

# MPBSE Class 10th Maths Question Paper With Solutions 2019

(1 \* 5 = 5)

**Question 1:** Choose the correct option and write it in your answer book.

**Question i:** The H.C.F. of 96 and 404 is :

- (a) 120                      (b) 4                      (c) 10                      (d) 3

**Answer:** (b)

**Question ii:** If  $\alpha$  and  $\beta$  are the zeroes of the quadratic polynomial  $ax^2 + bx + c$ , then the value of  $\alpha\beta$  is -

- (a)  $c / a$                       (b)  $a / c$                       (c)  $-c / a$                       (d)  $-a / c$

**Answer:** (a)

**Question iii:** The zeroes of the polynomial  $x^2 - 3$  will be -

- (a)  $\pm \sqrt{3}$                       (b)  $\pm 3$                       (c) 3                      (d) 9

**Answer:** (a)

**Question iv:** When  $a_1 / a_2 = b_1 / b_2 \neq c_1 / c_2$  then the system of equation  $a_1x + b_1y + c_1 = 0$  and  $a_2x + b_2y + c_2 = 0$

- (a) has two solutions                      (b) has no solution  
(c) has infinitely many solutions                      (d) has unique solution

**Answer:** (b)

**Question v:** Lines  $x - 2y = 0$  and  $3x + 4y - 20 = 0$  are:

- (a) Intersect                      (b) Coincide                      (c) Parallel                      (d) None

**Answer:** (a)

**Question 2: Fill in the blanks.**

**(1 \* 5 = 5)**

- [i] A quadratic equation  $ax^2 + bx + c = 0$  has no real root if – \_\_\_\_\_ ( $D < 0$ )
- [ii] The discriminant of the equation  $3x^2 - 2x + [1 / 3] = 0$  is \_\_\_\_\_ ( $D = 0$ ).
- [iii] In the AP,  $3 / 2, 1 / 2, -1 / 2, 3 / 2$  ..... the common difference  $d$  is \_\_\_\_\_.  
(-1)
- [iv] The sum of the probabilities of all the elementary events of an experiment is.  
\_\_\_\_\_. (1)
- [v] Formula of area of the sector of angle is \_\_\_\_\_. ( $\pi r^2 \theta / 360^\circ$ )

**Question 3: Write true / false in the following.**

**(1 \* 5 = 5)**

- [i] The perpendicular is drawn from the centre of a circle to a chord bisects the chord. [True]
- [ii] All squares are similar. [True]
- [iii] Area of right triangle =  $[1 / 2] \times \text{base} \times \text{altitude}$ . [True]
- [iv] A line intersecting a circle in two points is called a secant. [True]
- [v] The angle of elevation of an object viewed is the angle formed by the line of sight with the horizontal when we lower our head to look at the object. [False]

**Question 4: Write the answers in one word / sentence.**

**(1 \* 5 = 5)**

- [i] What will be the Arithmetic mean of 1, 2, 3, 4, 5?**

Solution: Mean = Sum of the observations / Number of observations  
 $= [1 + 2 + 3 + 4 + 5] / 5$   
 $= 15 / 5$   
 $= 3$

- [ii] Write the formula of the median.**

**Solution:**

$$\text{Median} = l + \left( \frac{\frac{n}{2} - cf}{f} \right) \times h$$

**[iii] Find the value of Probability of an EVENT E + Probability of the EVENT "not E".**

**Solution:**

Probability of an “event E” + probability of the event “not E” is  $P(E) + P(E')$ .  
 The sum of the probabilities of an event and its complement is always equal to 1.  
 Probability of an “event E” + probability of the event “not E” = 1  
 $P(E) + P(E') = 1$

**[iv] Write the formula of volume of a frustum of a cone.**

**Solution:**

Volume of a conical frustum =  $V = (1/3) * \pi * h * (r_1^2 + r_2^2 + (r_1 * r_2))$ .

**[v] How many parallel tangents of a circle?**

**Solution:**

A circle can have exactly two parallel tangents and they must pass through ends of a diameter. That is their point of contacts must be diametrically opposite.

**Question 5: Match the following.**

<b>Column A</b>	<b>Column B</b>
[i] $1 + \cot^2 \theta$	[a] $\sin \theta$
[ii] $\sec \theta$	[b] 0
[iii] $\sin^2 \theta + \cos^2 \theta$	[c] $\sqrt{3}$
[iv] $\tan 60^\circ$	[d] 1
[v] $\cos (90 - \theta)$	[e] $\operatorname{cosec}^2 \theta$
	[f] $1 / \cos \theta$
	[g] $1 / \sqrt{3}$

[i] – e

[ii] – f

[iii] – d

[iv] – c

[v] – a

**Question 6: [i] Find the LCM and HCF of 6 and 20 by the prime factorisation method.**

**OR**

**[ii] Find the H.C.F. of 6, 72 and 120 using the prime factorisation method.**

**Solution:**

$$[i] 6 = 2 \times 3$$

$$20 = 2 \times 2 \times 5$$

$$\text{HCF} = 2$$

$$\text{LCM} = 2 \times 2 \times 3 \times 5 = 60$$

$$[ii] 6 = 2 \times 3$$

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

$$120 = 2 \times 2 \times 2 \times 3 \times 5$$

$$\text{HCF} = 2 \times 3 = 6$$

$$\text{LCM} = 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$$

$$\text{HCF} = 6 \text{ and } \text{LCM} = 360$$

**Question 7: [i] Find a quadratic polynomial, the sum and product of whose zeroes are – 3 and 2 respectively.**

**OR**

**[ii] Divide  $2x^2 + 3x + 1$  by  $x + 2$ .**

**Solution:**

[i] Let  $\alpha$  and  $\beta$  be the two roots of the equation

$$\alpha + \beta = -3$$

$$\alpha \beta = 2$$

A quadratic polynomial can be written in the form:

$$P(x) = x^2 - (\text{sum of roots})x + \text{product of roots}$$

$$P(x) = x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$P(x) = x^2 - (-3)x + 2 = 0$$

$x^2 + 3x + 2 = 0$  is the polynomial.

