Solution:

[i] Let △ABC and △PQR be two similar triangles.

⇒ Perimeter of △ABC = 20 cm.

⇒ Perimeter of △PQR = 30 cm.

⇒ QR = 12 cm, BC = ?

In the similarity case

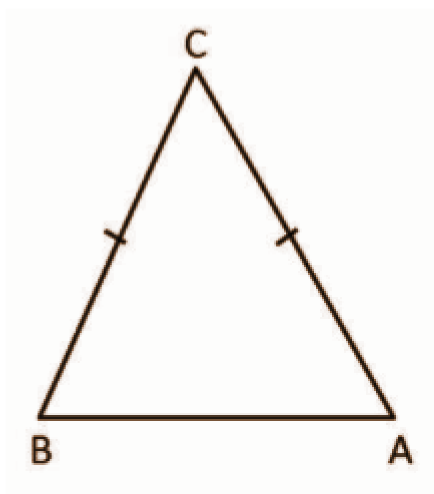
Perimeter of △ABC / Perimeter of △PQR = AB / PQ = BC / QR = AC / PR

⇒ 20 / 30 = BC / 12

⇒ BC = 8 cm

OR

[ii]



$AB^2 = 2AC^2$

$AB^2 = AC^2 + AC^2$

$AB^2 = AC^2 + BC^2$

AB will be the largest side that is the hypotenuse.

By Pythagoras theorem,

$H^2 = B^2 + P^2$

$AB^2 = AC^2 + BC^2$

$AB^2 = AC^2 + AC^2$

$AB^2 = AB^2$

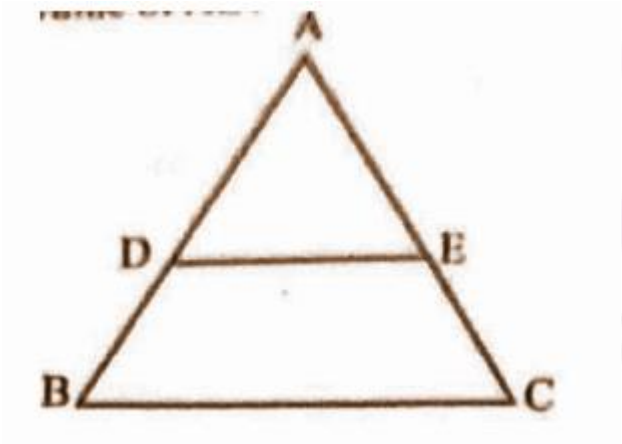
LHS = RHS

Since Pythagoras theorem is satisfied, ΔACB is a right-angled triangle.

Question 8: [i] Triangle ABC and triangle PQR are two similar triangles. The areas of these are 64 cm^2 and 100 cm^2, respectively. If QR = 12cm. Then find the value of side BC. **2M**

OR

[ii] In the figure given below DE ∥ BC, if AD / DB = 3 / 5 and side AC = 6cm. Then, find the value of AE.



Solution:

[i] $\Delta ABC \approx \Delta PQR$

Area of Δ ABC / Area of ΔPQR = $(AB / PQ)^2$ = $(BC / QR)^2$ = $(AC / PR)^2$

Area of Δ ABC = 64 cm^2

Area of Δ PQR = 100 cm^2

QR = 12 cm

BC = ?

Area of ΔABC / Area of Δ PQR = $(BC / QR)^2$

=> 64 / 100 = $(BC / 12)^2$

=> 16 / 25 = $BC^2 / 12^2$

=> 0.64 = BC^2 / 144

=> 92.16 = BC^2

=> BC = 9.6 cm

OR

[ii] DE || BC
Using B. P. T theorem
AD / DB = AE / EC
EC = AC – AE
AD / DB = AE / AC – AE
3 / 5 = AE / 6 – AE
0.6 = AE / 6 – AE
3.6 – 0.6 AE = AE
3.6 = AE + 0.6 AE
3.6 = 1.6AE
AE = 2.25 cm

Question 9: [i] Write any two properties of Arithmetic Mean. 2M

OR

[ii] Find the Median of the following values:
5, 10, 3, 7, 1, 9, 6, 2, 11

Solution:
[i] (a) The sum of deviations of the items from their arithmetic mean is always zero, i.e. $\sum(x - X) = 0$.
(b) The sum of the squared deviations of the items from Arithmetic Mean (A.M) is minimum, which is less than the sum of the squared deviations of the items from any other values.

