EXERCISE 4.1

1. (i) $8xy^3 + 12x^2y^2$
Solution:-
$8xy^3 + 12x^2y^2$
Take out common in both terms,
Then, $4xy^2 (2y + 3x)$
Therefore, HCF of $8xy^3$ and $12x^2y^2$ is $4xy^2$.

(ii) $15 ax^3 – 9ax^2$
Solution:-
$15 ax^3 – 9ax^2$
Take out common in both terms,
Then, $3ax^2 (5x - 3)$
Therefore, HCF of $15 ax^3$ and $9ax^2$ is $3ax^2$.

2.
(i) $21py^2 – 56py$
Solution:-
$21py^2 – 56py$
Take out common in both terms,
Then, $7py (3y - 8)$
Therefore, HCF of $21py^2$ and $56py$ is $7py$.

(ii) $4x^3 – 6x^2$
Solution:-
$4x^3 – 6x^2$
Take out common in both terms,
Then, $2x^2 (2x - 3)$
Therefore, HCF of $4x^3$ and $6x^2$ is $2x^2$.

3.
(i) $2\pi r^2 - 4\pi r$
Solution:-
$2\pi r^2 - 4\pi r$
Take out common in both terms,
Then, $2\pi r (r - 2)$

Therefore, HCF of $2\pi r^2$ and $4\pi r$ is $2\pi r$.

(ii) 18m + 16n
Solution:-
18m + 16n
Take out common in both terms,
Then, 2 (9m – 8n)
Therefore, HCF of 18m and 16n is 2.

4.
(i) $25abc^2 – 15a^2b^2c$
Solution:-
$25abc^2 – 15a^2b^2c$
Take out common in both terms,
Then, 5abc (5c – 3ab)
Therefore, HCF of $25abc^2$ and $15a^2b^2c$ is 5abc.

(ii) $28p^2q^2r – 42pq^2r^2$
Solution:-
$28p^2q^2r – 42pq^2r^2$
Take out common in both terms,
Then, $14pq^2r$ (2p – 3r)
Therefore, HCF of $28p^2q^2r$ and $42pq^2r^2$ is $14pq^2r$.

5.
(i) $8x^3 – 6x^2 + 10x$
Solution:-
$8x^3 – 6x^2 + 10x$
Take out common in both terms,
Then, $2x(4x^2 – 3x + 5)$
Therefore, HCF of $8x^3$, $6x^2$ and 10x is 2x.

(ii) 14mn + 22m – 62p
Solution:-
14mn + 22m – 62p
Take out common in both terms,
Then, 2 (7mn + 11m – 31p)

Therefore, HCF of 14mn, 22m and 62p is 2.

6.
(i) $18p^2q^2 – 24pq^2 + 30p^2q$
Solution:-
$18p^2q^2 – 24pq^2 + 30p^2q$
Take out common in both terms,
Then, 6pq (3pq – 4q + 5p)
Therefore, HCF of $18p^2q^2$, $24pq^2$ and $30p^2q$ is 6pq.

(ii) $27a^3b^3 – 18a^2b^3 + 75a^3b^2$
Solution:-
$27a^3b^3 – 18a^2b^3 + 75a^3b^2$
Take out common in both terms,
Then, $3a^2b^2$ (9a – 6b + 25a)
Therefore, HCF of $27a^3b^3$, $18a^2b^3$ and $75a^3b^2$ is $3a^2b^2$.

7.
(i) 15a (2p – 3q) – 10b (2p – 3q)
Solution:-
15a (2p – 3q) – 10b (2p – 3q)
Take out common in both terms,
Then, 5(2p – 3q) [3a – 2b]
Therefore, HCF of 15a (2p – 3q) and 10b (2p – 3q) is 5(2p – 3q).

(ii) $3a(x^2 + y^2) + 6b (x^2 + y^2)$
Solution:-
$3a(x^2 + y^2) + 6b (x^2 + y^2)$
Take out common in all terms,
Then, $3(x^2 + y^2)$ (a + 2b)
Therefore, HCF of $3a(x^2 + y^2)$ and $6b (x^2 + y^2)$ is $3(x^2 + y^2)$.

8.
(i) $6(x + 2y)^3 + 8(x + 2y)^2$
Solution:-
$6(x + 2y)^3 + 8(x + 2y)^2$
Take out common in all terms,

Then, $2(x + 2y)^2 [3(x + 2y) + 4]$
Therefore, HCF of $6(x + 2y)^3$ and $8(x + 2y)^2$ is $2(x + 2y)^2$.

(ii) $14(a – 3b)^3 – 21p(a – 3b)$
Solution:-
$14(a – 3b)^3 – 21p(a – 3b)$
Take out common in all terms,
Then, $7(a – 3b) [2(a – 3b)^2 – 3p]$
Therefore, HCF of $14(a – 3b)^3$ and $21p(a – 3b)$ is $7(a – 3b)$.

9.
(i) $10a(2p + q)^3 – 15b (2p + q)^2 + 35 (2p + q)$
Solution:-
$10a(2p + q)^3 – 15b (2p + q)^2 + 35 (2p + q)$
Take out common in all terms,
Then, $5(2p + q) [2a (2p + q)^2 – 3b (2p + q) + 7]$
Therefore, HCF of $10a(2p + q)^3$, $15b (2p + q)^2$ and $35 (2p + q)$ is $5(2p + q)$.

(ii) $x(x^2 + y^2 – z^2) + y(-x^2 – y^2 + z^2) – z (x^2 + y – z^2)$
Solution:-
$x(x^2 + y^2 – z^2) + y(-x^2 – y^2 + z^2) – z (x^2 + y – z^2)$
Take out common in all terms,
Then, $(x^2 + y^2 – z^2) [x – y - z]$
Therefore, HCF of $x(x^2 + y^2 – z^2)$, $y(-x^2 – y^2 + z^2)$ and $z (x^2 + y – z^2)$ is $(x^2 + y^2 – z^2)$

EXERCISE 4.2

1.
(i) $x^2 + xy - x - y$
Solution:-
$x^2 + xy - x - y$
Take out common in all terms,
$x(x + y) - 1(x + y)$
$(x + y) (x - 1)$

(ii) $y^2 - yz - 5y + 5z$
Solution:-
$y^2 - yz - 5y + 5z$
Take out common in all terms,
$y(y - z) - 5(y - z)$
$(y - z) (y - 5)$

2.
(i) $5xy + 7y - 5y^2 - 7x$
Solution:-
$5xy - 7x - 5y^2 + 7y$
Take out common in all terms,
$x(5y - 7) - y(5y - 7)$
$(5y - 7) (x - y)$

(ii) $5p^2 - 8pq - 10p + 16q$
Solution:-
$5p^2 - 8pq - 10p + 16q$
Take out common in all terms,
$p(5p - 8q) - 2(5p - 8q)$
$(5p - 8q) (p - 2)$

3.
(i) $a^2b - ab^2 + 3a - 3b$
Solution:-
$a^2b - ab^2 + 3a - 3b$
Take out common in all terms,

ab(a - b) + 3(a - b)
(a - b) (ab + 3)

(ii) $x^3 - 3x^2 + x - 3$
Solution:-
$x^3 - 3x^2 + x - 3$
Take out common in all terms,
$x^2 (x - 3) + 1(x - 3)$
$(x - 3) (x^2 + 1)$

4.
(i) $6xy^2 - 3xy - 10y + 5$
Solution:-
$6xy^2 - 3xy - 10y + 5$
Take out common in all terms,
$3xy(2y - 1) - 5(2y - 1)$
$(2y - 1) (3xy - 5)$

(ii) $3ax - 6ay - 8by + 4bx$
Solution:-
$3ax - 6ay - 8by + 4bx$
Take out common in all terms,
$3a(x - 2y) + 4b (x - 2y)$
$(x - 2y) (3a + 4b)$

5.
(i) $1 - a - b + ab$
Solution:-
$1 - a - b + ab$
Take out common in all terms,
$1(1 - a) - b(1 - a)$
$(1 - a) (1 - b)$

(ii) $a(a - 2b - c) + 2bc$
Solution:-
$a(a - 2b - c) + 2bc$
Above question can be written as,

$a^2 - 2ab - ac + 2bc$
Take out common in all terms,
$a(a - 2b) - c(a + 2b)$
$(a - 2b) (a - c)$

6.
(i) $x^2 + xy (1 + y) + y^3$
Solution:-
$x^2 + xy (1 + y) + y^3$
Above question can be written as,
$x^2 + xy + xy^2 + y^3$
Take out common in all terms,
$x(x + y) + y^2(x + y)$
$(x + y) (x + y^2)$

(ii) $y^2 - xy (1 - x) - x^3$
Solution:-
$y^2 - xy (1 - x) - x^3$
Above question can be written as,
$y^2 - xy + x^2y - x^3$
Take out common in all terms,
$y(y - x) + x^2 (y - x)$
$(y - x) (y + x^2)$