OR

[ii]

$$\begin{array}{r}
 \text{Quotient} \swarrow 2x - 1 \\
 x + 2 \overline{) 2x^2 + 3x + 1} \\
 \underline{2x^2 + 4x} \quad (-) \quad (-) \\
 -x + 1 \\
 \underline{-x - 2} \quad (+) \quad (+) \\
 3 \quad \swarrow \text{Remainder}
 \end{array}$$

**Question 8: [i] Find the distance between points (2, 3) and (4, 1).**

OR

**[ii] Find the area of a triangle whose vertices (1, -1), (-4, 6) and -3, -5).**

**Solution:**

[i] The distance between two points (2, 3) & (4, 1) is given by

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(4 - 2)^2 + (1 - 3)^2}$$

$$= \sqrt{2^2 + 2^2}$$

$$= \sqrt{8}$$

$$= 2\sqrt{2}$$

OR

$$[ii] \text{ Side } l^2 = (-4-1)^2 + (6-(-1))^2 = (-5)^2 + 7^2 = 25 + 49 = 74$$

$$\text{Side } 2^2 = (-3-1)^2 + (-5 -(-1))^2 = (-4)^2 + (-4)^2 = 16 + 16 = 32$$

$$\text{Side } 3^2 = (-3 -(-4))^2 + (-5 - 6)^2 = 1^2 + (-11)^2 = 1 + 121 = 122$$

$$\text{Side } 1 = \sqrt{74} = 8.60$$

$$\text{Side } 2 = \sqrt{32} = 5.66$$

$$\text{Side } 3 = \sqrt{122} = 11.05$$

$$S = (\text{Side } 1 + \text{Side } 2 + \text{Side } 3) / 2$$

$$= 25.31/2$$

$$= 12.66$$

$$\text{Area} = \sqrt{(s (s - \text{side } 1)(s - \text{side } 2)(s - \text{side } 3))}$$

$$= \sqrt{(12.66 * 4.06 * 7 * 1.61)}$$

$$= \sqrt{579.27}$$

$$= 24$$

**Question 9: [i] Two players Sangeeta and Reshma, play a tennis match. It is known that the probability of Sangeeta winning the match is 0.62. What is the probability of Reshma winning the match?**

**OR**

**[ii] A box contains 3 blue, 2 white and 4 red marbles. If a marble is drawn at random from the box, what is the probability that will be**

**(i) white?**

**(ii) blue?**

**(iii) red?**

**Solution:**

[i] The probability of Sangeeta winning the match is 0.62 and the probability of Reshma winning the match be P(not E) i.e it will be the probability of Sangeeta not winning the match.

$$P(E) + P(\text{not } E) = 1$$

$$\Rightarrow 0.62 + P(\text{not } E) = 1$$

$$\Rightarrow P(\text{not } E) = 0.38$$

Hence the probability of Reshma winning the same match is 0.38.

**OR**

[ii] Total number of marble =  $3 + 2 + 4 = 9$

(i) The probability of getting a white ball =  $2 / 9$

(ii) The probability of getting a blue ball =  $3 / 9 = 1 / 3$

(iii) The probability of getting a red ball =  $4 / 9$ .

**Question 10:** [i] If  $P(E) = 0.05$ , what is the probability of (“not E”)?

**OR**

[ii] One card is drawn from a well-shuffled deck of 52 cards. Calculate the probability, that the card will –

(i) be an ace

(ii) not be an ace.

**Solution:**

[i]  $P(\text{not } E) = 1 - P(E)$

$$= 1 - 0.05$$

$$= 0.95$$

[ii] (i)  $P(\text{getting an ace card}) = \text{Number of aces} / \text{Total number of cards}$

$$= 4 / 52$$

$$= 1 / 13$$

(ii)  $P(\text{not getting an ace card}) = 1 - P(\text{getting an ace card})$

$$= 1 - (1 / 13)$$

$$= 12 / 13$$

**Question 11:** [i] Prove that  $\sqrt{1 + \sin A} / 1 - \sin A = \sec A + \tan A$

**OR**

[ii] Evaluate the following :  $\sin 60^\circ \cdot \cos 30^\circ + \sin 30^\circ \cdot \cos 60^\circ$ .

**Solution:**

[i]  $LHS = \sqrt{1 + \sin A} / \sqrt{1 - \sin A}$

$$= \sqrt{1 + \sin A} \times \sqrt{1 + \sin A} / \sqrt{1 - \sin A} \times \sqrt{1 + \sin A}$$

$$= \sqrt{1 + \sin A}^2 / \sqrt{1 - \sin^2 A}$$

$$= (1 + \sin A) / \sqrt{\cos^2 A}$$

$$\begin{aligned}
&= (1 + \sin A) / \cos A \\
&= 1 / \cos A + \sin A / \cos A \\
&= \sec A + \tan A \\
&= \text{RHS}
\end{aligned}$$

**OR**

$$\begin{aligned}
&[\text{ii}] \sin 60^\circ \cdot \cos 30^\circ + \sin 30^\circ \cdot \cos 60^\circ \\
&= \sin 60 \cdot \cos (90 - 60) + \sin 30 \cdot \cos (90 - 30) \\
&= \sin 60 \cdot \sin 60 + \sin 30 \cdot \sin 30 \\
&= \sin^2 60 + \sin^2 30 \\
&= \left\{ \frac{\sqrt{3}}{2} \right\}^2 + \left[ \frac{1}{2} \right]^2 \\
&= \left[ \frac{3}{4} \right] + \left[ \frac{1}{4} \right] \\
&= (3 + 1) / 4 \\
&= 4 / 4 \\
&= 1
\end{aligned}$$

**Question 12: [i] Find the value of K, if the points A (2, 3), B (4, K) and C (6, -3) are collinear.**

**OR**

**[ii] Find the ratio in which the y-axis divides the line segment joining the points (5, -6) and (-1, -4) also find the point of intersection.**

**Solution:**

[i] The given points are A (2, 3), B (4, k) and C (6, -3).

