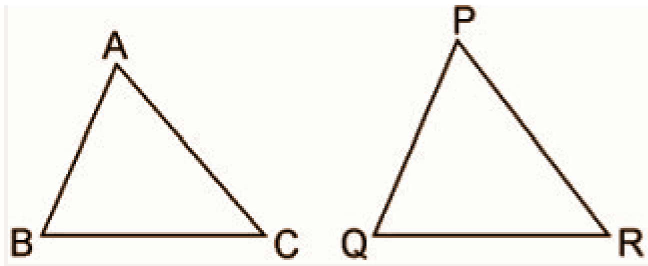
[ii]



Use the theorem that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides, then prove that they are congruent.

Given: $\Delta ABC \sim \Delta PQR$ and area of ΔABC = area of ΔPQR

To Prove: $\Delta ABC \cong \Delta PQR$

Proof:

Since $\Delta ABC \sim \Delta PQR$,

Area of ΔABC = area of ΔPQR (given)

$\Rightarrow AB^2 / PQ^2 = BC^2 / QR^2 = CA^2 / PR^2 = 1$ [Using the theorem of area of similar triangles]

$\Rightarrow AB = PQ$, $BC = QR$ & $CA = PR$

Thus, $\Delta ABC \cong \Delta PQR$ [BY SSS criterion of congruence]

Question 9: [i] Find the median of the following values of variate: 2M
15, 35, 18, 26, 19, 25, 29, 20, 27.

OR

[ii] Find the mean of all factors of 20.

Solution:

[i] Arranging in ascending order: 15, 18, 19, 20, 25, 26, 27, 29, 35

Number of observations = 9

Median = $\{[n + 1] / 2\}^{th}$ term

= 10 / 2

= 5^{th} term

= 25

25 is the median.

OR

[ii] Factors of 20 are 1 , 2 , 4 , 5 , 10 , 20.
The number of factors = 6.
Mean = sum of observations / number of observations
= [1 + 2 + 4 + 5 + 10 + 20] / 6
= 42 / 6
= 7

Question 10: [i] If the probability of raining tomorrow is 2 / 3 then what will be the probability of not raining tomorrow. 2M

OR

[ii] Two coins are tossed simultaneously. Find the probability of getting head on one coin and tail on another coin.

Solution:
[i] P (raining tomorrow) = 2 / 3
P (not raining tomorrow) = 1 - P (raining tomorrow)
= 1 - (2 / 3)
= [3 - 2] / 3
= 1 / 3

OR

[ii] When two different coins are tossed randomly, the sample space is given by
S = {HH, HT, TH, TT}
Therefore, n(S) = 4.
Let E = event of getting 1 head and 1 tail
Then, E = {HT, TH } and n (E) = 2.
P (getting head on one coin and tail on another coin) = P(E)
= n (E) / n (S)
= 2 / 4
= 1 / 2

Question 11: [i] Solve the following system of equation by elimination method:

3x + 2y = 11, 2x + 3y = 4. **4M**

OR

[ii] Solve the following system of equations by the Paravartya method of Vedic mathematics: 2x + y = 5, 3x – 4y = 2.

Solution:

[i] 3x + 2y = 11 ---- (1)

2x + 3y = 4

2 (3x + 2y = 11)

3 (2x + 3y = 4)

6x + 4y = 22

6x + 9y = 12

-5y = 10

---

y = 10 / -5

y = -2

Substitute the value of y in equation (1),

3x + 2 (-2) = 11

3x - 4 = 11

3x = 11 + 4

3x = 15

x = 15 / 3

x = 5

Therefore, x = 5 and y = -2.

OR

[ii] 2x + y = 5 ....(1)

3x − 4y = 2 ......(2)

Here $a_1 = 2$, $b_1 = 1$, $c_1 = 5$ and $a_2 = 3$, $b_2 = -4$, $c_2 = 2$

By Parvartya method of Vedic mathematics

$x = [b_1c_2 - b_2c_1] / [a_2b_1 - a_1b_2]$

= [1 * 2 - (- 4 * 5)] / [(3 * 1) - (2 * - 4)]

$= 22 / 11$
$= 2$
$y = [c_1a_2 - c_2a_1] / [a_2b_1 - a_1b_2]$
$= [(5 * 3) - (2 * 2)] / [(3 * 1) - (2 * - 4)]$
$= 11 / 11$
$= 1$
$x = 2, y = 1$

Question 12: [i] The sum of two numbers is 80 and the first number is 20 more than the second. Find the numbers. 4M

OR

[ii] The cost of 2 chairs and 3 tables is Rs. 800 and the cost of 4 chairs and 3 tables are Rs. 1000. Find the cost of 2 chairs and 2 tables.