7.
(i) $ab^2 + (a - 1)b - 1$
Solution:-
$ab^2 + (a - 1)b - 1$
Above question can be written as,
$ab^2 + ab - b - 1$
Take out common in all terms,
$ab(b + 1) - 1(b + 1)$
$(b + 1) (ab - 1)$

(ii) $2a - 4b - xa + 2bx$
Solution:-
$2a - 4b - xa + 2bx$

Take out common in all terms,
2(a – 2b) – x(a – 2b)
(a – 2b) (2 - x)

8.
(i) 5ph – 10qk + 2rph – 4qrk
Solution:-
5ph – 10qk + 2rph – 4qrk
Re-arranging the given question we get,
5ph + 2rph – 10qk – 4qrk
Take out common in all terms,
ph(5 + 2r) – 2qk(5 + 2r)
(5 + 2r) (ph – 2qk)

(ii) x^2 – x(a + 2b) + 2ab
Solution:-
x^2 – x(a + 2b) + 2ab
Above question can be written as,
x^2 – xa – 2xb + 2ab
Take out common in all terms,
x(x - a) – 2b(x - a)
(x - a) (x – 2b)

9.
(i) ab(x^2 + y^2) – xy(a^2 + b^2)
Solution:-
ab(x^2 + y^2) – xy(a^2 + b^2)
Above question can be written as,
abx^2 + aby^2 – xya^2 – xyb^2
Re-arranging the above we get,
abx^2 – xyb^2 + aby^2 – xya^2
Take out common in all terms,
bx(ax - by) + ay(by - ax)
bx(ax - by) – ay (ax - by)
(ax - by) (bx - ay)

(ii) $(ax + by)^2$ + $(bx - ay)^2$

Solution:-
By expanding the give question, we get,
$(ax)^2 + (by)^2 + 2axby + (bx)^2 + (ay)^2 - 2bxay$
$a^2x^2 + b^2y^2 + b^2x^2 + a^2y^2$
Re-arranging the above we get,
$a^2x^2 + a^2y^2 + b^2y^2 + b^2x^2$
Take out common in all terms,
$a^2 (x^2 + y^2) + b^2 (x^2 + y^2)$
$(x^2 + y^2) (a^2 + b^2)$

10.
(i) $a^3 + ab(1 - 2a) - 2b^2$
Solution:-
$a^3 + ab(1 - 2a) - 2b^2$
Above question can be written as,
$a^3 + ab - 2a^2b - 2b^2$
Re-arranging the above we get,
$a^3 - 2a^2b + ab - 2b^2$
Take out common in all terms,
$a^2(a - 2b) + b(a - 2b)$
$(a - 2b) (a^2 + b)$

(ii) $3x^2y - 3xy + 12x - 12$
Solution:-
$3x^2y - 3xy + 12x - 12$
Take out common in all terms,
$3xy(x - 1) + 12(x - 1)$
$(x - 1) (3xy + 12)$

11. $a^2b + ab^2 - abc - b^2c + axy + bxy$
Solution:-
$a^2b + ab^2 - abc - b^2c + axy + bxy$
Re-arranging the above we get,
$a^2b - abc + axy + ab^2 - b^2c + bxy$
Take out common in all terms,
$a(ab - bc + xy) + b(ab - bc + xy)$
$(a + b) (ab - bc + xy)$

12. ax² – bx² + ay² – by² + az² – bz²

Solution:-

$ax^2 - bx^2 + ay^2 - by^2 + az^2 - bz^2$

Re-arranging the above we get,

$ax^2 + ay^2 + az^2 - bx^2 - by^2 - bz^2$

Take out common in all terms,

$a(x^2 + y^2 + z^2) - b(x^2 + y^2 + z^2)$

$(x^2 + y^2 + z^2) (a - b)$

13. x – 1 – (x - 1)² + ax – a

Solution:-

$x - 1 - (x - 1)^2 + ax - a$

By expanding the above we get,

$X - 1 - (x^2 + 1 - 2x) + ax - a$

$x - 1 - x^2 - 1 + 2x + ax - a$

$2x - x^2 + ax - 2 + x - a$

Take out common in all terms,

$x(2 - x + a) - 1(2 - x + a)$

$(2 - x + a) (x - 1)$

EXERCISE 4.3

1. $4x^2 - 25y^2$
Solution:-
We know that, $a^2 - b^2 = (a + b) (a - b)$
So, $(2x)^2 - (5y)^2$
Then, $(2x + y) (2x - 5y)$

(ii) $9x^2 - 1$
Solution:-
We know that, $a^2 - b^2 = (a + b) (a - b)$
So, $(3x)^2 - 1^2$
Then, $(3x + 1) (3x - 1)$

2.
(i) $150 - 6a^2$
Solution:-
$150 - 6a^2$
Take out common in all terms,
$6(25 - a^2)$
$6(5^2 - a^2)$
We know that, $a^2 - b^2 = (a + b) (a - b)$
So, $6(5 + a) (5 - a)$

(ii) $32x^2 - 18y^2$
Solution:-
$32x^2 - 18y^2$
Take out common in all terms,
$2(16x^2 - 9y^2)$
$2((4x)^2 - (3y)^2)$
We know that, $a^2 - b^2 = (a + b) (a - b)$
$2(4x + 3y) (4x - 3y)$

3.
(ii) $(x - y)^2 - 9$
Solution:-
$(x - y)^2 - 9$

$(x - y)^2 - 3^2$
We know that, $a^2 - b^2 = (a + b) (a - b)$
$(x - y + 3) (x - y - 3)$

(ii) $9(x + y)^2 - x^2$
Solution:-
$9[(x + y)^2 - x^2]$
We know that, $a^2 - b^2 = (a + b) (a - b)$
$9[(x + y + x) (x + y - x)]$
So, $9(2x + y) y$
$9y(2x + y)$

4.
(i) $20x^2 - 45y^2$
Solution:-
$20x^2 - 45y^2$
Take out common in all terms,
$5(4x^2 - 9y^2)$
$5((2x)^2 - (3y)^2)$
We know that, $a^2 - b^2 = (a + b) (a - b)$
$5(2x + 3y) (2x - 3y)$

(ii) $9x^2 - 4(y + 2x)^2$
Solution:-
$9x^2 - 4(y + 2x)^2$
Above question can be written as,
$(3x)^2 - [2(y + 2x)]^2$
$(3x)^2 - (2y + 4x)^2$
We know that, $a^2 - b^2 = (a + b) (a - b)$
$(3x + 2y + 4x) (3x - 2y - 4x)$
$(7x + 2y) (-x - 2y)$

5.
(i) $2(x - 2y)^2 - 50y^2$
Solution:-
$2(x - 2y)^2 - 50y^2$
Take out common in all terms,

2[(x − 2y)^2 − 25y^2]
2[(x − 2y)^2 − (5y)^2]
We know that, a^2 − b^2 = (a + b) (a - b)
2[(x − 2y + 5y) (x − 2y − 5y)]
2[(x + 3y) (x − 7y)]
2(x + 3y) (x − 7y)

(ii) 32 − 2(x - 4)^2
Solution:-
32 − 2(x - 4)^2
Take out common in all terms,
2[16 − (x - 4)^2]
2[4^2- (x - 4)^2]
We know that, a^2 − b^2 = (a + b) (a - b)
2[(4 + x - 4) (4 − x + 4)]
2[(x) (8 - x)]
2x (8 - x)

6.
(i) 108a^2 − 3(b - c)^2
Solution:-
108a^2 − 3(b - c)^2
Take out common in all terms,
3[36a^2 − (b - c)^2]
3[(6a)^2 − (b - c)^2]
We know that, a^2 − b^2 = (a + b) (a - b)
3[(6a + b - c) (6a − b + c)]

(ii) πa^5 − π^{3}ab^2
Solution:-
πa^5 − π^{3}ab^2
Take out common in all terms,
πa(a^4 − π^{2}b^2)
πa((a^2)^2 − (πb)^2)
We know that, a^2 − b^2 = (a + b) (a - b)
πa(a^2 + πb) (a^2 - πb)

7.
(i) 50x² – 2(x - 2)²
Solution:-
$50x^2 - 2(x - 2)^2$
Take out common in all terms,
$2[25x^2 - (x - 2)^2]$
$2[(5x)^2 - (x - 2)^2]$
We know that, $a^2 - b^2 = (a + b) (a - b)$
$2[(5x + x - 2) (5x - x + 2)]$
$2[(6x - 2) (4x + 2)]$
$2(6x - 2) (4x + 2)$

(ii) (x - 2)(x + 2) + 3
Solution:-
We know that, $a^2 - b^2 = (a + b) (a - b)$
$(x^2 - 2^2) + 3$
$X^2 - 4 + 3$
$X^2 - 1$
Then,
$(x + 1) (x - 1)$

8.
(i) x – 2y – x² + 4y²
Solution:-
$x - 2y - x^2 + 4y^2$
$x - 2y - (x^2 + (2y)^2)$
We know that, $a^2 - b^2 = (a + b) (a - b)$
$x - 2y - [(x + 2y) (x - 2y)]$
Take out common in all terms,
$(x - 2y) (1 - (x + 2y))$
$(x - 2y) (1 - x - 2y)$

