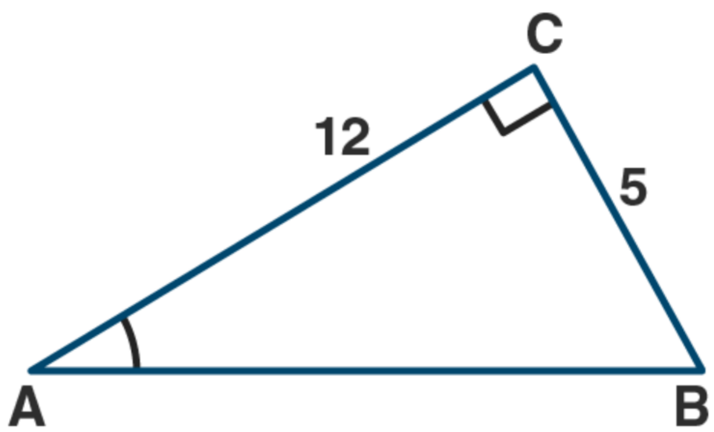
(ii) cos θ

(iii) tan θ

(iv) cot θ

(v) sec θ

(vi) cosec θ



Solution:

(a) From right angled triangle OMP,

By Pythagoras theorem, we get

$OP^2 = OM^2 + MP^2$

$MP^2 = OP^2 + OM^2$

$MP^2 = (15)^2 – (12)^2$

$MP^2 = 225 – 144$

$MP^2 = 81$

$MP^2 = 9^2$

MP = 9

(i) sin θ = MP/OP

= 9/15

= 3/5

(ii) cos θ = OM/OP

= 12/15

= 4/5

(iii) tan θ = MP/OP

= 9/12

= ¾

(iv) cot θ = OM/MP

= 12/9

= 4/3

(v) sec θ = OP/OM
= 15/12
= 5/4

(vi) cosec θ = OP/MP
= 15/9
= /3

(b) From right angled triangle ABC,
By Pythagoras theorem, we get
$AB^2 = AC^2 + BC^2$
$AB^2 = (12)^2 + (5)^2$
$AB^2 = 144 + 25$
$AB^2 = 169$
$AB^2 = 13^2$
AB = 13
(i) sin A = BC/AB
= 5/13

(ii) cos A = AC/AB
= 12/13

(iii) $\text{Sin}^2 A + \cos^2 A = (BC/AB)^2 + (AC/AB)^2$
$= (5/13)^2 + (12/13)^2$
= (25/169) + (144/169)
= (25 + 144)/ 169
= 169/169
= 1
$\text{Sin}^2 A + \cos^2 A = 1$

(iv) $\text{Sec}^2 A - \tan^2 A = (AB/AC)^2 - (BC/AC)^2$
$= (13/12)^2 - (5/12)^2$
= (169/144) - (25/144)
= (169 - 25)/ 144
= 144/144
= 1
$\text{Sec}^2 A - \tan^2 A = 1$

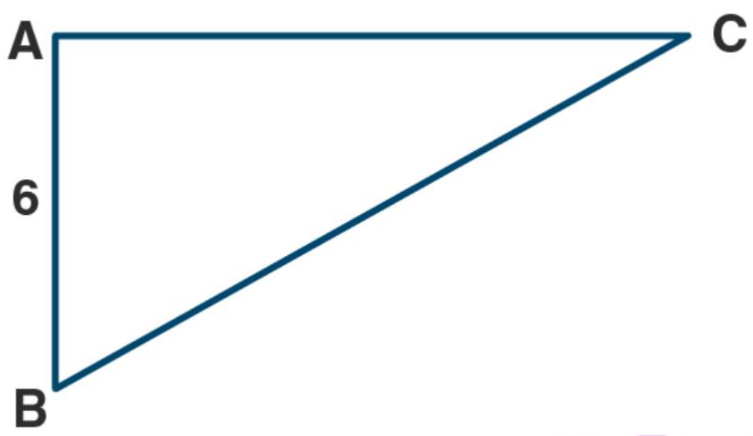
2. (a) From the figure (1) given below, find the values of:

(i) sin B

(i) cos C

(iii) sin B + sin C

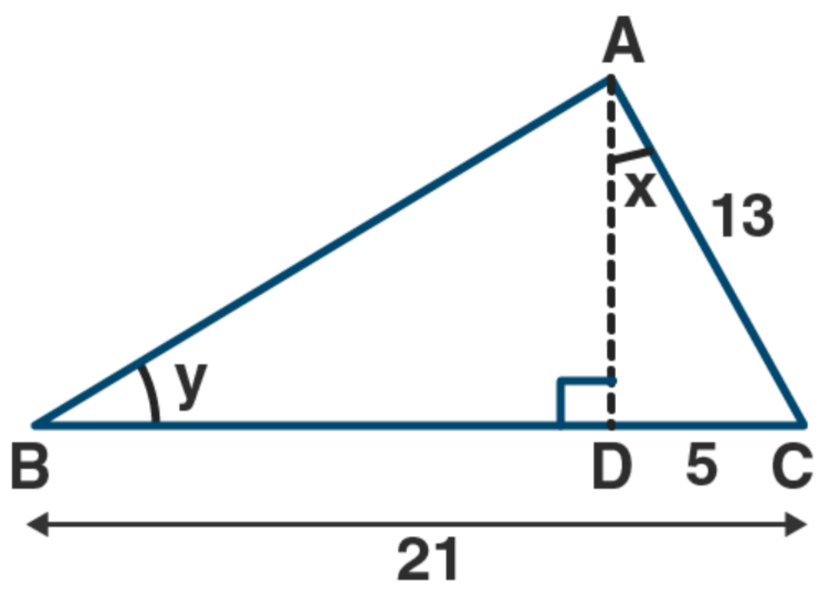
(iv) sin B cos C + sin C cos B



(b) From the figure (2) given below, find the values of……..

(1) sin B

(ii) cos C



Solution:

From right angled triangle ABC,

By Pythagoras theorem, we get

$BC^2 = AC^2 + AB^2$

$AC^2 = BC^2 - AB^2$

$AC^2 = (10)^2 - (6)^2$

$AC^2 = 100 - 36$

$AC^2 = 64$

$AC^2 = 8^2$

AC = 8

(i) sin B = perpendicular/ hypotenuse

= AC/BC

= 8/10

= 4/5

(ii) cos C = Base/hypotenuse

= AC/BC

= 8/10

= 4/5

(iii) sin B = Perpendicular/hypotenuse

= AC/BC

= 8/10

= 4/5

Sin C = perpendicular/hypotenuse

= AB/BC

= 6/10

= 3/5

Now,

Sin B + sin C = (4/5) + (3/5)

= (4 + 3)/5

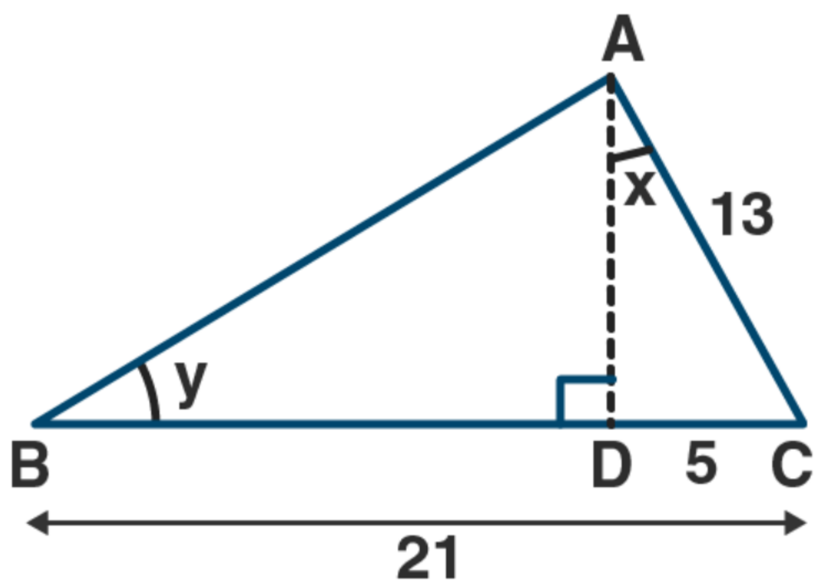
= 7/5

(iv) sin B = 4/5

Cos C = 4/5

Sin C = perpendicular/ hypotenuse

= AB/BC
= 6/10
= 3/5
Cos B = Base/Hypotenuse
= AB/BC
= 6/10
= 3/5
sin B cos C + sin C cos B
= (4/5) × (4/5) × (3/5) × (3/5)
= (26/25) × (9/25)
= (16+9)/25
= 25/25
= 1
From Figure
AC = 13, CD = 5, BC =21,
BD = BC – CD
= 21 – 5
= 16



From right angled ΔACD,
By Pythagoras theorem we get
$AC = AD^2 + CD^2$
$AD^2 = AC^2 – CD^2$

