PSEB Class 10 Mathematics Question Paper 2017 Paper A with Solutions

Paper Code - 04/A
PART - A

1. (i) For which value of p does the pair of equations given below have a unique solution.
$4x + py + 8 = 0$
$2x + 2y + 2 = 0$

Solution:
Given,
$4x + py + 8 = 0$
$2x + 2y + 2 = 0$
Comparing with the standard form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,
$a_1 = 4, b_1 = p, c_1 = 8$
$a_2 = 2, b_2 = 2, c_2 = 2$
Condition for a unique solution is:
$a_1/a_2 \neq b_1/b_2$
$4/2 \neq p/2$
$2 \neq p/2$
$p \neq 4$
Hence, p takes all the real values except 4.

(ii) For AP: 3/2, ½, -½, -3/2,.... Write the first term 'a' and common difference 'd'.

Solution:
Given,
3/2, ½, -½, -3/2,....
First term = a = 3/2
Common difference = d = (½) - (3/2)
= (1 - 3)/2
= -2/2
= -1
Therefore, a = 3/2 and d = -1.

(iii) Write the definition of Pythagoras theorem.

Solution:
Pythagoras theorem states that: "In a right-angled triangle, the square of the hypotenuse side is equal to the sum of squares of the other two sides".

(iv) Fill in the blank:
______ - $\cot^2\theta = 1$

Solution:

cosec²θ - cot²θ = 1

(v) Find the volume of a sphere whose radius is 3 cm.

Solution:
Given,
Radius of sphere = r = 3 cm
Volume of sphere = $(4/3)\pi r^3$
= (4/3) × (22/7) × 3 × 3 × 3
= (36 × 22)/7
= 792/7
= 113.14 cm^3

PART - B

2. Find the LCM and HCF of 6 and 20 by prime factorisation method.

Solution:
Prime factorisation of 6:
6 = 2 × 3
Prime factorisation of 20:
20 = 2 × 2 × 5
LCM = 2 × 2 × 3 × 5 = 60
HCF = 2

3. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by first polynomial.
$t^2 - 3$, $t^4 + 3t^3 - 2t^2 - 9t - 12$

Solution:

$$
\begin{array}{r|rrrrr}
 & t^2 & +3t & +1 & & \\
\hline
t^2-3 & t^4 & +3t^3 & -2t^2 & -9t & -12 \\
 - & t^4 & +0t^3 & -3t^2 & & \\
\hline
 & & 3t^3 & +t^2 & -9t & -12 \\
 & - & 3t^3 & +0t^2 & -9t & \\
\hline
 & & & t^2 & +0t & -12 \\
 & & - & t^2 & +0t & -3 \\
\hline
 & & & & & -9
\end{array}
$$

Here, the remainder is not equal to 0.

Hence, the first polynomial is not a factor of the second polynomial.

4. Find such value of k for quadratic equation $kx^2 - 2kx + 6 = 0$ so that they have two equal roots.

Solution:
Given,
$kx^2 - 2kx + 6 = 0$
Comparing with the standard form $ax^2 + bx + c = 0$,
$a = k, b = -2k, c = 6$
Condition of two equal roots of a quadratic equation is:
$b^2 - 4ac = 0$
$(-2k)^2 - 4(k)(6) = 0$
$4k^2 - 24k = 0$
$4k(k - 6) = 0$
$k = 0, k = 6$

5. Show that $a_1, a_2, a_3, \ldots a_n$ form an AP where $a_n = 3 + 4n$.

Solution:
Given,
$a_n = 3 + 4n$
Substituting n = 1,
$a_1 = 3 + 4(1) = 3 + 4 = 7$
$a_2 = 3 + 4(2) = 3 + 8 = 11$
$a_3 = 3 + 4(3) = 3 + 12 = 15$
The series is: 7, 11, 15,....
This is an AP with a = 7 and d = 4.

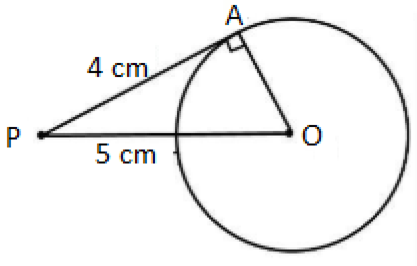
OR

For AP: 7, 11, 15, 19,.... Find the sum of the first 15 terms.