OR

[ii] Arrange the data
1, 2, 3, 5, 6, 7, 9, 10, 11
Median = $\{[n + 1] / 2\}^{th}$ term
= $\{[9 + 1] / 2\}^{th}$ term
= 5^{th} term
Median = 6

Question 10: [i] Write the probability of getting an odd number in a single throw of a die. **2M**

OR

[ii] If two coins are tossed simultaneously, find the probability of getting two heads.

Solution:

[i] Sample space of die = {1, 2, 3, 4, 5, 6}

Odd numbers = {1, 3, 5}

Probability (getting an odd number) = odd numbers / sample space

= 3 / 6

= 1 / 2

OR

[ii] If we toss two coins simultaneously, then possible outcomes (s), are

S = {HT, TH, HH, TT}

⇒ n (S) = 4

Let E be the favourable outcomes of getting two heads, then E = {H, H}

⇒ n (E) = 1

P (getting two heads) = n (E) / n (S)

= 1 / 4

Question 11: [i] Solve the following system of equation **4M**

3x - 2y = 4

y + 2x = 5

OR

[ii] Find the value of m, for which the system

2x+ my - 4 = 0

3x - 7y - 10 = 0 has

(i) a unique solution

(ii) no solution

Solution:

[i] 3x - 2y = 4 ---- (1)

$2x + y = 5$ ---- (2)

Multiply equation (1) by 2 and equation (2) by 3,

$6x - 4y = 8$

$6x + 3y = 15$

____

$-7y = -7$

$y = 1$

Put $y = 1$ in (2),

$2x + y = 5$

$2x + 1 = 5$

$2x = 4$

$x = 4 / 2$

$x = 2$

OR

[ii] $2x + my – 4 = 0$

$3x – 7y – 10 = 0$

Comparing with $ax + by + c = 0$

$a_1 = 2, b_1 = m, c_1 = - 4$

$a_2 = 3, b_2 = - 7, c_2 = - 10$

(1) for unique solution,

$a_1 / a_2 \neq b_1 / b_2$

$2 / 3 = m / -7$

$-14 = 3m$

$m = -14 / 3$

For all real values of m, [$m \neq -14 / 3$] the system has a unique solution.

(2) for no solution, the lines are parallel such that,

$a_1 / a_2 = b_1 / b_2 \neq c_1 / c_2$

$2 / 3 = m / -7 = -4 / -10$

$m \neq [-7 * 4] / -10$

$m \neq -14 / 5$

So, m takes every value except [-14 / 5].

Question 12: [i] The sum of two numbers is 7. If the sum of these numbers is seven times its difference. Find the number. **4M**

OR

[ii] If in ΔABC $\angle C = 2\angle B = \angle A + \angle B + 20°$, then find all the three angles of a triangle.

Solution:

[i] Let the numbers be x and y.

$x + y = 7$ ……….. (1)

$x + y = 7(x - y)$……(2)

$x + y = 7x – 7y$

$0 = 6x – 8y$

$0 = 3x – 4y$………… (3)

Solving (1) and (3)

$x + y = 7 \times 4$

$3x - 4y = 0$

$4x + 4y = 28$

____

$7x = 28$

$x = 4$

$y = 7 – x$ (from equation 1)

$= 7 – 4$

$= 3$

OR

[ii] Let $\angle A$ be x and $\angle B$ be y.

Given that $\angle C = 2\angle B$

$\angle C = 2y$

Also,

$\angle C = \angle A + \angle B + 20°$

$\angle C = x + y + 20$

Since ABC is a triangle, by angle sum property,

$\angle A + \angle B + \angle C = 180^o$

$x + y + 2y = 180^o$

$x + 3y = 180^o$ ---- (1)

$\angle A + \angle B + \angle C = 180^o$

$x + y + [x + y + 20] = 180^o$

$x + y + x + y + 20 = 180^o$

$2x + 2y = 160^o$

$x + y = 80^o$ ---- (2)

