Exercise 18.1

1. The length and breadth of a rectangular field are in the ratio 9 : 5. If the area of the field is 14580 square metres, find the cost of surrounding the field with a fence at the rate of ₹3.25 per metre.

Solution:

Let the length of rectangle be 9x and its breadth be 5x

So,

Area = l × b

⇒ 14580 = 9x × 5x

$45x^2 = 14580$

$x^2 = 14580/45 = 324$

x = √324

x = 18

Hence,

Length = 9 × 18 = 162 m and Breadth = 5 × 18 = 90 m

Now, Perimeter = 2(l + b)

= 2 (162 + 90) = 2(252)

= 504 m.

Therefore, cost for fencing the surrounding 504 m at the rate of ₹3.25 per metre = ₹(504 × 3.25) = ₹1638

2. A rectangle is 16 m by 9 m. Find a side of the square whose area equals the area of the rectangle. By how much does the perimeter of the rectangle exceed the perimeter of the square?

Solution:

Area of rectangle = $(16 \times 9)\ m^2 = 144\ m^2$

Given condition,

Area of square = Area of rectangle

∴ $(Side)^2 = 144$

Side = √144 = 12 m

Now,

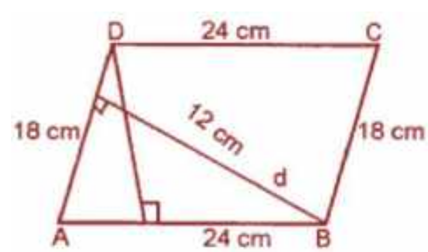
Perimeter of square = 4 × side = 4 × 12 = 48 m

Perimeter of rectangle = 2(l + b) = 2 (16 + 9) = 50 m

Hence, difference in their perimeters = 50 – 48 = 2 m

3. Two adjacent sides of a parallelogram are 24 cm and 18 cm. If the distance between longer sides is 12 cm, find the distance between shorter sides.

Solution:



Let take 24 cm as the base of parallelogram, then its height is 12 cm.

We know that,

Area of parallelogram = base × height

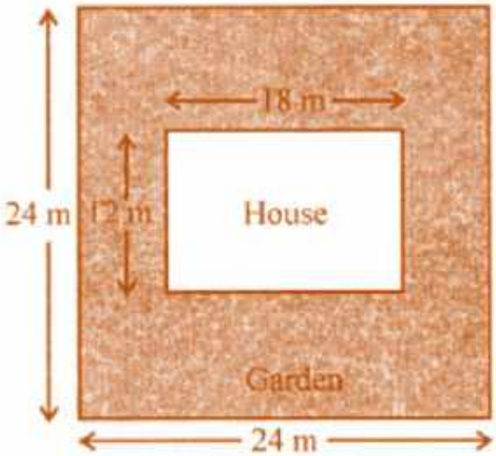
= $24 \times 12 = 288\ cm^2$

Let's consider d cm to be the distance between the shortest sides.

∴ Area of parallelogram = $(18 \times d)\ cm^2$

$18 \times d = 288$
$\Rightarrow d = 288/18 = 16$ cm
Therefore, the distance between the shorter sides is 16 cm.

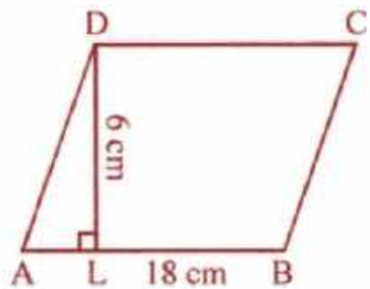
4. Rajesh has a square plot with the measurement as shown in the given figure. He wants to construct a house in the middle of the plot. A garden is developed around the house. Find the total cost of developing a garden around the house at the rate of ₹50 per m^2.



Solution:

Given,
Side of square plot = 24 m
Length of house (l) = 18 m
and breadth (b) = 12m
Now,
Area of square plot = $(24)^2$ m^2 = (24×24) m^2 = 576 m^2
And,
Area of house = $18 \times 12 = 216$ m^2
Remaining area of the garden = 576 m^2 – 216 m^2 = 360 m^2
The cost of developing the garden = ₹50 per m^2
Therefore, the total cost = ₹50 × 360 = ₹18000

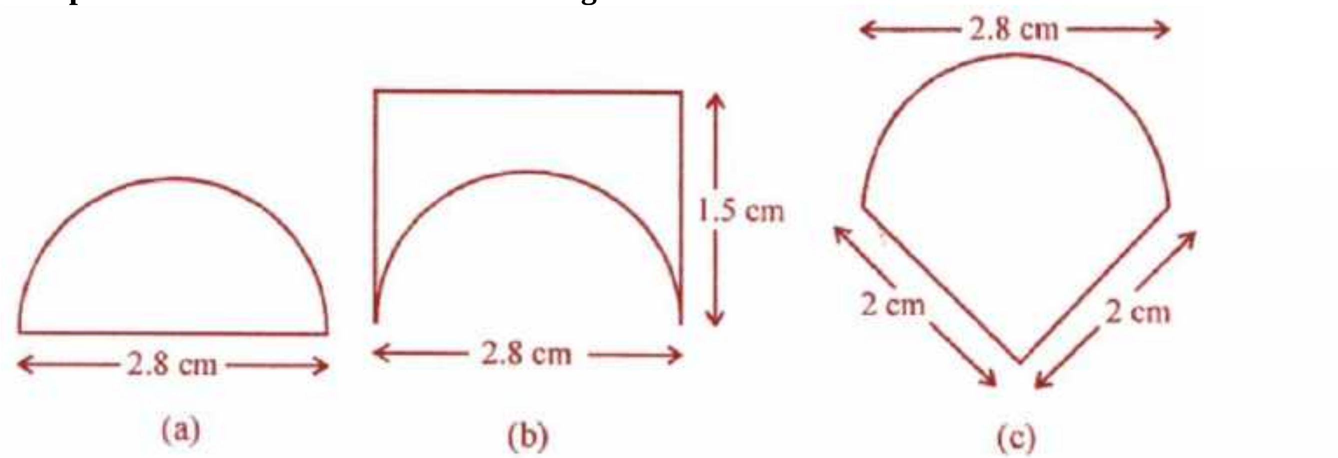
5. A flooring tile has a shape of a parallelogram whose base is 18 cm and the corresponding height is 6 cm. How many such tiles are required to cover a floor of area 540 m^2? (If required you can split the tiles in whatever way you want to fill up the comers).
Solution:



Given,
Base of the parallelogram-shaped flooring tile = 18 cm and its height = 6 cm
So,
Area of one tile = Base × Height
= 18 × 6
= 108 cm^2
We have the area of floor = 540 m^2

Hence, number of tiles = Total area/ Area of one tile
= (540 x 100 x 100)/108 [As, 1 m^2 = (100 x 100) cm^2]
= 50000

6. An ant is moving around a few food pieces of different shapes scattered on the floor. For which food piece would the ant have to take a longer round?



Solution:

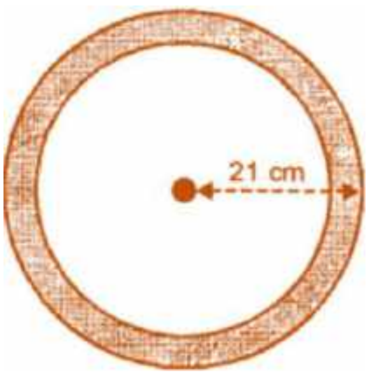
(a) Diameter of semicircle = 2.8 cm
∴ Perimeter = πr + 2r
= 22/7 × 2.8 + 2 × 2.8
= 8.8 + 5.6 cm
= 14.4 cm

(b) Total perimeter = 1.5 + 1.5 + 2.8 + semi circumference (πr, where r = 2.8/2 = 1.4 cm)
= 1.5 + 1.5 + 2.8 + (22/7 x 1.4)
= 5.8 + 8.8
= 14.6 cm

(c) Total perimeter = 2 + 2 + Semi circumference (πr, where r = 2.8/2 = 1.4 cm)
= 4 + 8.8
= 12.8 cm

Hence, it is clearly seen that distance of (b) i.e. 14.6 is the longest.

7. In the adjoining figure, the area enclosed between the concentric circles is 770 cm2. If the radius of the outer circle is 21 cm, calculate the radius of the inner circle.



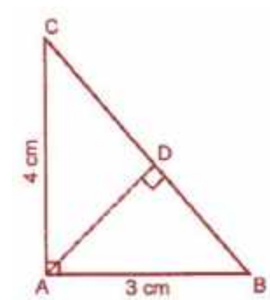
Solution:

Given,
Radius of outer circle (R) = 21 cm.
Radius of inner circle (r) = r cm.
Area of shaded portion = 770 cm^2
$\Rightarrow \pi (R^2 - r^2) = 770$
$(21^2 - r^2) = 770$
$441 - r^2 = 770 \times (7/22) = 35 \times 7 = 245$
$r^2 = 441 - 245$
$r^2 = 196$
$r = \sqrt{196}$
$\therefore r = 14$ cm

8. A copper wire when bent in the form of a square encloses an area of 121 cm2. If the same wire is bent into the form of a circle, find the area of the circle.
Solution:

Given,
Area of the square = 121 cm^2
So, side = $\sqrt{121}$ = 11 cm
Now,
Perimeter = 4 a = 4 × 11= 44 cm
And, circumference of the circle = 44 cm
∴ Radius = (44 x 7)/ (2 x 22) = 7cm
Therefore, area of the circle = $\pi r^2 = (7)^2$
= 22/7 × 7 × 7
= 154 cm^2

9. From the given figure, find
(i) the area of Δ ABC
(ii) length of BC
(iii) the length of altitude from A to BC



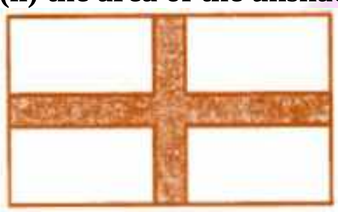
Solution:

(i) We have,
Base = 3 cm and height = 4 cm.
Hence,
Area = ½ × base × height
= ½ × 3 × 4
= 6 cm^2

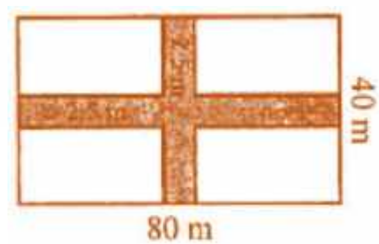
(ii) By Pythagoras theorem, we have
$BC^2 = AB^2 + AC^2$
$BC^2 = (3)^2 + (4)^2$
= 9 + 16 = 25
⇒ BC = √25 cm = 5 cm

(iii) Now,
Base = BC = 5 cm, h = AD =?
So,
Area = ½ × b × h
6 = ½ × 5 × h [∵ Area = 6 cm^2 as in part (i)]
⇒ h = 12/6 = 2.4 cm.

10. A rectangular garden 80 m by 40 m is divided into four equal parts by two cross-paths 2.5 m wide. Find
(i) the area of the cross-paths.
(ii) the area of the unshaded portion.



