Higher Mathematics 2018

Time : 3 Hours] Class : 12th [M. M. : 100

Instructions :

(1) All questions are compulsory.
(2) Read instructions carefully of the question paper and then write answers of the questions.
(3) Question paper has two Sections - SECTION - "A" and SECTION - "B"
(4) In the Section "A" Question Nos. 1 to 5 are Objective type. Each question carries 5 marks.
(5) In the Section "B" Question Nos. 6 to 26 have internal option.
(6) Question Nos. 6 to 10 carry 2 Marks Each.
(7) Question Nos. 11 to 14 carry 3 Marks Each.
(8) Question Nos. 15 to 21 carry 4 Marks Each.
(9) Question Nos. 22 to 26 carry 5 Marks Each.

SECTION-"A"

Q.1 Choose the correct options $5 \times 1 = 5$

(i) If $\frac{1}{(1+\sin x)(2+\sin x)} = \frac{a}{(1+\sin x)} + \frac{b}{(2+\sin x)}$ then $a \div b =$

(A) 0 (B) 1
(C) 2 (D) 3

(ii) The domain of $\sin^{-1} X$ is :

(A) $(-\pi, \pi)$ (B) $[-1, 1]$
(C) (0.2π) (D) $(-\infty, \infty)$

(iii) The distance of point (3, 4, 5) from XZ-plane is

(A) 4 (B) 3
(C) 5 (D) 0

(iv) Two lines $\frac{x}{1} - \frac{y}{2} = \frac{z}{3}$ and $\frac{x}{5} - \frac{y}{10} - \frac{z}{15}$ are mutually

(A) Perpendicular (B) Coincident
(C) Intersect (D) Parallel

(v) The shortest distance of point (a, b, c) from X-axis is :

(A) $\sqrt{b^2 + c^2}$ (B) $\sqrt{a^2 + b^2}$
(C) $\sqrt{c^2 + a^2}$ (D) $\sqrt{a^2 + b^2 + c^2}$

Q.2 Write true/false in the following statements : $5 \times 1 = 5$

(i) Equation of plane parallel to Y-axis is $ax + by + d = 0$.

(ii) The value of $\vec{a} \cdot (\vec{a} \times \vec{b})$ is zero.

(iii) The value of $\int \frac{dx}{\sin x}$ is $\log \tan \frac{x}{2} + c$.

(iv) The velocity of the particle at the maximum height is always zero.

(v) Work is a vector quantity.

Q.3 Fill in the blanks : $5 \times 1 = 5$

(1) Simpson's Rule is based upon the Principle

(2) If $y = a^x$ then $\frac{dy}{dx}$ is

(3) Differential coefficient of $x^{-5/7}$ is

(4) The coefficient of correlation is the mean of the regression coefficient.

(5) The direction cosine of the vector $3i - 2j + 6k$ are

Q.4 Give answers in one word/sentence : $5 \times 1 = 5$

(i) Write the Simpson's One-third Rule formula for $\int_a^b f(x)dx$.

(ii) In which interval does the root of equation $x^3 - x - 2 = 0$ lie?

(iii) Write the formula of Trapezoidal rule in numerical method.

(iv) Perfect Negative Correlation is

(v) Write Newton-Raphson's formula

Q.5 Match the column : http://www.mpboardonline.com $5 \times 1 = 5$

('A')	('B')
(a) $\int \frac{1}{x^2 - a^2} dx$	(i) $\frac{1}{2}\left[x\sqrt{a^2 + x^2} + a^2 \log\left(x + \sqrt{a^2 + x^2}\right)\right]$
(b) $\int \frac{1}{a^2 - x^2} dx$	(ii) $\log\left[x + \sqrt{x^2 + a^2}\right]$
(c) $\int \frac{1}{\sqrt{x^2 + a^2}} dx$	(iii) $\frac{1}{2a} \log \frac{x - a}{x + a}$
(d) $\int \frac{1}{\sqrt{x^2 - a^2}} dx$	(iv) $\frac{1}{2}\left[x\sqrt{x^2 - a^2} - a^2 \log\left(x + \sqrt{x^2 - a^2}\right)\right]$
(e) $\int \sqrt{a^2 + x^2} dx$	(v) $\log\left[x + \sqrt{x^2 - a^2}\right]$
	(vi) $\frac{1}{2a} \log \frac{a + x}{a - x}$

SECTION - "B"

Q.6 If $\vec{r} = 3\vec{i} - 4\vec{j} + 5\vec{k}$ then find the unit vector in the direction of $\vec{r}$ 2

(Or) Show that $\vec{a}-2\vec{b}+3\vec{c}, -2\vec{a}+3\vec{b}-4\vec{c}$ and $\vec{a}-3\vec{b}+5\vec{c}$ are coplanar.

Q.7 Prove that the sum of three vectors represented by the consecutive sides of triangle is zero vector. 2

(Or) If $\vec{a}=3\vec{i}-\vec{j}-4\vec{k}, \vec{b}=-2\vec{i}+4\vec{j}-3\vec{k}$, then find the magnitude of vector $3\vec{a}-2\vec{b}$

Q.8 Find the Vector Equation and Cartesian Equation of the sphere whose centre is (-1, 0, 1) and radius is 2. 2

(Or) Prove that vectors $2\vec{i}-3\vec{j}+5\vec{k}$ and $-2\vec{i}+2\vec{j}+2\vec{k}$ are mutually perpendicular.