Consider the equations (1) and (2),

$x + 3y = 180^o$

$x + y = 80^o$

$2y = 100$

$y = 100 / 2$

$y = 50^o$

$\angle C = 2y$

$= 2 * 50^o$

$= 100^o$

Put y = 50 in (1),

$x + 3 * 50 = 180^o$

$x = 180^o - 150^o$

$x = 30^o$

$\angle A = 30^o$

$\angle B = 50^o$

$\angle C = 100^o$

Question 13: [i] If x / [b + c] = y / [c + a] = z / [a + b], then prove that (b - c) x + (c - a) y + (a - b) z = 0. **4M**

OR

[ii] What should be subtracted from 11, 20, 26 and 50 so that the remaining numbers are in proportion?

Solution:

[i] Let x / [b + c] = y / [c + a] = z / [a + b] = k

x = (b + c) k ...(1)

$y = (c + a) k$ ...(2)

$z = (a + b) k$ ...(3)

To prove $(b - c) x + (c - a) y + (a - b) z = 0$

LHS $= (b - c) x + (c - a) y + (a - b) z$

$= (b - c) (b + c) k + (c - a) (c + a) k + (a - b) (a + b) k$

$= k [(b - c) (b + c) + (c - a) (c + a) + (a - b) (a + b)]$

$= k [b^2 – c^2 + c^2 – a^2 + a^2 – b^2]$

$= k [0]$

$= 0$

= RHS

OR

[ii] Let the number be subtracted be x so, $(11 – x) : (20 – x) : : (26 – x) : (50 – x)$

$11 - x / 20 - x = 26 - x / 50 - x$

$(11 – x) (50 – x) = (26 – x) (20 – x)$

$550 – 11x – 50x + x^2 = 520 – 26x – 20x + x^2$

$550 – 61x = 520 – 46 x$

$550 – 520 = - 46x + 61x$

$30 = 15x$

$x = 30 / 15$

$= 2$

Question 14: [i] Solve the equation $3x - 1 / x = 2$ using the formula method. 4M

OR

[ii] Find the nature of the roots of the following equation, $6x^2 - x - 2 = 0$.

Solution:

[i] $3x - [1 / x] = 2$

$3x^2 - 1 = 2x$

$3x^2 - 2x - 1 = 0$

$a = 3, b = -2, c = -1$

$x = −b ± \sqrt{b^2 - 4ac} / [2a]$

$= (2) ± \sqrt{(−2)^2 - 4 * 3 * (-1)} / [2 * 3]$

$= 2 \pm \sqrt{16} / 6$
$= 2 \pm 4 / 6$
$x = 2 + 4 / 6$
$= 6 / 6$
$x = 1$
$x = 2 - 4 / 6$
$= -2 / 6$
$= -1 / 3$
$x = 1, -1 / 3$

OR

[ii] The discriminant formula ($b^2 - 4ac$) is used.

- If discriminant > zero, then it has real and unequal roots.
- If discriminant = zero, then it has real and equal roots.
- If discriminant < zero, then it has unreal roots.

$6x^2 - x - 2 = 0$
$a = 6 ; b = -1 ; c = -2$
$\Delta = b^2 - 4ac$
$= 1 - 4 * 6 * (-2)$
$= 1 + 48$
$= 49$
So, $\Delta > 0$
It has real and unequal roots.

Question 15: [i] From the top of the hill, the angle of depression of the top and the bottom of the 16m high building is 30° and 60° respectively. Find the height of the hill.
4M

OR

[ii] An aeroplane is flying at a height of 8,000m. The angle of depression of the control tower of the airport from the aeroplane is 30°. Find the horizontal distance between the control tower and the aeroplane.

Solution:

[i] Let the height of building 'h' m = AD
Height of the tall building = 16 m = BE
The angle of depression of top of the tall building from the multistoried building = 30°
The angle of depression of bottom of the tall building from the multistoried building = 60°
So, BC = x and CD = 16 m [As BCDE forms a rectangle]
AD = AC + CD
So, AC = (h – 16) m
In ΔBCA
tan 30º = AC/ BC
$1 / \sqrt{3} = (h - 16)/x$
$x = \sqrt{3}(h - 16)$…… (i)
In ΔADE
tan 60º = AD / ED
1.732 = h / x
h = x
$h = \sqrt{3}(h - 16)$ [using (i)]
$h = \sqrt{3}h - 16\sqrt{3}$
$(\sqrt{3} - 1)h = 16\sqrt{3}$
$h = 16\sqrt{3}/ (\sqrt{3} - 1)$