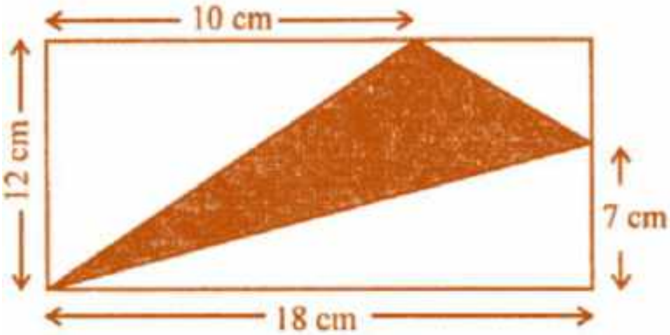
Solution:



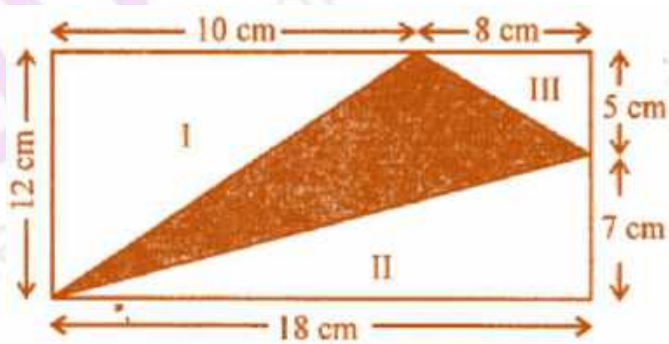
Given,
Length of rectangular garden = 80 m
and breadth = 40 m
Width of crossing path 2.5 m
So,
Area of length wise path = 80 × 2.5 = 200 m^2
and
Area of breadth wise path = 40 × 2.5 = 100 m^2
(i) Total area of both paths
= 200 + 100 – 2.5 × 2.5 m^2
= 300 – 6.25 = 293.75 m^2

(ii) Area of unshaded portion
= Area of garden – Area of paths
= 80 × 40 – 293.75 m^2
= 3200 – 293.75 m^2
= 2906.25 m^2

11. In the given figure, ABCD is a rectangle. Find the area of the shaded region.

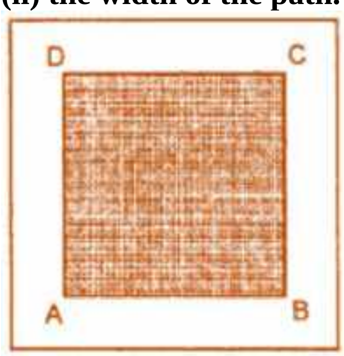


Solution:

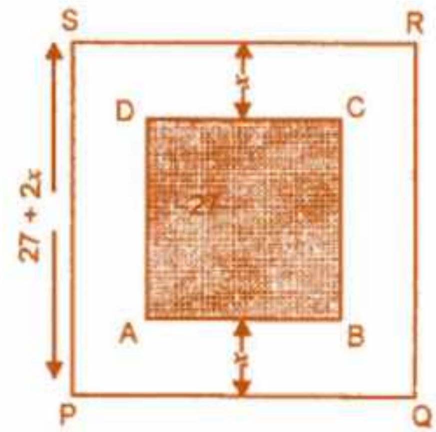


In the given figure, we have
Length of rectangle = 18 cm and breadth = 12 cm
∴ Area = l × b = 18 × 12 cm^2 = 216 cm^2
Area of triangle I = ½ × 12 × 10 = 60 cm^2
Area of triangle III = ½ × 18 × 7 = 63 cm^2
Thus,
Area of shaded portion
= Area of rectangle – Area of 3 triangles
= 216 – (60 + 63 + 20)
= 216 – 143 cm2
= 73 cm^2

12. In the adjoining figure, ABCD is a square grassy lawn of area 729 m^2. A path of uniform width runs all around it. If the area of the path is 295 m^2, find
(i) the length of the boundary of the square field enclosing the lawn and the path.
(ii) the width of the path.



Solution:



Given,
Area of square ABCD = 729 m^2
So, its side = $\sqrt{729}$ = 27 m
Let's take the width of path = x m
Then,
Side of outer field = 27 + x + x = (27 + 2x) m
And, area of square PQRS = $(27 + 2x)^2$ m^2
Now,
Area of PQRS – Area of ABCD = Area of path
⇒ $(27 + 2x)^2 m^2 - 729 m^2 = 295 m^2$
$729 + 4x^2 + 108x - 729 = 295$
$4x^2 + 108x - 295 = 0$
By using the quadratic formula, we have
a = 4, b = 108 and c = -295

$$\Rightarrow x = \frac{-108 \pm \sqrt{(108)^2 - 4 \times (4) \times (-295)}}{8} \qquad \left(\because x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$$

$$= \frac{-108 \pm \sqrt{11664 + 4720}}{8}$$

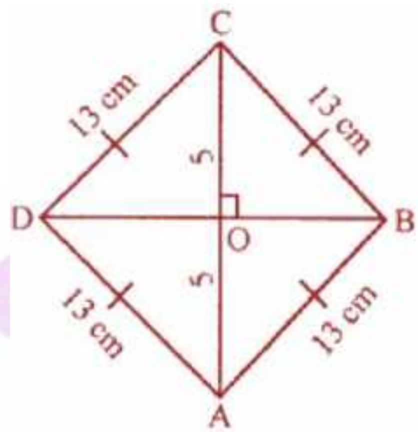
$$= \frac{-108 \pm \sqrt{16384}}{8} = \frac{-108 \pm 128}{8}$$

$$= \frac{20}{8} = 2 \cdot 5 \quad \text{(Taking the positve root)}$$

Hence,
Width of the path is 2.5 m
Now, side of square field PQRS = 27 + 2x
= (27 + 2 × 2.5) m
= 32 m
Therefore,
Length of boundary = 4 × side = 32 × 4 = 128 m

Exercise 18.2

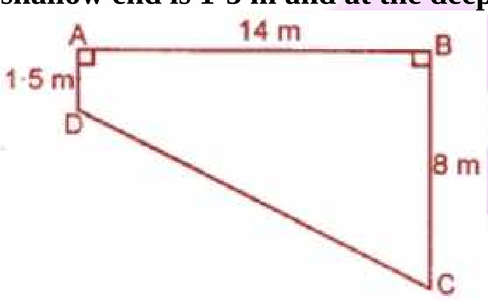
1. Each sides of a rhombus is 13 cm and one diagonal is 10 cm. Find
(i) the length of its other diagonal
(ii) the area of the rhombus
Solution:



(i) Given,
Side of rhombus = 13 cm.
Length of diagonal AC = 10 cm.
∴ OC = 5 cm.
Since, the diagonals of rhombus bisect each other at right angles
So, ∆BOC is rt. angled.
Then, by Pythagoras Theorem we have
$BC^2 = OC^2 + OB^2$
$13^2 = 5^2 + OB^2$
$OB^2 = 169 – 25 = 144$
⇒ OB = √144 = 12 cm
Hence,
Diagonal BD = 2 × OB = 2 × 12 = 24 cm

(ii) Area of rhombus = ½ × d_1 × d_2
= ½ × 10 × 24 = $120cm^2$

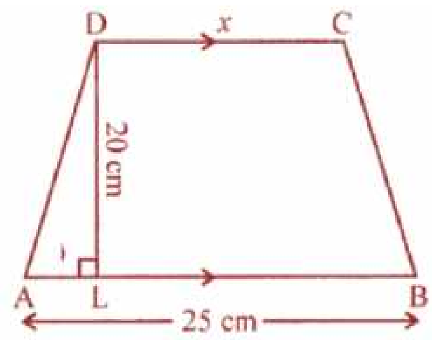
2. The cross-section ABCD of a swimming pool is a trapezium. Its width AB = 14 m, depth at the shallow end is 1-5 m and at the deep end is 8 m. Find the area of the cross-section.



Solution:

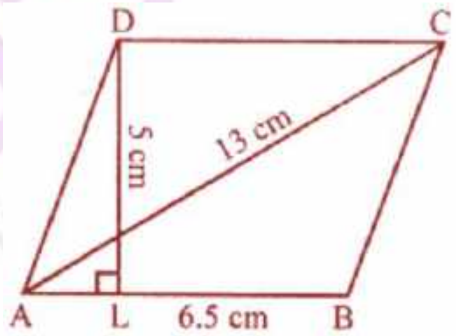
Here, AD and BC are the two parallel sides of trapezium
And, distance between them is 14 m.
∴ Area of trapezium = ½ (1·5 + 8) × 14
= ½ × 9·5 × 14
= 66 × 5 m^2

3. The area of a trapezium is 360 m^2, the distance between two parallel sides is 20 m and one of the parallel side is 25 m. Find the other parallel side.
Solution:



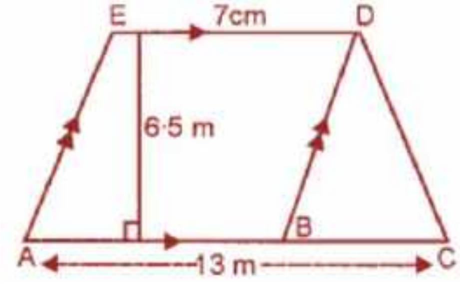
Given,
Area of a trapezium = 360 m^2
Distance between two parallel lines = 20 m
One parallel side = 25 m
Now,
Let's assume the second parallel side to be x m
So, Area = (25 + x) × 20
⇒ 360 = (25 + x) × 20
∴ x = 36 – 25 = 11 m
Therefore, the second parallel side is 11 m.

4. Find the area of a rhombus whose side is 6.5 cm and altitude is 5 cm. If one of its diagonal is 13 cm long, find the length of other diagonal.
Solution:



Given,
Side of rhombus = 6.5 cm
And altitude = 5 cm
So,
Area of a rhombus = Side × Altitude = 6.5 × 5 = 32.5 cm^2
We have, one diagonal = 13 cm
Hence,
Length of other diagonal = (2 x Area)/ One diagonal
= (32.5 x 2)/ 13
= 5 cm

5. From the given diagram, calculate
(i) the area of trapezium ACDE
(ii) the area of parallelogram ABDE
(iii) the area of triangle BCD.