Here,  $(x_1 = 2, y_1 = 3)$ ,  $(x_2 = 4, y_2 = k)$  and  $(x_3 = 6, y_3 = -3)$ .

It is given that the points A, B and C are collinear.

Then,

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

$$2(k + 3) + 4(-3 - 3) + 6(3 - k) = 0$$

$$2k + 6 - 24 + 18 - 6k = 0$$

$$-4k = 0$$

$$k = 0$$

**OR**



[ii] Let the coordinates of the point be  $(0, y)$  and the  $y$ -axis divides the line segment in the ratio  $m:n$ .

So,

$$x = [mx_2 + nx_1] / [m + n]$$

$$x = 0 \text{ (given)}$$

$$0 = m * 5 + n * -1 / [m + n]$$

$$0 = 5m - n$$

$$5m = n$$

$$m / n = 1 / 5$$

$$m : n = 1 : 5$$

$$y = [my_2 + ny_1] / [m + n]$$

$$y = 1 * -6 + 5 * -4 / 1 + 5$$

$$y = -6 - 20 / 6$$

$$y = -26 / 6$$

$$y = -13 / 3$$

The coordinates of the point are  $(0, -13 / 3)$ .

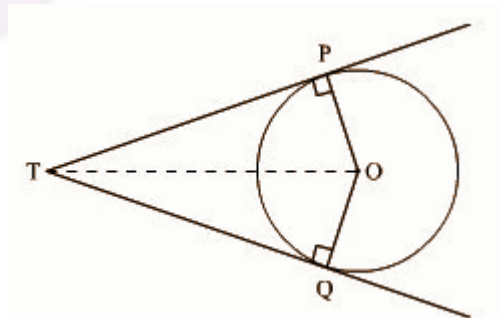
**Question 13: [i] The length of tangents drawn from an external point to a circle is equal.**

**OR**

**[ii] Prove that the tangents drawn at the ends of a diameter of a circle are parallel.**

**Solution:**

[i]



Let the two tangents be  $PT$  and  $QT$ . They are drawn to the circle of centre  $O$  as shown in the figure.

The tangents PT and QT touch to the circle at P and Q respectively.

To prove: Length of PT = Length of QT

Construction:- Draw a line segment OT, from centre O to external point T  
{touching point of two tangents}.

Now  $\Delta POT$  and  $\Delta QOT$  are obtained.

The tangents make right angles with the radius of the circle.

PO and QO are radii.

So,  $\angle OPT = \angle OQT = 90^\circ$

Now, it is clear that both the triangles  $\Delta POT$  and  $\Delta QOT$  are right-angled triangles and there exists a common hypotenuse OT.

In  $\Delta POT$  and  $\Delta QOT$ ,

$\angle OPT = \angle OQT = 90^\circ$

Common hypotenuse OT and  $OP = OQ$  [OP and OQ are radii]

So, by RHS rule of similarity,

$\Delta POT \sim \Delta QOT$

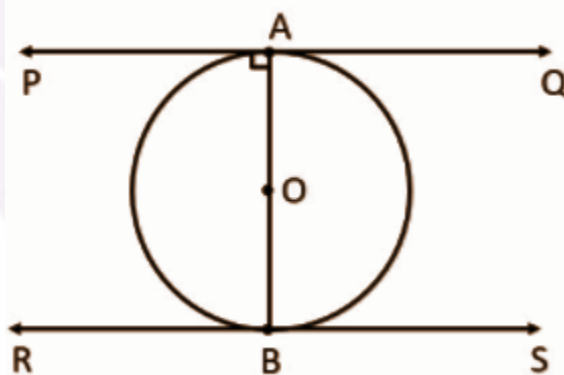
Hence,  $OP / OQ = PT / QT = OT / OT$

$PT / QT = 1$

$PT = QT$

OR

[ii]



Given: A circle with centre O and diameter AB.

Let PQ be the tangent at point A & RS be the tangent at point B.

To Prove:  $PQ \parallel RS$

Since PQ is tangent at point A,  $OA \perp PQ$  (Tangent at any point of a circle is perpendicular to the radius through the point of contact)

$$\angle OAP = 90^\circ \dots (i)$$

Similarly

RS is a tangent at point B,  $OB \perp RS$  (Tangent at any point of a circle is perpendicular to the radius through the point of contact)

$$\angle OBS = 90^\circ \dots (ii)$$

From (i) and (ii)

$$\angle OAP = 90^\circ \text{ and } \angle OBS = 90^\circ$$

$$\angle OAP = \angle OBS$$

$$\angle BAP = \angle ABS$$

For lines PQ and RS and transversal AB,  $\angle BAP = \angle ABS$  i.e., both alternate angles are equal.