(ii) 4a² – b² + 2a + b
Solution:-
$4a^2 - b^2 + 2a + b$
$(2a)^2 - b^2 + 2a + b$
We know that, $a^2 - b^2 = (a + b) (a - b)$

((2a + b) (2a - b)) + 1(2a + b)
Take out common in all terms,
(2a + b) (2a – b + 1)

9.
(i) a(a - 2) – b(b - 2)
Solution:-
a(a - 2) – b(b - 2)
Above question can be written as,
$a^2 - 2a - b^2 - 2b$
Rearranging the above terms, we get,
$a^2 - b^2 - 2a - 2b$
We know that, $a^2 - b^2 = (a + b)(a - b)$
[(a + b)(a - b)] – 2(a - b)
Take out common in all terms,
(a - b) (a + b - 2)

(ii) a(a - 1) – b(b - 1)
Solution:-
a(a - 1) – b(b - 1)
Above question can be written as,
$a^2 - a - b^2 + b$
Rearranging the above terms, we get,
$a^2 - b^2 - a + b$
We know that, $a^2 - b^2 = (a + b)(a - b)$
[(a + b)(a - b)] – 1(a - b)
Take out common in all terms,
(a - b) (a + b - 1)

10.
(i) $9 - x^2 + 2xy - y^2$
Solution:-
$9 - x^2 + 2xy - y^2$
$9 - x^2 + 2xy - y^2$
Above terms can be written as,
$9 - x^2 + xy + xy - y^2$
Now,

9 – x² + xy + 3x – 3x + 3y – 3y + xy – y²
Rearranging the above terms, we get,
9 – 3x + 3y + 3x – x² + xy + xy – 3y – y²
Take out common in all terms,
3(3 – x + y) + x(3 – x + y) + y (-3 – y + x)
3(3 – x + y) + x(3 – x + y) – y(3 – x + y)
(3 – x + y) (3 + x - y)

(ii) 9x⁴ – (x² + 2x + 1)
Solution:-
9x⁴ – (x² + 2x + 1)
Above terms can be written as,
(3x²)² – (x + 1)²   … [because (a + b)² = a² + 2ab + b²]
We know that, a² – b² = (a + b) (a - b)
So, (3x² + x + 1) (3x² – x - 1)

11.
(i) 9x⁴ – x² - 12x - 36
Solution:-
9x⁴ – x² - 12x - 36
Above terms can be written as,
9x⁴ – (x² + 12x + 36)
We know that, (a + b)² = a² + 2ab + b²
(3x²)² – (x² + (2 × 6 × x) + 6²)
So, (3x²)² – (x + 6)²
We know that, a² – b² = (a + b) (a - b)
(3x² + x + 6) (3x² – x - 6)

(ii) x³ – 5x² – x + 5
Solution:-
x³ – 5x² – x + 5
Take out common in all terms,
x²(x - 5) – 1(x - 5)
(x - 5) (x² - 1)
(x - 5) (x² – 1²)
We know that, a² – b² = (a + b) (a - b)
(x - 5) (x + 1) (x - 1)

12.
(i) $a^4 - b^4 + 2b^2 - 1$
Solution:-
$a^4 - b^4 + 2b^2 - 1$
Above terms can be written as,
$a^4 - (b^4 - 2b^2 + 1)$
We know that, $(a - b)^2 = a^2 - 2ab + b^2$
$a^4 - ((b^2)^2) - (2 \times b^2 \times 1) + 1^2)$
$(a^2)^2 - (b^2 - 1)^2$
We know that, $a^2 - b^2 = (a + b)(a - b)$
$(a^2 + b^2 - 1)(a^2 - b^2 + 1)$

(ii) $x^3 - 25x$
Solution:-
$x^3 - 25x$
Take out common in all terms,
$x(x^2 - 25)$
Above terms can be written as,
$x(x^2 - 5^2)$
We know that, $a^2 - b^2 = (a + b)(a - b)$
$x(x + 5)(x - 5)$

13.
(i) $2x^4 - 32$
Solution:-
$2x^4 - 32$
Take out common in all terms,
$2(x^4 - 16)$
Above terms can be written as,
$2((x^2)^2 - 4^2)$
We know that, $a^2 - b^2 = (a + b)(a - b)$
$2(x^2 + 4)(x^2 - 4)$
$2(x^2 + 4)(x^2 - 2^2)$
$2(x^2 + 4)(x + 2)(x - 2)$

(ii) $a^2(b + c) - (b + c)^3$
Solution:-

$a^2(b + c) - (b + c)^3$
Take out common in all terms,
$(b + c) (a^2 - (b + c)^2)$
We know that, $a^2 - b^2 = (a + b) (a - b)$
$(b + c) (a + b + c) (a - b - c)$

14.
(i) $(a + b)^3 - a - b$
Solution:-
$(a + b)^3 - a - b$
Above terms can be written as,
$(a + b)^3 - (a + b)$
Take out common in all terms,
$(a + b) [(a + b)^2 - 1]$
$(a + b) [(a + b)^2 - 1^2]$
We know that, $a^2 - b^2 = (a + b) (a - b)$
$(a + b) (a + b + 1) (a + b - 1)$

(ii) $x^2 - 2xy + y^2 - a^2 - 2ab - b^2$
Solution:-
$x^2 - 2xy + y^2 - a^2 - 2ab - b^2$
Above terms can be written as,
$(x^2 - 2xy + y^2) - (a^2 + 2ab + b^2)$
We know that, $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$
$(x^2 - (2 \times x \times y) + y^2) - (a^2 + (2 \times a \times b) + b^2)$
$(x - y)^2 - (a + b)^2$
We know that, $a^2 - b^2 = (a + b) (a - b)$
$[(x - y) + (a + b)] [(x - y) - (a + b)]$
$(x - y + a + b) (x - y - a - b)$

15.
(i) $(a^2 - b^2) (c^2 - d^2) - 4abcd$
Solution:-
$(a^2 - b^2) (c^2 - d^2) - 4abcd$
$a^2(c^2 - d^2) - b^2 (c^2 - d^2) - 4abcd$
$a^2c^2 - a^2d^2 - b^2c^2 + b^2d^2 - 4abcd$
$a^2c^2 + b^2d^2 - a^2d^2 - b^2c^2 - 2abcd - 2abcd$

Rearranging the above terms, we get,

$a^2c^2 + b^2d^2 - 2abcd - a^2d^2 - b^2c^2 - 2abcd$

We know that, $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$

$(ac - bd)^2 - (ad - bc)^2$

$(ac - bd + ad - bc)(ac - bd - ad + bc)$

(ii) $4x^2 - y^2 - 3xy + 2x - 2y$

Solution:-

$4x^2 - y^2 - 3xy + 2x - 2y$

Above terms can be written as,

$x^2 + 3x^2 - y^2 - 3xy + 2x - 2y$

Rearranging the above terms, we get,

$(x^2 - y^2) + (3x^2 - 3xy) + (2x - 2y)$

We know that, $a^2 - b^2 = (a + b)(a - b)$ and take out common terms,

$(x + y)(x - y) + 3x(x - y) + 2(x - y)$

$(x - y)[(x + y) + 3x + 2]$

$(x - y)(x + y + 3x + 2)$

$(x - y)(4x + y + 2)$

16.

(i) $x^2 + 1/x^2 - 11$

Solution:-

$x^2 + 1/x^2 - 11$

Above terms can be written as,

$x^2 + (1/x^2) - 2 - 9$

Then, $(x^2 + (1/x^2) - 2) - 3^2$

We know that, $(a - b)^2 = a^2 - 2ab + b^2$,

$(x^2 - (2 \times x^2 \times (1/x^2)) + (1/x)^2)$

$(x - 1/x)^2 - 3^2$

We know that, $a^2 - b^2 = (a + b)(a - b)$

$(x - 1/x + 3)(x - 1/x - 3)$

(ii) $x^4 + 5x^2 + 9$

Solution:-

$x^4 + 5x^2 + 9$

$x^4 + 6x^2 - x^2 + 9$

$(x^4 + 6x^2 + 9) - x^2$

$((x^2)^2 + (2 \times x^2 \times 3) + 3^2)$

We know that, $(a + b)^2 = a^2 + 2ab + b^2$,

$((x^2)^2 + (2 \times x^2 \times 3) + 3^2)$

So, $(x^2 + 3)^2 - x^2$

We know that, $a^2 - b^2 = (a + b) (a - b)$

$(x^2 + 3 + x) (x^2 + 3 - x)$

17.
(i) $a^4 + b^4 - 7a^2b^2$
Solution:-
$a^4 + b^4 - 7a^2b^2$
Above terms can be written as,
$a^4 + b^4 + 2a^2b^2 - 9a^2b^2$
We know that, $(a + b)^2 = a^2 + 2ab + b^2$,
$[(a^2)^2 + (b^2)^2 + (2 \times a^2 \times b^2)] - (3ab)^2$
$(a^2 + b^2)^2 - (3ab)^2$
We know that, $a^2 - b^2 = (a + b) (a - b)$
$(a^2 + b^2 + 3ab) (a^2 + b^2 - 3ab)$