$AD^2 = (13)^2 – (5)^2$
$AD^2 = 169 – 25$
$AD^2 = 144)$
$AD^2 = (12)^2$
AD = 12
From right angled ΔABD,
By Pythagoras angled ΔABD
By Pythagoras theorem we get
$AB^2 = AD^2 + BD^2$
$AB^2 = 400$
$AB^2 = (20)^2$
AB = 20

(i) tan x = perpendicular/Base (in right angled ΔACD)
=CD/AD
= 5/12

(ii) cos y = Base/Hypotenuse (in right angled ΔABD)
= BD/AB
= (20)/12 – (5/3)
Cot y = Base/Perpendicular (in right ΔABD)
=BD/AB
= 16/20 = 4/5

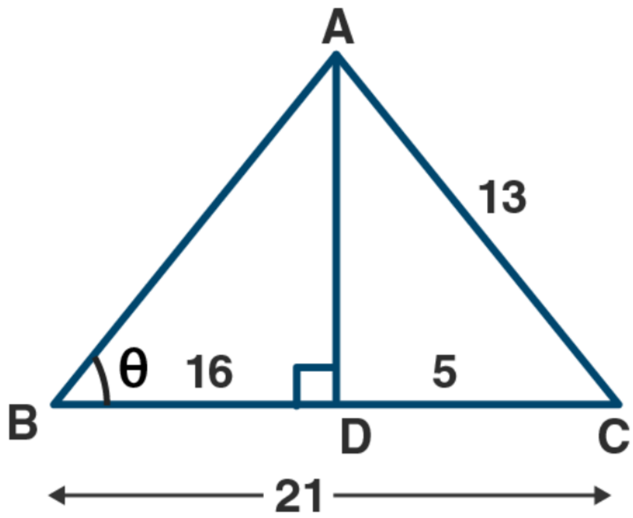
(iii)) cos y = Hypotenuse/ perpendicular (in right angled ΔABD)
BD/AB
= 20/12
= 5/3
Cot y = Base/Perpendicular (in right ΔABD)
AB/AD
= 16/12
= 4/3
$Cosec^2 y - cot^2 y = (5/3)^2 – (4/3)^2$
= (25/9) – (16/9)
= (25-16)/9
= 9/9
= 1

Hence, $cosec^2 y - cot^2 y = 1$

(iv) sin x = Perpendicular/Hypotenuse (in right angled ΔACD)
= AD/AB
= 12/20
= 3/5
Cot y = Base/Perpendicular (in right angled ΔABD)
= BD/AD
= 16/12
= 4/3
(5/sin x) + (3/sin y) – 3cot y
= 5/(5/13) + 3/(3/5) – 3 × 4/3
= 5 × 13/5 + 3 × 5/3 – 3 × 4/3
= 1 × 13/1 + 1 × 5/1 – 1 × 4/1
= 13 + 5 – 4 = 18 – 4
= 14
Hence 5/sin x + 3/sin y – 3cot y = 14

3. From the figure (1) given below, find the value of sec …
(b) From the figure (2) given below, find the values of
(i) sin x
(ii) cot x
(iii) $cot^2 x - cosec^2 x$
(iv) sec y
(v) tan ……

Solution:
(a) From the figure, Sec θ = AB / BD
But in ΔADC, $\angle D = 90^o$



$AC^2 = AD^2 + DC^2$ (Pythagoras Theorem)

$(13)^2 = AD^2 + 25$

$AD^2 = 169 - 25$

$= 144$

$= (12)^2$

AD = 12

(in right ΔABD)