Solution:

(i) Area of trapezium ACDE = ½ × (AC + DE) × h
= ½ × (13 + 7) × 6.5
= ½ × 20 × 6.5
= 65 m^2

(ii) Area of parallelogram ABDE = ½ × b × h

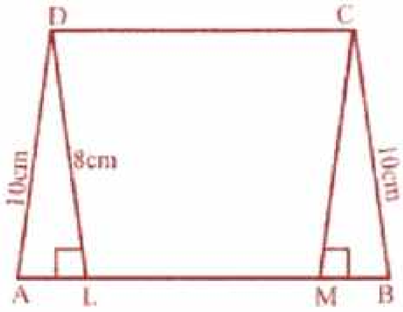
$= \frac{1}{2} \times 6 \times 6.5$
$= 15.5\ m^2$

(iii) Area of $\Delta BCD = \frac{1}{2} \times$ base $\times$ height
$= \frac{1}{2} \times BC \times$ height
$= \frac{1}{2} \times 6 \times 6.5$ [$\because BC = AC - AB = 13 - 7 = 6$ m]
$= 19.5\ m^2$

6. The area of a rhombus is equal to the area of a triangle whose base and the corresponding altitude are 24.8 cm and 16.5 cm respectively. If one of the diagonals of the rhombus is 22 cm, find the length of the other diagonal.
Solution:

Given,
Base of triangle = 24.8 cm and altitude = 16.5 cm
Area = ½ × base × altitude
$= \frac{1}{2} \times 24.8 \times 16.5\ cm^2$
$= 204.6\ cm^2$
Now, Area of Δ = Area of rhombus
But, area of rhombus = $204.6\ cm^2$
Length of one diagonal = 22 cm
Area of rhombus = (First diagonal × Second diagonal)
Thus,
Second diagonal = (2 × Area)/ First diagonal
= (204.6 × 2)/ 22
= 18.6 cm

7. The perimeter of a trapezium is 52 cm. If its non-parallel sides are 10 cm each and its altitude is 8 cm, find the area of the trapezium.
Solution:



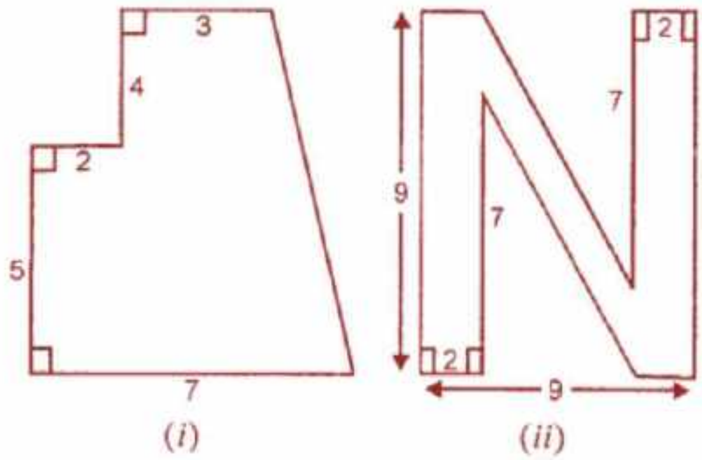
Given,
Perimeter of a trapezium = 52 cm
Length of each non-parallel side = 10 cm
Altitude DL = 8 cm
Now,
In right ΔDAL, by Pythagoras Theorem we have
$DA^2 = DL^2 + AL^2$
$(10)^2 = (8)^2 + AL^2$
$100 = 64 + AL^2$
$AL^2 = 100 - 64 = 36 = (6)^2$
∴ AL = 6 cm
Similarly,
BM = 6 cm and DC = LM
Also, we have
Perimeter = AB + BC + CD + DA

and CD = DA
So, CD + DA = 2DA
But,
AB + CD = Perimeter – 2 AD
= 52 – 2 × 10
= 52 – 20
= 32 cm
Thus, area of trapezium = ½ × (sum of parallel sides) × altitude
= ½ × 32 × 8
= 128 cm^2

8. The area of a trapezium is 540 cm^2. If the ratio of parallel sides is 7 : 5 and the distance between them is 18 cm, find the lengths of parallel sides.
Solution:

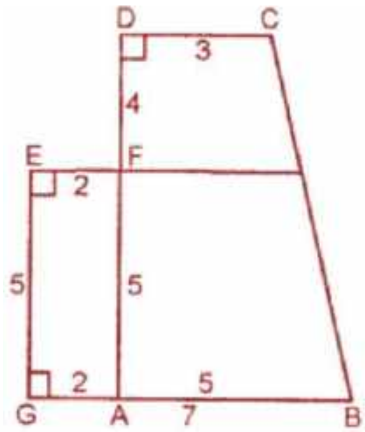
Let's assume the two parallel sides of trapezium to be 7x and 5x.
Height = 18 cm
Now,
Area of trapezium = ½ × [Sum of || gm sides × height]
⇒ 540 = ½ × (7x + 5x) × 18
540 = ½ × 12x × 18
540 = 108x
x = 540/108
x = 5 cm
Hence, the two parallel sides are:
7x = 7 × 5 = 35 cm and 5x = 5 × 5 = 25 cm

9. Calculate the area enclosed by the given shapes. All measurements are in cm.



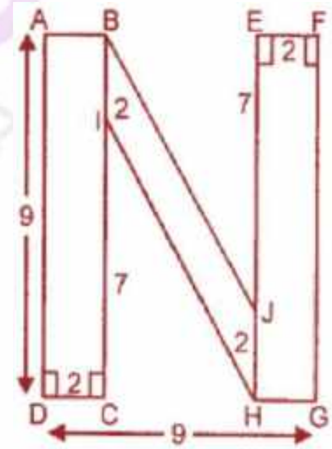
Solution:

(i) Firstly,
Area of trapezium ABCD
= (Sum of opposite ||gm sides) × height



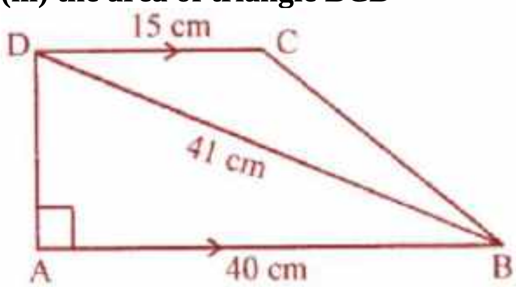
= [(AB + CD) × (AF + FD)]
= [(AB + CD) × (AF + FD)
= [(5 + 3) × (5 + 4)]
= (5 + 3) × 9
= 36 cm^2
Secondly,
Area of rectangle GAFE = Length × Breadth
= 2 × 5 = 10 cm^2
Hence,
Total area of the figure = Area of trapezium ABCD + Area of rectangle GAFE
= (36 + 10) cm^2
= 46 cm^{2+}

(ii) It's seen that,
Area of given figure = Area of rect. ABCD + Area of || gm BIHJ + Area of rectangle EFGH



Area of rectangle ABCD = Length × Breadth
= AD × DC
= 9 × 2 = 18 cm^2
And,
Area of rectangle EFGH = Length × Breadth
= (EJ + JH) × EF
= (7 + 2) × 2
= 9 × 2 = 18 cm^2
Now,
Area of parallelogram BIHJ = 2 × 5 = 10 cm^2
[Since, distance between BI and HJ = 9 – 2 – 2 = 5 cm]
Hence,
Total area of the figure = (18 + 18 + 10) cm^2 = 46 cm^2

10. From the adjoining sketch, calculate
(i) the length AD
(ii) the area of trapezium ABCD
(iii) the area of triangle BCD



Solution:

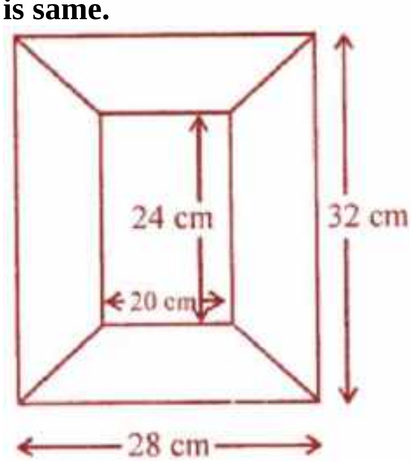
(i) In right angled Δ ABD, by Pythagoras Theorem we have

$BD^2 = AD^2 + AB^2$
$\Rightarrow AD^2 = BD^2 - AB^2$
$= (41)^2 - (40)^2$
$= 1681 - 1600$
$= 81$
$\therefore AD = \sqrt{81} = 9$ cm

(ii) Area of trapezium ABCD
= (Sum of opposite || gm lines) × height
= (AB + CD) × AD
= (40+ 15) × 9
= 247.5 cm^2

(iii) Area of triangle BCD = Area of trapezium ABCD – Area of Δ ABD
= (247.5 – × 40 × 9) cm^2
= (247.5 – 180) cm^2
= 67·5 cm^2

11. Diagram of the adjacent picture frame has outer dimensions = 28 cm × 32 cm and inner dimensions 20 cm × 24 cm. Find the area of each section of the frame, if the width of each section is same.

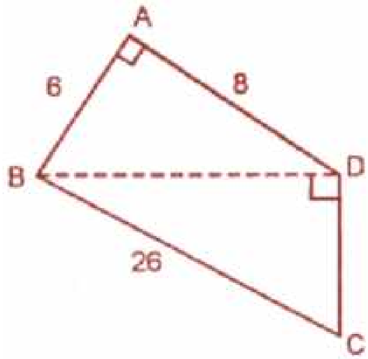


Solution:

Given,
Outer length of the frame = 32 cm and outer breadth = 28 cm
Inner length = 24 cm and outer breadth = 20 cm
So, width of the frame = (32 - 24)/ 2 = 4 cm
⇒ Height = 4 cm
Now, area of each portion of length side
= ½ × (24 + 32) × 4
= ½ × 56 × 4
= 112 cm^2
And,
Area of each portion of breadth side

= ½ × (20 + 28) × 4
= ½ × 48 × 4
= 96 cm^2
Therefore,
Area each section are 112 cm^2, 96 cm^2, 112 cm^2, 96 cm^2.

12. In the given quadrilateral ABCD, ∠BAD = 90° and ∠BDC = 90°. All measurements are in centimetres. Find the area of the quadrilateral ABCD.

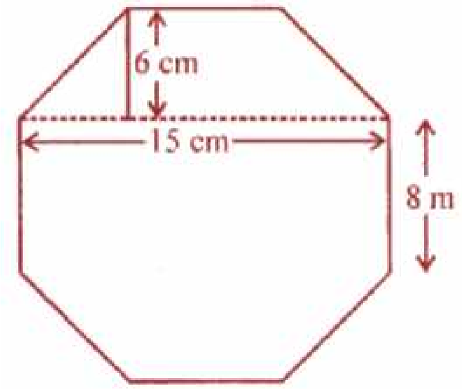


Solution:

In right angled triangle ABD, by Pythagoras Theorem we have
$BD^2 = AB^2 + AD^2 = (6)^2 + (8)^2$
= 36 + 64
= 100 cm^2
∴ BD = √100 = 10 cm
Now, Area of ΔABD = ½ × b × h
= ½ × 6 × 8
= 24cm^2 …(i)
In Δ BDC, we have
BD = 10 cm, BC = 26 cm
DC = ?
By Pythagoras theorem,
$BC^2 = BD^2 + DC^2$
$(26)^2 = (10)^2 + DC^2$
$676 – 100 = DC^2$
⇒ DC = √576 = 24 cm.
Now,
Area of Δ BDC = ½ × b × h
= ½ × 24 × 10
= 12 cm^2 …(ii)
Adding (i) and (ii), we get
Area of ΔABD + Area of ΔBDC = (24 + 120) cm^2
Hence,
Area of quadrilateral ABCD = 144 cm^2

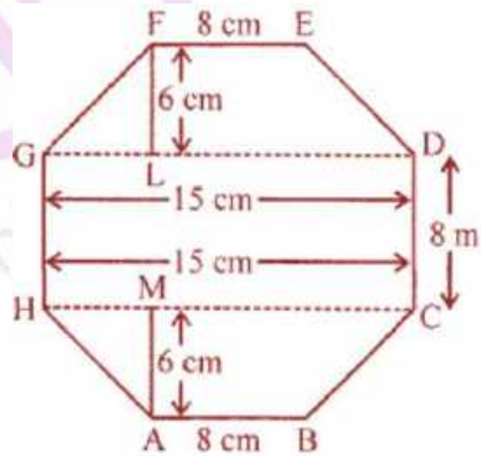
13. Top surface of a raised platform is in the shape of a regular octagon as shown in the given

figure. Find the area of the octagonal surface.



Solution:

The raised surface of platform is in the shape of regular octagon ABCDEFGH of each side = 8 cm.
Join HC.