Solution:
Given,
7, 11, 15, 19,....
First term = a = 7
Common difference = d = 11 - 7 = 4
n = 15
Sum of first n terms
$S_n = n/2\ [2a + (n - 1)d]$
$S_{15} = (15/2) \times [2 \times 7 + (15 - 1)4]$
$= (15/2) \times [14 + 14 \times 4]$
$= (15/2) \times (14 + 56)$
$= (15/2) \times 70$
$= 15 \times 35$
$= 525$
Hence, the sum of the first 15 terms of the given AP is 525.

6. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Solution:
Let PA be the tangent drawn from an external point P to the circle with centre O.
We know that the radius (OA) is perpendicular to the tangent (PA) through the point of contact (A).

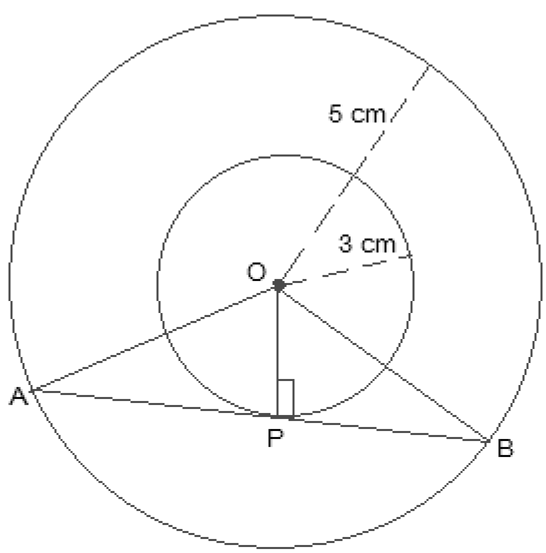


In right triangle OAP,
$OP^2 = OA^2 + PA^2$
$(5)^2 = OA^2 + (4)^2$
$OA^2 = 25 - 16$
$= 9$
$OA = 3$
Therefore, the radius of the circle is 3 cm.

OR

Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Solution:
Let O be the centre of two concentric circles.
AB be the chord of the larger circle which touches the smaller circle at P.

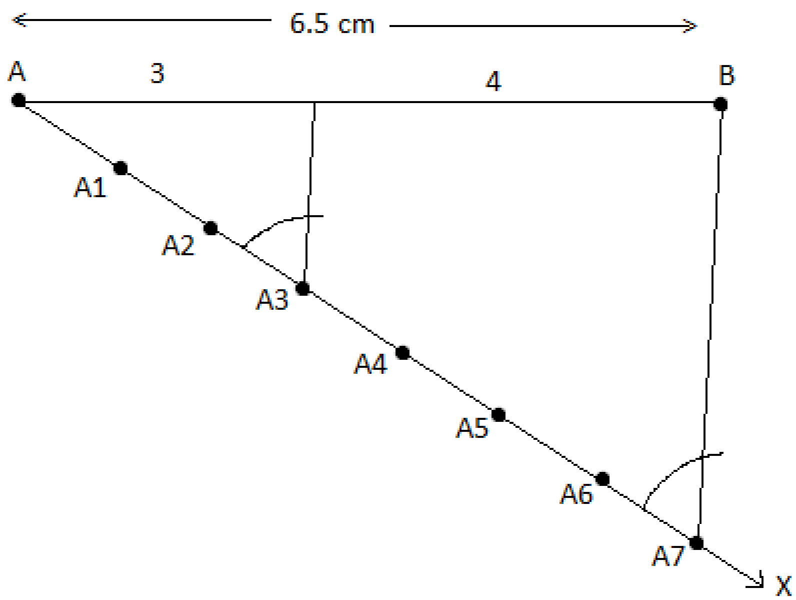


OP = Radius of smaller circle = 3 cm
OA = OB = Radius of the larger circle = 5 cm
$\therefore$ OP $\perp$ AB
In right triangle OPA,

By Pythagoras theorem,
$OA^2 = AP^2 + OP^2$
$5^2 = AP^2 + 3^2$
$\Rightarrow AP^2 = 25 - 9 = 16$
$\Rightarrow AP = 4$
Also, the perpendicular from the center of the circle bisects the chord, i.e. AP will be equal to PB.
Thus, $AB = 2AP = 2 \times 4 = 8$ cm
Hence, the length of the chord of the larger circle is 8 cm.

