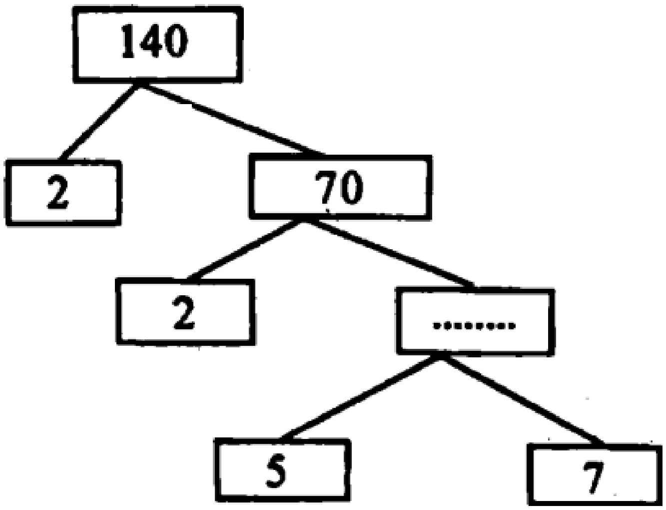
# PSEB Class 10 Mathematics 2019 Question Paper with Solutions

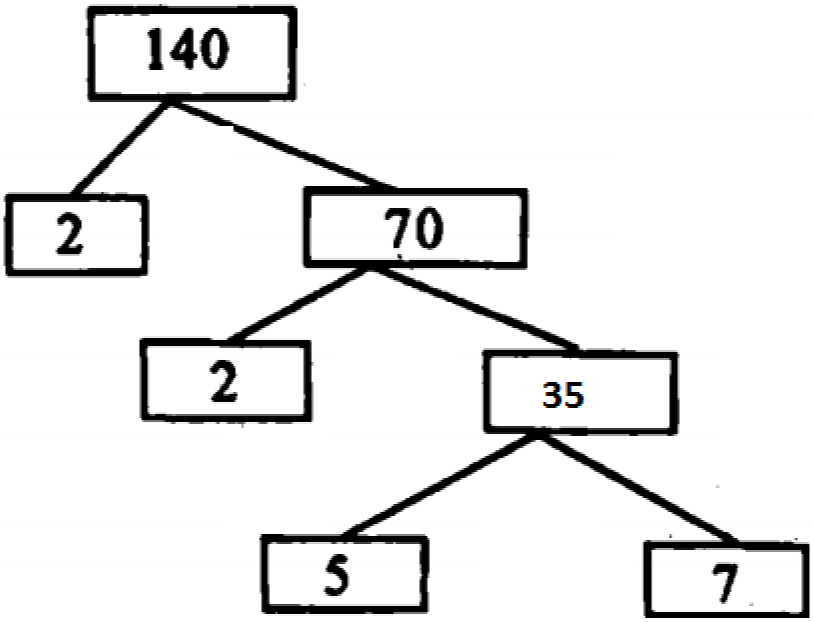
**PART - A**

**1.** Complete the prime factor tree:

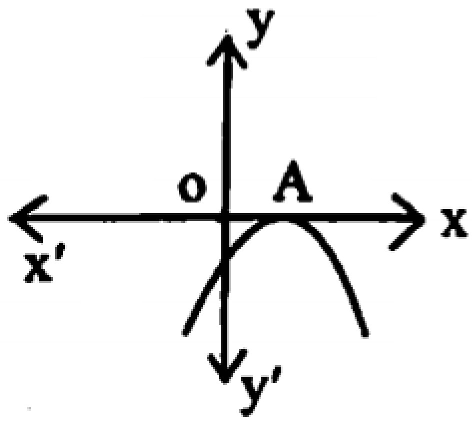



**Solution:**

70 = 2 × 35




**2.** The graph of y = p(x) is given. Find the number of zeroes of p(x).




(a) 0
(b) 1
(c) 2
(d) 3

**Solution:**
Correct answer: (b)

The graph of y = p(x) touches the x-axis at only one point.
Hence, the number of zeroes of p(x) = 1

**3.** Write first four terms of the AP when the first term a = 4 and common difference d = -3.

**Solution:**
Given,
First term = a = 4
Common difference = d = -3
Second term = a + d = 4 - 3 = 1
Third term = a + 2d = 4 + 2(-3) = 4 - 6 = -2
Fourth term = a + 3d = 4 + 3(- 3) = 4 - 9 = -5
Therefore, the four terms of the AP are 4, 1, -2, -5.