So, lines are parallel

$$\therefore PQ \parallel RS$$

**Question 14: [i] Find the area of the sector of a circle with radius 4 cm and angle  $30^\circ$ . Also, find the area of the corresponding major sector. (Use  $\pi = 3.14$ )**

**OR**

**[ii] Find the area of a sector of a circle with radius 6 cm whose angle of the sector is  $60^\circ$ .**

**Solution:**

$$[i] \text{ Area of Sector} = \left[ \frac{\theta}{360} \right] \times \pi \times r^2$$

$$\theta = 30$$

$$r = 4 \text{ cm}$$

Area of Sector.....

$$= \left[ \frac{30}{360} \right] \times \left[ \frac{22}{7} \right] \times (4)^2$$

$$= \left[ \frac{1}{12} \right] \times \left[ \frac{22}{7} \right] \times 16$$

$$= 352 / 84$$

$$= 4.19 \text{ cm}^2$$

Area of the major corresponding sector = Area of Circle - Area of Sector

$$\begin{aligned}
&= \pi r^2 - 4.19 \text{ cm}^2 \\
&= (22 / 7 \times 16) - 4.19 \text{ cm}^2 \\
&= 50.29 - 4.19 \text{ cm}^2 \\
&= 46.09 \text{ cm}^2
\end{aligned}$$

**OR**

[ii] Area of sector =  $[\Phi / 360^\circ] * \pi r^2$   
 $\Phi$  = angle of sector =  $60^\circ$   
Area =  $6 * 6 * [22 / 7] * 6$   
 $= 792 / 42$   
 $= 18.85$   
Area of sector =  $18.85 \text{ cm}^2$

**Question 15: [i] Prove that  $5 - \sqrt{3}$  is an irrational number.**

**OR**

**[ii] Show that any positive odd integer is of the form  $4q + 1$  or  $4q + 3$  where  $q$  is an integer.**

**Solution:**

[i] Let us assume the given number be rational and we will write the given number in  $p/q$  form

$$\Rightarrow 5 - \sqrt{3} = p / q$$

$$\Rightarrow \sqrt{3} = [5q - p] / q$$

LHS is irrational and RHS is rational, which is not possible.

This is a contradiction.

Hence our assumption that the given number is rational is false.

$$\Rightarrow 5 - \sqrt{3} \text{ is irrational.}$$

**OR**

[ii] Let  $a$  be any positive integer and  $b = 4$ . Then by Euclid's algorithm,  
 $a = 4q + r$  for some integer  $q \geq 0$ , and  $r = 0, 1, 2, 3$

Case 1: When  $r = 0$ ,

$$a = 4q$$

Case 2: When  $r = 1$ ,

$$a = 4q + 1$$

Case 3: When  $r = 2$ ,

$$a = 4q + 2$$

When  $r = 3$ ,

$$a = 4q + 3$$

Since,  $4q$  and  $4q + 2$  are multiples of 2, they are even numbers.

Now,  $4q$  that is  $2(2q)$  is an even number.

$\therefore 4q + 1$  is an odd number.

$4q + 2$  that is  $2(2q + 1)$  which is also an even number.

$\therefore (4q + 2) + 1 = 4q + 3$  is an odd number.

Thus, it is proved that any odd integer can be written in the form  $4q + 1$  or  $4q + 3$  where  $q$  is some integer.

**Question 16: [i] Find the zeros of the quadratic polynomial  $x^2 + 7x + 10$  and verify the relationship between the zeros and the coefficients.**

**OR**

**[ii] Divide  $3x^3 + x^2 + 2x + 5$  by  $1 + 2x + x^2$ .**

**Solution:**

[i] Let  $p(x) = x^2 + 7x + 10$

To zeroes of the polynomial,  $p(x) = 0$

$$\Rightarrow x^2 + 7x + 10 = 0$$

$$\Rightarrow x^2 + 2x + 5x + 10 = 0$$

$$\Rightarrow x(x + 2) + 5(x + 2) = 0$$

$$\Rightarrow (x + 2)(x + 5) = 0$$

$$\Rightarrow x + 2 = 0 \text{ or } x + 5 = 0$$

$$\Rightarrow x = -2 \text{ or } x = -5$$

The zeroes of  $p(x)$  are  $-2, -5$ .

i) Compare  $x^2 + 7x + 10$  with  $ax^2 + bx + c$ ,

$$a = 1, b = 7, c = 10$$

Sum of the zeroes =

$$-2 - 5 = (-b / a)$$

$$-7 = -7 / 1$$

$$-7 = -7$$

ii) Product of the zeroes

$$-2 * -5 = c / a$$

$$10 = (10) / (1)$$

$$10 = 10$$

OR

[ii]

$$\begin{array}{r}
 \phantom{x^2 + 2x + 1} \overline{3x - 5} \text{ ← Quotient} \\
 x^2 + 2x + 1 \overline{) 3x^3 + x^2 + 2x + 5} \\
 \underline{3x^3 + 6x^2 + 3x} \phantom{+ 5} \\
 (-) \quad (-) \quad (-) \\
 \hline
 -5x^2 - x + 5 \\
 \underline{-5x^2 - 10x - 5} \\
 (+) \quad (+) \quad (+) \\
 \hline
 9x + 10 \text{ ← Remainder}
 \end{array}$$

**Question 17:** [i] If the sum of the first 14 terms of an A.P. is 1050 and its first term is 10. find the 20<sup>th</sup> term.

OR

[ii] Find the 31<sup>st</sup> term of an A.P. whose 11<sup>th</sup> term is 38 and 16<sup>th</sup> term is 73.

**Solution:**

$$[i] S_{14} = 1050$$

$$n = 14, a = 10$$

$$S_n = [n / 2] [2a + (n - 1)d]$$

$$1050 = [14 / 2] [20 + 13d]$$

$$1050 = 140 + 91d$$

$$910 = 91d$$

$$d = 10$$

Therefore,

$$a_{20} = 10 + (20 - 1) \times 10$$

$$= 200$$

The 20<sup>th</sup> term is 200.