GD = HC = 15 cm, FL = AM = 6 cm

Now, in each trapezium parallel sides are 15 cm and 6 cm and height = 6 cm

So, Area of each trapezium FEDG = ½ (GD + FE) × FL

= ½ (15 + 8) × 6

= 23 × 3 cm^2

= 69 cm^2

Also,

Area of trapezium FEDG = Area of trapezium ABCH = 69 cm^2

And area of rectangle HCDG = HC × CD

= 15 × 8

= 120 cm^2

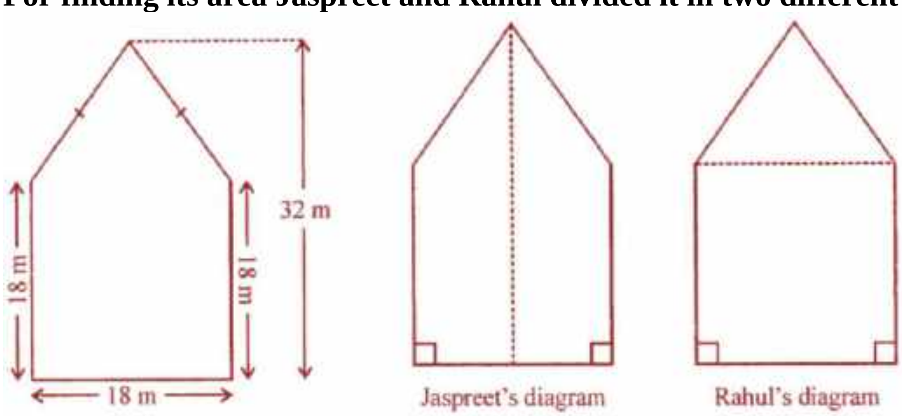
Hence,

Total area = Area of trapezium FEDG + Area of trapezium ABCH + Area of rectangle HCDG.

= 69 + 69 + 120

= 258 cm^2

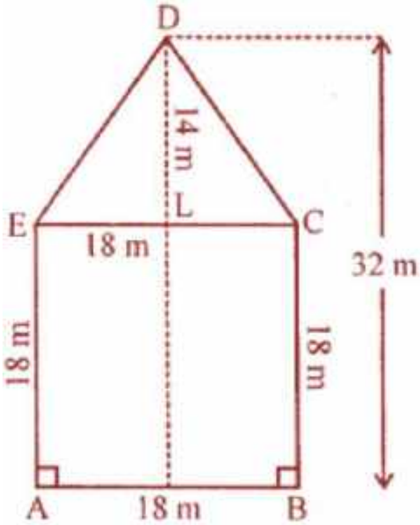
14. There is a pentagonal shaped park as shown in the following figure:
For finding its area Jaspreet and Rahul divided it in two different ways.



Find the area of this park using both ways. Can you suggest some other way of finding its area?
Solution:

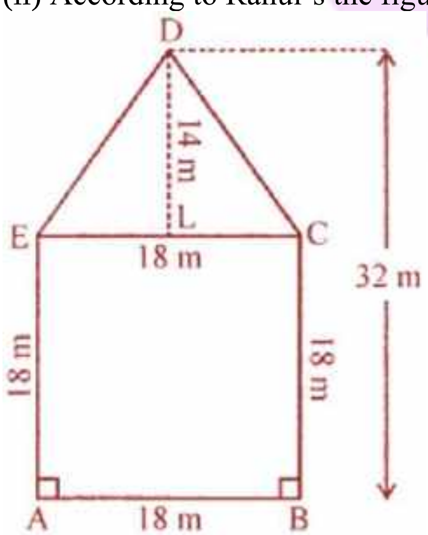
The pentagonal shaped park is shown in the given figure.
In which DL ⊥ CE and is produced to M.
So, DM = 32 m
LM = CB = 18 m
∴ DL = 32 – 18 = 14 m

(i) According to Jaspreet's the figure is divided into two equal trapezium in area: DEAM and DCBM



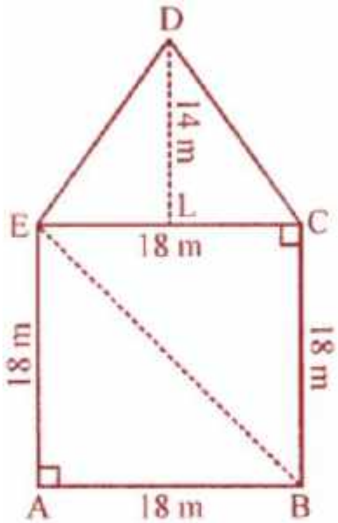
Now,
Area of trapezium DEAM = ½ (AE + DM) × AM
= ½ (32 + 18) × 9
= (50 x 9)/ 2
= $225m^2$

(ii) According to Rahul's the figure is divided into shapes: one square and on isosceles triangle.



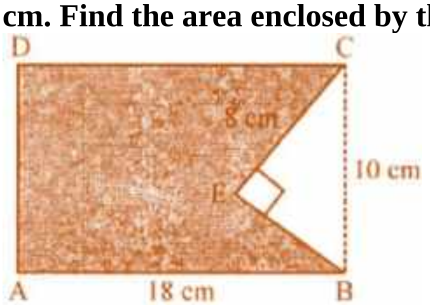
Area of square ABCE = $(\text{Side})^2$
$= (18)^2$
$= 324\ m^2$
And, area of isosceles ΔEDC = ½ × EC × DC
= ½ × 18 × 14
$= 126\ m^2$
∴ Total area = 225 × 2 = $450\ m^2$

The third way to find out the area of given figure is as follow:



Here, DL ⊥ ED and DL = 14 m
Area of ΔDEC = ½ × EC × LD
= ½ × 18 × 14
$= 126\ m^2$
Area of ΔAEB = ½ × AB × AE
= ½ × 18 × 18
$= 162\ m^2$
Area of ΔBEC = ½ × BC × EC
= ½ × 18 × 18 = $162\ m^2$
Hence, area of pentagon ABCDE = Area ΔDEC + Area of ΔAEB + Area of ΔBEC
$= (126 + 162 + 162)\ m^2$
$= 450\ m^2$

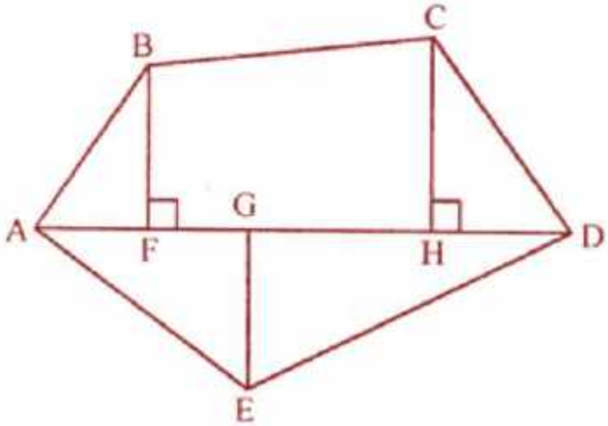
15. In the diagram, ABCD is a rectangle of size 18 cm by 10 cm. In Δ BEC, ∠E = 90° and EC = 8 cm. Find the area enclosed by the pentagon ABECD.



Solution:

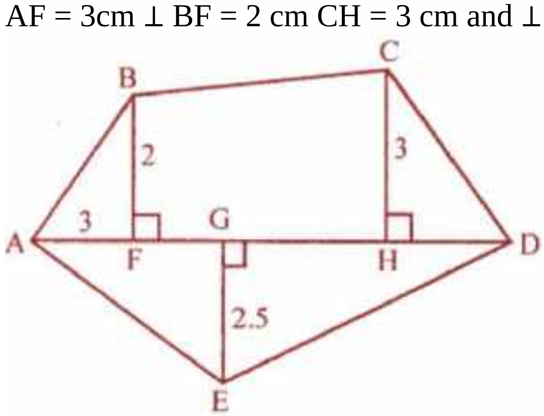
Area of rectangle ABCD = Length × Breadth
= 18 × 10
= 180 cm^2
In right angled Δ BEC,
By Pythagoras theorem, we have
$BC^2 = CE^2 + BE^2$
$(10)^2 = 8^2 + BE^2$
$BE^2 = 100 – 64 = 36$
⇒ BE = √36 = 6 cm.
So,
Area of rt. Δ BEC = ½ × 6 × 8
= 24cm^2
Area of pentagon ABECD = Area of rectangle – area of Δ
= (180 – 24) cm^2
= 156 cm^2

16. Polygon ABCDE is divided into parts as shown in the given figure. Find its area if AD = 8 cm, AH = 6 cm, AG = 4 cm, AF = 3 cm and perpendiculars BF = 2 cm, CH = 3 cm, EG = 2.5 cm.



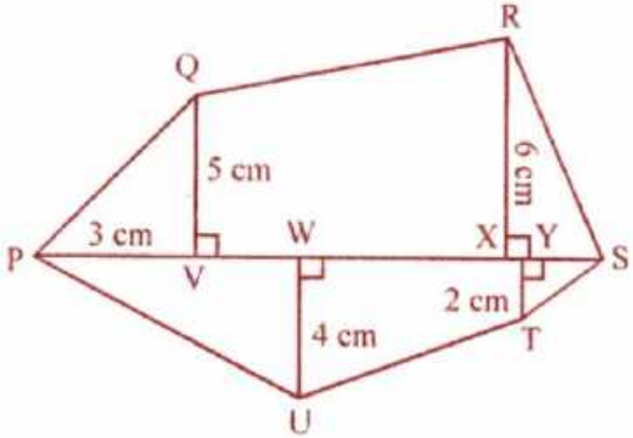
Solution:

In the given figure, ABCDE, AD = 8 cm, AH = 6 cm, AG = 4 cm,
AF = 3cm ⊥ BF = 2 cm CH = 3 cm and ⊥ EG = 2.5 cm



The given figure, consists of 3 triangles and one trapezium.
Now,
Area of $\Delta AED = ½ \times AD \times GE$
$= ½ \times 8 \times 2.5$
$= 10 \text{ cm}^2$
Area of $\Delta ABF = ½ \times AF \times BF$
$= ½ \times 3 \times 2$
$= 3 \text{ cm}^2$
Area of $\Delta CDH = ½ \times HD \times CH$
$= ½ \times (AD - AH) \times 3$
$= ½ \times (8 - 6) \times 3$
$= ½ \times 2 \times 3$
$= 3 \text{ cm}^2$
Area of trapezium BFHC $= ½ \times (BF + CH) \times FH$
$= ½ \times (2 + 3) \times (AH - AF)$
$= ½ \times 5 \times (6 - 3)$
$= ½ \times 5 \times 3$
$= 7.5 \text{ cm}^2$
Hence,
Total area of the figure = Area of ΔAED + Area of ΔABF + Area of ΔCDH + Area of trapezium BFHC
$= 10 + 3 + 3 + 7.5$
$= 23.5 \text{ cm}^2$

17. Find the area of polygon PQRSTU shown in 1 the given figure, if PS = 11 cm, PY = 9 cm, PX = 8 cm, PW = 5 cm, PV = 3 cm, QV = 5 cm, UW = 4 cm, RX = 6 cm, TY = 2 cm.