7. Draw a line segment of length 6.5 cm and divide it in the ratio 3 : 4.

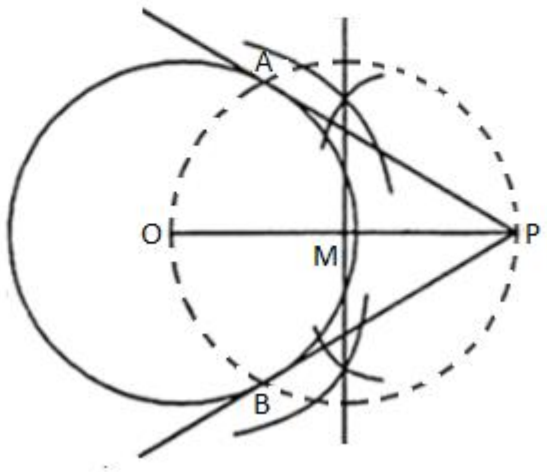
Solution:



OR

Draw a circle of radius 4 cm. From a point 7 cm away from a centre, construct the pair of tangents to the circle and measure its length.

Solution:



Hence, PA and PB are the required tangents to the circle.
Length of the tangents = PA = PB = 5.75 cm (approx.)

8. A bag contains 3 red and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is
(i) red
(ii) not red

Solution:
Given,
3 red balls ad 5 black balls
Total = 3 + 5 = 8
Total number of outcomes = 8
(i) Number of red balls = 3
P(getting a red ball) = $\frac{3}{8}$
(ii) P(not getting a red ball) = 1 - P(getting a red ball)
= 1 - ($\frac{3}{8}$)
= (8 - 3)/8
= $\frac{5}{8}$

PART - C

9. The cost of 1 pencil and 3 erasers is Rs. 10 and the cost of 4 pencils and 6 erasers is Rs. 28. Find the cost of 5 pencils and 4 erasers.

Solution:
Let x be the cost (in Rs) of one pencil and y be the cost (in Rs) of one eraser.
According to the given,
x + 3y = 10…..(i)
4x + 6y = 28….(ii)
From (i),
x = 10 - 3y….(iii)
Substituting (iii) in (ii),
4(10 - 3y) + 6y = 28
40 - 12y + 6y = 28

⇒ 6y = 40 - 28
⇒ 6y = 12
⇒ y = 2
Substituting y = 2 in (iii),
x = 10 - 3(2) = 10 - 6 = 4
Thus, the cost of one pencil is Rs. 4 and the cost of one eraser is Rs. 2.
The cost of 5 pencils and 4 erasers = 5(Rs. 4) + 4(Rs. 2)
= Rs. 20 + Rs. 8
= Rs. 28

10. Find the coordinates of the points of trisection (i.e. point dividing the three equal parts) of the segment joining the points A(2, -2) and B(-7, 4).

Solution:
Let the given points be:
A = (x_1, y_1) = (2, -2)
B = (x_2, y_2) = (-7, 4)

P divides AB in the ratio 1 : 2.
Here, m : n = 1 : 2
Using the section formula,
P(x, y) = $[(mx_2 + nx_1)/ (m + n), (my_2 + ny_1)/ (m + n)]$
= [(-7 + 4)/ (1 + 2), (4 - 4)/ (1 + 2)]
= (-3/3, 0/3)
= (-1, 0)
Q is the midpoint of PB.
Q = [(-1 - 7)/2, (0 + 4)/2]
= (-8/2, 4/2)
= (-4, 2)
Hence, the required points are (-1, 0) and (-4, 2).