$AB^2 = AD^2 + BD^2$

$= (12)^2 + (16)^2$

$= 144 + 256$

$= 400$

$= (20)^2$

AB = 20

Now, Sec θ = AB / BD

= 20/16

= 5/4

(b) let given ΔABC

BD = 3, AC = 12, AD = 4

In right angled ΔABD

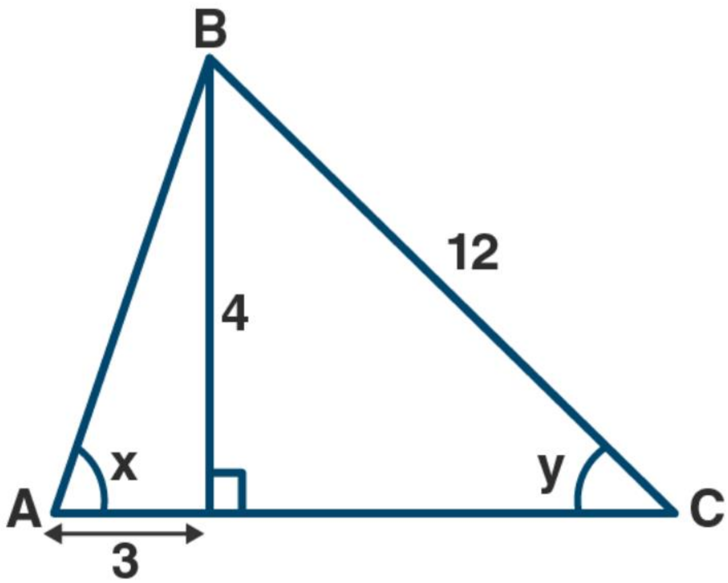
By Pythagoras theorem

$AB^2 = AD^2 + BD^2$

$AB^2 = (4)^2 + (3)^2$

$AB^2 = 16 + 9$

$AB^2 = 25$
$AB^2 = (5)^2$
AB = 5



In right angled triangle ACD
By Pythagoras theorem,
$AC^2 = AD^2 + CD^2$
$CD^2 = AC^2 – AD^2$
$CD^2 = (12)^2 – (4)^2$
$CD^2 = 144 - 16$
$CD^2 = 128$
CD = √128
CD = √64 × 2 CD
= 8√2

(i) sin x = perpendicular/Hypotenuse
= AD/AB
= 4/5

(ii) cot x = Base/Perpendicular
= BD/AD
= ¾

(iii) cot x = Base/ Perpendicular

BD/AD
= 3/4
cosec x = Hypotenuse / Perpendicular
AB/BD
= 5/4
$\text{Cot}^2 x - \text{cosec}^2 x$
$= (3/4)^2 - (5/4)^2$
= 9/16 – 25/16
(9 -25)/16
= -16/16
= -1
Perpendicular = Hypotenuse/Base (in right angled ΔACD)
= AD/CD
= 12/(8 √2)
= 3/(2 √2)
Cot y = Base/ Hypotenuse
= AD/CD
= 4/8 √ 2
= 1/2 √2
Cot y = Base / Hypotenuse ((in right angled ΔACD)
= CD/AC
= 8√2/12
= 2√/3
Now $\tan^2 y - 1/\cot^2 y$
$= (1/2\sqrt{2})^2 - 1/(2\sqrt{2}/3)^2$
= ¼ × - ¼ × 2
= (1/8) – (9/8)
= (1-9)/8
= -8/8
= -1
$\tan^2 y - 1/\cot^2 y = -1.$

4. In a right-angled triangle, it is given that angle A is an acute angle and that Tan A=5/12 Find the values of:
(i) cos A
(ii) cosec A- cot A.

Solution:

Here, ABC is right angled triangle

∠A is an acute angle and ∠C = 90^o

tan A = 5/12

BC/AC =5/12

Let BC = 5x and AC = 12x

From right angled ΔABC

By Pythagoras theorem, we get

$AB^2 = (5x)^2 + (12x)^2$

$AB^2 = 25x^2 + 144x^2$

$AB^2 = 169x^2$

(i) cos A = Base/ Hypotenuse

= AC / AB

= 12x/13x

=12/13

(ii) cosec A = Hypotenuse/perpendicular

= AC / BC

= 13x /5x

= 13/5

cot A = 13/5 – 12/5

= (13-12)/5

= 1/5

6. In triangle ABC, AB = 15 cm, AC = 15 cm and BC = 18 cm.

Solution :

Here ABC is a triangle in which

AB = 15 cm, AC = 15 cm and BC = 18 cm

Draw AD perpendicular to BC , D is mid-point of BC.

Then, BD – DC = 9 cm

in right angled triangle ABD,

By Pythagoras theorem, we get

$AB^2 = AD^2 + BD^2$

$AD^2 = AB^2 – BD^2$

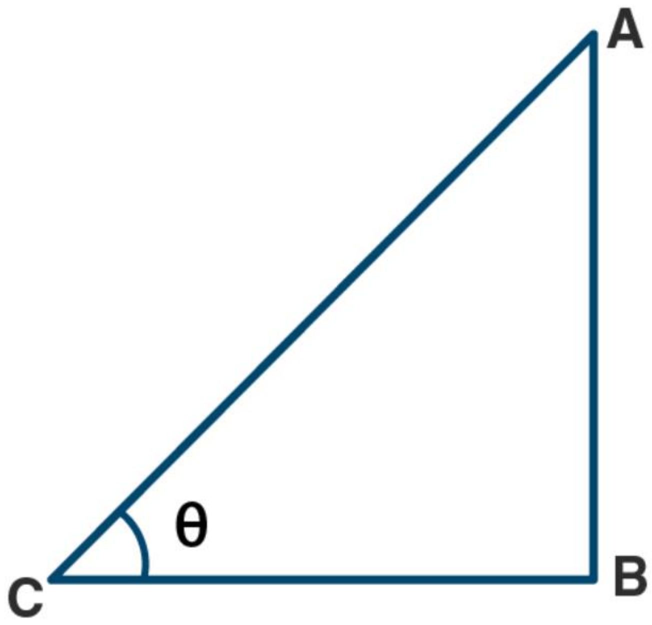
$AD^2 = (15)^2 – (9)^2$
$AD^2 = 225 – 81$
$AD^2 = 144$
AD – 12 cm