**4.** Which point lies on the x-axis from the following:
(a) (1, 1)
(b) (2, 0)
(c) (0, 3)
(d) (-4, -2)

**Solution:**
Correct answer (b)

We know that the value of y-coordinate is 0 on the x-axis.
Hence, the point (2, 0) lies on the x-axis.

**5.** Fill in the blank.
The hypotenuse is the______ side in the right triangle.

**Solution:**
The hypotenuse is the **longest** side in the right triangle.

**6.** Write the formula for finding the area of the sector of a circle with angle θ.

**Solution:**
Area of the sector of a circle = $(\theta/360°) \times \pi r^2$
Where,
r = Radius of the circle
θ = Angle made by the sector

**7.** Write True/False
The formula for finding the surface area of the sphere is $4/3\ \pi r^3$.

**Solution:**
False.
Reason: Surface area of sphere is $4\pi r^2$

**8.** Write True/False
Probability of an event E + Probability of the event 'not E' = 1

**Solution:**
True
P(E) + P(not E) = 1

**PART - B**

**9.** Express 5005 as a product of its prime factors.

**Solution:**
Prime factorization of 5005 is:
5005 = 5 × 7 × 11 × 13

**10.** Find the zeroes of the quadratic polynomial $3x^2 - x - 4$ and verify the relationship between the zeroes and the coefficients.

**Solution:**
Given polynomial is:
$3x^2 - x - 4$
To find the zeroes of the polynomial:
$3x^2 - x - 4 = 0$
$3x^2 + 3x - 4x - 4 = 0$
$3x(x + 1) - 4(x + 1) = 0$
$(3x - 4)(x + 1) = 0$
$3x - 4 = 0,\ x + 1 = 0$
$x = 4/3, -1$
Therefore, the zeroes of the polynomial are 4/3 and -1.
Sum of the zeroes = (4/3) - 1
= (4 - 3)/3
= ⅓

= -(-1)/3
= -Coefficient of x/ Coefficient of $x^2$
Product of the zeroes = (4/3)(-1)
= -4/3
= Constant term/ Coefficient of $x^2$
Hence, verified the relationship between the zeroes and the coefficients.

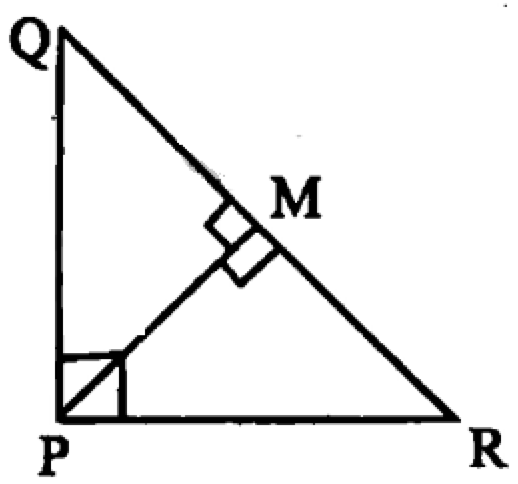
**11.** The coach of a cricket team buys 3 bats and 6 balls for Rs. 3900. Later, he buys another bat and 3 more balls of the same kind for Rs. 1300. Represent this situation algebraically.

**Solution:**
Let x be the cost (in Rs) of one bat.
Let y be the cost (in Rs) of one ball.
The algebraic representation of the given information is:
$3x + 6y = 3900$
$x + 3y = 1300$

**12.** Check whether the equation $x(2x + 3) = x^2 + 1$ is a quadratic equation.

**Solution:**
Given,
$x(2x + 3) = x^2 + 1$
$2x^2 + 3x - x^2 - 1 = 0$
$x^2 + 3x - 1 = 0$
The highest power is 2.
Hence, the given equation is a quadratic equation.