11. If tan (A + B) = √3 and tan (A - B) = 1/√3; 0° < A + B ≤ 90°; A > B, then find the value of A and B.

Solution:
Given,
tan (A + B) = √3
tan (A + B) = tan 60°
A + B = 60°….(i)
And
tan (A - B) = 1/√3
tan (A - B) = tan 30°
A - B = 30°….(ii)
Adding (i) and (ii),
2A = 90°
A = 90°/2 = 45°
Substituting A = 45° in (i),
45° + B = 60°
B = 60° - 45° = 15°

Therefore, A = 45° and B = 15°.

OR

Prove that √[(1 + sin A)/(1 - sin A)] = sec A + tan A; A < 90°.

Solution:

$$\text{LHS} = \sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1-\sin A}{1-\sin A}}$$

$$= \sqrt{\frac{1-\sin^2 A}{(1-\sin A)^2}}$$

$$= \frac{\cos A}{1-\sin A}$$

$$= \frac{\cos A(1+\sin A)}{(1-\sin A)(1+\sin A)}$$

$$= \frac{\cos A(1+\sin A)}{1-\sin^2 A}$$

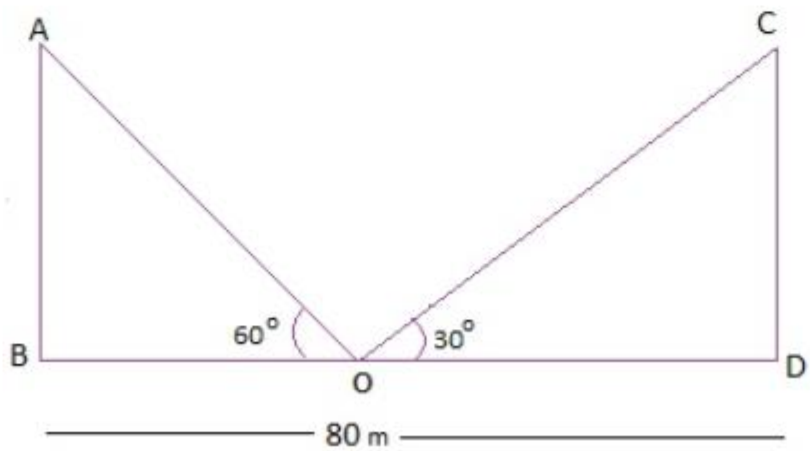
$$= \frac{1+\sin A}{\cos A}$$

$$= \sec A + \tan A$$

$$= \text{RHS}$$

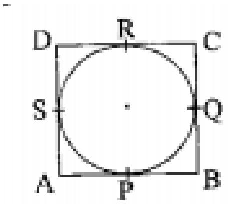
12. Two poles of equal heights are standing opposite each other on either side of the road which is 80 m wide. From a point between them on the road the angles of elevation of top of the poles are 60° and 30°. Find the height of the poles and distance of the point from the poles.

Solution:
Let AB and CD be the two poles of equal heights.
BD = Distance between two poles.



OB + OD = 80 m
In right triangle CDO,
tan 30° = CD/OD
1/√3 = CD/OD
CD = OD/√3....(i)
In right ΔABO,
tan 60° = AB/OB
√3 = AB/(80 - OD)
AB = √3(80 - OD)
AB = CD (Given)
√3(80 - OD) = OD/√3 [From (i)]
3(80 - OD) = OD
240 - 3 OD = OD
4 OD = 240
OD = 60
Substituting OD = 60 in (i),
CD = OD/√3
CD = 60/√3
CD = 20√3 m
Also,
OB + OD = 80 m
⇒ OB = 80 - 60 = 20 m
Hence, the height of the poles is 20√3 m and distances from the point of elevation are 20 m and 60 m respectively.

13. In the figure given below a quadrilateral ABCD is drawn to circumscribe a circle. Prove that: AB + CD = AD + BC



Solution:
From the given,

DR = DS ....(i) (tangents from D)
BP = BQ ....(ii) (tangents from B)
AP = AS ....(iii) (tangents from A)
CR = CQ ....(iv) (tangents from C)
Adding (i), (ii), (iii) and (iv),
DR + BP + AP+ CR = DS + BQ + AS + CQ
By rearranging them,
(DR + CR) + (BP + AP) = (CQ + BQ) + (DS + AS)
AD + BC = CD + AB
Hence proved.

14. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find
(i) the length of the arc
(ii) area of sector

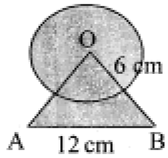
Solution:
Given,
Radius = 21 cm
Angle of the sector = θ = 60°

(i) Length of an arc = θ/360° × 2πr
= (60°/360°) × 2 × (22/7) × 21
= (1/6) × 2 × (22/7) × 21
= 22 cm

(ii) Area of sector = (θ/360°) × πr^2
= (60°/360°) × (22/7) × 21 × 21
= 11 × 21
= 231 cm^2

OR

In the figure given below, find the area of the shaded region where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.



Solution:
From the given,
OAB is an equilateral triangle since each angle is 60°.
Area of the sector is common in both.
Also,
Radius of the circle = r = 6 cm
Side of the triangle = 12 cm
Area of the equilateral triangle = √3/4 × $(OA)^2$
= √3/4 × $(12)^2$
= = √3/4 × 144

$= 36\sqrt{3}\ cm^2$

Area of the circle = πr^2

$= (22/7) \times (6)^2$

$= 792/7\ cm^2$

Area of the sector making angle 60° = $(60°/360°) \times \pi r^2$

$= (1/6) \times (22/7) \times (6)^2$

$= 132/7\ cm^2$

Area of the shaded region = Area of the equilateral triangle + Area of the circle - Area of the sector

$= 36\sqrt{3} + (792/7) - (132/7)$

$= 36\sqrt{3} + (792 - 132)/7$

$= (36\sqrt{3} + 660/7)\ cm^2$

15. The following distribution gives the daily income of 50 workers of a factory.

Daily income (in Rs.)	100 - 120	120 - 140	140 - 160	160 - 180	180 - 200
No. of workers	12	14	8	6	10

Convert the above distribution to a less than type cumulative frequency distribution and draw its ogive.

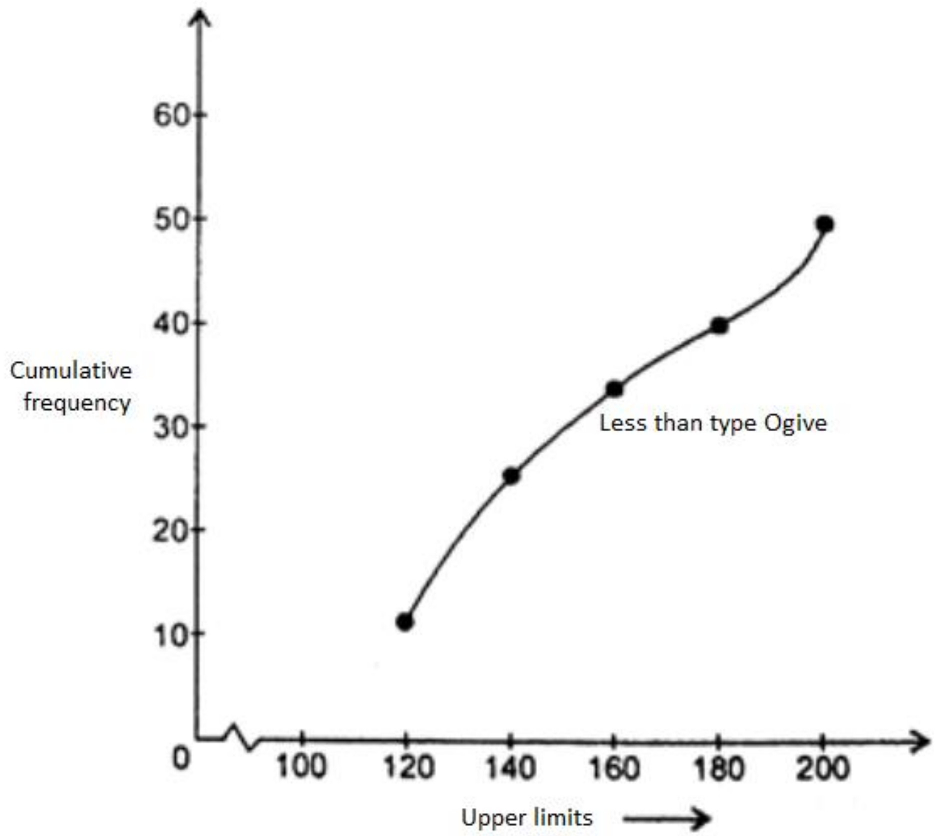
Solution:

Less than type cumulative frequency distribution table:

Daily income (in Rs.)	Number of workers
Less than 120	12
Less than 140	26
Less than 160	34
Less than 180	40
Less than 200	50

Plot the points (120, 12), (140, 26), (160, 34), (180, 40) and (200, 50).