Solution:
[i] Suppose the first number is x and the second number is y
According to the question:
$x + y = 80$ ...... (1) and
$x = y + 20$
$\Rightarrow x - y = 20$ ....(2)
Adding equation (1)and (2)
$x + y = 80$
$x - y = 20$
____
$2x = 100$
$x = 50$
Putting the value of x in equation (1)
$50 + y = 80$
$\Rightarrow y = 80 - 50$
$\Rightarrow y = 30$
The required numbers are 50 and 30.

OR

[ii] Let x, y denote chairs & tables.

$2x + 3y = 800$ ---- (1)

$4x + 3y = 1000$ ---- (2)

____

$- 2x + 0 = -200$

$2x = 200$

$x = 200 / 2$

$x = 100$

Then the cost of 2 chairs ,

$2(x) = 2 (100)$

The cost of two chairs is Rs.200.

Put $x = 100$ in (1)

$2 (100) + 3y = 800$

$200 + 3y = 800$

$3y = 800 - 200$

$3y = 600$

$y = 600 / 3$

$y = 200$

Then the cost of 2 tables is,

$2y = 2 * 200$ = Rs.400

cost of 2 chairs + 2 tables

= 200 + 400

= Rs. 600

Question 13: [i] If $x / a = y / b = z / c$ then prove that $x^3 / a^3 - y^3 / b^3 + z^3 / c^3 = xyz / abc$. **4M**

OR

[ii] If q is the mean proportional of p and r then prove that
$p^2 - q^2 + r^2 = q^4 [(1 / p^2) - (1 / q^2) + (1 / r^2)]$.

Solution:

[i] Let $x / a = y / b = z / c = k$ then

$x = ak$ ..... (1)

$y = bk$ ...... (2)

$z = ck$ ..... (3)

LHS = $x^3 / a^3 - y^3 / b^3 + z^3 / c^3$
= $(x / a)^3 - (y / b)^3 + (z / c)^3$
From equations (1), (2) and (3),
= $(ak / a)^3 - (bk / b)^3 + (ck / c)^3$
= $k^3 - k^3 + k^3$
= k^3
RHS = xyz / abc
= (x / a) . (y / b) . (z / c)
From equations (1), (2) and (3),
= (ak / a) . (bk / b) . (ck / c)
= k . k .k
= k^3
LHS = RHS
Hence $x^3 / a^3 - y^3 / b^3 + z^3 / c^3 = xyz / abc$.

OR

[ii] Since, q is the mean proportional of p and r.
Hence, $q^2 = pr$
RHS = $q^4[1 / p^2 – 1 / q^2 + 1 / r^2]$
= $q^4[1/p^2 – 1/pr + 1/r^2]$
= $q^4[r^2 – pr + p^2/p^2r^2]$
= $q^4[p^2 – pr + r^2/(pr)^2]$
= $q^4[p^2 – pr + r^2/q^4]$
= $p^2 – pr + r^2$
= $p^2 – q^2 + r^2$
= L.H.S

Question 14: [i] Solve the equation $x^2 - 5x - 6 = 0$ by the formula method. 4M

OR

[ii] Find the value of p in equation $2py^2 - 8y + p = 0$ so that the equation has equal roots.

Solution:
[i] $x = [(- b) \pm \sqrt{b^2 - 4ac}] / [2a]$

$x^2 - 5x - 6 = 0$

a = 1, b = -5, c = -6

$x = (5) \pm \sqrt{(-5)^2 - 4 * (1) * (-6)} / [2 * 1]$

$= 5 \pm \sqrt{25 + 24} / [2]$

$= 5 \pm \sqrt{49} / 2$

$= 5 \pm 7 / 2$

$x = 5 + 7 / 2$

$= 12 / 2$

$= 6$

$x = 5 - 7 / 2$

$= -2 / 2$

$= -1$

$x = 6, -1$

OR

[ii] The discriminant formula ($b^2 - 4ac$) is used.

If discriminant > zero, then it has real and unequal roots.

If discriminant = zero, then it has real and equal roots.