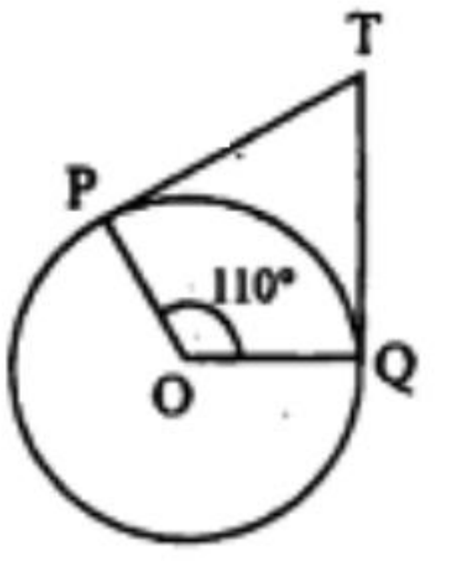
**13.** PQR is a triangle right angle at P and M is a point on QR such that PM ⊥ QR. Show that $PM^2$ = QM . MR.




**Solution:**
Given,
PM ⊥ QR
In ΔQMP and ΔQPR,
∠QMP = ∠QPR (each equal to 90°)

∠Q = ∠Q (common)
⇒ ΔQMP ~ ΔQPR....(i)  (by AA similarity)
In ΔPMR and ΔQPR,
<PMR = <QPR (each equal to 90°)
∠R = ∠R (common)
⇒ ΔRMP ~ ΔQPR....(ii) (by AA similarity)
From (i) and (ii),
ΔQMP ~ ΔPMR
⇒ QM/PM = PM/RM
⇒ QM.RM = PM. PM
⇒ $PM^2$ = QM.RM

**14.** In the given figure, if TP and TQ are the two tangents to a circle with centre O so that ∠POQ = 110°, find the angle ∠PTQ.




**Solution:**
Given,
∠POQ = 110°
TP and PQ are the tangents to the circle.
We know that radius is perpendicular to the tangent through the point of contact.
⇒ ∠OPT = ∠OQT = 90°
In quadrilateral OPTQ,
∠OPT + ∠POQ + ∠OQT + ∠PTQ = 360°
90° + 110° + 90° + ∠PTQ = 360°
∠PTQ = 360° - (90° + 110° + 90°)
∠PTQ = 360° - 290°
∠PTQ = 70°

**15.** The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

| | | | | | |
|---|---|---|---|---|---|
| Literacy rate (in %) | 45 - 55 | 55 - 65 | 65 - 75 | 75 - 85 | 85 - 95 |

| | | | | | |
|---|---|---|---|---|---|
| Number of cities | 3 | 10 | 11 | 8 | 3 |

**Solution:**

| Literacy rate (in %) | Number of cities ($f_i$) | Mid Value ($x_i$) | $d_i = x_i - a$ | $f_i d_i$ |
|---|---|---|---|---|
| 45 - 55 | 3 | 50 | -20 | -60 |
| 55 - 65 | 10 | 60 | -10 | -100 |
| 65 - 75 | 11 | 70 = a | 0 | 0 |
| 75 - 85 | 8 | 80 | 10 | 80 |
| 85 - 95 | 3 | 90 | 20 | 60 |
| | $\sum f_i = 35$ | | | $\sum f_i d_i = -20$ |

Assumed mean = a = 70
Mean = a + ($\sum f_i d_i / \sum f_i$) × h
= 70 + (-20/35) × 10
= 70 - (200/35)
= 70 - 5.7
= 64.3
Hence, the mean literacy rate is 64.3%.

**16.** One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting:
(i) a spade
(ii) the queen of diamond

**Solution:**
Total number of cards = 52
(i) Number of spades = 13
P(getting a spade) = 13/52 = 1/4
(ii) Number of queen of diamonds = 1
P(getting the queen of diamond) = 1/52

**PART - C**

**17.** The sum of the reciprocals of Rehman's ages (in years) 3 years ago and 5 years from now is 1/3. Find his present age.

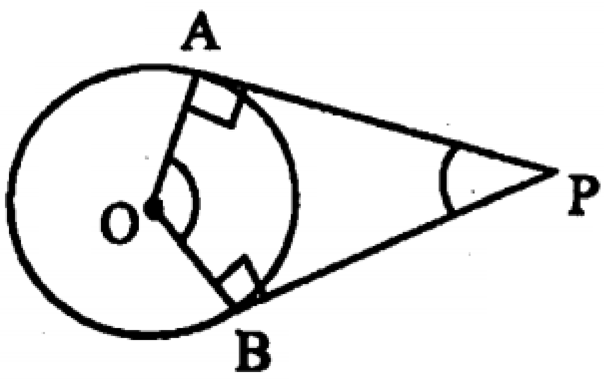
**Solution:**
Let x be the present age (in years) of Rehman.
According to the given,
[1/(x - 3)] + [1/(x + 5)] = 1/3
[x + 5 + x - 3]/ [(x - 3)(x + 5)] = 1/3

$3(2x + 2) = x^2 + 5x - 3x - 15$
$6x + 6 = x^2 + 2x - 15$
$x^2 + 2x - 15 - 6x - 6 = 0$
$x^2 - 4x - 21 = 0$
$x^2 - 7x + 3x - 21 = 0$
$x(x - 7) + 3(x - 7) = 0$
$(x + 3)(x - 7) = 0$
$x = -3, x = 7$
Age cannot be negative.
Therefore, $x = 7$
Hence, the present age of Rehman is 7 years.