(ii) $x^4 - 14x^2 + 1$
Solution:-
$x^4 - 14x^2 + 1$
Above terms can be written as,
$x^4 + 2x^2 + 1 - 16x^2$
We know that, $(a + b)^2 = a^2 + 2ab + b^2$,
So, $[(x^2)^2 + (2 \times x^2 \times 1) + 1^2] - 16x^2$
$(x^2 + 1)^2 - (4x)^2$
We know that, $a^2 - b^2 = (a + b) (a - b)$
$(x^2 + 1 + 4x) (x^2 + 1 - 4x)$

18.
(i) $(x^2 - 5x + 7) (x^2 + 5x + 7)$
Solution:-
$(x^2 - 5x + 7) (x^2 + 5x + 7)$
Rearranging the above terms, we get,
$((x^2 + 7) - 5x) ((x^2 + 7) + 5x)$
As, we know that, $a^2 - b^2 = (a + b) (a - b)$

So, (x² + 7)² – (5x)²
(x² + 7)² -25x²

(ii) (x² – 5x + 7) (x² – 5x - 7)
Solution:-
(x² – 5x + 7) (x² – 5x - 7)
[(x² – 5x) + 7) ((x² – 5x) - 7)
As, we know that, a² – b² = (a + b) (a - b)
(x² – 5x)² – 7²
(x² – 5x)² – 49

(iii) (x² + 5x - 7) (x² – 5x + 7)
Solution:-
(x² + 5x - 7) (x² – 5x + 7)
[x² + (5x – 7)] [x² – (5x - 7)]
As, we know that, a² – b² = (a + b) (a - b)
x² – (5x - 7)²
We know that, (a - b)² = a² - 2ab + b²,
X² – [(5x)² – (2 × 5x × 7) + 7²]
X² – (25x² – 70x + 49)
X² – 25x² + 70x – 49
-24x² + 70x – 49

19.
(i) x – 1/x = 5/6
Solution:-
x – 1/x = 5/6
Now, squaring on both side we get,
X² + (1/x²) – 2 = 25/36
Transposing – 2 from LHS to RHS then it becomes 2.
x² + (1/x²) = (25/36) + 2
x² + (1/x²) = (25 + 72)/36
x² + 1/x² = 97/36

(ii) (x + 1/x)² = (x² + (1/x²) + 2)
Solution:-
(x + 1/x)² = (x² + (1/x²) + 2)

From (i), $x^2 + 1/x^2 = 97/36$

So, $(x + 1/x)^2 = (97/36) + 2$

$(x + 1/x)^2 = (97 + 72)/36$

$(x + 1/x)^2 = 169/36$

$x + (1/x) = \sqrt{(169/36)}$

$x + (1/x) = 13/6$

(iii) $(x + 1/x)^3 = x^3 + (1/x^3) + 3(x + (1/x))$

Solution:-

$(x + 1/x)^3 = x^3 + (1/x^3) + 3(x + (1/x))$

From (ii), $x + (1/x) = 13/6$

$(13/6)^3 = x^3 + (1/x^3) + 3(13/6)$

$2197/216 = x^3 + (1/x^3) + 39/6$

$x^3 + (1/x^3) = (2197/216) - (39/6)$

$x^3 + (1/x^3) = (2197 - 1404)/216$

$x^3 + (1/x^3) = 793/216$

(iv) $x^4 - 1/x^4$

Solution:-

$x^4 - 1/x^4 = (x^2 + 1/x^2)(x^2 - 1/x^2)$

$\qquad\qquad = (x^2 + 1/x^2)(x + 1/x)(x - 1/x)$

From (i), (ii) and (iii) substitute the values we get,,

$\qquad\qquad = (97/36) \times (13/6) \times (5/6)$

$\qquad\qquad = 6305/1296$

20. If $x + 1/x = \sqrt{3}$, find the values of

(i) $x^3 + 1/x^3$

Solution:-

From question, $x + 1/x = \sqrt{3}$

$x + 1/x = \sqrt{3}$

Cubing on both LHS and RHS we get,

$(x + 1/x)^3 = (\sqrt{3})^3$

$x^3 + (1/x^3) + 3(x + (1/x)) = 3\sqrt{3}$

$x^3 + 1/x^3 + 3(\sqrt{3}) = 3\sqrt{3}$

By transposing we get,

$x^3 + 1/x^3 + 3\sqrt{3} - 3\sqrt{3} = 0$

$x^3 + 1/x^3 = 0$

(ii) x⁶ – 1/x⁶
Solution:-
$x^6 - 1/x^6$
Above terms can be written as,
$= (x^3)^2 - (1/x^3)^2$
We know that, $(a - b)^2 = a^2 - 2ab + b^2$,
$(x^3 + (1/x^3)) (x^3 - (1/x^3))$
From (i) $x^3 + 1/x^3 = 0$
So, $0 \times (x^3 - 1/x^3)$

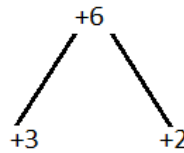## EXERCISE 4.4

1.
(i) $x^2 + 5x + 6$
Solution:-
$x^2 + 5x + 6$
$x^2 + 3x + 2x + 6$
Take out common in all terms we get,
$x(x + 3) + 2 (x + 3)$
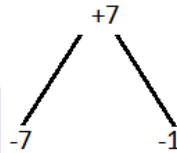$(x + 3) (x + 2)$

(ii) $x^2 - 8x + 7$
Solution:-
$x^2 - 8x + 7$
$x^2 - 7x - x + 7$
Take out common in all terms we get,
$x(x - 7) - 1(x - 7)$
$(x - 7) (x - 1)$
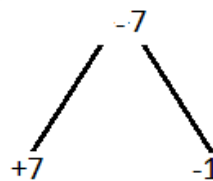
2.
(i) $x^2 + 6x - 7$
Solution:-
$x^2 + 6x - 7$
$x^2 + 7x - x - 7$
Take out common in all terms we get,
$x(x + 7) - 1(x + 7)$
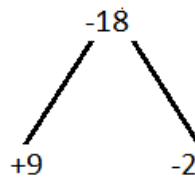$(x + 7) (x - 1)$

(ii) $y^2 + 7y - 18$
Solution:-
$y^2 + 7y - 18$
$y^2 + 9y - 2y - 18$
Take out common in all terms we get,
$y(y + 9) - 2(y + 9)$
$(y + 9) (y - 2)$

3.

(i) $y^2 - 7y - 18$

Solution:-

$y^2 - 7y - 18$

$y^2 + 2y - 9y - 18$

Take out common in all terms we get,

$y(y + 2) - 9(y + 2)$

$(y + 2) (y - 9)$

(ii) $a^2 - 3a - 54$

Solution:-

$a^2 - 3a - 54$

$a^2 + 6a - 9a - 54$

Take out common in all terms we get,

$a(a + 6) - 9(a + 6)$

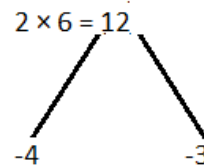So, $(a + 6) (a - 9)$

4.

(i) $2x^2 - 7x + 6$

Solution:-

$2x^2 - 7x + 6$

$2x^2 - 4x - 3x + 6$

Take out common in all terms we get,

$2x(x - 2) - 3(x - 2)$

$(x - 2) (2x - 3)$
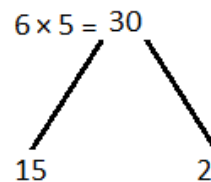
$2 \times 6 = 12$

$-4 \qquad -3$

(ii) $6x^2 + 13x - 5$

Solution:-

$6x^2 + 13x - 5$

$6x^2 + 15x - 2x - 5$

Take out common in all terms we get,

$3x(2x + 5) - 1(2x + 5)$

$(2x + 5) (3x - 1)$

$6 \times 5 = 30$

$15 \qquad 2$

5.

(i) $6x^2 + 11x - 10$

Solution:-

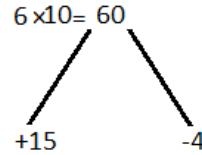$6x^2 + 11x - 10$

$6x^2 + 15x - 4x - 10$

Take out common in all terms we get,

$3x(2x + 5) - 2(2x + 5)$

$(2x + 5) (3x - 2)$
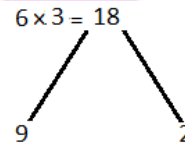
6 ×10= 60

+15   -4

(ii) $6x^2 - 7x - 3$

Solution:-

$6x^2 - 7x - 3$

$6x^2 - 9x + 2x - 3$

Take out common in all terms we get,

$3x(2x - 3) + 1(2x - 3)$

$(2x - 3) (3x + 1)$

6 × 3 = 18

9   2

6.