(i) cos ∠ABC = Base/ / Hypotenuse
(In right angled ΔABD, ∠ABC = ∠ABD)
= BD / AB
= 9/15
= 3/5

(ii) sin ∠ACB = sin ∠ACD
= perpendicular/ Hypotenuse
= AD/AC
= 12/15
= 4/5

7. If sin …….=3/5 and …….

Solution:
Let Δ ABC be a right angled at B
Let ∠ACB = θ
Given that, sin θ = 3/5
AB/AC = 3/5
Let AB = 3x
then AC = 5x
In right angled Δ ABC,
By Pythagoras theorem,
We get



$(5x)^2 = (3x)^2 + BC^2$
$BC^2 = (5x)^2 – (3x)^2$
$BC^2 = (2x)^2$
BC = 4x

(i) cos θ = Base/ Hypotenuse
= BC / AC
= 4x /5x
= 4/5

(ii) tan θ = perpendicular/Base
= AB/BC
= 3x/4x
= ¾

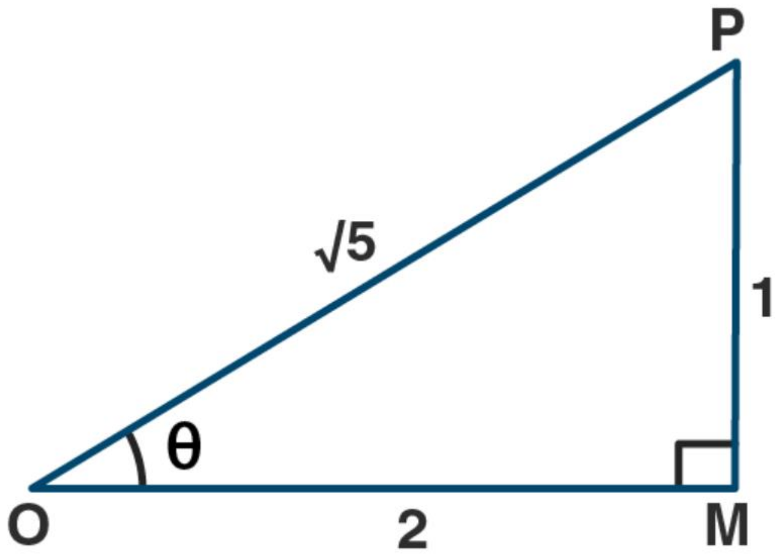
8. If tan = 4/3
find the value of sin .. cos … (both sin and cos are Positive)

Solution :
Let ΔABC be a right angled
∠ACB = θ
Given that, tan θ = 4/3

(AB/BC = 4/3)
Give that, tan θ = 4/3
(AB/BC = 4/3)
Let AB = 4x,
then BC = 3x
In right angled ΔABC
By Pythagoras theorem, we get
$AC^2 = AB^2 + BC^2$
$AC^2 = AB^2 + BC^2$
$AC^2 = AB^2 + BC^2$
$AC^2 = AB^2 + BC^2$
$(AC^2 = (4x)^2 + (3x)^2$
$AC^2 = 16x^2 + 9x^2$
$AC^2 = 25x^2$
$AC^2 = (5x)^2$
AC = 5x
Sin θ = perpendicular/Hypotenuse
= AB/AC
= 4x/5x
= 4/5
Cos θ = Base/Hypotenuse
= BC/AC
= 3x/5x
= 3/5
Sin θ + cos θ
= 4/5 + 3/5
= (4 + 3)/5
= 7/5
Hence, Sin θ + cos θ = 7/5 = 1 (2/5)