Solution:

In the figure PQRSTU, we have
PS = 11 cm, PY = 9 cm, PX = 8 cm, PW = 5 cm, PV = 3 cm, QV = 5 cm, UW = 4 cm, RX = 6 cm and TY = 2 cm
And,
The figure consists of 4 triangle and 2 trapeziums
From the figure its seen that,
VX = PX – PV

= 8 – 3
= 5 cm
XS = PS – PX
= 11 – 8
= 3 cm
YS = PS – PY
= 11 – 9
= 2 cm
WY = PY – PW
= 9 – 5
= 4 cm
Now,
Area ΔPQV = ½ × PV × QV
= ½ × 3 × 5 = 15/2
= 7.5 cm^2
Area of ΔRXS = ½ × XS × RX
= ½ × 3 × 6
= 9 cm^2
Area of ΔPUW = ½ × PW × UW
= ½ × 5 × 4
= 10 cm^2
Area ΔYTS = ½ × YS × TY
= ½ × 2 × 2
= 2 cm^2
Area of trapezium ΔVX R = ½ × (QV + RX) × VX
= ½ × (5 + 6) × 5
= ½ × 11 × 5 cm^2
= 55/7
= 27.5 cm^2
Area of trapezium WUTY = ½ × (UW + TY) × WY
= ½ × (4 + 2) × 4
= ½ × 6 × 4
= 12 cm^2
Hence,
Area of the figure = (7.5 + 9 + 10 + 2 + 27.5 + 12) cm^2 = 68 cm^2.

Exercise 18.3

1. The volume of a cube is 343 cm^3, find the length of an edge of cube.
Solution:

Given,
Volume of a cube = 343 cm^3
Let's consider 'a' to be the edge of cube, then
$V = a^3 = 343 = (7)^3$
∴ a = 7 cm

2. Fill in the following blanks:

	Volume of cuboid	Length	Breadth	Height
(i)	90 cm^3	-	5 cm	3 cm
(ii)	-	15 cm	8 cm	7 cm
(iii)	62.5 m^3	10 cm	5 cm	-

Solution:

Volume of cuboid = length x Breadth x Height
(i) 90 cm^3 = length x 5 cm x 3 cm
Length = 90/(5 x 3) = 90/15 = 6 cm

(ii) Volume = 15 cm x 8 cm x 7 cm
= 840 cm^3

(iii) 62.5 m^3 = 10 m x 5 m x height
Height = 62.5/(10 x 5) = 1.25 m

	Volume of cuboid	Length	Breadth	Height
(i)	90 cm^3	6 cm	5 cm	3 cm
(ii)	840 cm^3	15 cm	8 cm	7 cm
(iii)	62.5 m^3	10 cm	5 cm	1.25 m

3. Find the height of a cuboid whose volume is 312 cm^3 and base area is 26 cm^2.
Solution:

Given,
Volume of a cuboid = 312 cm^3
Base area = l × b = 26 cm^2
∴ Height= Volume/Base area = 312/26 = 12cm

4. A godown is in the form of a cuboid of measures 55 m × 45 m × 30 m. How many cuboidal boxes can be stored in it if the volume of one box is 1.25 m^3?
Solution:

Given,

Length of a godown (l) = 55 m
Breadth (b) = 45 m
Height (h) = 30 m
So,
Volume = l × b × h
= $(55 \times 45 \times 30)\ m^3$
= $74250\ m^3$
Also given, volume of one box = $1.25\ m^3$
Thus,
Number of boxes = 74250/1.25 = 59400 boxes

5. A rectangular pit 1.4 m long, 90 cm broad and 70 cm deep was dug and 1000 bricks of base 21 cm by 10.5 cm were made from the earth dug out. Find the height of each brick.
Solution:

Here l = 1.4 m = 140 cm, b = 90 cm and h = 70 cm
Volume of rectangular pit = l × b × h
= $(140 \times 90 \times 70)\ cm^3$
= $882000\ cm^3$
Volume of brick = 21 × 10.5 × h
Now,
Number of bricks = Volume of pit/ Volume of brick
1000 = 882000/ (21 × 10.5 × h)
h = 882000/ (21 × 10.5 × 1000)
= 4 cm
Thus, the height of each brick is 4 cm.

6. If each edge of a cube is tripled, then find how many times will its volume become?
Solution:

Let's consider the edge of a cube to be x
Then, it's volume = x^3
Now, if the edge is tripled
Edge = 3x
So, volume = $(3x)^3 = 27x^3$
∴ Its volume is 27 times the volume of the given cube.

7. A milk tank is in the form of cylinder whose radius is 1.4 m and height is 8 m. Find the quantity of milk in litres that can be stored in the tank.
Solution:

Given,
Radius of the milk cylindrical tank = 1.4 m and height (h) = 8 m
Hence,
Volume of milk in the tank = $\pi r^2 h$
= $(22/7) \times 1.4 \times 1.4 \times 8\ m^3$

= 49.28 m^3
= 49.28 × 1000 litres
= 49280 litres

Therefore, the quantity of the tank is 49280 litres.

8. A closed box is made of 2 cm thick wood with external dimension 84 cm × 75 cm × 64 cm. Find the volume of the wood required to make the box.
Solution:

Given,
Thickness of the wood used in a closed box = 2 cm
External length of box (L) = 84 cm
Breadth (b) = 75 cm and height (h) = 64 cm
So, internal length (l) = 84 – (2 × 2)
= 84 – 4
= 80 cm
Breadth (b) = 75 – (2 × 2)
= 75 – 4
= 71 cm
and height (h) = 64 – (2 × 2)
= 64 – 4
= 60 cm
Hence, Volume of wood used = 84 × 75 × 64 – 80 × 71 × 60 cm^3
= 403200 – 340800 cm^3
= 62400 cm^3

9. Two cylindrical jars contain the same amount of milk. If their diameters are in the ratio 3 : 4, find the ratio of their heights.
Solution:

Given,
Ratio in diameters of two cylindrical jars = 3 : 4
But their volumes are same.
Let's assume h_1 and h_2 to be the heights of the two jars respectively.
Let radius of the first jar (r_1) = 3x/2
and radius of the second jar (r_2) = 4x/2
Then according to the condition in the problem, we have

$$\pi r_1^2 h_1 = \pi r_2^2 h_2$$

$$\pi\left(\frac{3x}{2}\right)^2 h_1 = \pi\left(\frac{4x}{2}\right)^2 h_2$$

$$\frac{9}{4}x^2 h_1 = \frac{16}{4}x^2 h_2$$

$$\frac{h_1}{h_2} = \frac{16}{4}x^2 \times \frac{4}{9x^2} = \frac{16}{9}$$

Therefore, the ratio in their heights =16 : 9

10. The radius of the base of a right circular cylinder is halved and the height is doubled. What is the ratio of the volume of the new cylinder to that of the original cylinder?
Solution:

Let's consider the radius of a cylinder to be r
And height = h
So,
Volume = $\pi r^2 h$
Now, its radius is halved and height is doubled, then
Volume = $\pi (r/2)^2 \times (2h)$
= $\pi r^2 h/ 2$
Thus, the ratio in the volumes of the new cylinder to old one is
= $\pi r^2 h/ 2 : \pi r^2 h$
= 1 : 2

11. A rectangular piece of tin of size 30 cm × 18 cm is rolled in two ways, once along its length (30 cm) and once along its breadth. Find the ratio of volumes of two cylinders so formed.
Solution:

Given,
Size of rectangular tin plate = 30 cm × 18 cm
(i) When rolled along its length (30 cm),
Then, the circumference of the circle so formed = 30 cm
Radius(r_1) = $C/2\pi$ = (30 x 7)/ (2 x 22) = 105/22 cm
And height (h_1) = 18 cm
Then, volume = $\pi r_1^2 h_1 = \pi$ x $(105/22)^2$ x (18) cm^3
If it is rolled along its breadth (18 cm) then,
Circumference = 18 cm
So, radius (r_2) = $C/2\pi$ = (18 x 7)/ (2 x 22) = 63/22 cm
And height (h_2) = 30 cm
Then, volume = $\pi r_2^2 h_2 = \pi$ x $(63/22)^2$ x (30) cm^3
Now, ratio between the two volumes
= π x $(105/22)^2$ x (18) : π x $(63/22)^2$ x (30)
= $(105/22)^2$ x (18) : $(63/22)^2$ x (30)
= 5 : 3

12. Water flows through a cylindrical pipe of internal diameter 7 cm at 5 m per sec. Calculate
(i) the volume in litres of water discharged by the pipe in one minute.
(ii) the time in minutes, the pipe would take to fill an empty rectangular tank of size 4 m × 3 m × 2.31 m.
Solution:

Given,
Speed of water flow through cylindrical pipe = 5 m/sec.
Internal diameter of the pipe = 7 cm
So, radius (r) = 7/2 cm
Now, length of water flow in 1 minutes (h) = 5 × 60 = 300 m
∴ Volume of water = $\pi r^2 h$
= 22/7 x 7/2 x 7/2 x 300 x 100 cm^3
= 1155000 cm^3 = 1155 litres
(i) the volume of water = 1155000 cm^3
Volume of rectangular tank of size = 4m × 3m × 2.31m
= 27.72 m^3
Also given, speed of water = 4 m/sec.
Radius of pipe = 7/2 cm
Volume of water in 1 sec = 22/7 x 7/2 x 7/2 x 5 x 100 cm^3
= 19250 cm^3
Hence,
(ii) Time taken to empty the tank = 27.72 m^3/ 19250
= (2772 x 100 x 100 x 100)/(100 x 19250) sec
= 1440 sec
= 1440/60 = 24 minutes

13. Two cylindrical vessels are filled with milk. The radius of one vessel is 15 cm and height is 40 cm, and the radius of other vessel is 20 cm and height is 45 cm. Find the radius of another cylindrical vessel of height 30 cm which may just contain the milk which is in the two given vessels.
Solution:

Given,
Radius of one cylinder (r_1) = 15 cm
And height (h_1) = 40 cm
Radius of second cylinder (r_2) = 20 cm
And height (h_2) = 45 cm
Now,
Volume of first cylinder = $\pi r_1^2 h_1$
= 22/7 x 15 x 15 x 40 cm^3
= 198000/7 cm^3
And,
Volume of second cylinder = $\pi r_2^2 h_2$
= 22/7 x 20 x 20 x 45
= 396000/7 cm^3
So, total volume = (198000/7 + 396000/7) cm^3
= 594000/7 cm^3
Now, volume of third cylinder = 594000/7 cm^3
And height = 30 cm
Thus,

$$\text{Radius} = \sqrt[3]{\frac{594000}{7} \times \frac{7}{22} \times \frac{1}{30}}$$

= √900 = 30 cm

∴ Radius of the third cylinder = 30 cm

14. A wooden pole is 7 m high and 20 cm in diameter. Find its weight if the wood weighs 225 kg per m^3.

Solution:

Given,

Height of pole (h) = 7 m

Diameter = 20 cm

So, radius (r) = 20/2 = 10 cm = 10/100 = 1/10 m

And,

Volume = $\pi r^2 h$

= 22/7 x 1/10 x 1/10 x 7 m^3

= 22/100 m^3

Weight of wood = 225 kg per m^3

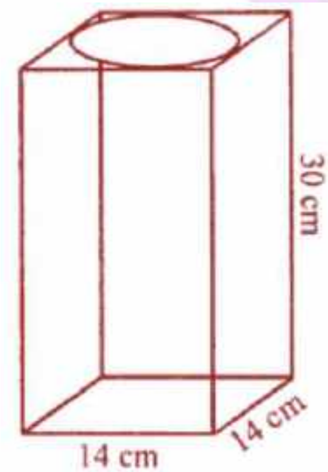
Hence,

Total weight = 225 × (22/100) = 99/2 = 49.5 kg

15. A cylinder of maximum volume is cut from a wooden cuboid of length 30 cm and cross-section a square of side 14 cm. Find the volume of the cylinder and the volume of the wood wasted.