If discriminant < zero, then it has unreal roots.

For equal roots, $\Delta = b^2 - 4ac = 0$

$2py^2 - 8y + p = 0$

a = 2p, b = -8, c = 1

$\Delta = b^2 - 4ac = 0$

$0 = 8^2 - 4 * (2p) * (p)$

$0 = 64 - 8p^2$

$8p^2 = 64$

$p^2 = 64 / 8$

$p^2 = 8$

$p = \pm 2\sqrt{2}$

Question 15: [i] At a distance of 30 m away from the tower, the angle of elevation of the top of the tower is 30º. Find the height of the tower. 4M

OR

[ii] From the top of a 60m high house, the angle of depression of the ship is 60º. Find the distance between the ship and the foot of the lighthouse.

Solution:

[i] Let AB be the tower and O be the point of observation.

OA = 30 m and $\angle AOB = 30^o$

$\therefore$ In $\triangle OAB$, $\tan 30^o = AB / OA$

$\Rightarrow 1 / \sqrt{3} = AB / 30$

$\Rightarrow AB = 30 / \sqrt{3}$

$\therefore AB = (30 / \sqrt{3}) * (\sqrt{3} / \sqrt{3})$

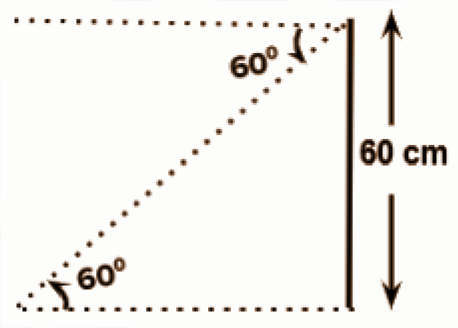
= 10 x 1.73

= 17.3

$\therefore$ Height of the tower is 17.3 m.

OR

[ii]



$\tan 60^o = 60 / x$

$\sqrt{3} = 60 / x$

$\sqrt{3}x = 60$

$x = 60 / \sqrt{3}$

= 34.64m

Question 16: [i] In a circle an arc subtends an angle 45 at the centre. If the length of an arc is 11 cm then find the radius of the circle. **4M**

OR

[ii] If V is the volume of a cuboid whose length is 'a', breadth is 'b' and height is 'c' and 's' is its surface area then prove that: (1 / V) = (2 / S) (1 / a + 1 / b + 1 / c).

Solution:

[i] Length of arc is 11cm.

The subtended angle by arc at the centre of circle = 45^o

$2\pi r\theta / 360$ = Length of arc

$\Rightarrow 2 \times r \times 45^o \times (22 / 7) / 360 = 11$

$\Rightarrow r = 11 \times 360^o \times 7 / 2 \times 22 \times 45^o$

r = 14 cm

∴ Radius of the circle = 14 cm.

OR

[ii] Volume of a cuboid = a × b × c

Surface area of cuboid = 2 (ab + bc + ac)

[2 / S] (1 / a + 1 / b + 1 / c) = [2 / S] ((bc + ac + ab) / abc)

It can be written as [2 / S] (1 / a + 1 / b + 1 / c) = [2 / S] (S / 2V)

On further calculation [2 / S] (1 / a + 1 / b + 1 / c) = 1 / V

1 / V = 2 / S (1 / a + 1 / b + 1 / c)

Therefore, it is proved that 1 / V = 2 / S (1 / a + 1 / b + 1 / c).

Question 17: [i] The radius of a cone is 7 cm and its height is 9 cm. The volume of this cone is equal to the lateral surface area of another cone which has the same radius. Find the slant height of the cone. 4M

OR

[ii] A cylinder of height 90 cm and base diameter 8 cm is melted and recast into spheres of diameter 12 cm. Find the number of spheres.

Solution:

[i] Radius of cone r = 7 cm and its height is h = 9 cm.

The slant height of the second cone = t cm

The lateral surface of the second cone = Volume of the first cone

$\pi rl = [1 / 3]\, \pi r^2 h$

$\pi \times 7 \times l = [1 / 3]\, \pi \times 7^2 \times 9$

$l = [1 / 3] \times [7 \times 3 \times 9] / 7$

$= 21$

Thus the required slant height of the second cone = 21cm.