**18.** How many terms of the AP 9, 17, 25,.... must be taken to give a sum of 636?

**Solution:**
Given AP:
9, 17, 25,...
Sn = 636
First term = a = 9
Common difference = d = 17 - 9 = 8
Sum of first n terms
$S_n = n/2 [2a + (n - 1)d]$
$636 = n/2 [2(9) + (n - 1)8]$
$636 \times 2 = n[18 + 8n - 8]$
$1272 = 10n + 8n^2$
$8n^2 + 10n - 1272 = 0$
$4n^2 + 5n - 636 = 0$
$4n^2 - 48n + 53n - 636 = 0$
$4n(n - 12) + 53(n - 12) = 0$
$(4n + 53)(n - 12) = 0$
$n = -53/4, n = 12$
Number of terms of AP cannot be negative or fraction.
n = 12
Hence, the sum of 12 terms of the AP is 636.

**19.** Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

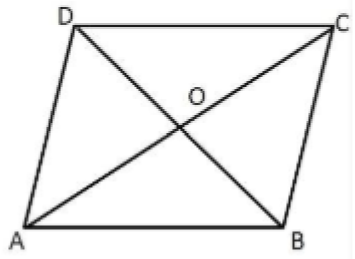


**Solution:**
Given,
PA and PB are the tangents to the circle.
We know that radius is perpendicular to the tangent through the point of contact.
$\angle OAP = \angle OBP = 90°$
In quadrilateral OAPB,
$\angle OAP + \angle APB + \angle OBP + \angle AOB = 360°$
$90° + \angle APB + 90° + \angle AOB = 360°$
$\angle APB + \angle AOB = 360° - (90° + 90°)$
$\angle APB + \angle AOB = 360° - 180°$
$\angle APB + \angle AOB = 180°$
Hence proved.

**OR**

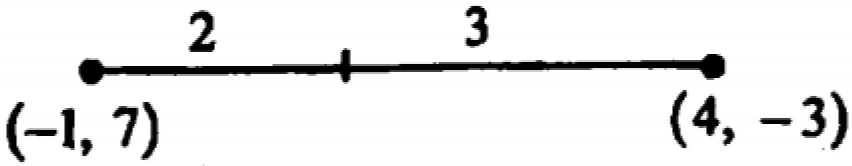
Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

**Solution:**
Given,
ABCD is the rhombus in which diagonals AC and BD intersect each other at O.
To prove: $(AB^2 + BC^2 + CD^2 + DA^2) = AC^2 + BD^2$




Proof:
OA = 1/2 AC
OB = 1/2 BD
In right triangle AOB,
$AB^2 = OA^2 + OB^2$
$AB^2 = (1/2\ AC)^2 + (1/2\ BD)^2$
$AB^2 = 1/4\ (AC^2 + BD^2)$
$4AB^2 = AC^2 + BD^2$
$AB^2 + AB^2 + AB^2 + AB^2 = AC^2 + BD^2$
$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$ (sides are equal in rhombus)
Hence proved.

**20.** Find the coordinates of the point which divides the join of (-1, 7) and (4, -3) in the ratio 2 : 3.



**Solution:**
Let P(x, y) be the point which divides the line segment joining the points (-1, 7) and (4, -3) in the ratio 2 : 3.
$(x_1, y_1) = (-1, 7)$
$(x_2, y_2) = (4, -3)$
m : n = 2 : 3
Using section formula,
$P(x, y) = [(mx_2 + nx_1)/ (m + n), (my_2 + ny_1)/ (m + n)]$
= [(8 - 3)/(2 + 3), (-6 + 21)/(2 + 3)]
= (5/5, 15/5)
= (1, 3)
Hence, the required point is (1, 3).