(i) $2x^2 - x - 6$

Solution:-

$2x^2 - x - 6$

$2x^2 - 4x + 3x - 6$

Take out common in all terms we get,

$2x(x - 2) + 3(x - 2)$

$(x - 2) (2x + 3)$

(ii) $1 - 18y - 63y^2$

Solution:-

$1 - 18y - 63y^2$

$1 - 21y + 3y - 63y^2$

Take out common in all terms we get,

$1(1 - 21y) + 3y(1 - 21y)$

$(1 - 21y) (1 + 3y)$

7.

(i) $2y^2 + y - 45$

Solution:-

$2y^2 + y - 45$

$2y^2 + 10y - 9y - 45$

Take out common in all terms we get,

$2y (y + 5) - 9(y + 5)$

$(y + 5) (2y - 9)$

(ii) 5 – 4x – 12x²

Solution:-

$5 - 4x - 12x^2$

$5 - 10x + 6x - 12x^2$

Take out common in all terms we get,

$5(1 - 2x) + 6x(1 - 2x)$

$(1 - 2x) (5 + 6x)$

8.

(i) x(12x + 7) – 10

Solution:-

$x(12x + 7) - 10$

Above terms can be written as,

$12x^2 + 7x - 10$

$12x^2 + 15x - 8x - 10$

Take out common in all terms we get,

$3x(4x + 5) - 2(4x + 5)$

$(4x + 5) (3x - 2)$

(ii) (4 - x)² – 2x

Solution:-

$(4 - x)^2 - 2x$

We know that, $(a - b)^2 = a^2 - 2ab + b^2$

So, $(4^2 - (2 \times 4 \times x) + x^2) - 2x$

$16 - 8x + x^2 - 2x$

$x^2 - 10x + 16$

$x^2 - 8x - 2x + 16$

Take out common in all terms we get,

$x(x - 8) - 2(x - 8)$

$(x - 8) (x - 2)$

9.
(i) $60x^2 - 70x - 30$
Solution:-
$60x^2 - 70x - 30$
Take out common in all terms we get,
$10(6x^2 - 7x - 3)$
$10(6x^2 - 9x + 2x - 3)$
Again, take out common in all terms we get,
$10(3x(2x - 3) + 1(2x - 3))$
$10(2x - 3)(3x + 1)$

(ii) $x^2 - 6xy - 7y^2$
Solution:-
$x^2 - 6xy - 7y^2$
$x^2 - 7xy + xy - 7y^2$
Take out common in all terms we get,
$x(x - 7y) + y(x - 7y)$
$(x - 7y)(x + y)$

10.
(i) $2x^2 + 13xy - 24y^2$
Solution:-
$2x^2 + 13xy - 24y^2$
$2x^2 + 16xy - 3xy - 24y^2$
Take out common in all terms we get,
$2x(x + 8y) - 3y(x + 8y)$
$(x + 8y)(2x - 3y)$

(ii) $6x^2 - 5xy - 6y^2$
Solution:-
$6x^2 - 5xy - 6y^2$
$6x^2 - 9xy + 4xy - 6y^2$
Take out common in all terms we get,
$3x(2x - 3y) + 2y(2x - 3y)$
$(2x - 3y)(3x + 2y)$

11.
(i) $5x^2 + 17xy - 12y^2$
Solution:-
$5x^2 + 17xy - 12y^2$
$5x^2 + 20xy - 3xy - 12y^2$
Take out common in all terms we get,
$5x(x + 4y) - 3y(x + 4y)$
$(x + 4y) (5x - 3y)$

(ii) $x^2y^2 - 8xy - 48$
Solution:-
$x^2y^2 - 8xy - 48$
$x^2y^2 - 12xy + 4xy - 48$
Take out common in all terms we get,
$xy(xy - 12) + 4(xy - 12)$
$(xy - 12) (xy + 4)$

12.
(i) $2a^2b^2 - 7ab - 30$
Solution:-
$2a^2b^2 - 7ab - 30$
$2a^2b^2 - 12ab + 5ab - 30$
Take out common in all terms we get,
$2ab(ab - 6) + 5 (ab - 6)$
$(ab - 6) (2ab + 5)$

(ii) $a(2a - b) - b^2$
Solution:-
$a(2a - b) - b^2$
Above terms can be written as,
$2a^2 - ab - b^2$
$2a^2 - 2ab + ab - b^2$
Take out common in all terms we get,
$2a(a - b) + b(a - b)$
$(a - b) (2a + b)$

13.

(i) (x - y)² – 6(x - y) + 5
Solution:-
(x - y)² – 6(x - y) + 5
Above terms can be written as,
(x - y)² – 5(x - y) – (x - y) + 5
(x - y) (x – y - 5) – 1(x – y - 5)
Then,
(x – y - 5) (x – y - 1)

(ii) (2x - y)² – 11(2x - y) + 28
Solution:-
(2x - y)² – 11(2x - y) + 28
Above terms can be written as,
(2x - y)² – 7(2x - y) – 4(2x - y) + 28
(2x - y) (2x – y - 7) – 4(2x – y - 7)
(2x – y - 7) (2x – y - 4)

14.
(i) 4(a - 1)² – 4(a - 1) – 3
Solution:-
4(a - 1)² – 4(a - 1) – 3
Above terms can be written as,
4(a - 1)² – 6(a - 1) + 2(a - 1) - 3
Take out common in all terms we get,
2(a - 1) [2(a - 1) - 3] + 1[2(a - 1) - 3]
(2(a - 1) - 3) (2(a - 1) + 1)
(2a – 2 - 3) (2a – 2 + 1)
(2a - 5) (2a - 1)

(ii) 1 – 2a – 2b – 3(a + b)²
Solution:-
1 – 2a – 2b – 3(a + b)²
Above terms can be written as,
1 – 2(a + b) – 3(a + b)²
1 – 3(a + b) + (a + b) – 3(a + b)²
Take out common in all terms we get,
1(1 – 3(a + b)) + (a + b) (1 – (a + b))

(1 – 3(a + b)) (1 + (a + b))
(1 – 3a + 3b) (1 + a + b)

15.
(i) 3 – 5a – 5b – 12(a + b)²
Solution:-
3 – 5a – 5b – 12(a + b)²
Above terms can be written as,
3 – 5(a + b) – 12(a + b)²
3 – 9(a + b) + 4(a + b) – 12(a + b)²
Take out common in all terms we get,
3(1 – 3(a + b)) + 4(a + b) (1 – 3(a + b))
(1 – 3(a + b)) (3 + 4(a + b))
(1 – 3a – 3b) (3 + 4a + 4b)

(ii) a⁴ – 11a² + 10
Solution:-
a⁴ – 11a² + 10
Above terms can be written as,
a⁴ – 10a² – a² + 10
Take out common in all terms we get,
a² (a² - 10) – 1(a² - 10)
(a² - 10) (a² - 1)

16.
(i) (x + 4)² – 5xy -20y – 6y²
Solution:-
(x + 4)² – 5xy -20y – 6y²
Above terms can be written as,
(x + 4)² – 5y(x + 4) – 6y²
(x + 4)² – 6y(x + 4) + y(x + 4) – 6y²
Take out common in all terms we get,
(x + 4) (x + 4 – 6y) + y(x + 4 – 6y)
(x – 6y + 4) (x + 4 + y)

(ii) (x² – 2x²) – 23(x² – 2x) + 120
Solution:-

$(x^2 – 2x^2) – 23(x^2 – 2x) + 120$
Above terms can be written as,
$(x^2 – 2x)^2 – 15(x^2 – 2x) – 8(x^2 – 2x) + 120$
Take out common in all terms we get,
$(x^2 – 2x) (x^2 – 2x - 15) – 8(x^2 – 2x - 15)$
$(x^2 – 2x - 15) (x^2 – 2x - 8)$

17. $4(2a - 3)^2 – 3(2a - 3) (a - 1) – 7 (a - 1)^2$
Solution:-
$4(2a - 3)^2 – 3(2a - 3) (a - 1) – 7 (a - 1)^2$
Let us assume, $2a – 3 = p$ and $a – 1 = q$
So, $4p^2 – 3pq – 7q^2$
Then, $4p^2 – 7pq + 4pq – 7q^2$
Take out common in all terms we get,
$P(4p – 7q) + q(4p – 7q)$
$(4p – 7q) (p + q)$
Now, substitute the value of p and q we get,
$(4(2a - 3) – 7(a - 1)) (2a – 3 + a - 1)$
$(8a – 12 – 7a + 7) (3a - 4)$
$(a - 5) (3a - 4)$

18. $(2x^2 + 5x) (2x^2 + 5x - 19) + 84$
Solution:-
$(2x^2 + 5x) (2x^2 + 5x - 19) + 84$
Let us assume, $2x^2 + 5x = p$
So, $(p) (p - 19) + 84$
$p^2 – 19p + 84$
$p^2 – 12p – 7p + 84$
$p(p - 12) – 7(p - 12)$
$(p - 12) (p - 7)$
Now, substitute the value of p we get,
$(2x^2 + 5x - 12) (2x^2 + 5x - 7)$