OR

[ii] Given that: for sphere D = 12cm = R= 12 / 2 = 6cm

For cylinder d = 8 cm

$\Rightarrow r = 8 / 2 = 4$cm and h = 90 cm

Let the number of spheres made = n

According to the question,

The volume of n sphere = Volume of cylinder

$\Rightarrow \pi \times [4 / 3 \times \pi r^2 = \pi r^2 h$

$\Rightarrow \pi \times [4 / 3]\pi(6)^3 = \pi \times 4^2 \times 90$

$\Rightarrow$ n = 5 spheres

Question 18: [i] Find cyclic factors: $x(y^2 + z^2) + y(z^2 + x^2) + z(x^2 + y^2) + 2xyz$.

5M

OR

[ii] Which rational expression should be subtracted from $[x^2 + 1] / [x - 1]$ to get $[x - 3] / [x + 1]$?

Solution:

[i] $x(y^2 + z^2) + y(z^2 + x^2) + z(x^2 + y^2) + 2xyz$

Expanding the equation,

$xy^2 + xz^2 + yz^2 + yx^2 + zx^2 + zy^2 + 2xyz$

Rearranging the above equation,

$xy^2 + zy^2 + xyz + yx^2 + zx^2 + xyz + xz^2 + yz^2$

$(xy^2 + zy^2 + xyz) + (yx^2 + zx^2 + xyz) + (xz^2 + yz^2)$

Taking commons,

$y(xy + zy + xz) + x(xy + xz + yz) + z^2(x+y)$

Taking $(xy + zy + xz)$ common from first two terms,

$(y + x)(xy + zy + xz) + z^2(x + y)$
$(x + y)(xy + zy + xz) + z^2(x + y)$
Taking $(x + y)$ common
$(x + y)(xy + zy + xz + z^2)$
$(x + y)((xy + zy) + (xz + z^2))$
Taking y and z common
$(x + y)(y(x + z) + z(x + z))$
Taking $(x + z)$ common,
$(x + y)(y + z)(x + z)$
Hence the cyclic factors of the given equation are $(x + y)$, $(y + z)$ and $(x + z)$.

OR

[ii] $[x^2 + 1] / [x - 1] - z = [x - 3] / [x + 1]$
$[x^2 + 1] / [x - 1] - [x - 3] / [x + 1] = z$
$[x^3 + x + x^2 + 1 - (x^2 - 3x - x + 3)] / [x^2 - 1] = z$
$[x^3 + x + x^2 + 1 - x^2 + 4x - 3] / [x^2 - 1] = z$
$x^3 + 5x - 2 / x^2 - 1 = z$

Question 19: [i] If α and β are roots of quadratic equation $ax^2 + bx + c = 0$, then find α / β + β / α. **5M**

OR

[ii] The length of the rectangle is 5 cm more than its breadth. If the area of the rectangle is 150 sq. cm. Find the sides of the rectangle.

Solution:
[i] $ax^2 + bx + c = 0$
$\alpha + \beta = - b / a$
$\alpha \beta = c / a$
$\alpha / \beta + \beta / \alpha = [\alpha^2 + \beta^2] / \alpha \beta$
$= [\alpha + \beta]^2 - 2 \alpha \beta / \alpha \beta$
$= (b^2 - a^2) - 2 * (c / a) / [c / a]$
$= (b^2 - a^2) - (2c / a) / [c / a]$
$= b^2 - 2ac / a^2 / [c / a]$
$= [b^2 - 2ac] / ac$

OR

[ii] Area of a rectangle = Length × Breadth
Length of a rectangle is 5 cm more than its breadth.
Assume the breadth to be x.
Length becomes = 5 + x
Putting values in the above
$150 = x \times (5+x)$
$150 = x^2 + 5x$
$x^2 + 5x - 150 = 0$
$x^2 + 15x - 10x - 150 = 0$
$x (x + 15) - 10 (x + 15) = 0$
$(x + 15) (x - 10) = 0$
$x = - 15$ and $x = 10$
(As the breadth sides of the rectangle cannot be negative. Thus ignore x = -15)
Take x = 10
Length = 10 + 5
= 10 + 5
= 15 cm
Therefore when x = 10cm, then the other side is 15cm.

Question 20: [i] Find the compound interest on Rs. 8,000 for the period of 1 (1 / 2) years at the rate of interest 10% per annum, if the interest is compounded half-yearly.
5M

OR

[ii] A watch is sold for Rs. 960 cash or for Rs. 480 cash down payment and two monthly instalments of Rs. 245 each. Find the rate of interest charged under the instalment plan.