Ogive:



OR

A survey conducted on 20 households in a locality by a group of students resulted in the following frequency table for the number of family members in a household.

Family size	1 - 3	3 - 5	5 - 7	7 - 9	9 - 11
No.of families	7	8	2	2	1

Find the mode of this data.

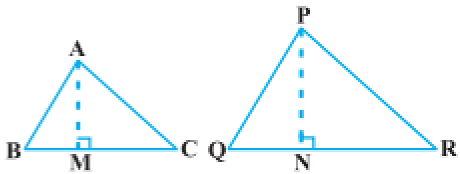
Solution:
From the given,
Maximum frequency is 8 which lies in the class interval 3 - 5.
Modal class = 3 - 5
Lower limit of the modal class = l = 3
Frequency of the modal class = $f_1 = 8$
Frequency of the class preceding the modal class = $f_0 = 7$
Frequency of the class succeeding the modal class = $f_2 = 2$
Class height = h = 2
Mode = $l + [(f_1 - f_0)/ (2f_1 - f_0 - f_2)] \times h$
$= 3 + [(8 - 7)/ (2 \times 8 - 7 - 2)] \times 2$
$= 3 + [1/ (16 - 9)] \times 2$

= 3 + (1/7) × 2
= 3 + 0.286
= 3.286
Hence, the mode is 3.286.

PART - D

16. Prove that the ratio of the areas of two similar triangles is equal to the squares of the ratio of their corresponding sides.

Solution:
Given,
Two triangles ABC and PQR such that ΔABC ~ ΔPQR
Draw altitudes AM and PN of two triangles ABC and PQR respectively.



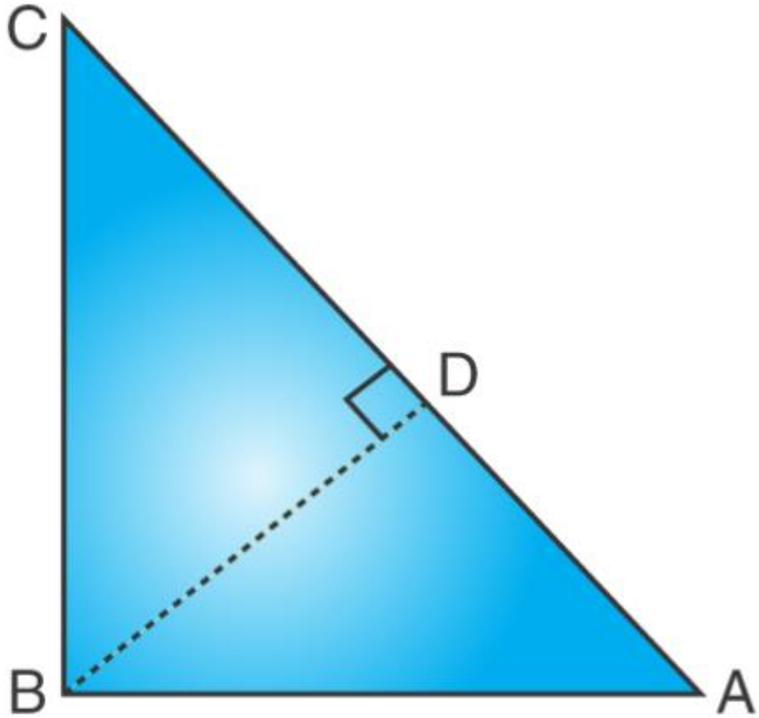
ar(ΔABC) = 1/2 BC × AM
ar(ΔPQR) = 1/2 QR × PN
Now,
ar(ΔABC)/ar(ΔPQR) = [(1/2) BC × AM] / [(1/2) QR × PN]
= (BC × AM)/ (QR × PN) ....(i)
In ΔABM and ΔPQN,
∠B = ∠Q (ΔABC ~ ΔPQR)
∠M = ∠N (each 90°)
Thus, ΔABM ~ ΔPQN (by AA similarity criterion)
⇒ AM/PN = AB/PQ ....(ii)
ΔABC ~ ΔPQR (given)
AB/PQ = BC/QR = CA/RP ....(iii)
From (i) and (iii),
ar(ΔABC)/ar(ΔPQR) = (AB/PQ) × (AM/PN)
= (AB/PQ) × (AB/PQ) [From (ii)]
= $(AB/PQ)^2$
Similarly,
ar(ΔABC)/ar(ΔPQR) = $(AB/PQ)^2 = (BC/QR)^2 = (CA/RP)^2$
Hence proved.