9. 1f cosec = √5 and is less than 90, find the value of cot - Cos

Solution:
Given cosec θ = √5/1 = OP/PM
OP = √5 and PM = 1



Now $OP^2 = OM^2 + PM^2$ using Pythagoras theorem

$(\sqrt{5})^2 = OM^2 + 1^2$

$5 = OM^2 + 1$

$OM^2 = 5 – 1$

$OM^2 = 4$

$OM = 2$

Now $\cot \theta = OM/PM$

$= 2/1$

$= 2$

$\cos \theta = OM/OP$

$= 2/\sqrt{5}$

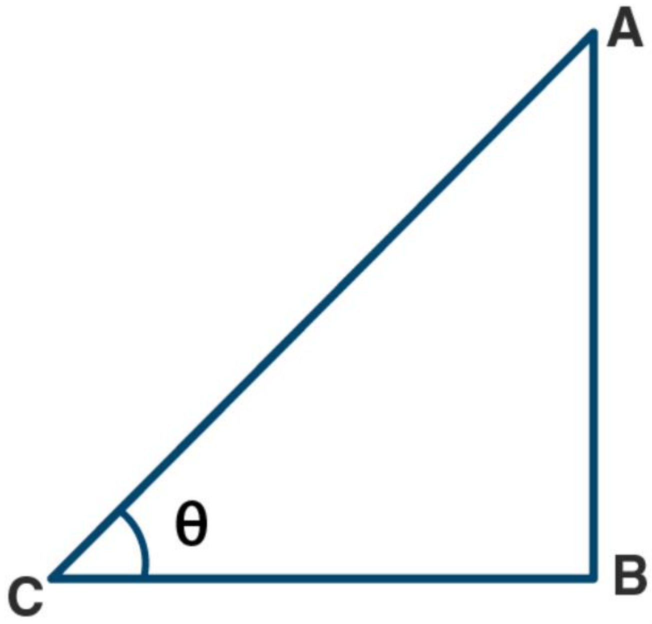
Now $\cot \theta - \cos \theta = 2 – (2/\sqrt{5})$

$= 2 (\sqrt{5} – 1)/ 2/\sqrt{5}$

10. Given sin = p/q find sin + cos

Solution:

Given that $\sin \theta = p/q$



Which implies,
AB/AC = p/q
Let AB = px
And then AC = qx
In right angled triangle ABC
By Pythagoras theorem,
We get
$AC^2 = AB^2 + BC^2$
$BC^2 = AC^2 – AB^2$
$BC^2 = q^2x^2 – p^2x^2$
$BC^2 = (q^2 – p^2)x^2$
$BC = \sqrt{(q^2 – p^2)}x$
In right angled triangle ABC,
Cos θ = base/ hypotenuse
= BC/AC
$= \sqrt{(q^2 – p^2)}x/qx$
$= \sqrt{(q^2 – p^2)}/ q$
Now,
$\text{Sin } \theta + \cos \theta = p/q + \sqrt{(q^2 – p^2)}/ q$
$= [p + \sqrt{(q^2 – p^2)}]/ q$

11. If O is an acute angle and tan = 8/15 then find sec + cosec

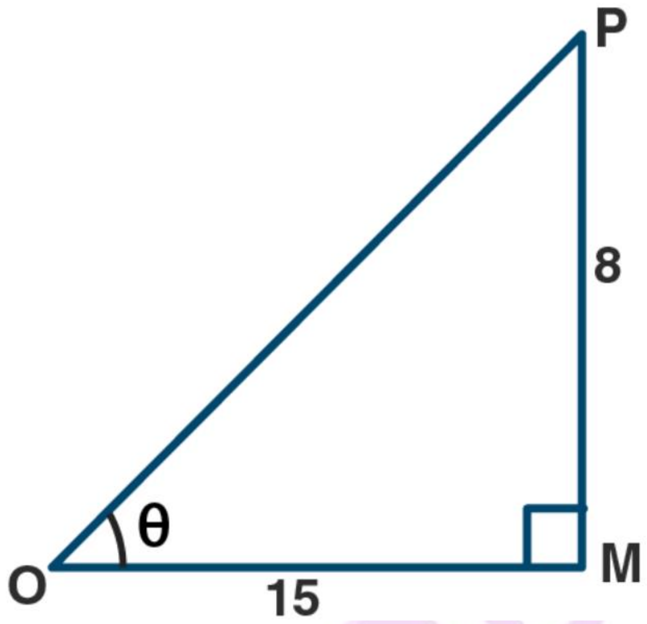
Solution:

Given tan θ = 8/15

θ is an acute angle

in the figure triangle OMP is a right angled triangle,

∠M = 90^o and ∠Q = θ



Tan θ = PM/OL = 8/15

Therefore, PM = 8, OM = 15

But $OP^2 = OM^2 + PM^2$ using Pythagoras theorem,

$= 15^2 + 8^2$

= 225 + 64

= 289

$= 17^2$

Therefore, OP = 17

Sec θ = OP/OM

= 17/15

Cosec θ = OP/PM

= 17/8

Now,

Sec θ + cosec θ = (17/15) + (17/8)