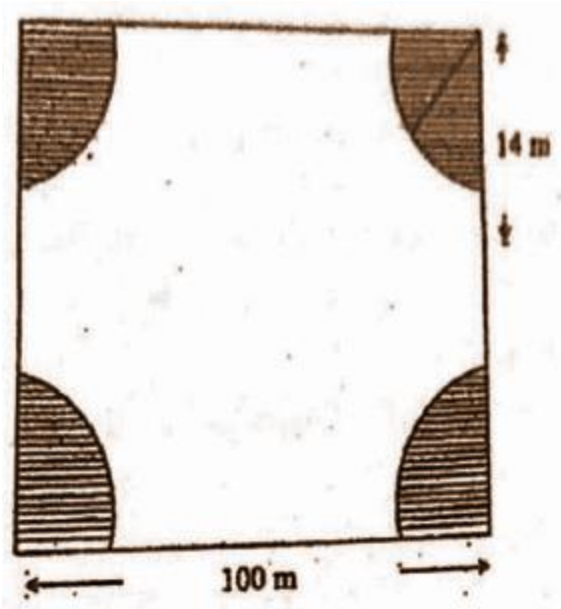
Rationalising the denominator by $(\sqrt{3} + 1)$, we have
$h = 16\sqrt{3}(\sqrt{3} + 1) / (3 - 1)$
$h = 16\sqrt{3}(\sqrt{3} + 1) / 2$
h = 48 + 16 / 2
= 64 / 2
= 32m
The height of the building is 32m.

OR

[ii] Let the horizontal distance between the control tower and the aeroplane be x.

tan 30^o = 8000 / x
x = 8000 / tan 30^o
x = 13856.4m

Question 16: [i] A quadrants shaped flower bed is made of radius 14m in a square garden of side 100m, in all the four corners. Find the area of the remaining part of the square garden. 4M



OR

[ii] A rocket is in the form of a closed cylinder from below, and the upper part surmounted by a cone of equal radius. The radius of the cylinder is 2 m and the height is 21 m. The height of the cone is 8.4m. Find the volume of the rocket.

Solution:
[i] Side of square = 100cm
Radius of quadrant, r = 14cm
Angle measure, θ = 90° (angle of each corner of the square is 90°)
Area of square = (side)^2
= $(100)^2$
= 10000 cm^2
Area of each quadrant = [1 / 4] * **π** * r^2

$= 1 / 4 * 22 / 7 * 14 * 14$
$= 154 cm^2$
Area of remaining part of square = Area of the square - 4 * Area of each quadrant
$= 10000 - 4 * 154$
$= 10000 - 616$
$= 9384 cm^2$

OR

[ii] Volume of Rocket = Volume of cone + Volume of cylinder
$= [1 / 3]\pi r^2h + \pi R^2h$
$= [1 / 3] * [22 / 7] * 2^2 * (8.4) + [22 / 7] * 2^2 * 21$
$= 35.2 + 264$
$= 299.2m^3$

Question 17: [i] The area of three adjacent faces of a cuboid are x, y and z. If the volume of the cuboid is V, then prove that $V^2 = xyz$. 4M

OR

[ii] An iron sphere of radius 8 cm is melted then recast into small spheres each of radius 1 cm. Find the number of small spheres.

Solution:
[i] Let the dimensions of the cuboid are
Length = l
Breadth = b
Height = h
Given the area of three faces x, y and z are
lb = x --(1)
bh = y ---(2)
lh = z ---(3)
Multiply (1), (2) and (3)
$lb \times bh \times lh = xyz$
$l^2 \times b^2 \times h^2 = xyz$
$(lbh)^2 = xyz$
$V^2 = xyz$ [Since the volume of the cuboid = V = lbh]

OR

[ii] Radius of bigger sphere (R) = 8 cm
Radius of small spheres (r) = 1 cm
Let the number of small spheres = n
n × volume of a small spheres = volume of bigger sphere
n * [4 / 3]πr^3 = [4 / 3] πR^3
n = R^3 / r^3
= $8^3 / 1^3$
= 512

Question 18: [i] Find cyclic factors ab (a - b) + bc (b - c) + ca (c - a). 5M

OR

[ii] Multiply the rational expressions $x^2 - 7x + 10 / (x - 4)^2$ and $x^2 - 7x + 12 / x - 5$ and express the product in its lowest terms.