EXERCISE 4.5

1.
(i) 8x³ + y³
Solution:-
$8x^3 + y^3$
Above terms can be written as,
$(2x)^3 + y^3$
We know that, $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
Where, $a = 2x$, $b = y$
Then, $(2x)^3 + y^3 = (2x + y)((2x)^2 - (2x \times y) + y^2)$
$$= (2x + y)(4x^2 - 2xy + y^2)$$

(ii) 64x³ – 125y³
Solution:-
$64x^3 - 125y^3$
Above terms can be written as,
$(4x)^3 - (5y)^3$
We know that, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
Where, $a = 4x$, $b = 5y$
Then, $(4x)^3 - (5y)^3 = (4x - 5y)((4x)^2 + (4x \times 5y) + 5y^2)$
$$= (4x - 5y)(16x^2 + 2oxy + 25y^2)$$

2.
(i) 64x³ + 1
Solution:-
$64x^3 + 1$
Above terms can be written as,
$(4x)^3 + 1^3$
We know that, $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
Where, $a = 4x$, $b = 1$
Then, $(4x)^3 + 1^3 = (4x + 1)((4x)^2 - (4x \times 1) + 1^2)$
$$= (4x + 1)(16x^2 - 4x + 1)$$

(ii) 7a³ + 56b³
Solution:-
$7a^3 + 56b^3$

Take out common in all terms we get,

$7(a^3 + 8b^3)$

Above terms can be written as,

$7(a^3 + (2b)^3)$

We know that, $a^3 + b^3 = (a + b) (a^2 - ab + b^2)$

Where, a = a, b = 2b

Then, $7[(a)^3 + (2b)^3] = 7[(a + 2b) ((a)^2 - (a \times 2b) + (2b)^2)]$

$= 7(a + 2b) (a^2 - 2ab + 4b^2)$

3.

(i) $(x^6/343) + (343/x^6)$

Solution:-

$(x^6/343) + (343/x^6)$

Above terms can be written as,

$(x^2/7)^3 + (7/x^2)^3$

We know that, $a^3 + b^3 = (a + b) (a^2 - ab + b^2)$

Where, a = $(x^2/7)$, b = $(7/x^2)$

Then, $(x^2/7)^3 + (7/x^2)^3 = [(x^2/7) + (7/x^2)] [(x^2/7)^2 - ((x^2/7) \times (7/x^2)) + (7/x^2)^2]$

$= [(x^2/7) + (7/x^2)] [(x^4/49) - 1 + (49/x^4)]$

(ii) $8x^3 - 1/27y^3$

Solution:-

$8x^3 - 1/27y^3$

Above terms can be written as,

$(2x)^3 - (1/3y)^3$

We know that, $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$

Where, a = 2x, b = $(1/3y)$

Then, $(2x)^3 - (1/3y)^3 = (2x - (1/3y)) ((2x)^2 + (2x \times (1/3y)) + (3y)^2)$

$= (2x - (1/3y)) (4x^2 + (2x/3y) + 9y^2)$

4.

(i) $x^2 + x^5$

Solution:-

$x^2 + x^5$

Take out common in all terms we get,

$x^2(1 + x^3)$

$x^2(1^3 + x^3)$

We know that, $a^3 + b^3 = (a + b) (a^2 - ab + b^2)$
Where, a = 1, b = x

$= x^2 [(1 + x) (1^2 - (1 \times x) + x^2)]$
$= x^2 (1 + x) (1 - x + x^2)$

(ii) $32x^4 - 500x$

Solution:-

$32x^4 - 500x$

Take out common in all terms we get,

$4x(8x^3 - 125)$

Above terms can be written as,

$4x((2x)^3 - 5^3)$

We know that, $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$

Where, a = 2x, b = 5

$= 4x(2x - 5) ((2x)^2 + (2x \times 5) + 5^2)$
$= 4x(2x - 5) (4x^2 + 10x + 25)$

5.
(i) $27x^3y^3 - 8$

Solution:-

$27x^3y^3 - 8$

Above terms can be written as,

$(3xy)^3 - 2^3$

We know that, $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$

Where, a = 3xy, b = 2

$= (3xy - 2) ((3xy)^2 + (3xy \times 2) + 2^2)$
$= (3xy - 2) (9x^2y^2 + 6xy + 4)$

(ii) $27(x + y)^3 + 8(2x - y)^3$

Solution:-

$27(x + y)^3 + 8(2x - y)^3$

Above terms can be written as,

$3^3(x + y)^3 + 2^3(2x - y)^3$

$(3(x + y))^3 + (2(x - y))^3$

We know that, $a^3 + b^3 = (a + b) (a^2 - ab + b^2)$

Where, a = 3(x + y), b = 2(x - y)

$= [3(x + y) + 2(2x - y)] [(3(x + y))^3 - (3(x + y) \times 2(2x - y)) + (2(2x - y))^2]$

= [3x + 3y + 4x − 2y] [9(x + y)² − 6(x + y)(2x - y) + 4(2x - y)²]
= (7x - y) [9(x² + y² + 2xy) − 6(2x² − xy + 2xy − y²) + 4(4x² + y² − 4xy)]
= (7x - y) [9x² + 9y² + 18xy − 12x² − 6xy − 6y² + 16x² + 4y² − 16xy]
= (7x - y) [13x² − 4xy + 19y²]

6.
(i) a³ + b³ + a + b
Solution:-
a³ + b³ + a + b
(a³ + b³) + (a + b)
We know that, a³ + b³ = (a + b) (a² - ab + b²)
[(a + b) (a² − ab + b²)] + (a + b)
(a + b) (a² − ab + b² + 1)

(ii) a³ − b³ − a + b
Solution:-
a³ − b³ − a + b
(a³ − b³) − (a - b)
We know that, a³ - b³ = (a - b) (a² + ab + b²)
[(a - b) (a² + ab + b²)] - (a - b)
(a - b) (a² + ab + b² - 1)

7.
(i) x³ + x + 2
Solution:-
x³ + x + 2
Above terms can be written as,
x³ + x + 1 + 1
Rearranging the above terms, we get
(x³ + 1) (x + 1)
(x³ + 1³) (x + 1)
We know that, a³ + b³ = (a + b) (a² - ab + b²)
[(x + 1) (x² − x + 1)] + (x + 1)
(x + 1) (x² − x + 1 + 1)
(x + 1) (x² − x + 2)

(ii) a³ − a − 120

Solution:-

$a^3 - a - 120$

Above terms can be written as,

$a^3 - a - 125 + 5$

Rearranging the above terms, we get

$a^3 - 125 - a + 5$

$(a^3 - 125) - (a - 5)$

$(a^3 - 5^3) - (a - 5)$

We know that, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$[(a - 5)(a^2 + 5a + 5^2)] - (a - 5)$

$(a - 5)(a^2 + 5a + 25) - (a - 5)$

$(a - 5)(a^2 + 5a + 25 - 1)$

$(a - 5)(a^2 + 5a + 24)$

8.

(i) $x^3 + 6x^2 + 12x + 16$

Solution:-

$x^3 + 6x^2 + 12x + 16$

$x^3 + 6x^2 + 12x + 8 + 8$

Above terms can be written as,

$(x^3 + (3 \times 2 \times x^2) + (3 \times 2^2 \times x) + 2^3) + 8$

We know that, $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

Now $a = x$ and $b = 2$

So, $(x + 2)^3 + 2^3$

We know that, $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$\qquad(x + 2 + 2)((x + 2)^2 - (2 \times (x + 2)) + 2^2)$

$\qquad(x + 4)(x^2 + 4 + 4x - 2x - 4 + 4)$

$\qquad(x + 4)(x^2 + 2x + 4)$

(ii) $a^3 - 3a^2b + 3ab^2 - 2b^3$

Solution:-

$a^3 - 3a^2b + 3ab^2 - 2b^3$

Above terms can be written as,

$a^3 - 3a^2b + 3ab^2 - b^3 - b^3$

We know that, $(a - b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$

So, $(a - b)^3 + b^3$

We also know that, $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$
Where, $a = a - b$, $b = b$
$(a - b - b) ((a - b)^2 + (a - b)b + b^2)$
$(a - 2b) (a^2 + b^2 - 2ab + ab - b^2 + b^2)$
$(a - 2b) (a^2 + b^2 - ab)$

9.
(i) $2a^3 + 16b^3 - 5a - 10b$
Solution:-
$2a^3 + 16b^3 - 5a - 10b$
Above terms can be written as,
$2(a^3 + 8b^3) - 5(a + 2b)$
$2(a^3 + (2b)^3) - 5(a + 2b)$
We know that, $a^3 + b^3 = (a + b) (a^2 - ab + b^2)$
$2[(a + 2b) (a^2 - 2ab + 4b^2)] - 5(a + 2b)$
$(a + 2b) (2a^2 - 4ab + 8b^2 - 5)$

(ii) $a^3 - (1/a^3) - 2a + 2/a$
Solution:-
$a^3 - (1/a^3) - 2a + 2/a$
$(a^3 - (1/a)^3) - 2a + 2/a$
We know that, $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$
$[(a - 1/a) - (a^2 + (a \times 1/a) + (1/a)^2] - 2(a - 1/a)$
$(a - 1/a) (a^2 + 1 + 1/a^2) - 2(a - 1/a)$
$(a - 1/a) (a^2 + 1 + 1/a^2 - 2)$
$(a - 1/a) (a^2 + (1/a^2) - 1)$

10.
(i) $a^6 - b^6$
Solution:-
$a^6 - b^6$
Above terms can be written as,
$(a^2)^3 - (b^2)^3$
We know that, $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$
So, $a = a^2$, $b = b^2$
$(a^2 - b^2) ((a^2)^2) + a^2b^2 + (b^2)^2)$
$(a^2 - b^2) (a^4 + a^2b^2 + b^4)$

(ii) $x^6 – 1$

Solution:-

$x^6 – 1$

Above terms can be written as,

$(x^2)^3 – 1^3$

We know that, $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$

So, $a = x^2$, $b = 1$

$(x^2 - 1) ((x^2)^2 + (x^2 \times 1) + 1^2)$

$(x^2 - 1) (x^4 + x^2 + 1)$

11.