**OR**

$$[ii] a_{11} = 38 \text{ and } a_{16} = 73$$

$$a_n = a + (n - 1)d$$

$$a_{11} = a + 10d = 38 \text{ and } a_{16} = a + 15d = 73$$

Subtracting 11<sup>th</sup> term from 16<sup>th</sup> term,

$$a + 15d - a - 10d = 73 - 38$$

$$5d = 35$$

$$d = 7$$

Substituting the value of d in 11<sup>th</sup> term,

$$a + 10 \times 7 = 38$$

$$a + 70 = 38$$

$$a = 38 - 70$$

$$= -32$$

$$a_{31} = a + 30d$$

$$= -32 + 30 \times 7$$

$$= -32 + 210$$

$$= 178$$

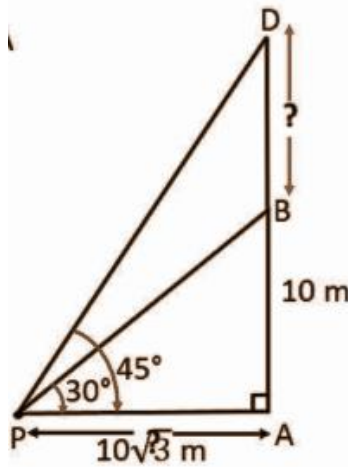
**Question 18: [i] From a point P on the ground the angle of elevation of the top of a 10-meter tall building is 30°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45°. Find the length of the flagstaff and the distance of the building from point P.**

**OR**

**[ii] The angle of depression of the top and the bottom of an 8 m. tall building from the top of a multi-storeyed building is 30° and 45° respectively. Find the height of the multi-storeyed building and distance between the two buildings.**

**Solution:**

**[i]**



In  $\triangle BAP$ ,

$$\tan 30^\circ = BA / PA$$

$$1 / \sqrt{3} = 10 / PA$$

$$1 * PA = 10 * \sqrt{3}$$

$$PA = 10\sqrt{3} \text{ m}$$

$$PA = 10 * 1.732$$

$$= 17.32 \text{ m}$$

IN  $\triangle DPA$ ,

$$\tan 45^\circ = AD / AP$$

$$1 = DB + BA / 10\sqrt{3}$$

$$\Rightarrow 10\sqrt{3} = 10 + BD$$

$$\Rightarrow BD = 10\sqrt{3} - 10$$

$$\Rightarrow BD = 10 [\sqrt{3} - 1]$$

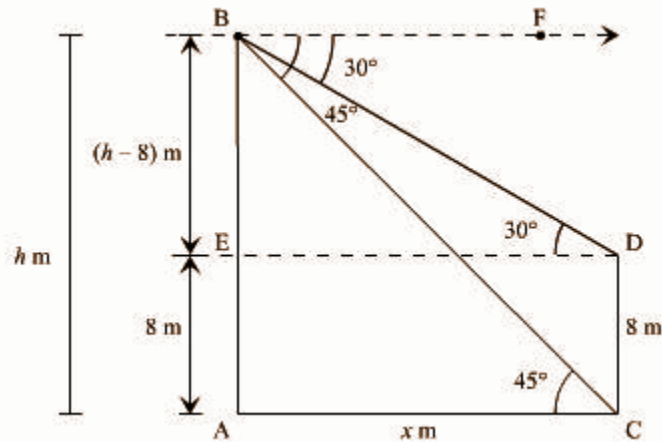
$$BD = 10 * (1.732 - 1)$$

$$= 7.32 \text{ m}$$

**OR**

[ii]





Let AB and CD be the multi-storeyed building and the building respectively.

Let the height of the multi-storeyed building = 'h' m and the distance between the two buildings = 'x' m.

$$AE = CD = 8 \text{ m}$$

$$BE = AB - AE = (h - 8) \text{ m and } AC = DE = x \text{ m [Given]}$$

$$\angle FBD = \angle BDE = 30^\circ \text{ (Corresponding angles)}$$

$$\angle FBC = \angle BCA = 45^\circ \text{ (Corresponding angles)}$$

In  $\triangle ACB$ ,

$$\tan 45^\circ = AB / AC$$

$$1 = h / x$$

$$x = h \text{ ---- (i)}$$

In  $\triangle BDE$ ,

$$\tan 30^\circ = BE / ED$$

$$1 / \sqrt{3} = [h - 8] / x$$

$$x = \sqrt{3} [h - 8] \text{ ----- (ii)}$$

From (i) and (ii),

$$h = \sqrt{3}h - 8\sqrt{3}$$

$$\sqrt{3}h - h = 8\sqrt{3}$$

$$h = 8\sqrt{3} / \sqrt{3} - 1$$

$$h = [8\sqrt{3} / \sqrt{3} - 1] * [\sqrt{3} + 1 / \sqrt{3} + 1]$$

$$= 4\sqrt{3} (\sqrt{3} + 1)$$

$$= 12 + 4\sqrt{3} \text{ m}$$

Distance between the two-building,  $x = 12 + 4\sqrt{3} \text{ m}$ .

**Question 19: [i] Use the elimination method to find all possible solutions of the following pair of linear equations :**

$$2x + 3y = 8$$

$$4x + 6y = 7$$

**OR**

**[ii] The cost of 5 oranges and 3 apples is Rs. 35 and the cost of 2 oranges and 4 apples are Rs. 28. Let us find the cost of an orange and an apple.**

**Solution:**

$$[i] \ 2x + 3y = 8 \ \text{.....(1)}$$

$$4x + 6y = 7 \ \text{.....(2)}$$

Step 1: Multiply equation (1) by 2 and equation (2) by 1 to make the coefficients of x equal.