Solution:
[i] ab (a - b) + bc (b - c) + ca (c - a)
$a^2b - ab^2 + b^2c - bc^2 + ac^2 - a^2c$
= $a^2(b - c) - a (b - c) (b + c) + bc (b - c)$
= $(b - c) [a^2 - a (b + c) + bc]$
= $(b - c) [a^2 - ab - ac + bc]$
= $(b - c) [a (a - b) - c (a - b)]$
= $(b - c) (a - b) (a - c)$

OR

[ii]$\{x^2 - 7x + 10\}/ \{(x - 4)^2\} * \{x^2 - 7x + 12\} / \{x - 5\}$
= $[x^2 - 5x - 2x - 10] / (x - 4)^2 * (x^2 - 4x - 3x - 12) / (x - 5)$
= $(x - 5) (x - 2) / (x - 4)^2 * (x - 4) (x - 3) / (x - 5)$
= $(x - 2) (x - 3) / (x - 4)$

Question 19: [i] If α and β are the roots of quadratic equations $3x^2 - 5x - 7 = 0$, then find the value of α / β + β / α. **5M**

OR

[ii] The length of the side forming the right angle of a right angled triangle are x cm and (x + 1) cm. If the area of the triangle is 10 cm^2, then find the sides of the triangle.

Solution:

[i] $3x^2 - 5x - 7 = 0$

a = 3 ; b = -5 ; c = -7

α + β = -b / a = 5 / 3

αβ = c / a = -7 / 3

$\alpha / \beta + \beta / \alpha = \alpha^2 + \beta^2 / \alpha\beta$

$= (\alpha + \beta)^2 - 2\alpha\beta / \alpha\beta$

$= [(5 / 3)^2 - 2 * (-7 / 3)] / [-7 / 3]$

= [(25 / 9) + (14 / 3) / [-7 / 3]

= [67 / 9] / [-7 / 3]

= -67 / 21

OR

[ii] Area of a triangle = (1 / 2) * bh

[1 / 2] (x (x + 1)) = 10

$[1 / 2] [x^2 + x] = 10$

$x^2 + x - 20 = 0$

$x^2 + x = 20$

$x^2 + 2 \times x \times 1 / 2 = 20$

$x^2 + 2 \times x \times 1 / 2 + (1 / 2)^2 = 20 + (1 / 2)^2$

$(x + 1 / 2)^2 = 20 + 1 / 4$

$(x + 1 / 2)^2 = (80 + 1) / 4$

$(x + 1 / 2)^2 = 81 / 4$

$x + 1 / 2 = \pm \sqrt{(81 / 4)}$

x + 1 / 2 = ± 9 / 2

x = -1 / 2 ± 9 / 2

x = (-1 / 2 + 9 / 2) or x = (-1 / 2 - 9 / 2)

x = 8 / 2 or x = -10 / 2
x = 4 or x = -5
Hence, sides are 4 cm and 5 cm.

Question 20: [i] Find the compound interest on Rs, 2000 at the rate of interest 4% per annum for 2 years. 5M

OR

[ii] A sewing machine is available for Rs. 1600 Cash or for Rs. 1200 cash down payment and Rs. 460 to be paid after 6 months. Find the rate of interest charged under the instalment plan.

Solution:

[i] Principle = Rs 2000 = P
Interest rate = 4% per annum = R
Time = 2 Years = n
Amount = P $(1 + R / 100)^n$
=> Amount = 2000 $(1 + 4 / 100)^2$
=> Amount = 2000 * 1.04^2
=> Amount = 2,163.2
Compound Interest = Amount - Principal
= 2163.2 - 2000
= 163.2 Rs
163.2 Rs is the compound interest on rupees 2000 for 2 years at 4% per annum compounded annually.