(i) $64x^6 – 729y^6$

Solution:-

$64x^6 – 729y^6$

Above terms can be written as,

$(2x)^6 – (3y)^6$

$[(2x)^2]^3 – [(3y)^2]^3$

We know that, $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$

So, $a = (2x)^2$, $b = (3y)^2$

$[(2x)^2 – (3y)^2] [((2x)^2)^2 + ((2x)^2 \times (3y)^2) + ((3y)^2)^2]$

$(4x^2 – 9y^2) [16x^4 + (4x^2 \times 9y^2) + (9y^2)^2]$

$(4x^2 – 9y^2) [16x^4 + 36x^2y^2 + 81y^4]$

$[(2x)^2 – (3y)^2] [16x^4 + 36x^2y^2 + 81y^4]$

$(2x + 3y) (2x – 3y) (16x^4 + 36x^2y^2 + 81y^4)$

(ii) $x^3 – (8/x)$

Solution:-

$x^3 – (8/x)$

Above terms can be written as,

$(1/x) (x^3 - 8)$

$(1/x) [(x)^3 – (2)^3]$

We know that, $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$

So, $a = x$, $b = 2$

$(1/x) (x - 2) (x^2 + 2x + 4)$

12.

(i) 250 (a - b)³ + 2
Solution:-
250 (a - b)³ + 2
Take out common in all terms we get,
2(125(a - b)³ + 1)
2[(5(a - b))³ + 1³]
We know that, $a^3 + b^3 = (a + b) (a^2 - ab + b^2)$
$\qquad\qquad = 2[(5a - 5b + 1) ((5a - 5b)^2 - (5a - 5b)1 + 1^2)]$
$\qquad\qquad = 2(5a - 5b + 1) (25a^2 + 25b^2 - 50ab - 5a + 5b + 1)$

(ii) 32a²x³ − 8b²x³ − 4a²y³ + b²y³
Solution:-
$32a^2x^3 - 8b^2x^3 - 4a^2y^3 + b^2y^3$
Take out common in all terms we get,
$8x^3(4a^2 - b^2) - y^3(4a^2 - b^2)$
$(4a^2 - b^2) (8x^3 - y^3)$
Above terms can be written as,
$((2a)^2 - b^2) ((2x)^3 - y^3)$
We know that, $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$ and $(a^2 - b^2) = (a + b) (a - b)$
$(2a + b) (2a - b) [(2x - y) ((2x)^2 + 2xy + y^2)]$
$(2a + b) (2a - b) (2x - y) (4x^2 + 2xy + y^2)$

13.
(i) x⁹ + y⁹
Solution:-
$x^9 + y^9$
Above terms can be written as,
$(x^3)^3 + (y^3)^3$
We know that, $a^3 + b^3 = (a + b) (a^2 - ab + b^2)$
Where, $a = x^3, b = y^3$
$(x^3 + y^3) ((x^3)^2 - x^3y^3 + (y^3)^2)$
$(x^3 + y^3) (x^6 - x^3y^3 + y^6)$
Then, $(x^3 + y^3)$ in the form of $(a^3 + b^3)$
$(x + y)(x^2 - xy + y^2) (x^6 - x^3y^3 + y^6)$

(ii) x⁶ − 7x³ − 8
Solution:-

$X^6 – 7x^3 – 8$

Above terms can be written as,

$(x^2)^3 – 7x^3 – x^3 + x^3 – 8$

$(x^2)^3 – 8x^3 + x^3 – 2^3$

$(((x^2)^3) – (2x)^3) + (x^3 – 2^3)$

We know that, $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$

$(x^2 – 2x) ((x^2)^2 + (x^2 × 2x) + (2x)^2) + (x - 2) (x^2 + 2x + 2^2)$

$(x^2 – 2x) (x^4 + 2x^3 + 4x^2) + (x - 2) (x^2 + 2x + 4)$

$x(x - 2) x^2(x^2 + 2x + 4) + (x - 2) (x^2 + 2x + 4)$

Take out common in all terms we get,

$(x - 2) (x^2 + 2x + 4) ((x × x^2) + 1)$

$(x - 2) (x^2 + 2x + 4) (x^3 + 1)$

So, above terms are in the form of $a^3 + b^3$

Therefore, $(x - 2) (x^2 + 2x + 4) (x + 1) (x^2 – x + 1)$

CHAPTER TEST

1.
(i) 15(2x - 3)³ – 10(2x - 3)
Solution:-
$15(2x - 3)^3 – 10(2x - 3)$
Take out common in both terms,
Then, $5(2x – 3) [3(2x - 3)^2 – 2]$

(ii) a(b - c) (b + c) – d(c - b)
Solution:-
$a(b - c) (b + c) – d(c - b)$
Above terms can be written as,
$a(b - c) (b + c) + d(b - c)$
Take out common in both terms,
$(b - c) [a(b + c) + d]$
$(b - c) (ab + ac + d)$

2.
(i) 2a²x – bx + 2a² – b
Solution:-
$2a^2x – bx + 2a^2 – b$
Rearrange the above terms we get,
$2a^2x + 2a – bx – b$
Take out common in both terms,
$2a^2(x + 1) – b(x + 1)$
$(x + 1) (2a^2 - b)$

(ii) p² – (a + 2b)p + 2ab
Solution:-
$p^2 – (a + 2b)p + 2ab$
Above terms can be written as,
$p^2 – ap - 2bp + 2ab$
Take out common in both terms,
$p(p - a) - 2b(p - a)$
$(p – a) (p – 2b)$

3.
(i) $(x^2 – y^2)z + (y^2 – z^2)x$
Solution:-
$(x^2 – y^2)z + (y^2 – z^2)x$
Above terms can be written as,
$zx^2 – zy^2 + xy^2 – xz^2$
Rearrange the above terms we get,
$zx^2 – xz^2 + xy^2 – zy^2$
Take out common in both terms,
$zx(x - z) + y^2(x - z)$
$(x - z) (zx + y^2)$

(ii) $5a^4 – 5a^3 + 30a^2 – 30a$
Solution:-
$5a^4 – 5a^3 + 30a^2 – 30a$
Take out common in both terms,
$5a(a^3 – a^2 + 6a - 6)$
$5a[a^2(a - 1) + 6(a - 1)]$
$5a(a - 1) (a^2 + 6)$

4.
(i) $b(c -d)^2 + a(d - c) + 3c – 3d$
Solution:-
$b(c -d)^2 + a(d - c) + 3c – 3d$
Above terms can be written as,
$b(c - d)^2 – a(c - d) + 3c – 3d$
$b(c - d)^2 – a(c - d) + 3(c - d)$
Take out common in both terms,
$(c - d) [b(c - d) – a + 3]$
$(c - d) (bc – bd – a + 3)$

(ii) $x^3 – x^2 – xy + x + y – 1$
Solution:-
$x^3 – x^2 – xy + x + y – 1$
Rearrange the above terms we get,
$x^3 – x^2 – xy + y + x – 1$
Take out common in both terms,

$x^2(x - 1) - y(x - 1) + 1(x - 1)$
$(x - 1) (x^2 - y + 1)$

5.
(i) x(x + z) – y (y + z)
Solution:-
$x(x + z) - y (y + z)$
$x^2 + xz - y^2 - yz$
Rearrange the above terms we get,
$x^2 - y^2 + xz - yz$
We know that, $(a^2 - b^2) = (a + b) (a - b)$
So, $(x + y) (x - y) + z(x - y)$
$(x - y) (x + y + z)$

(ii) a¹²x⁴ – a⁴x¹²
Solution:-
$a^{12}x^4 - a^4x^{12}$
Take out common in both terms,
$a^4x^4 (a^8 - x^8)$
$a^4x^4((a^4)^2 - (x^4)^2)$
We know that, $(a^2 - b^2) = (a + b) (a - b)$
$a^4x^4 (a^4 + x^4) (a^4 - x^4)$
$a^4x^4 (a^4 + x^4) ((a^2)^2 - (x^2)^2)$
$a^4x^4(a^4 + x^4) (a^2 + x^2) (a^2 - x^2)$
$a^4x^4 (a^4 + x^4) (a^2 + x^2) (a + x) (a - x)$

6.
(i) 9x² + 12x + 4 – 16y²
Solution:-
$9x^2 + 12x + 4 - 16y^2$
Above terms can be written as,
$(3x)^2 + (2 \times 3x \times 2) + 2^2 - 16y^2$
Then, $(3x + 2)^2 + (4y)^2$
$(3x + 2 + 4y) (3x + 2 - 4y)$

(ii) x⁴ + 3x² + 4
Solution:-

$x^4 + 3x^2 + 4$

Above terms can be written as,

$(x^2)^2 + 3(x^2) + 4$

$(x^2)^2 + (2)^2 + 4x^2 – x^2$

$(x^2 + 2)^2 – (x^2)$

We know that, $(a^2 – b^2) = (a + b) (a - b)$

$(x^2 + 2 + x) (x^2 + 2 - x)$

$(x^2 + x + 2) (x^2 – x + 2)$

7.