Solution:

A cylinder of the maximum volume is cut from a wooden cuboid of length 30 cm and cross-section a square side 14 cm.



So,

Diameter of the cylinder = 14 cm

⇒ Radius (r) = 14/2 = 7 cm

and height (h) = 30 cm

Volume of cuboid = $30 \times 14 \times 14 = 5880 \text{ cm}^3$

Volume of cylinder = $\pi r^2 h$

$= 22/7 \times 7 \times 7 \times 30$

$= 4620 \text{ cm}^3$

Hence,

The wastage of wood = $5880 - 4620 = 1260 \text{ cm}^3$

Exercise 18.4

1. The surface area of a cube is 384 cm^2. Find
(i) the length of an edge
(ii) volume of the cube.
Solution:

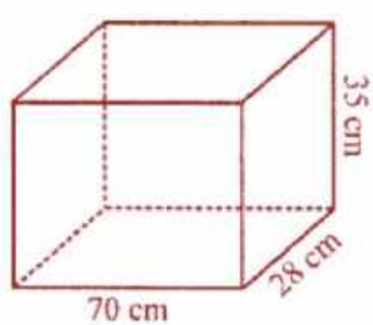
Given,
Surface area of a cube = 384 cm^2
(i) Surface area of cube = $6(side)^2$
Hence, edge (side) = √(surface area/6)
= √(384/6)
= √64
= 8 cm

(ii) Volume = $(Edge)^3 = (8)^3 = 8 \times 8 \times 8\ cm^3 = 512\ cm^3$

2. Find the total surface area of a solid cylinder of radius 5 cm and height 10 cm. Leave your answer in terms of n.
Solution:

Given,
Radius of a solid cylinder (r) = 5 cm
Height (h) = 10 cm
Hence,
Total surface area = $2\pi rh + 2\pi r^2$
$= 2r\pi(h + r)$
$= 2\pi \times 5(10 + 5)$
$= \pi \times 10 \times 15$
$= 150\pi\ cm^2$

3. An aquarium is in the form of a cuboid whose external measures are 70 cm × 28 cm × 35 cm. The base, side faces and back face are to be covered with coloured paper. Find the area of the paper needed.
Solution:



Given, a cuboid shaped aquarium
Length (l) = 70 cm
Breadth (b) = 28 cm
and height (h) = 35 cm
Now,
Area of base = $70 \times 28\ cm^3$
$= 1960\ cm^3$
Area of side face = $(28 \times 35) \times 2\ cm^2$
$= 1960\ cm^2$
Area of back face = $70 \times 35\ cm^2$

= 2450 cm
Thus, the total area = 1960 + 1960 + 2450 = 6370 cm^2
Hence, area of paper required is 6370 cm^2.

4. The internal dimensions of rectangular hall are 15 m × 12 m × 4 m. There are 4 windows each of dimension 2 m × 1.5 m and 2 doors each of dimension 1.5 m × 2.5 m. Find the cost of white washing all four walls of the hall, if the cost of white washing is ₹5 per m^2. What will be the cost of white washing if the ceiling of the hall is also white washed?
Solution:

Given,
Internal dimension of rectangular hall = 15m × 12 m × 4 m
Now,
Area of 4-walls = 2(l + b) × h
= 2(15 + 12) × 4
= 2 × 27 × 4 m^2
= 216 m^2
Area of 4 windows of size = (2 × 1.5) × 4 = 12 m^2
Area of 2 door of size = 2 × (1.5 × 2.5) = 7.5 m^2
So, area of remaining hall = 216 – (12 + 7.5) = 216 – 19.5 m^2 = 196.5 m^2
And,
Cost of white washing the walls all four halls of the house is at the rate of ₹5 per m^2
= 196.5 × 5 = ₹982.50
Area of ceiling = l × b = 15 × 12 = 180 m^2
Cost of white washing = 180 × 5 = ₹900
Therefore, the total cost for white washing = ₹982.50 + 900.00
= ₹1882.50

5. A swimming pool is 50 m in length, 30 m in breadth and 2.5 m in depth. Find the cost of cementing its floor and walls at the rate of ₹27 per square metre.
Solution:

Given,
Length of swimming pool = 50 m
Breadth of swimming pool = 30 m
Depth (Height) of swimming pool = 2·5 m
Now,
Area of floor = 50 × 30 = 1500 m^2
Area of four walls = 2 (50 + 30) × 2.5 = 160 × 2.5 = 400 m^2
So, the area to be cemented = 1500 m^2 + 400 m^2 = 1900 m^2
Cost of cementing 1m^2 = ₹27
Hence,
Cost of cementing 1900m^2 = ₹27 × 1900 = ₹51300

6. The floor of a rectangular hall has a perimeter 236 m. Its height is 4·5 m. Find the cost of painting its four walls (doors and windows be ignored) at the rate of Rs. 8.40 per square metre.

Solution:

Given,
Perimeter of Hall = 236 m.
Height = 4.5 m
Perimeter = 2 (l + b) = 236 m
Area of four walls = 2 (l + b) × h
= 236 × 4.5
= 1062 m^2
We have, cost of painting 1 m^2 = ₹8.40
Hence,
Cost of painting 1062 m^2 = ₹8.40 × 1062 = ₹8920.80

7. A cuboidal fish tank has a length of 30 cm, a breadth of 20 cm and a height of 20 cm. The tank is placed on a horizontal table and it is three-quarters full of water. Find the area of the tank which is in contact with water.
Solution:

Given,
Length of tank = 30 cm
Breadth of tank = 20 cm
Height of tank = 20 cm
As the tank is three-quarters full of water
So, the height of water in the tank = (20 x 3)/4 = 15 cm
Hence,
Area of the tank in contact with the water = Area of floor of Tank + Area of 4 walls upto 15 cm
= 30 × 20 + 2 (30 + 20) × 15
= 600 + 2 × 50 × 15
= 600 + 1500 = 2100 cm^2

8. The volume of a cuboid is 448 cm^3. Its height is 7 cm and the base is a square. Find
(i) a side of the square base
(ii) surface area of the cuboid.
Solution:

Given,
Volume of a cuboid = 448 cm^3
Height = 7 cm
So, area of base = 448/7 = 64 cm^2
Thus, the base is a square.
(i) Side of square base = √64 = 8 cm

(ii) Surface area of the cuboid = 2 [lb + bh + hl]
= 2[8 × 8 + 8 × 7 + 7 × 8] cm^2
= 2[64 + 56 + 56]
= 2 × 176 = 352 cm^2

9. The length, breadth and height of a rectangular solid are in the ratio 5 : 4 : 2. If its total surface area is 1216 cm^2, find the volume of the solid.
Solution:

Given that the ratio in length, breadth and height of a rectangular solid = 5 : 4 : 2
Total surface area =1216 cm^2
Let's assume the length = 5x, breadth = 4x and height = 2x
Total surface area = 2[5x × 4x + 4x × 2x + 2x × 5x]
= $2[20x^2 + 8x^2 + 10x^2]$
= $2 \times 38x^2$
= $76x^2$
So, $76x^2 = 1216$
$\Rightarrow x^2 = 1216/76 = 16 = (4)^2$
$\therefore x = 4$
Hence, the dimensions of the rectangular solid are
Length = 5 × 4 = 20 cm
Breadth = 4 × 4 = 16 cm
Height = 2 × 4 = 8 cm
and volume = lbh = 20 × 16 × 8 = 2560 cm^3

10. A rectangular room is 6 m long, 5 m wide and 3.5 m high. It has 2 doors of size 1·1 m by 2 m and 3 windows of size 1.5 m by 1.4 m. Find the cost of whitewashing the walls and the ceiling of the room at the rate of ₹5.30 per square metre.
Solution:

Given, Length of room = 6 m
Breadth of room = 5 m
Height of room = 3.5 m
So, Area of four walls = 2 (l + b) × h
= 2 (6 + 5) × 3.5
= 77 m^2
Area of 2 doors and 3 windows = (2 × 1.1 × 2 + 3 × 1.5 × 1.4)
= (44 + 6.3) m^2
= 10.7 m^2
Area of ceiling = l × b = 6 × 5 = 30 m^2
Thus,
Total area for white washing = (77 – 10.7 + 30) m^2 = 96.3 m^2
Hence, the cost of white washing = ₹(96.3 × 5.30) = ₹510.39

11. A cuboidal block of metal has dimensions 36 cm by 32 cm by 0·25 m. It is melted and recast into cubes with an edge of 4 cm.
(i) How many such cubes can be made?
(ii) What is the cost of silver coating the surfaces of the cubes at the rate of ₹0·75 per square centimetre?
Solution:

(i) Given, Length of cuboid = 36 cm
Breadth of cuboid = 32 cm
Height of cuboid = 0·25 × 100 = 25 cm
So,
Volume of cuboid = lbh
= (36 × 32 × 25) cm^3
= 28800 cm^3
Volume of cube = $(side)^2$
= $(4)^2$
= 64 cm^2
Hence, the number of cubes recasting from cuboid = 28800/64 = 450

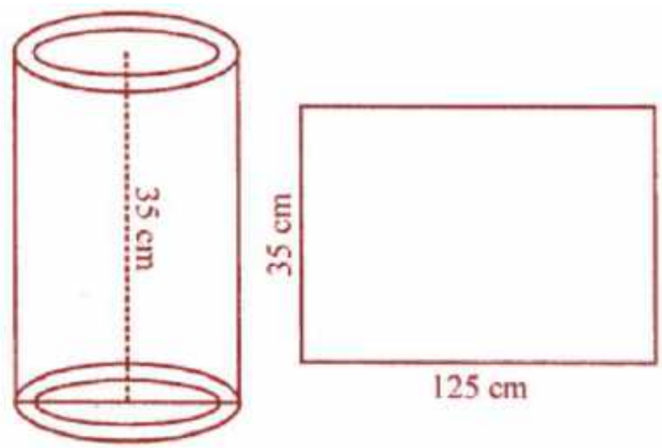
(ii) Surface area of 1 cube = 6 × a^2
= 6 × 16
= 96 cm^2
So, the surface area of 450 cubes = 96 × 450
= 43200 cm^2
Hence, the cost of silver coating on cubes = ₹0.75 × 43200
= ₹32400

12. Three cubes of silver with edges 3 cm, 4 cm and 5 cm are melted and recast into a single cube, find the cost of coating the surface of the new cube with gold at the rate of ₹3.50 per square centimetre?
Solution:

Let's consider a cm to be the edge of new cube
Then, according to given conditions in the question
$a^3 = 3^3 + 4^3 + 5^3$
= 27 + 64 + 125
= 216 cm^3
a = $\sqrt[3]{216}$
∴ a = 6 cm
So, the surface area of new cube = 6 × $(side)^2$
= 6 × $(6)^2$
= 216 cm^2
Hence, the cost of coating the surface of new cube = ₹3.50 × 216 = ₹156

13. The curved surface area of a hollow cylinder is 4375 cm^2, it is cut along its height and formed a rectangular sheet of width 35 cm. Find the perimeter of the rectangular sheet.
Solution:

Given, curved surface area of a hollow cylinder = 4375 cm^2
By cutting it from the height,



It becomes a rectangular sheet whose width = 35 cm

So, the height of cylinder = 35 cm
And, length of sheet = Area/Height
= 4375/35
= 125 cm
Hence, Perimeter of the sheet = 2(l + b)
= 2 × (125 + 35)
= 2 × 160
= 320 cm

14. A road roller has a diameter 0.7 m and its width is 1.2 m. Find the least number of revolutions that the roller must take in order to level a playground of size 120 m × 44 m.
Solution:

Given,
Diameter of a road roller = 0.7 m = 70 cm
So, radius (r) = 70/2 cm = 35 cm = 35/100 m
and width (h) = 1.2 m
Now,
Curved surface area = 2πrh
= (2 × 22/7 × 35/100 × 1.2) m^2
= 264/100 m^2
Area of playground = 120 m × 44 m
= 120 × 44 m^2
= 5280 m^2
Hence, the number of revolution made by the road roller = (5280/264) × 100
= 2000 revolutions

15. A company packages its milk powder in cylindrical container whose base has a diameter of 14 cm and height 20 cm. Company places a label around the surface of the container (as shown in the figure). If the label is placed 2 cm from top and bottom, what is the area of the label?