Q.9 Evaluate : $\int \sqrt{1+\cos x}\, dx$. 2

(Or) Evaluate : $\int \frac{dx}{x^2-6x+13}$

Q.10 Evaluate : $\int x \cdot \log x\, dx$. 2

(Or) Integrate with respect to x : $\int \frac{e^{\cos^{-1}x}}{\sqrt{1-x^2}}$

Q.11 Find the angle between the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane $3x+y+z=7$. 3

(Or) Obtain the equation of the sphere described on the joining of the point A(2, -3, 4) and B(-5, 6, 7) as a diameter.

Q.12 Find the centroid of the triangle whose vertices are $A(X_1, Y_1, Z_1)$, $B(X_2, Y_2, Z_2)$ and $C(X_3, Y_3, Z_3)$. 3

(Or) Prove that the distance between two parallel planes $2x-2y+z+3=0$ and $4x-4y+2z+5=0$ is $1/6$.

Q.13 Prove by vector method that $\cos(\alpha+\beta)=\cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$. 3

(Or) Find the angle between the vectors $3\hat{i}+\hat{j}+2\hat{k}$ and $2\hat{i}-2\hat{j}+4\hat{k}$.

Q.14 If G is the centroid of any triangle then show that (prove that)

$\vec{GA}+\vec{GB}+\vec{GC}=\vec{0}$

(Or) If $\vec{a} = 3\hat{i}+2\hat{j}+2\hat{k}$, $\vec{b} = -\hat{i}+3\hat{j}-\hat{k}$ and $\vec{c} = \hat{i}+\hat{j}+\hat{k}$ then find $\vec{a}\times(\vec{b}\times\vec{c})$.

Q.15 Resolve $\frac{x^2+7x}{x^2+2x-8}$ into Partial fractions. 4

(Or) Resolve $\frac{2x+1}{(x-1)(x^2+1)}$ into Partial fractions.

Q.16 Prove that : 4

$$\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}=\pi/4$$

(Or) Prove that

$$\frac{1}{2}\cos^{-1}\frac{1-x}{1+x}=\tan^{-1}\sqrt{x}$$

Q.17 Differentiate sinX by First Principle. 4

(Or) Differentiate $\log\sqrt{\frac{1+\cos x}{1-\cos x}}$ with respect to x.

Q.18 If $y = a\cos(\log x)+ b\sin(\log x)$ then prove that $x^2y_2 + xy_1 + y = 0$. 4

(Or) If $x^y = e^{y-x}$, prove that $\frac{dy}{dx}=\frac{2-\log x}{(1-\log x)^2}$.

Q.19 Verify Rolle's theorem for function $f(x) = x^2 - 1$ in the interval [-1, 1]. 4

(Or) A particle moves according to the law $S = 5e^{-t}\cos t$, find its (a) velocity and (b) acceleration when $t = \pi/2$

Q.20 Prove that : Value of correlation coefficient ρ lies between -1 to +1. 5

(Or) Find the coefficient of correlation from the following data :

x	2	3	5	7	3
y	15	17	4	5	4

Q.21 An article costs Rs.70 at Gwalior, find the corresponding most appropriate value at Bhopal using the following data : 5

	Gwalior	Bhopal
Mean Value	65	67
Standard Deviation	2.5	3.5

The correlation coefficient between the values of the two cities is 0.8

(Or) Two lines of regression are $x + 2y = 5$ and $2x + 3y = 8$, find :

(i) $\bar{x}$ and $\bar{y}$ (b) b_{yx} and b_{xy}

(c) ρ_{xy}

Q.22 Find the equation of the sphere passing through the points (1, 0, 0) (0, 1, 0) and (0, 0, 1) and whose centre lies on the plane $3x - y + z = 2$. 5

(Or) Find the angle between the lines whose direction cosine are given by the relation $2l + 2n - m = 0$ and $ml + mn + nl = 0$.

Q.23 Evaluate : $\lim_{x \to \pi/4} \dfrac{\sin X - \cos X}{X - \frac{\pi}{4}}$ 5

(Or) Prove that the function is discontinuous :

$$g(x) = \begin{cases} 3x, & x < 3 \\ 3, & x = 3 \\ x^2, & x > 3 \end{cases}$$

Q.24 Evaluate : $\int \dfrac{dx}{5 + 4\cos x}$. 5

(Or) Find the value of $\int_0^{\pi} \dfrac{x \sin x}{1 + \cos^2 x}\, dx$.

Q.25 Solve the differential equation $\dfrac{dy}{dx} = \dfrac{x^2 + 5xy + 4y^2}{x^2}$ 5

(Or) Solve the $(1 + X^2) \dfrac{dY}{dX} + 2XY - 4X^2 = 0$.

Q.26 Find the probability that a leap year selected at random will contain 53 Sundays. 5

(Or) A husband and his wife attended an interview for a post. Probabilith of the husband being selected $\frac{1}{4}$. whereas that of the wife is $\frac{1}{6}$. Find the probability that none of them will be selected.