= (136 + 255)/ 120

= 391/120

12. Given A is an acute angle and 13 sin A 5, evaluate: (5 sin A – 2 cos A)/ tan A

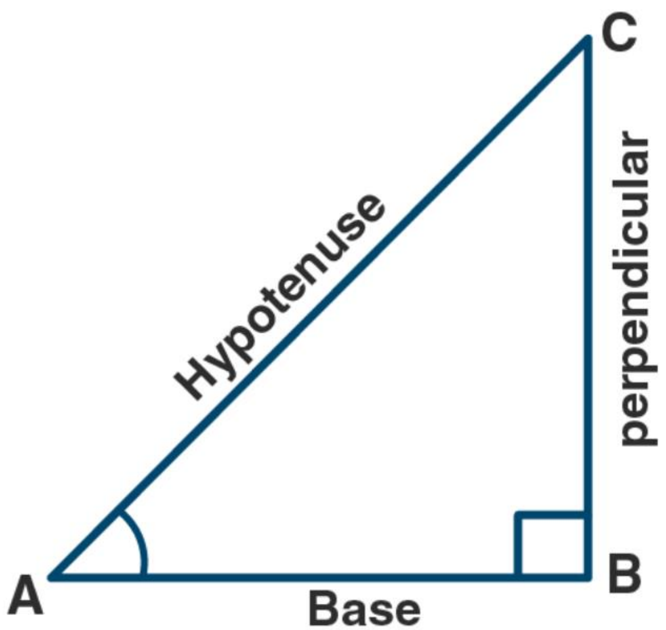
Solution:

Let triangle ABC be a right angled triangle at B and A is an acute angle

Given that 13 sin A = 5

Sin A = 5/13

AB/Ac = 5/13



Let AB = 5x

AC = 13 x

In right angled triangle ABC,

Using Pythagoras theorem,

We get

$AC^2 = AB^2 + BC^2$

$BC^2 = AC^2 – BC^2$

$BC^2 = (13x)^2 – (5x)^2$

$BC^2 = 169x^2 – 25x^2$

$BC^2 = 144x^2$

BC = 12x

Sin A = 5/13

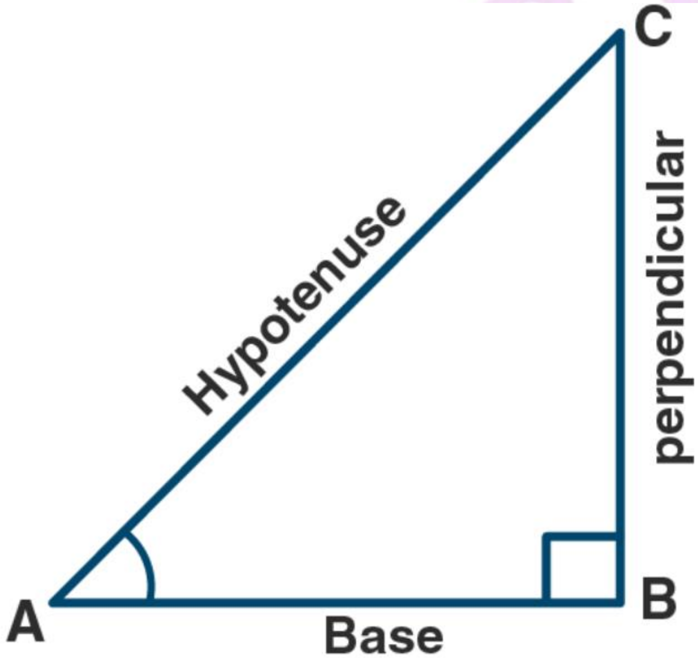
Cos A = base/ hypotenuse

= BC/AC

= 12x/ 13x
= 12/13
Tan A = perpendicular/ base
= AB/BC
= 5x/ 12x
= 5/ 12
Now,
(5 sin A – 2 cos A)/ tan A = [(5) (5/13) – (2) (12/13)]/ (5/12)
= (1/13)/ (5/12)
= 12/65
Hence (5 sin A – 2 cos A)/ tan A = 12/65

13. Given A is an acute angle and cosec A = √2, find the value of 2 $\sin^2$ A + 3 $\cot^2$ A/ ($\tan^2$ A – $\cos^2$ A)

Solution:
Let triangle ABC be a right angled at B and A is a acute angle.
Given that cosec A = √2