**OR**

Find the value of y for which the distance between the points P(2, -3) and Q(10, y) is 10 units.

**Solution:**
Given,
P(2, -3) and Q(10, y)
PQ = 10 units
$PQ^2 = (10)^2$
$(10 - 2)^2 + (y + 3)^2 = 100$
$(8)^2 + y^2 + 6y + 9 - 100 = 0$
$y^2 + 6y - 91 + 64 = 0$
$y^2 + 6y - 27 = 0$
$y^2 + 9y - 3y - 27 = 0$
y(y + 9) - 3(y + 9) = 0
(y - 3)(y + 9) = 0
y = 3, y = -9

**21.** Match the following:
(i) sin (90° - A) (a) sin A
(ii) cos 0° (b) 0
(iii) sin 0° (c) 1
(iv) cos (90° - A) (d) cos A

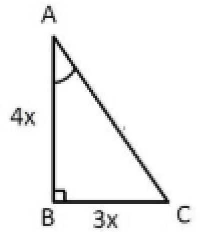
**Solution:**
(i) sin (90° - A) = (d) cos A
(ii) cos 0° = (c) 1
(iii) sin 0° = (b) 0
(iv) cos (90° - A) = (a) sin A

**OR**

If 3 cot A = 4, check whether $(1 - \tan^2 A)/ (1 + \tan^2 A) = \cos^2 A - \sin^2 A$ or not.

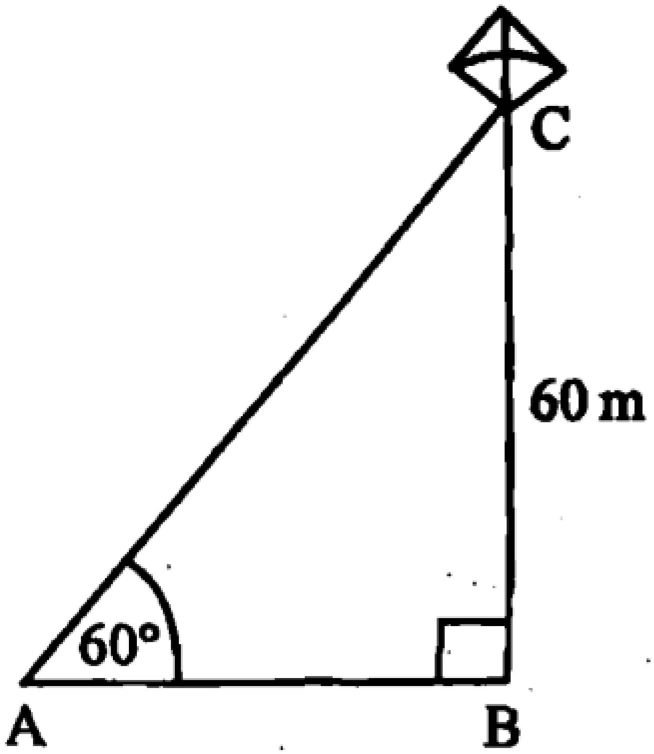
**Solution:**
Given,
3 cot A = 4
cot A = 4/3

tan A = 1/cot A = 1/ (4/3) = ¾
tan2A = (¾)2 = 9/16
$(1 - \tan^2 A)/ (1 + \tan^2 A) = [1 - (9/16)]/ [1 + (9/16)]$
= (16 - 9)/ (16 + 9)
= 7/25
Now,
cot A = side adjacent to A/side opposite to A = 4/3




By Pythagoras theorem,
$AC^2 = AB^2 + BC^2$
$= (4x)^2 + (3x)^2$
$= 16x^2 + 9x^2$
$= 25x^2$
AC = 5x
sin A = 3x/5x = ⅗
cos A = 4x/5x = 4/5
$\cos^2 A - \sin^2 A = (4/5)^2 - (3/5)^2$
= (16 - 9)/25
= 7/25

