Exercise 8

1. (i) $(81/16)^{-3/4}$
Solution:

$(81/16)^{-3/4}$
$= [(3^4/2^4)]^{-3/4}$
$= [(3/2)^4]^{-3/4}$
$= (3/2)^{-3/4 \times 4}$
$= (3/2)^{-3}$
$= (2/3)^3$
$= 2^3/3^3$
$= (2 \times 2 \times 2)/(3 \times 3 \times 3)$
$= 8/27$

(ii) $(1\frac{61}{64})^{-\frac{2}{3}}$
Solution:

$$(1\tfrac{61}{64})^{-\frac{2}{3}} = (\frac{125}{64})^{-\frac{2}{3}} = (\frac{5^3}{4^3})^{-\frac{2}{3}}$$

$= (5/4)^{3 \times -2/3}$
$= (5/4)^{-2}$
$= (4/5)^2$
$= 16/25$

2. (i) $(2a^{-3}b^2)^3$
Solution:

$(2a^{-3}b^2)^3$
$= 2^3 \, a^{-3x3} \, b^{2x3}$
$= 8a^{-1}b^6$

(ii) $(a^{-1} + b^{-1})/(ab)^{-1}$
Solution:

$$\frac{a^{-1}+b^{-1}}{(ab)^{-1}} = \frac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{ab}} = \frac{a+b}{ab} \times \frac{ab}{1} = a+b$$

3. (i) $(x^{-1} y^{-1})/(x^{-1} + y^{-1})$
Solution:

$$\frac{x^{-1}y^{-1}}{x^{-1}+y^{-1}} = \frac{(xy)^{-1}}{\frac{1}{x}+\frac{1}{y}}$$
$$= \frac{\frac{1}{xy}}{\frac{xy}{x+y}} = \frac{1}{xy} \times \frac{xy}{x+y}$$
$$= \frac{1}{x+y}$$

(ii) $(4 \times 10^7)(6 \times 10^{-5})/(8 \times 10^{10})$
Solution:

$$\frac{(4 \times 10^7)(6 \times 10^{-5})}{8 \times 10^{10}}$$
$$= \frac{4 \times 6 \times 10^7 \times 10^{-5}}{8 \times 10^{10}}$$
$$= \frac{24 \times 10^{7+(-5)}}{8 \times 10^{10}}$$
$$= 3 \times \frac{10^2}{10^{10}} = 3 \times 10^{2-10} = 3 \times 10^{-8}$$

4. (i) $3a/b^{-1} + 2b/a^{-1}$
Solution:

$3a/b^{-1} + 2b/a^{-1}$
$= 3a/(1/b) + 2b/(1/a)$
$= (3a \times b)/1 + (2b \times a)/1$
$= 3ab + 2ab = 5ab$

(ii) $5^0 \times 4^{-1} + 8^{1/3}$
Solution:

$5^0 \times 4^{-1} + 8^{1/3}$
$= 1 \times (1/4) + (2)^{3 \times 1/3}$
$= ¼ + 2$
$= (1 + 8)/4$
$= 9/4 = 2¼$

5. (i) $(8/125)^{-1/3}$
Solution:

$(8/125)^{-1/3}$

$= [(2 \times 2 \times 2)/(5 \times 5 \times 5)]^{-1/3}$
$= (2^3/5^3)^{-1/3}$
$= (2/5)^{3 \times -1/3}$
$= (2/5)^{-1}$
$= 5/2 = 2½$

(ii) $(0.027)^{-1/3}$
Solution:

$(0.027)^{-1/3}$
$= (27/1000)^{-1/3}$
$= [(3 \times 3 \times 3)/(10 \times 10 \times 10)]^{-1/3}$
$= (3^3/10^3)^{-1/3}$
$= (3/10)^{3 \times -1/3}$
$= (3/10)^{-1}$
$= 10/3$

6. (i) $(-1/27)^{-2/3}$
Solution:

$(-1/27)^{-2/3}$
$= (-1/3^3)^{-2/3}$
$= (-1/3)^{3 \times -2/3}$
$= (-1/3)^{-2}$
$= (-3)^2$
$= 9$

(ii) $(1\frac{61}{64})^{-\frac{2}{3}}$
Solution:

$$(1\tfrac{61}{64})^{-\frac{2}{3}} = (\frac{125}{64})^{-\frac{2}{3}} = (\frac{5^3}{4^3})^{-\frac{2}{3}}$$

$= (5/4)^{3 \times -2/3}$
$= (5/4)^{-2}$
$= (4/5)^2$
$= 16/25$