Then we get the equations as :

$$4x + 6y = 16 \ \text{.....(3)}$$

$$4x + 6y = 7 \ \text{.....(4)}$$

Step 2 : Subtracting equation (4) from equation (3),

$$(4x - 4x) + (6y - 6y) = 16 - 7$$

$0 = 9$ , which is a false statement.

Therefore, the pair of equations has no solution.

**OR**

[ii] Let the cost of 1 orange be x and the cost of 1 apple be y.

According to question

$$5x + 3y = 35 \ \text{--- -- (i)}$$

$$2x + 4y = 28 \ \text{--- -- (ii)}$$

Multiply by 2 in equation (i) and by 5 in equation (ii),

$$\Rightarrow 10x + 6y = 70 \ \text{--- -- (i)}$$

$$\Rightarrow 10x + 20y = 140 \ \text{--- -- (ii)}$$

By subtraction equation (i) from (ii)

$$\Rightarrow 14y = 70$$

$$\Rightarrow y = 70 / 14$$

$$\Rightarrow y = 5$$

Putting the value of y in equation (i)

$$\Rightarrow 5x + (3 * 5) = 35$$

$$\Rightarrow 5x = 35 - 15$$

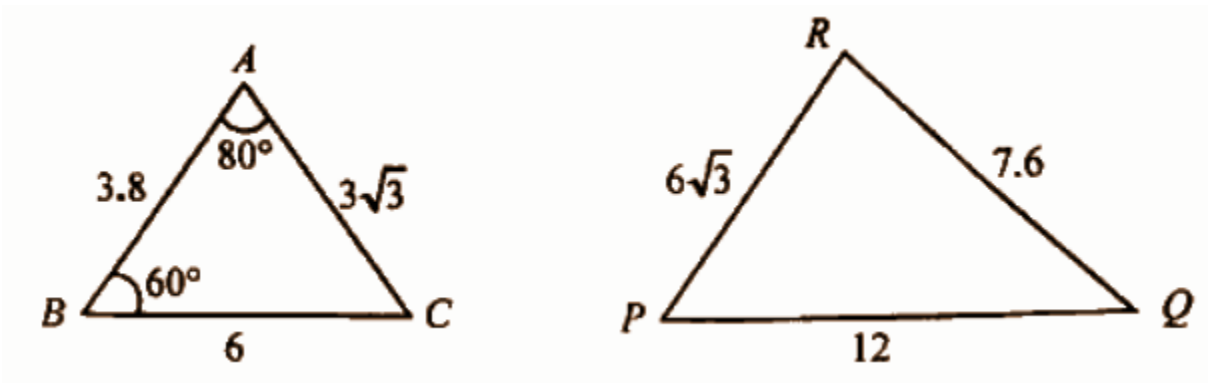
$$\Rightarrow x = 20 / 5$$

$$\Rightarrow x = 4$$

Hence

The cost of an orange = ₹4 and the cost of an apple = ₹5.

**Question 20: [i] Observe the figure and find  $\angle P$ .**



**OR**

**[ii]  $\triangle ABC$  is an Isosceles triangle right angled at  $C$ . Prove that  $AB^2 = 2 AC^2$ .**

**Solution:**

[i] In  $\triangle ABC$  and  $\triangle PQR$ ,

$$AB / QR = AC / RP = BC / PQ = 1/2 \text{ (each).}$$

If in two triangles, the ratio of sides is equal then triangles are similar.

$$\triangle ABC \sim \triangle RQP$$

$$\angle A = \angle R = 80^\circ$$

$$\angle B = \angle Q = 60^\circ$$

$$\angle C = \angle P = 180^\circ - (80^\circ + 60^\circ)$$

$$= 180^\circ - 140^\circ$$

$$= 40^\circ$$

**OR**

[ii] Given:  $\triangle ABC$  is a right triangle.

Also,  $\triangle ABC$  is an isosceles triangle.

To prove:  $AB^2 = 2 AC^2$

Here,

Hypotenuse = AB

Also, as it is given that,  $\triangle ABC$  is isosceles,  $AC = BC$  [equal sides of isosceles  $\triangle$ ]

Using Pythagoras theorem,

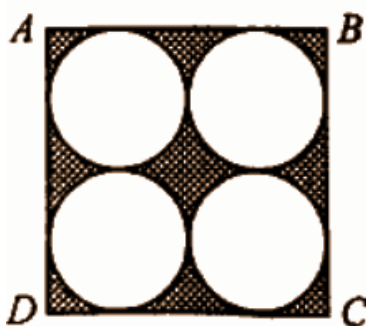
In  $\triangle ABC$ ,

$$AB^2 = AC^2 + BC^2$$

$$AB^2 = AC^2 + AC^2 [AC = BC]$$

$$AB^2 = 2 AC^2$$

**Question 21:** [i] Find the area of the shaded region in the given figure where ABCD is a square of side 14 cm.



OR

[ii] In a circle of radius 21 cm, an arc subtends an angle of  $60^\circ$  at the centre find;

(i) the length of the arc

(ii) area of the sector.