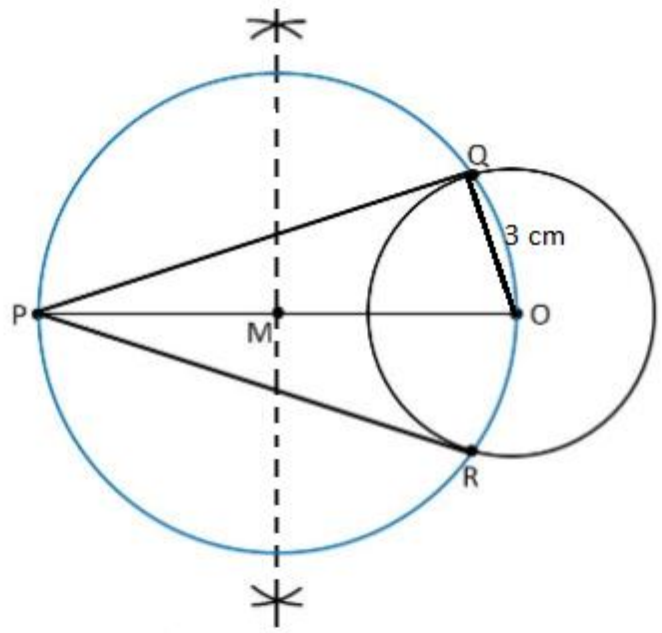
**22.** A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.




**Solution:**
From the given,
AC = Length of the string.
In right triangle ABC,
sin 60° = BC/AC
$\sqrt{3}/2 = 60/AC$
AC = (60 × 2)/3
$= 120/\sqrt{3}$
$= 40\sqrt{3}$ m

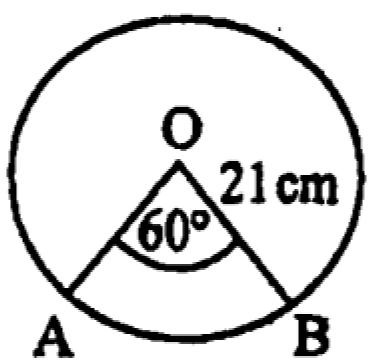
**23.** Draw a circle of radius 3 cm. Take a point outside the circle. Construct the pair of tangents from this point to the circle.

**Solution:**




Hence, PQ and PR are the required tangents to the circle.

**24.** In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find




(i) the length of the arc
(ii) area of the sector formed by the arc

**Solution:**
Given,
Radius = r = 21 cm
Angle of the sector = $\theta = 60°$
(i) Length of an arc = $(\theta/360°) \times 2\pi r$
$= (60°/360°) \times 2 \times (22/7) \times 21$
$= (1/6) \times 2 \times (22/7) \times 21$
$= 22$ cm
(ii) Area of the sector = $(\theta/360°) \times \pi r^2$
$= (60°/360°) \times (22/7) \times 21 \times 21$
$= 22 \times 21$

= 231 $cm^2$

## PART - D

**25.** Check graphically whether the pair of equations x + 3y = 6 and 2x - 3y = 12 is consistent. If so, solve them graphically.

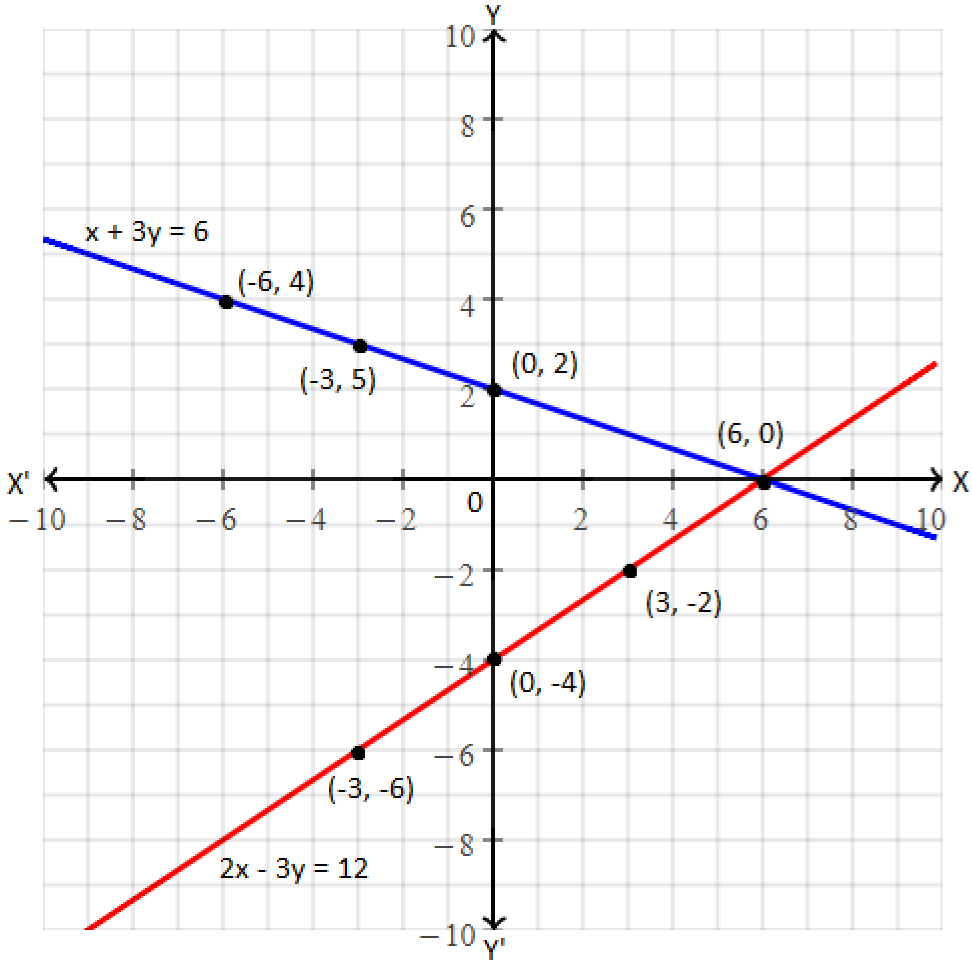
**Solution:**
Given,
x + 3y = 6
2x - 3y = 12
Comparing with the standard form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,
$a_1 = 1$, $b_1 = 3$, $c_1 = -6$
$a_2 = 2$, $b_2 = -3$, $c_2 = 12$
$a_1/a_2 = 1/2$
$b_1/b_2 = 3/-3 = -1$
$a_1/a_2 \neq b_1/b_2$
Thus, the given system of equations has a unique solution.
Consider the first equation:
x + 3y = 6
3y = -x + 6
y = -(1/3)x + 2