OR

Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution:
Given:
In a right triangle ABC, ∠B = 90°
To prove:
$AC^2 = AB^2 + BC^2$

Construction:
Draw a perpendicular BD onto the side AC.



We know that,
△ADB ~ △ABC
Therefore, AD/AB = AB/AC (by similarity)
$AB^2 = AD \times AC$ .....(i)
Also, △BDC ~△ABC
Therefore, CD/BC = BC/AC (by similarity)
$BC^2 = CD \times AC$ ....(ii)
Adding (i) and (ii),
$AB^2 + BC^2 = AD \times AC + CD \times AC$
$AB^2 + BC^2 = AC (AD + CD)$
Since, AD + CD = AC
Therefore, $AC^2 = AB^2 + BC^2$
Hence proved.

17. A cylindrical bucket 32 cm high and with a radius of base, 18 cm is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

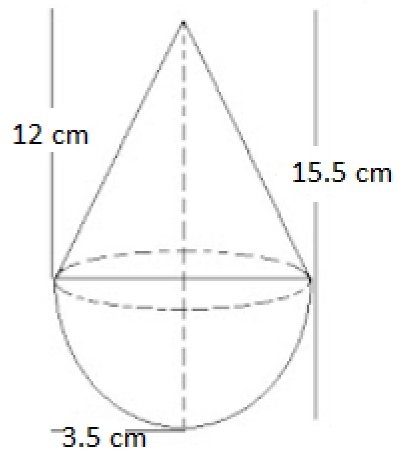
Solution:
Given,
Height of the cylindrical bucket = H = 32 cm
Radius of the base of cylindrical bucket = R = 18 cm
Height of the conical heap = h = 24 cm
Let r be the radius of the base of conical heap.
Volume of sand in cylindrical bucket = Volume of sand of conical heap
$\pi R^2 H = (1/3)\pi r^2 h$

$18 \times 18 \times 32 = (1/3) \times r^2 \times 24$
$r^2 = (18 \times 18 \times 32 \times 3)/24$
$r^2 = 18 \times 3 \times 8 \times 3$
$r^2 = 1296$
$r = 36$ cm
Slant height = $l = \sqrt{(r^2 + h^2)}$
$= \sqrt{(36^2 + 24^2)}$
$= \sqrt{(1296 + 576)}$
$= \sqrt{(1872)}$
$= 12\sqrt{13}$ cm

OR

A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of the same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Solution:



Given,
Radius of cone = Radius of hemisphere = r = 3.5 cm = 7/2 cm
Total height of the toy = 15.5 cm
Height of the cone = h = 15.5 - r
= 15.5 - 3.5
= 12 cm
Slant height of cone = $l = \sqrt{(r^2 + h^2)}$
$= \sqrt{[(12)^2 + (7/2)^2]}$
$= \sqrt{[144 + (49/4)]}$
$= \sqrt{[(576 + 49)/4]}$
$= \sqrt{(625/4)}$
= 25/2
l = 25/2 cm
Total surface area of the toy = CSA of cone + CSA of hemisphere
$= \pi rl + 2\pi r^2$
$= (22/7) \times (7/2) \times (25/2) + 2 \times (22/7) \times (7/2) \times (7/2)$
= 137.5 + 77
= 214.5 cm^2