OR

[ii] Cash price of sewing machine = 1600 Rs
Down payment = 1200 Rs
Balance due = 1600 – 1200 = 400 Rs
No. of instalment = 1
Amount of instalment = 460 Rs
Interest paid = 460 – 400 = 60 Rs
SI = PRT / 100
60 = [1600 * R * 6] / [100 * 12]

R = 7.5 %

Question 21: [i] The side of the triangle is 4cm, 6cm and 8cm. Draw the circumcircle of the triangle and write the steps of construction. 5M

OR

[ii] Construct a cyclic quadrilateral in which AC = 6cm, ∠B = 90°, AB = 3cm and AD = 4cm. Write the steps for construction also.

Solution:

[i] Steps for construction:

I) Construction of triangle

1) Let the base of the triangle be QR=8cm.

2) Taking Q as the centre and radius 4cm on the compass, draw an arc on the upper side of QR.

3) Taking R as the centre and radius 6cm on the compass, draw an arc on the upper side of QR, intersecting the previous arc. Mark this point of intersection as point P.

4) Join points P to Q and P to Q to complete the △PQR.

II) Construction of perpendicular bisectors

1) Taking Q as the centre and radius more than half of QR, mark arcs below and above the line.

2) Now, with R as the centre and same radius, draw arcs above and below the line to intersect the already drawn arcs. Name the new points as X and Y.

3) Join points X and Y. This line XY is the required perpendicular bisector of side QR.

4) Similarly, draw perpendicular bisectors of sides PQ and PR.

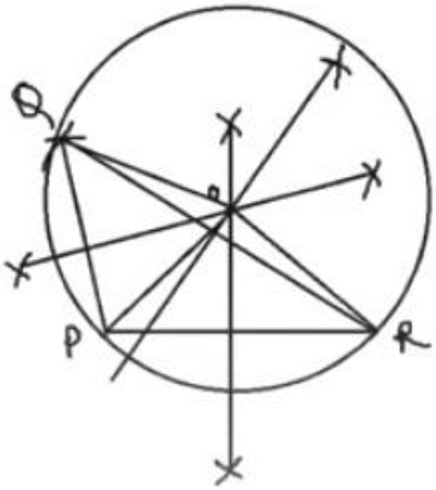
III) Construction of circumcircle

1) All the perpendicular bisectors of △PQR will intersect at one point in the interior of the triangle.

Name that point as O.

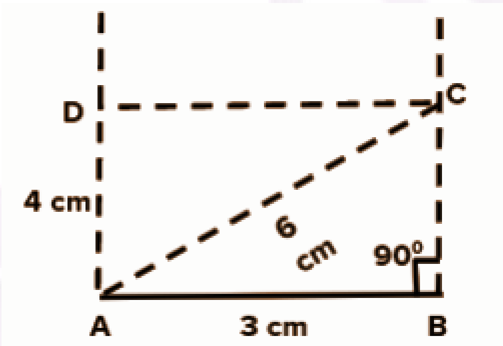
2) Taking point O as the centre and OP as the radius draw a circle. Points Q and R should also lie on this circle.

3) This is the required circumcircle.



OR

[ii]



(i) Draw AB = 3 cm

(ii) Draw ∠B + ∠D = 180^o

(iii) From A, draw an arc of 6 cm cutting BC.

(iv) Draw ∠A = 90^o.

(v) Cut an arc of 4 cm from A.

(vi) ABCD is the required cyclic quadrilateral.

Question 22: [i] Prove that $\sin^2\theta + \cos^2\theta = 1$. **5M**

OR

[ii] Show whether the following is the identity or not: tan θ + sin θ / tan θ – sin θ = sec θ + 1 / sec θ – 1.

Solution:

[i] Consider a triangle of height L, Base B, Hypotenuse H.

sin θ = L / H

cos θ = B / H

$L^2 + B^2 = H^2$

LHS = $\sin^2 \theta + \cos^2 \theta$

= $[L / H]^2 + [B / H]^2$

= $L^2 + B^2 / H^2$

= 1

= RHS

OR

[ii] LHS = (tan θ + sin θ) / (tan θ - sin θ)

= (sin θ / cos θ + sin θ) / (sin θ / cos θ - sin θ)

= (sin θ + sin θ . cos θ) / (sin θ - sin θ . cos θ)

= sin θ (1 + cos θ) / sin θ (1 - cos θ)