Solution:
[i] Principle P = Rs. 8000
Time t = 1 (1 / 2) = 3 / 2 years
But, as the interest is compounded half-yearly t = [3 / 2] × 2 = 3years

Rate of interest R = 10%

$A = P (1 + R / 100)^t$

$A = 8000 (1 + 10 / 100)^3$

$A = 8000 (110 / 100)^3$

$A = 8000 \times (1.1)^3$

$A = 8000 \times 1.331$

A = 10648

CI = A − P

CI = 10648 − 8000

CI = Rs. 2648

OR

[ii] Cash price of watch = Rs. 960

Cash down payment = Rs. 480

Balance due = 960 − 480 = Rs. 480

Total payment = 245 × 2 = Rs. 490

Total interest paid in instalment

= Rs. 490 − Rs. 480

= Rs. 10

Principal for 1st month = Rs. 480

Principal for 2nd month = Rs. 480 − 245

= Rs. 235

Total principal for 1 month = Rs.480 + Rs. 235

= Rs. 715

∴ Rate of interest in the instalment

= [Interest * 100] / [Principal * time]

= [10 × 100] / [715 × (1 / 12)]

= 1000 × 12 / 715

= 16.78%

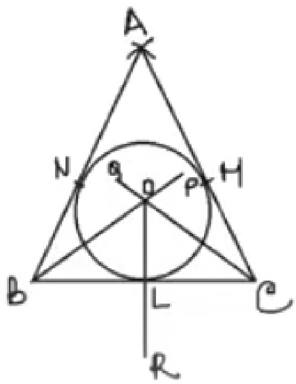
Question 21: [i] Construct the incircle of the equilateral triangle whose one side is 8 cm and write the steps of construction also. 5M

OR

[ii] Construct a cyclic quadrilateral ABCD in which vertical angle B = 65°, AB = 4 cm. AC = 6 cm. AD = 4 cm. Write the steps of construction also.

Solution:

[i]



Construction of △ABC -

(1) Draw a line segment BC=8 cm.

(2) Draw an arc of radius =8 cm taking B as the centre.

(3) Draw another arc of radius =8 cm taking B as the centre.

(4) Draw another arc of radius =8 cm. taking C as the centre with intersects the previous arc at a point A.

(5) Join AB and AC.

Thus the require △ABC is constructed.

Construction of the incircle of △ABC.

(1) Draw BP and CQ the bisectors of angles $\angle B$ and $\angle C$ respectively and which intersect each other at point O.

(2) Draw $OR \perp BC$ which intersects BC at L.

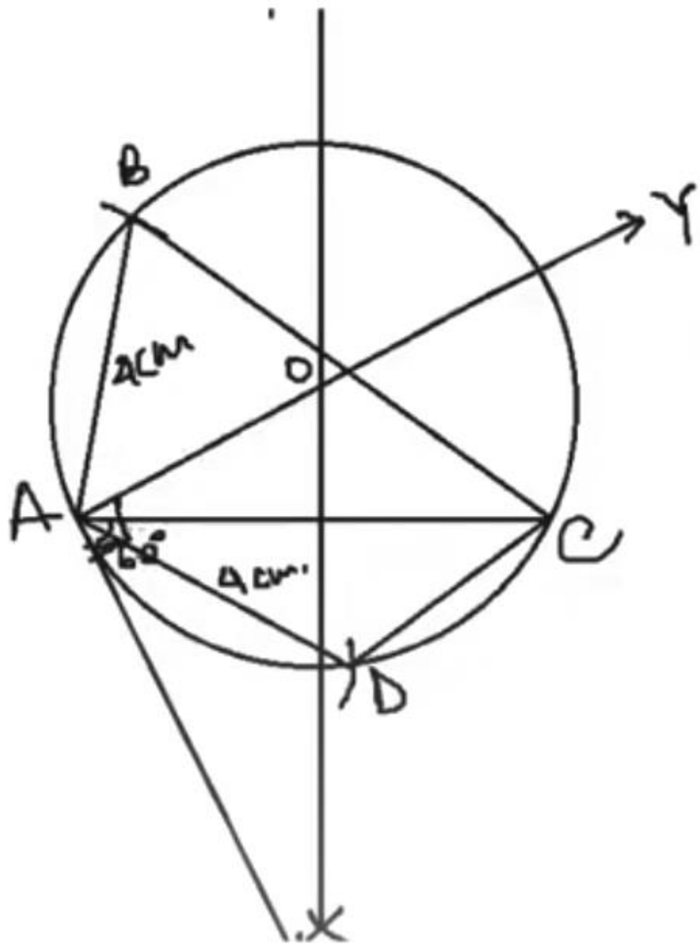
(3) Taking O as the centre and OL as radius draw a circle which touches the sides AB, BC and CA at points N, L and M respectively.