**Solution:**

$$[i] \text{ Area of square } ABCD = a^2$$

$$= 14 \times 14$$

$$= 196 \text{ cm}^2$$

$$\text{Diameter of each circle} = \text{side of square} / 2$$

$$= 14 / 2$$

$$= 7 \text{ cm}$$

$$\text{So, the radius of each circle} = 7 / 2 \text{ cm}$$

$$\text{So, area of 4 circles} = 4\pi r^2$$

$$= 4 \times [22 / 7] \times [7 / 2] \times [7 / 2]$$

$$= 4 \times [154 / 4] \text{ cm}^2$$

$$= 154 \text{ cm}^2$$

$$\begin{aligned}
 \text{Area of shaded region} &= \text{Area of a square} - \text{Area of 4 circles} \\
 &= 196 - 154 \\
 &= 42 \text{ cm}^2
 \end{aligned}$$

Hence, the area of the shaded region is  $42 \text{ cm}^2$ .

**OR**

[ii] Radius (r) of circle = 21 cm

The angle subtended by the given arc =  $60^\circ$

Length of an arc of a sector of angle =  $(2\pi r\theta / 360^\circ)$

Length of arc ACB =  $[60^\circ / 360^\circ] * 2 * (22 / 7) * (21)$

$$= [1 / 6] * 2 * 22 * 3$$

$$= 22 \text{ cm}$$

Area of sector OACB =  $(\pi r^2\theta / 360^\circ)$

$$= [60^\circ / 360^\circ] * (22 / 7) * (21)^2$$

$$= [1 / 2] * 22 * 3 * 7$$

$$= 11 * 3 * 7$$

$$= 231 \text{ cm}^2$$

**Question 22: [i] Find the roots of the following equation:  $x + [1 / x] = 3$ ,  $x \neq 0$ .**

**OR**

**[ii] Find two consecutive odd positive integers, the sum of whose squares is 290.**

**Solution:**

$$[i] x + [1 / x] = 3$$

$$x^2 + 1 = 3x$$

$$x^2 - 3x + 1 = 0$$

Here  $a = 1$ ,  $b = -3$ ,  $c = 1$

By quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{9 - 4 * 1 * 1}}{2 * 1}$$

$$= 3 \pm \sqrt{5} / 2$$

$$x = 3 + \sqrt{5} / 2 \text{ and } x = 3 - \sqrt{5} / 2$$

**OR**

[ii] Let the two consecutive odd positive integers be  $x$  and  $x + 2$

The sum of the squares is 290.

$$\Rightarrow x^2 + (x + 2)^2 = 290$$

$$\Rightarrow x^2 + x^2 + 4x + 4 = 290$$

$$\Rightarrow 2x^2 + 4x + 4 - 290 = 0$$

$$\Rightarrow 2x^2 + 4x - 286 = 0$$

Dividing by 2,

$$\Rightarrow x^2 + 2x - 143 = 0$$

$$\Rightarrow x^2 + 13x - 11x - 143 = 0$$

$$\Rightarrow x(x + 13) - 11(x + 13) = 0$$

$$\Rightarrow (x - 11)(x + 13) = 0$$

$$\Rightarrow x = -13, 11$$

In the question, we are given with positive integers.

Hence the number is 11 and the consecutive odd number is 13.

**Question 23: [i] If  $\sin A = 3 / 4$ , calculate  $\cos A$  and  $\tan A$ .**

**OR**

**[ii] Evaluate the following:  $2 \tan^2 - 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$ .**

**Solution:**

[i] The three fundamental trigonometric ratios are stated as:

$\sin \theta = \text{Opposite side} / \text{Hypotenuse side}$

$\tan \theta = \text{Opposite side} / \text{Adjacent side}$

$\cos \theta = \text{Adjacent side} / \text{Hypotenuse side}$

$$\sin A = 3 / 4$$

To calculate the length of the third side of the triangle, use the Pythagorean theorem to determine the two remaining trigonometric ratios.

The length of the third side will be:

$$a = \sqrt{4^2 - 3^2}$$

$$a = \sqrt{7}$$

Therefore:

$$\tan A = 3 / \sqrt{7}$$

$$\cos A = \sqrt{7} / 4$$

**OR**

$$[ii] 2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$$

$$\tan 45^\circ = 1$$

$$\cos 30^\circ = (\sqrt{3} / 2)$$

$$\sin 60^\circ = (\sqrt{3} / 2)$$

So put the values in the equation,

$$\begin{aligned} & 2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ \\ &= 2 (1)^2 + (\sqrt{3} / 2)^2 - (\sqrt{3} / 2)^2 \\ &= 2 \times 1 + (3 / 4) - (3 / 4) \\ &= 2 + (3 - 3) / 4 \\ &= 2 + (0 / 4) \\ &= 2 + 0 \\ &= 2 \\ & 2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ = 2 \end{aligned}$$

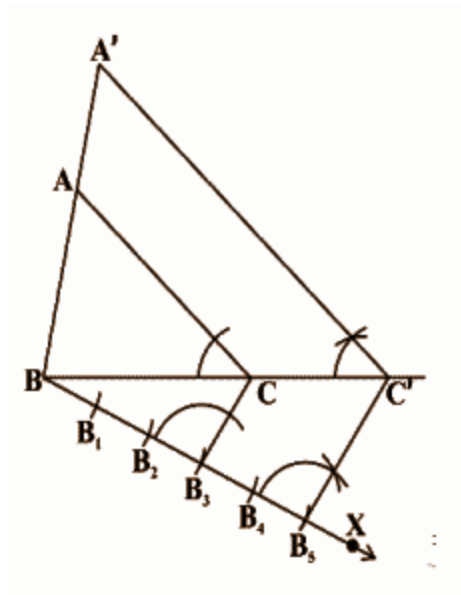
**Question 24: [i] Construct a triangle similar to a given triangle ABC with its side equal to  $5 / 3$  of the corresponding sides of the triangle ABC.**

**OR**

**[ii] Draw a circle with the help of a bangle. Take a point outside the circle. Construct the pair of tangents from this point to the circle.**