= (1 + cos θ) / (1 - cos θ)

= (1 + 1 / sec θ) / (1 - 1 / sec θ)

= (sec θ + 1) / (sec θ - 1)

= RHS

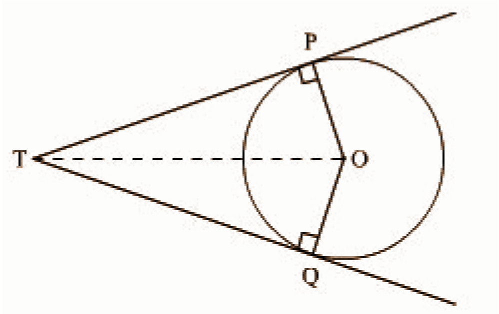
Question 23: [i] Prove that the length of two tangents drawn from an external point to a circle is equal. 6M

OR

[ii] Write the definition of a cyclic quadrilateral. Prove that the sum of pairs of opposite angles of a cyclic quadrilateral is 180°.

Solution:

[i]



Construction: Draw a line segment from centre O to external point T {touching point of two tangents}.
The tangents make a right angle with the radius of the circle.
PO and QO are radii.

So, $\angle OPT = \angle OQT = 90°$

Both the triangles ΔPOT and QOT are right−angled triangles with a common hypotenuse OT.

ΔPOT and ΔQOT

$\angle OPT = OQT = 90°$

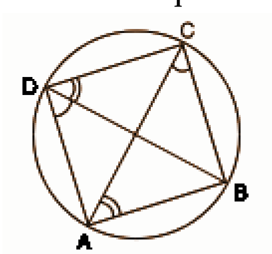
Common hypotenuse OT and OP = OQ [OP and OQ are radii]
So, RHS rule of similarity,

$\Delta POT \sim \Delta QOT$

Hence, OP / OQ = PT / QT = OT / OT
PT / QT = 1
PT = QT

OR

[ii] A cyclic quadrilateral is a quadrilateral which has all its four vertices lying on a circle. It is also sometimes called inscribed quadrilateral.



Construction: Draw AC and DB

Proof: $\angle ACB = \angle ADB$ and $\angle BAC = \angle BDC$ [Angles in the same segment]

$\therefore \angle ACB + \angle BAC = \angle ADB + \angle BDC = \angle ADC$

Adding $\angle ABC$ on both the sides,

$\angle ACB + \angle BAC + \angle ABC = \angle ADC + \angle ABC$

But $\angle ACB + \angle BAC + \angle ABC = 180°$ [Sum of the angles of a triangle]

$\therefore \angle ADC + \angle ABC = 180°$

$\therefore \angle BAD + \angle BCD = 360° - (\angle ADC + \angle ABC) = 180°$.

Question 24: Compute the mean by the short cut method of the following frequency distribution: 6M

Marks Obtained	**10 - 20**	**20 - 30**	**30 - 40**	**40 - 50**	**50 - 60**	**60 - 70**	**70 - 80**
Number of students	**6**	**8**	**13**	**7**	**3**	**2**	**1**

OR

Calculate the cost of living index number for the year 1999 on the basis of 1996 of a medium family from the following information.

Items	**Quantity**	**Price in the Year 1996**	**Price in the year 1999**
A	**8**	**22**	**25**
B	**12**	**35**	**40**
C	**5**	**25**	**30**
D	**15**	**20**	**25**

E	**10**	**15**	**20**

Solution:

[i]

Marks Obtained	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Number of students	6	8	13	7	3	2	1
x_i	15	25	35	45	55	65	75
$f_i x_i$	90	200	455	315	165	130	75

Mean = $\sum f_i x_i / \sum f_i$

= 1430 / 40

= 35.75

OR

[ii]

Items	**Quantity**	**Price in the Year 1996**	**Price in the year 1999**	**Total cost in 1996**	**Total cost in 1999**
A	8	22	25	176	200
B	12	35	40	420	480
C	5	25	30	125	150
D	15	20	25	300	375
E	10	15	20	150	200
				1171	**1405**

Cost of living index in 1999 = [Total Expense in 1999 / Total Expense in 1996] × 100

= [1405 / 1171] * 100

= 119.98

= 120