OR

[ii] Construction:

(a) Draw a line segment AC = 6cm.

(b) Draw a ray AX from point A making an angle equal to ∠B = 65° downward with line segment AC.
(c) Draw another ray AY ⊥ AX.
(d) Draw a line PQ perpendicular bisector of line segment AC which intersects the ray AY at O.
(e) Taking O as the centre, draw a circle of radius OA = OC which passes through A and C.
(f) Taking A as the centre, draw an arc of radius AB = 4cm, which intersects the upper part of the circle at B.
(g) Taking A as a centre, draw another arc of radius AD = 4cm which intersects the lower part of the circle at D.
(h) Join AB, BC, CD and DA
Thus the required cyclic quadrilateral is obtained.



Question 22: [i] Prove the following identity: $\sqrt{[1 - \sin\theta] / [1 + \sin\theta]} = \sec\theta - \tan\theta$.

5M

OR

[ii] Simplify: $(\sec\theta + \tan\theta)(1 - \sin\theta)$

Solution:

[i] $\sqrt{[1 - \sin\theta] / [1 + \sin\theta]} = \sec\theta - \tan\theta$

By squaring on both sides,

$[1 - \sin\theta] / [1 + \sin\theta] = (\sec\theta - \tan\theta)^2$

Consider RHS $= (\sec\theta - \tan\theta)^2$

$= (1 / \cos\theta) - (\sin\theta / \tan\theta)^2$

$= (1 - \sin\theta)(1 - \sin\theta) / (1 - \sin^2\theta)$

$= (1 - \sin\theta)(1 - \sin\theta) / (1 - \sin\theta)(1 + \sin\theta)$

$= [1 - \sin\theta] / [1 + \sin\theta]$

= LHS

So, $\sqrt{[1 - \sin\theta] / [1 + \sin\theta]} = \sec\theta - \tan\theta$

OR

[ii] $(\sec\theta + \tan\theta)(1 - \sin\theta)$

$= (1 / \cos\theta + \sin\theta / \cos\theta)(1 - \sin\theta)$

$= (1 + \sin\theta / \cos\theta)(1 - \sin\theta)$

$= 1 - \sin^2\theta / \cos\theta$

$= \cos^2\theta / \cos\theta$ [Since $1 - \sin^2\theta = \cos^2\theta$]

$= \cos\theta$

Question 23: [i] If PAB is a secant to a circle with centre O intersecting the circle at A and B and PT is a tangent segment, then prove that $PA * PB = PT^2$.

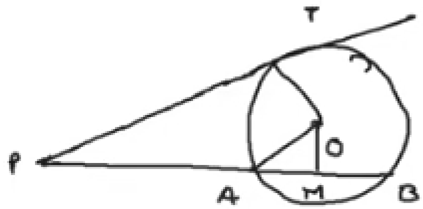
6M

OR

[ii] In a circle of radius 5 cm, AB and AC are the two chords such that AB = AC = 6 cm. Find the length of chord BC.

Solution:

[i]



Construction: $OM \perp AB$ is drawn OA, OP, OT are joined.

PA = PM – AM; PB = PM + MB

AM = BM [perpendicular drawn from the centre of the circle to a chord is also a bisector of the chord]

PA * PB = (PM − AM) * (PM + AM)

PA * PB = $PM^2 - AM^2$

Also $OM \perp AB$,

By Pythagoras theorem in ΔOMP,

$PM^2 = OP^2 - OM^2$

Now apply Pythagoras theorem in ΔOMA

$AM^2 = OA^2 - OM^2$

PA * PB = $PM^2 - AM^2$

PA * PB = $(OP^2 - OM^2) - (OA^2 - OM^2)$

PA * PB = $OP^2 - OM^2 - OA^2 + OM^2$

PA* PB = $OP^2 - OA^2$

PA * PB = $OP^2 - OT^2$

Since OA = OT (radii), as the radius is perpendicular to the tangent this will form a right-angled triangle.