Solution:

Given,
Diameter of cylindrical container = 14 cm
So, radius (r) = 14/2 = 7 cm
And, height (h) = 20 cm
Now,
Width of label = 2 0 – (2 + 2) cm = 20 – 4 = 16 cm
Hence, area of label = $2\pi rh$
$= 2 \times (22/7) \times 7 \times 16$
$= 704\ cm^2$

16. The sum of the radius and height of a cylinder is 37 cm and the total surface area of the cylinder is 1628 cm^2. Find the height and the volume of the cylinder.
Solution:

Given,
Sum of height and radius of a cylinder = 37 cm
Total surface area = 1628 cm^2
Let's consider the radius to be r
Then, height = (37 – r) cm
Now,
Total surface area = $2\pi(h + r)$

$$1628 = \frac{2 \times 22}{7} \times r(37)$$

$$\Rightarrow \frac{1628 \times 7}{2 \times 22 \times 37} = r$$

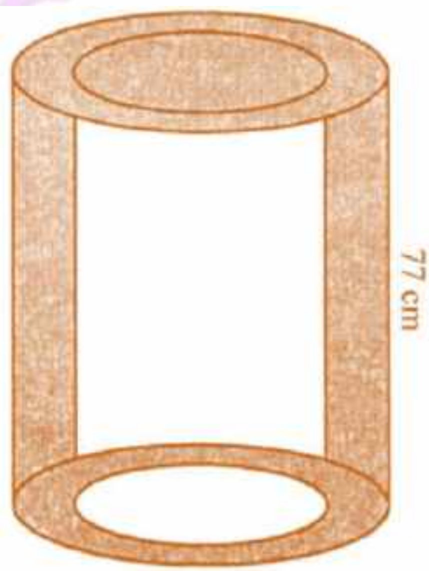
$\Rightarrow$ r = 7 cm
Height = 37 – 7 = 30 cm
Hence,
Volume = $\pi r^2 h$
$= (22/7) \times 7 \times 7 \times 30\ cm^3$
$= 4620\ cm^3$

17. The ratio between the curved surface and total surface of a cylinder is 1 : 2. Find the volume of the cylinder, given that its total surface area is 616 cm^3.
Solution:

Given that ratio between curved surface and total surface area of a cylinder = 1 : 2
Total surface area = 616 cm^2
So, curved surface area = 616/2 = 308 cm^2
And,
Area of two circular faces = 616 – 308 = 308 cm^2
Area of one circular face = 308/2 = 154 cm^2
Now, let's consider the radius to be r

$\pi r^2 = 154$
$(22/7) \times r^2 = 154$
$r^2 = (154 \times 7)/ 22 = 49$
$\Rightarrow r = 49 = \sqrt{7}$ cm
Hence, the volume = $\pi r^2 h = (22/7) \times 7 \times 7 \times 7 = 1078$ cm^2

18. The given figure shown a metal pipe 77 cm long. The inner diameter of cross section is 4 cm and the outer one is 4.4 cm.
Find its
(i) inner curved surface area
(ii) outer curved surface area
(iii) total surface area.
Solution:



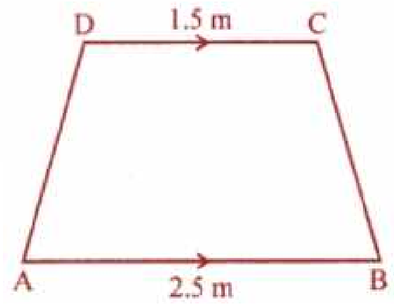
Given,
Length of metal pipe (h) = 77 cm
Inner diameter = 4 cm
and outer diameter = 4.4 cm
So, inner radius (r) = 4/2 = 2 cm
And outer radius (R) = 4.4/2 = 2.2 cm
(i) Inner curved surface area = $2\pi rh$
= $2 \times 22/7 \times 2 \times 77$ cm^2
= 968 cm^2
(ii) Outer surface area = $2\pi RH$
= $2 \times 22/7 \times 2.2 \times 77$ cm^2
= 1064.8 cm^2
(iii) Surface area of upper and lower rings = $2[\pi R^2 - \pi r^2]$
= $2 \times 22/7\ (2.2^2 - 2^2)$ cm^2
= $44/7 \times 4.2 \times 0.2$
= 5.28 cm^2
Hence,
Total surface area = (968 + 1064.8 + 5.28) cm^2 = 2038.08 cm^2

Check Your Progress

1. A square field of side 65 m and rectangular field of length 75 m have the same perimeter. Which field has a larger area and by how much?
Solution:

Given,
Side of a square field = 65 m
So, perimeter = 4 × Side
= 4 × 65
= 260 m
Now, perimeter of a rectangular field = 260 m
And given, length (l) = 75 m
Perimeter of rectangle = 2(l + b)
⇒ 260 = 2(75 + b)
260 = 150 + 2b
2b = 260 – 150
b = 110/2 = 55 m
Then,
Area of square field = $(\text{side})^2$
= $(65)^2$ m^2
= 4225 m^2
And, area of rectangular field = l × b
= 75 × 55 m
= 4125 m^2
Hence, it is clear that area of square field is greater.
Difference = 4225 – 4125
= 100 m^2

2. The shape of a top surface of table is a trapezium. Find the area if its parallel sides are 1.5 m and 2.5 m and perpendicular distance between them is 0.8 m.
Solution:



Shape of the top of a table is trapezium and parallel sides are 1.5 m and 2.5 m and the perpendicular distance between them = 0.8m
Hence,
Area = ½ × (Sum of parallel sides) × height
= ½ × (1.5 + 2.5) × 0.8
= ½ × 4 × 0.8 m^2
= 1.6 m^2

3. The length and breadth of a hall of a school are 26 m and 22 m respectively. If one student requires 1.1 sq. m area, then find the maximum number of students to be seated in this hall.
Solution:

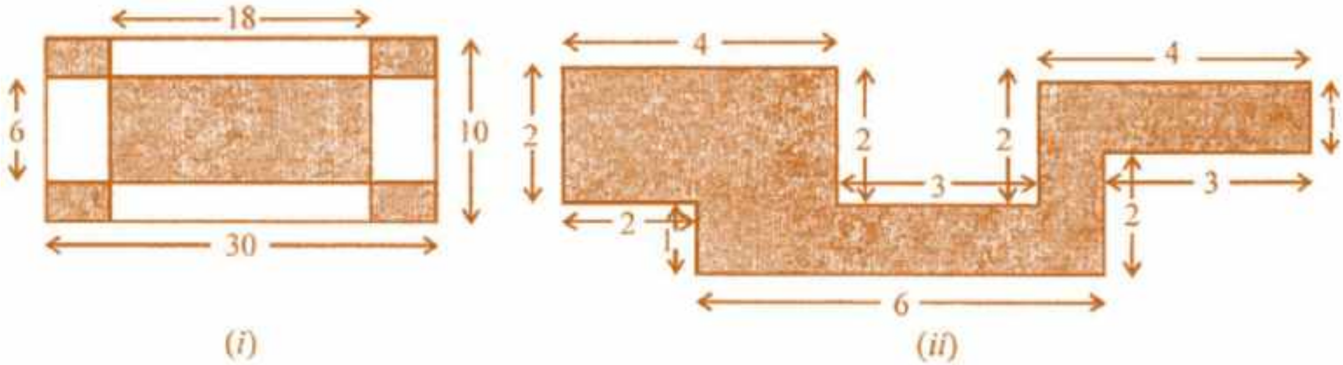
Given, length of a school hall (l) = 26 m and breadth (b) = 22 m

So, area = l × b
= 26 × 22 m^2
= 572 m^2
One student requires 1.1 sq. m area
Hence, number of students = 572/1.1
= (572 × 10)/11
= 520 students

4. It costs ₹936 to fence a square field at ₹7.80 per metre. Find the cost of levelling the field at ₹2.50 per square metre.
Solution:

Given,
Cost of fencing the square field at ₹7.80 per metre = ₹936.
So, total fence required will be = 936/7.80 = 120
Thus, the perimeter of the field = 120 m
⇒ 4 × Side = 120 m [Since, it's a square field]
⇒ Side = 120/4
∴ Side = 30 m
Hence, Area of square field = $(30)^2$ = 900 m^2
= 900 × 2.50 = ₹2250

5. Find the area of the shaded portion in the following figures all measurements are given in cm.



Solution:

(i) Outer length = 30 cm
Breadth = 10 cm
Side of each rectangle of the corner (l) = (30 - 18)/ 2 = 6 cm
and b = (10 – 6)/2 = 4/2 = 2 cm
So,
Area of 4 comer = (6 × 2) × 4
= 48 cm^2
And area of inner rectangle = 18 × 6
= 108 cm^2
Therefore,

Area of shaded portion = 108 + 48 = 156cm^2

(ii) Area of rectangle I = 4 × 2 = 8 cm^2
Area of rectangle II = 4 × 1 = 4 cm^2
Area of rectangle III = 6 × l = 6 cm^2
and area of square IV = 1 × 1 = 1 Cm^2
∴ Total area of shaded portion = 8 + 4 + 6 + 1 = 19 cm^2

6. Area of a trapezium is 160 sq. cm. Lengths of parallel sides are in the ratio 1:3. If smaller of the parallel sides is 10 cm in length, then find the perpendicular distance between them.
Solution:

Given,
Area of trapezium = 160 cm^2
Ratio of the length of its parallel sides = 1 : 3
Smaller parallel side = 10 cm
Then,
Length of greater side = 10 × 3 = 30 cm
Now, distance between them = h

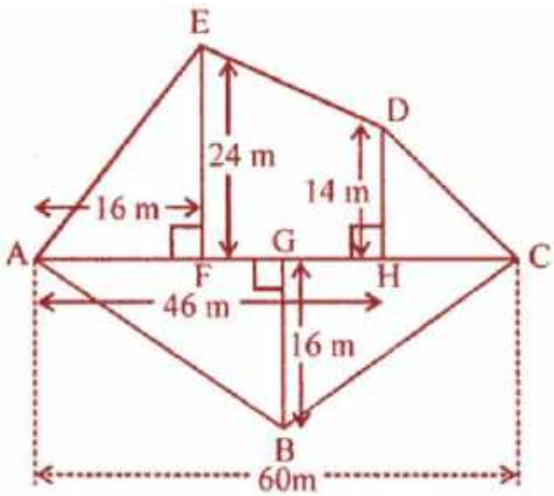
$$\therefore\ h = \frac{\text{Area} \times 2}{\text{Sum of parallel sides}} = \frac{160 \times 2}{10 + 30}$$

$$= \frac{160 \times 2}{40} = 8 \text{ cm}$$

7. The area of a trapezium is 729 cm^2 and the distance between two parallel sides is 18 cm. If one of its parallel sides is 3 cm shorter than the other parallel side, find the lengths of its parallel sides.
Solution:

Given,
Area of a trapezium = 729 cm^2
Distance between two parallel sides (Altitude) = 18 cm
So, the sum of parallel sides = (Area × 2)/ Altitude
= (729 × 2)/ 18
= 81 cm
One parallel side is shorter than the second by 3 cm
Let the longer side be taken as x
Then, shorter side = x – 3
According to the question, we have
⇒ x + x – 3 = 81
2x = 81 + 3 = 84
x = 84/2 = 42
Hence, the longer side = 42 cm and shorter side = 42 – 3 = 39 cm

8. Find the area of the polygon given in the figure:

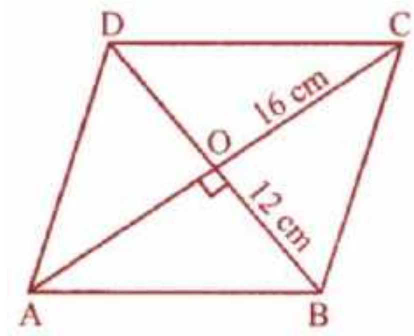


Solution:

In the given figure,
AC = 60 m, AH = 46 m, AF = 16 m, EF = 24 m, DH = 14 m, BG = 16 m
∴ FH = AH – AF = 46 – 16 = 30
And, HC = AC – AH = 60 – 46 = 14
In the figure, there are 3 triangles and one trapezium.
Now,
Area of ΔABC = ½ AC × BG
= ½ × 60 × 16
= 480 m^2
Area of ΔAEF = ½ AF × EF
= ½ × 16 × 24
= 192 m^2
Area of ΔDHC = ½ HC × DH
= ½ × 14 × 14
= 98 m^2
Area of trapezium EFHD = ½ (EF + DH) × FH
= ½ (24 + 14) × 30
= ½ × 38 × 30
= 570 m^2
Therefore, total area of the figure = Area of ΔABC + area ΔAEF + area ΔDHC + area trapezium EFHD
= 480 + 192 + 98 + 570
= 1340 m^2

9. The diagonals of a rhombus are 16 m and 12 m, find:
(i) its area
(ii) length of a side
(iii) perimeter.
Solution:

Diagonals of a rhombus are d_1 = 16 cm and d_2 = 12 cm



(i) Area = $(d_1 \times d_2) / 2$
$= (16 \times 12)/ 2$
$= 96\ cm^2$
(ii) As the diagonals of rhombus bisect each other at right angles, we have
AO = OC and BO = OD
AO = 16/2 = 8 cm and BO = 12/2 = 6 cm
Now, in right ΔAOB
$AB^2 = AO^2 + BO^2$ [By Pythagoras Theorem]
$= 8^2 + 6^2$
$= 64 + 36$
$= 100 = (10)^2$
∴ AB = 10 cm
Therefore, the side of rhombus = 10 cm

10. The area of a parallelogram is 98 cm^2. If one altitude is half the corresponding base, determine the base and the altitude of the parallelogram.
Solution:

Given,
Area of a parallelogram = 98 cm^2
One altitude = Half of its corresponding base
Let's consider the base as x cm
Then altitude = x/2 cm
So, area = Base × Altitude
⇒ 98 = x × (x/2
$x^2 = 98 \times 2$
$= 196$
$= (14)^2$
∴ x = 14
Therefore, Base = 14 cm and altitude = = 7 cm

11. Preeti is painting the walls and ceiling of a hall whose dimensions are 18 m × 15 m × 5 m. From each can of paint 120 m^2 of area is painted. How many cans of paint does she need to paint the hall?
Solution:

Given,
Length of a hall (l) = 18 m
Breadth (b) = 15m and
height (h) = 5 m
So,
Area of 4-wall and ceiling = 2(l + b)h + lb
$= 2(18 + 15) \times 5 + 18 \times 15\ m^2$
$= 2 \times 33 \times 5 + 270$

= 330 + 270
= 600 m^2
From 1 can an area of 120 m^2 can be painted
Hence, total number of cans required to paint the area of 600 m^2 = 600/120 = 5 cans

12. A rectangular paper is size 22 cm × 14 cm is rolled to form a cylinder of height 14 cm, find the volume of the cylinder. (Take π = 22/7)
Solution:

Given,
Length of a rectangular paper = 22 cm and breadth = 14 cm
By rolling it a cylinder is formed whose height is 14 cm
And, circumference of the base = 22 cm
We know that, circumference = 2πr
So, radius (r) = C/2π = (22 × 7)/(2 × 22) = 7/2 cm
Hence,
Volume of the cylinder so formed = $\pi r^2 h$
= 22/7 × 7/2 × 7/2 × 14
= 539cm^3

13. A closed rectangular wooden box has inner dimensions 90 cm by 80 cm by 70 cm. Compute its capacity and the area of the tin foil needed to line its inner surface.
Solution:

Given,
Inner length of rectangular box = 90 cm
Inner breadth of rectangular box = 80 cm
Inner height of rectangular box = 70 cm
Now,
Capacity of rectangular box = Volume of rectangular box
= l × b × h
= 90 cm × 80 cm × 70 cm
= 504000 cm^3
And,
Required area of tin foil = 2 (lb + bh + lh)
= 2(90 × 80 + 80 × 70 + 90 × 70) cm^2
= 2(7200 + 5600 + 6300) cm^2
= 2 × 19100 cm^2
= 38200 cm^2

14. The lateral surface area of a cuboid is 224 cm^2. Its height is 7 cm and the base is a square. Find
(i) side of the square base
(ii) the volume of the cuboid.
Solution:

Given,
Lateral surface area of a cuboid is 224 cm^2
Height (h) = 7 cm
(i) $2(l + b) \times h = 224$
$\Rightarrow 2(l + b) \times 7 = 224$
$l + b = 224/14 = 16$ cm
But $l = b$ [Since, the base of cuboid is a square]
So, $2 \times$ side = 16 cm
$\Rightarrow$ Side = 16/2 = 8 cm

(ii) Volume of cuboid = lbh = $8 \times 8 \times 7\ cm^3 = 448\ cm^3$

15. The inner dimensions of a closed wooden box are 2 m by 1.2 m by 0.75 m. The thickness of the wood is 2.5 cm. Find the cost of wood required to make the box if 1 m^3 of wood costs ₹5400.
Solution:

Given,
Inner dimensions of wooden box are 2 m, 1.2 m, 0.75 m
Thickness of the wood = 2.5 cm = 2.5/100 m = 0.025 m
Now,
External dimensions of wooden box are
= $(2 + 2 \times 0.025), (1.2 + 2 \times 0.025), (0.75 + 2 \times 0.025)$
= $(2 + 0.05), (1.2 + 0.05), (0.75 + 0.5)$
= 2.05, 1.25, 0.80
Thus,
Volume of solid = External volume of box – Internal volume of box
= $2.05 \times 1.25 \times 0.80\ m^3 - 2 \times 1.2 \times 0.75 m^3$
= $2.05 - 1.80 = 0.25\ m^3$
Given, the cost of wood = ₹5400 for 1 m^3
Hence,
Total cost = ₹5400 × 0.25 = ₹5400 × 25/100
= ₹54 × 25
= ₹1350

16. A car has a petrol tank 40 cm long, 28 cm wide and 25 cm deep. If the fuel consumption of the car averages 13.5 km per litre, how far can the car travel with a full tank of petrol?
Solution:

Given,
Capacity of car tank = 40 cm × 28 cm × 25 cm = $(40 \times 28 \times 25)\ cm^3$
= $(40 \times 28 \times 25)/1000$ litre [$\because$ 1000 cm^3 = 1 litre]
Average fuel consumption of car = 13.5 km per litres
Then, the distance travelled by car is given by

$= \frac{40 \times 28 \times 25}{1000} \times 13.5$ km

$= \frac{(40 \times 25) \times 28}{1000} \times \frac{135}{10}$ km

$= \frac{1 \times 28}{1} \times \frac{135}{10}$ km $= \frac{14 \times 135}{5}$ km

= 14 × 27 km

= 378 km

Hence, the car can travel 378 km with a full tank of petrol.

17. The diameter of a garden roller is 1.4 m and it is 2 m long. How much area it will cover in 5 revolutions?

Solution:

Given,

Diameter of a garden roller = 1.4 m

So, its radius (r) = 1.4/2 = 0.7 m = 70 cm

and length (h) = 2m

Now, Curved surface area = 2πrh

= 2 × 22/7 × 70 × 200 cm^2

= 88000 cm^2

Hence, area covered in 5 revolutions = (88000 × 5)/10000 m^2 = 44 m^2

18. The capacity of an open cylindrical tank is 2079 m^3 and the diameter of its base is 21m. Find the cost of plastering its inner surface at ₹40 per square metre.

Solution:

Given, capacity of an open cylindrical tank = 2079 m^3

Diameter of base = 21 m

So, radius (r) = 21/2 m

Let h be the height, then we have

$\pi r^2 h = 2079$

22/7 × 21/2 × 21/2 × h = 2079

h = (2079 × 2)/ (11 × 3 × 21) = 6 m

Now,

Curved Surface Area + Base area = $2\pi rh + \pi r^2$

$= 2 \times \frac{22}{7} \times \frac{7}{2} \times 6 + \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}\ m^2$

$= 396 + \frac{693}{2} = 396 + 346.5 = 742.5$

Hence, the cost of plastering the surface = ₹40 × 742.5 = ₹29700

19. A solid right circular cylinder of height 1.21 m and diameter 28 cm is melted and recast into 7 equal solid cubes. Find the edge of each cube.
Solution:

Given,
Height of solid right circular cylinder = 1.21 m = 121 cm
and diameter = 28 cm
So, radius (r) = 28/2 = 14 cm
Now,
Volume of the metal used = $\pi r^2 h$
= 22/7 × 14 × 14 × 121 cm^3
= 74536 cm^3
Thus, the volume of 7 solid cubes = 74536 cm^3
And, volume of 1 cube = 74536/7 = 10648 cm^3

Edge of the cube = $\sqrt[3]{10648}$

2	10648
2	5324
2	2662
11	1331
11	121
11	11
	1

= $\sqrt[3]{2\times2\times2\times11\times11\times11}$

= 2 × 11 = 22 cm

Hence, the edge of each cube is 22 cm.

20. (i) How many cubic metres of soil must be dug out to make a well 20 m deep and 2 m in diameter?
(ii) If the inner curved surface of the well in part (i) above is to be plastered at the rate of ₹50 per m^2, find the cost of plastering.
Solution:

(i) Given,
Depth of a well (h) = 20 m
and diameter = 2 m
Radius (r) = 2/2 = 1 m
Volume of earth dug out = $\pi r^2 h$
= 22/7 × 1 × 1 × 20
= 440/7 m^3
(ii) Inner curved surface area = $2\pi rh$
= 2 × 22/7 × 1 × 20
= 880/7 m^2

The cost of plastering at the rate of ₹50 per m^2 = ₹ 880/7 × 50

= ₹ 44000/7

= ₹ 6285.70