(i) $21x^2 – 59xy + 40y^2$

Solution:-

$21x^2 – 59xy + 40y^2$

By multiplying the first and last term we get, $21 × 40 = 840$

Then, $(-35) × (-24) = 840$

So, $21x^2 – 35xy – 24xy + 40y^2$

$7x(3x – 5y) – 8y(3x – 5y)$

$(3x – 5y) (7x – 8y)$

(ii) $4x^3y – 44x^2y + 112xy$

Solution:-

$4x^3y – 44x^2y + 112xy$

Take out common in all terms,

$4xy(x^2 – 11x + 28)$

Then, $4xy (x^2 – 7x – 4x + 28)$

$4xy [x(x - 7) – 4(x + 7)]$

$4xy (x - 7) (x - 4)$

8.

(i) $x^2y^2 – xy - 72$

Solution:-

$x^2y^2 – xy – 72$

$x^2y^2 – 9xy + 8xy – 72$

Take out common in all terms,

$xy(xy - 9) + 8(xy - 9)$

$(xy - 9) (xy + 8)$

(ii) 9x³y + 41x²y² + 20xy³
Solution:-
9x³y + 41x²y² + 20xy³
Take out common in all terms,
xy(9x² + 41xy + y²)
Above terms can be written as,
xy (9x² + 36xy + 5xy + 20y²)
xy [9x(x + 4y) + 5y(x + 4y)]
xy (x + 4y) (9x + 5y)

9.
(i) (3a – 2b)² + 3(3a – 2b) - 10
Solution:-
(3a – 2b)² + 3(3a – 2b) - 10
Let us assume, (3a – 2b) = p
p² + 3p – 10
p² + 5p – 2p – 10
Take out common in all terms,
p(p + 5) – 2(p + 5)
(p + 5) (p - 2)
Now, substitute the value of p
(3a – 2b + 5) (3a – 2b - 2)

(ii) (x² – 3x) (x² – 3x + 7) + 10
Solution:-
(x² – 3x) (x² – 3x + 7) + 10
Let us assume, (x² – 3x) = q
q (q + 7) + 10
q² + 7q + 10
q² + 5q + 2q + 10
q(q + 5) + 2(q + 5)
(q + 5) (q + 2)
Now, substitute the value of q
(x² – 3x + 5) (x² – 3x + 2)

10.

(i) (x² - x) (4x² – 4x - 5) - 6
Solution:-
(x² - x) (4x² – 4x - 5) – 6
(x² - x) [(4x² – 4x) - 5] – 6
(x² - x) [4(x² - x) - 5] – 6
Let us assume x² – x = q
So, q[4q - 5] – 6
4q² – 5q – 6
4q² – 8q + 3q – 6
4q(q - 2) + 3(q - 2)
(q - 2) (4q + 3)
Now, substitute the value of q
(x² – x - 2) (4(x² - x) + 3)
(x² – x – 2) (4x² – 4x + 3)
(x² – 2x + x - 2) (4x² – 4x + 3)
[x(x - 2) + 1(x - 2)] (4x² – 4x + 3)
(x - 2) (x + 1) (4x² – 4x + 3)

(ii) x⁴ + 9x²y² + 81y⁴
Solution:-
x⁴ + 9x²y² + 81y⁴
Above terms can be written as,
x⁴ + 18x²y² + 81y⁴ – 9x²y²
((x²)² + (2 × x² × 9y²) + (9y²)²) – 9x²y²
We know that, (a + b)² = a² + 2ab + b²
(x² + 9y²)² – (3xy)²
(x² + 9y² + 3xy) (x² + 9y² – 3xy)

11.
(i) (8/27)x³ – (1/8)y³
Solution:-
(8/27)x³ – (1/8)y³
Above terms can be written as,
((2/3)x)³ – (½y)³
We know that, a³ - b³ = (a - b) (a² + ab + b²)
((2/3)x – ½y) [(2/3)x + (2/3)x (1/2)y + ((1/2)y)²]
((2/3)x – (1/2)y) [(4/9)x² + (xy/3) + (y²/4)]

(ii) $x^6 + 63x^3 - 64$
Solution:-
$x^6 + 63x^3 - 64$
Above terms can be written as,
$x^6 + 64x^3 - x^3 - 64$
Take out common in all terms,
$x^3 (x^3 + 64) - 1(x^3 + 64)$
$(x^3 + 64) (x^3 - 1)$
$(x^3 + 4^3) (x^3 - 1^3)$
We know that, $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$ and $a^3 + b^3 = (a + b) (a^2 - ab + b^2)$
So, $(x + 4) [x^2 - 4x + 4^2] (x - 1) [x^2 + x + 1^2]$
$(x + 4) (x^2 - 4x + 16) (x - 1) (x^2 + x + 1)$

12.
(i) $x^3 + x^2 - (1/x^2) + (1/x^3)$
Solution:-
$x^3 + x^2 - (1/x^2) + (1/x^3)$
Rearranging the above terms, we get,
$x^3 + (1/x^3) + x^2 - (1/x^2)$
We know that, $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$ and $(a^2 - b^2) = (a + b) (a - b)$
$(x + 1/x) (x^2 - 1 + 1/x^2) + (x + 1/x) (x - 1/x)$
$(x + 1/x) [x^2 - 1 + 1/x^2 + x - 1/x]$

(ii) $(x + 1)^6 - (x - 1)^6$
Solution:-
$(x + 1)^6 - (x - 1)^6$
Above terms can be written as,
$((x + 1)^3)^2 - ((x - 1)^3)^2$
We know that, $(a^2 - b^2) = (a + b) (a - b)$
$[(x + 1)^3 + (x - 1)^3] [(x + 1)^3 - (x - 1)^3]$
$[(x + 1) + (x - 1)][(x + 1)^2 - (x - 1) (x + 1) + (x - 1)^2] [(x + 1) - (x - 1)][(x + 1)^2 + (x - 1) (x + 1) + (x - 1)^2]$
$(x + 1 + x - 1) [x^2 + 2x + 1 - x^2 + 1 + x^2 + 1 - 2x(x + 1) - x + 1] [x^2 + 2x + 1 + x^2 - 1 + x^2 - 2x + 1]$
By simplifying we get,
$2x(x^2 + 3) 2(3x^2 + 1)$
$4x(x^2 + 3) (3x^2 + 1)$

13. Show that (97)³ + (14)³ is divisible by 111

Solution:-

From the question,

$(97)^3 + (14)^3$

We know that, $a^3 + b^3 = (a + b) (a^2 - ab + b^2)$

So, $(97 + 14) [(97)^2 – (97 × 14) + (14)^2]$

$111 [(97)^2 – (97 × 14) + (14)^2]$

Therefore, it is clear that the given expression is divisible by 111.

14. a⁴ + a²b² + b⁴

Solution:-

$a^4 + a^2b^2 + b^4$

Above terms can be written as,

$a^4 + 2a^2b^2 + b^4 – a^2b^2$

$(a^2)^2 + 2a^2b^2 + (b^2)^2 – (ab)^2$

$(a^2 + b^2)^2 – (ab)^2$

$(a^2 + b^2 + ab) (a^2 + b - ab)$

$a + b = 8, ab = 15$

So, $(a + b)^2 = 8^2$

$a^2 + 2ab + b^2 = 64$

$a^2 + 2(15) + b^2 = 64$

$a^2 + b^2 + 30 = 64$

By transposing,

$a^2 + b^2 = 64 – 30$

$a^2 + b^2 = 34$

Then, $a^4 + a^2b^2 + b^4$

$\quad = (a^2 + b^2 + ab) (a^2 + b^2 - ab)$

$\quad = (34 + 15) (34 - 15)$

$\quad = 49 × 19$

$\quad = 931$