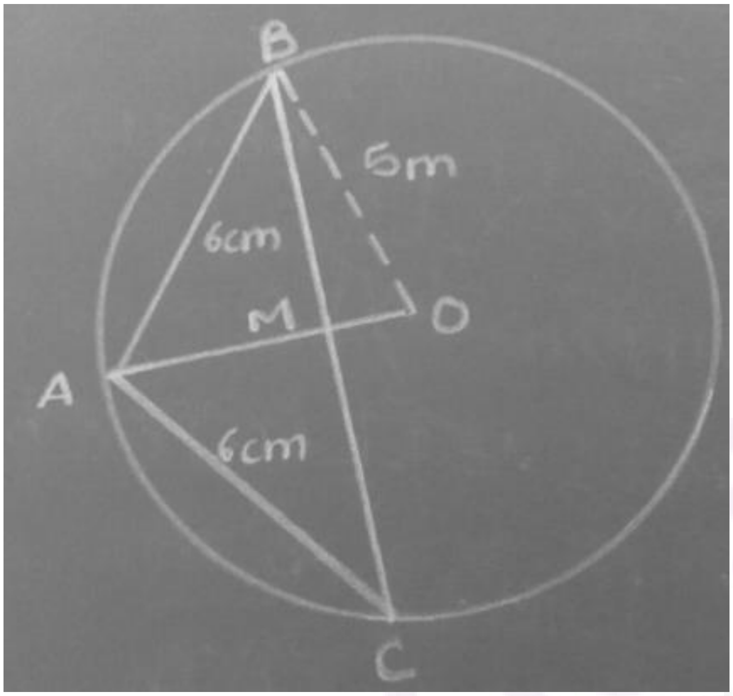
By applying Pythagoras theorem in ΔOPT,

$PT^2 = OP^2 - OT^2$

By putting this value in the above equation, PA * PB = PT^2.

OR

[ii]



In ΔABM,

$AB^2 = BM^2 + AM^2$

$AB^2 - AM^2 = BM^2$

$6^2 - AM^2 = BM^2$ _(i)

In ΔBMO

$BO^2 = BM^2 + OM^2$

$5^2 - OM^2 = BM^2$ _(ii)

From (i) and (ii),

$5^2 - OM^2 = 6^2 - AM^2$

$AM^2 = 36 - 25 + OM^2$

Since OM = AO - AM

$AM^2 = 9 + (AO - AM)^2$

$AM^2 = 9 + (5 - AM)^2$

$AM^2 = 9 + 25 + AM^2 - 10AM$

10AM = 36

AM = 3.6 cm

In ΔAMC

$AC^2 = AM^2 + CM^2$

$6^2 = 3.6^2 + CM^2$

$36 - 12.96 = CM^2$

$\sqrt{23.04} = CM$

$4.8 = CM$

Since AO is the perpendicular bisector of chord BC.

BM = CM

BM + CM = BC

2CM = BC

2(4.8) = BC

9.6 cm = BC

Question 24: [i] Find the mode of the following frequency table: 6M

Class interval	140 - 150	150 - 160	160 - 170	170 - 180	180 - 190	190 - 200
Frequency	4	6	10	12	9	3

OR

Calculate the cost of living index number for the year 1995 on the basis of the year 1990 from the following data:

Item	Quantity	Cost per kg in the year 1990	Cost per kg in the year 1995
A	8	30	45
B	5	28	36
C	12	6	11
D	40	9	15
E	18	10	12

Solution:

[i] The class interval with maximum frequency is 170 - 180.

$Z = L_1 + (F_1 - F_0) / (2F_1 - F_0 - F_2) * i$

$= 170 + \{(12 - 10) / (2 * 12 - 10 - 9)\} * 10$

= 170 + (2 / 5) * 10
= 174

OR

[ii]

Item	**Quantity**	**Cost per kg in the year 1990**	**Cost per kg in the year 1995**	**Total cost in 1990**	**Total cost of 1995**
A	8	30	45	240	360
B	5	28	36	140	180
C	12	6	11	72	132
D	40	9	15	360	600
E	18	10	12	180	216
				992	**1488**

Cost of living index in 1995 = Total Expense in 1995 / Total Expense in 1990
= [1488 / 992] × 100
= 150