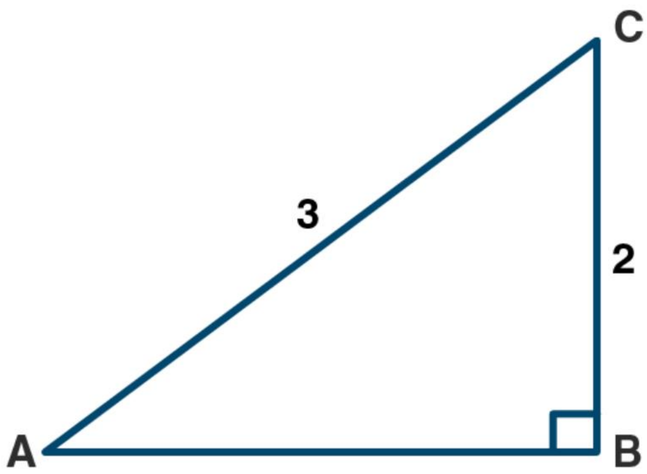
Which implies,
AC/BC = √2/1
Let AC = √2x

Then BC = x
In right angled triangle ABC
By using Pythagoras theorem,
We get
$AC^2 = AB^2 + BC^2$
$(\sqrt{2}x)^2 = AB^2 + x^2$
$AB^2 = 2x^2 - x^2$
AB = x
Sin A = perpendicular/ hypotenuse
= BC/AC
$= 1/\sqrt{2}$
Cot A = base/ perpendicular
= x/x
= 1
Tan A = perpendicular/ base
= BC/AB
= x/x
= 1
Cos A = base/ hypotenuse
= AB/AC
$= x/\sqrt{2}x$
$= 1/\sqrt{2}$
Substituting these values we get
$2 \sin^2 A + 3 \cot^2 A/ (\tan^2 A - \cos^2 A) = 8$

CHAPTER TEST

1. (a)From the figure (i) given below, calculate all the six t-ratios for both acute………
(b)From the figure (ii) given below, find the values of x and y in terms of t-ratios

Solution:



(a) From right angled triangle ABC,
By Pythagoras theorem, we get
$AC^2 = AB^2 + BC^2$
$AB^2 = AC^2 - BC^2$
$AB^2 = (3)^2 – (2)^2$
$AB^2 = 9 – 4$
$AB^2 = 5$
AB = √5

(i) sin A = perpendicular/ hypotenuse
= BC/AC
= 2/3

(ii) cos A = base/ hypotenuse
= AB/AC
= √5/3

(iii) tan A = perpendicular/ base
= BC/AB

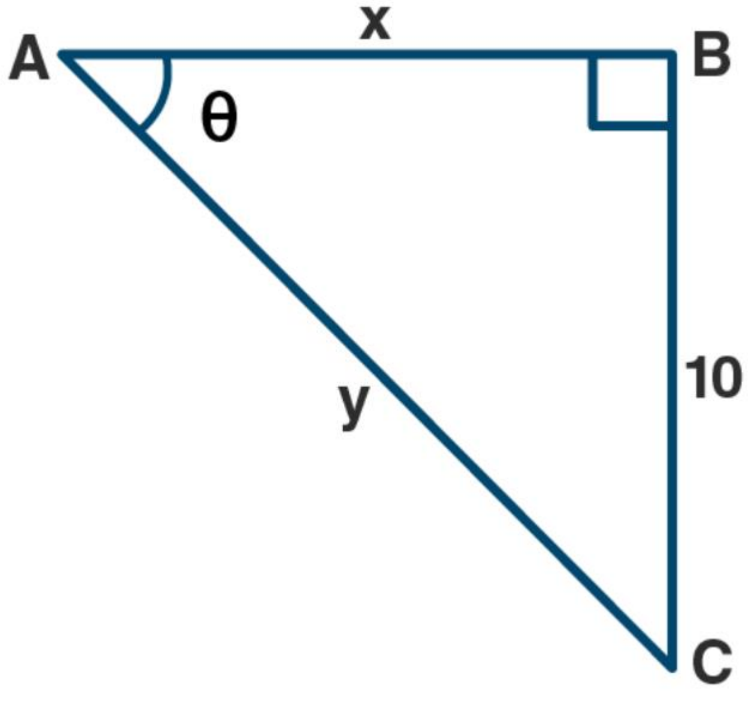
= 2/ √5

(iv) cot A = base/perpendicular
= AB/ BC
= √5/2

(v) sec A = hypotenuse/ base
= AC/AB
= 3/ √5

(vi) cosec A = hypotenuse/perpendicular
= AC/BC
= 3/2

(b) From right angled triangle ABC,



∠BAC = θ
Then we know that,
Cot θ = base/ perpendicular
= AB/ BC
= x/ 10
x = 10 cot θ

also cosec θ = hypotenuse/ perpendicular
= AC/ BC
= y/ 10
y = 10 cosec θ
hence x = 10 cot θ and y = 10 cosec θ