**Solution:**

**[i]**



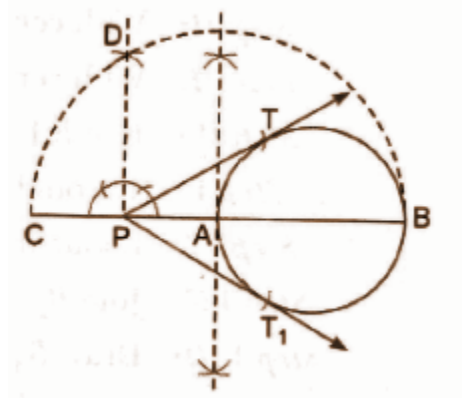
**Steps of construction:**

- (i) Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.
  - (ii) From B cut off 5 arcs  $B_1, B_2, B_3, B_4, B_5$  on BX such that  $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$ .
  - (iii) Join  $B_3$  to C and draw a line through  $B_5$  parallel to  $B_3C$ , intersecting the extended line segment BC at  $C'$ .
  - (iv) Draw a line through  $C'$  parallel to CA intersecting the extended line segment BA at  $A'$ .
- Then  $A'B'C'$  is the required triangle.

**OR**

[ii]





**Steps of construction:**

- (i) Draw a circle with the help of a bangle.
  - (ii) Let P be the external point from where the tangents are to be drawn to the given circle. Through P, draw a secant PAB to intersect the circle at A and B (say).
  - (iii) Produce PA to C, such that  $AP = PC$  that is P is the midpoint of AC.
  - (iv) Draw a semicircle with BC as diameter.
  - (v) Draw  $PD \perp BC$ , intersecting the semicircle at D.
  - (vi) With P as centre and PD as radius, draw two arcs to intersect the given circle at T and  $T_1$ .
  - (vii) Join PT and  $PT_1$ .
- So, PT and  $PT_1$  are the required tangents.

**Question 25: [i] The radii of the ends of a frustum of a cone 45 cm high are 28 cm and 7 cm respectively, find the volume.**

**OR**

**[ii] A hemispherical tank full of water is emptied by a pipe at the rate of 3 (4 / 7) litres per second. How much time will it take to empty half the tank, if it is 3 m in diameter?**

**Solution:**

$$\begin{aligned}
 \text{[i] Volume of the frustum of the cone} &= \left[ \frac{1}{3} \right] \pi h (r_1^2 + r_2^2 + r_1 r_2) \\
 &= \left( \frac{1}{3} \right) * \left( \frac{22}{7} \right) * 45 * [28^2 + 7^2 + 28 * 7] \\
 &= 48510 \text{ cm}^3
 \end{aligned}$$

**OR**

[ii] The rate of flow of the pipe is  $25 / 7$  litres per second.

The diameter of the base of the tank is 3 metres.

The radius of the base of the tank is:  $= (\text{diameter} / 2)$

$$= (3 / 2)$$

$$= 1.5 \text{ metres}$$

$$\text{Volume of the tank} = [2 / 3] \pi r^3$$

$$= [2 / 3] \times [22 / 7] \times (1.5)^3$$

$$= 7.071 \text{ metre}^3$$

$$= 7071 \text{ litres [Term of unit conversion is } 1000 \text{ litres} = 1 \text{ metre cube]}$$

$$\text{Half capacity of the tank} = \text{Total capacity of the tank} / 2$$

$$= 7071 / 2$$

$$= 3535.5 \text{ litres}$$

The pipe can empty  $[25 / 7]$  litres in 1 second.

$$1 \text{ litre in} = 1 \times [7 / 25] \text{ seconds}$$

Time required to empty 3535.5 litres is,

$$= 3535.5 \times [7 / 25]$$

$$= 989.94 \text{ seconds}$$

$$= 16.5 \text{ minutes}$$

So, the pipe will take approximately 16.5 minutes to empty half of the tank.

**Question 26: [i] A survey conducted on 20 households in a locality by a group of students resulted in the following frequency table for the number of a family member in a household:**

Family size	1 - 3	3 - 5	5 - 7	7 - 9	9 - 11
Number of families	7	8	2	2	1

**OR**

**[ii] In given data, find the mean.**

Class Interval	Number of students
----------------	--------------------

<b>10 - 25</b>	<b>2</b>
<b>25 - 40</b>	<b>3</b>
<b>40 - 55</b>	<b>7</b>
<b>55 - 70</b>	<b>6</b>
<b>70 - 85</b>	<b>6</b>
<b>85 - 100</b>	<b>6</b>

**Solution:**

[i] Here the maximum class frequency is 8, and the class corresponding to this frequency is 3 – 5.

So, the modal class is 3 – 5.

Modal class = 3 – 5

Lower limit (l) of modal class = 3

Class size (h) = 2

Frequency ( $f_1$ ) of the modal class = 8

Frequency ( $f_0$ ) of class preceding the modal class = 7

Frequency ( $f_2$ ) of class succeeding the modal class = 2.

On substituting the values in the formula,

$$\text{Mode} = l + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$= 3 + [(8 - 7) / [2 * 8 - 7 - 2]] * 2$$

$$= 3 + (2 / 7)$$

$$= 3.286$$

Therefore, the mode of the data above is 3.286.

**OR**

[ii]

<b>Class Interval</b>	<b>Number of students (<math>f_i</math>)</b>	<b>Midpoint (<math>x_i</math>)</b>	<b><math>f_i x_i</math></b>
10 - 25	2	17.5	35

25 - 40	3	32.5	97.5
40 - 55	7	47.5	332.5
55 - 70	6	62.5	375
70 - 85	6	77.5	465
85 - 100	6	92.5	555

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

$$= 1860 / 30$$

$$= 62$$