| | | | | |
|---|---|---|---|---|
| x | -6 | -3 | 0 | 6 |
| y | 4 | 5 | 2 | 0 |

Now, consider another equation:
2x - 3y = 12
3y = 2x - 12
y = (2/3)x - 4

| | | | | |
|---|---|---|---|---|
| x | -3 | 0 | 3 | 6 |
| y | -6 | -4 | -2 | 0 |

Graph:




Hence, the solution of the given pair of equations is x = 6 and y = 0.

**OR**

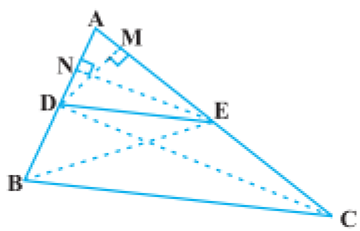
2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

**Solution:**
Let x be the number of days taken by 1 woman to complete the work.
Let y be the number of days taken by 1 man to complete the work.
Thus,
Work done by 1 woman in 1 day = 1/x
Work done by 1 man in 1 day = 1/y
According to the given,
4[(2/x) + (5/y)] = 1
(8/x) + (20/y) = 1....(i)
And
3[(3/x) + (6/y)] = 1
(9/x) + (18/y) = 1....(ii)
(i) × 9 - (ii) × 8,
(72/x) + (180/y) - (72/x) - (114/y) = 9 - 8

$(180 - 144)/y = 1$
$y = 36$
Substituting $y = 36$ in (ii),
$(9/x) + (18/36) = 1$
$9/x = 1 - (1/2)$
$9/x = 1/2$
$x = 18$
Hence, the time taken by 1 woman alone to finish the work is 18 days and the time taken by 1 man alone is 36 days.

**26.** If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. Prove it.

**Solution:**
Let ABC be the triangle and DE || BC.
Join BE and CD and then draw DM ⊥ AC and EN ⊥ AB.



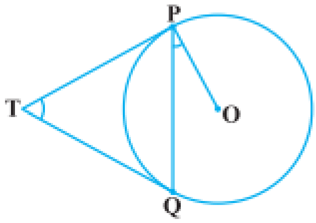

ar(ΔADE) = (1/2) × AD × EN
ar(ΔBDE) = (1/2) × DB × EN
Also,
ar(ΔADE) = (1/2) × AE × DM
ar(ΔDEC) = (1/2) × EC × DM
Now,
ar(ΔADE)/ar(ΔBDE) = [(1/2) × AD × EN]/ [(1/2) × DB × EN] = AD/DB....(i)
ar(ΔADE)/ar(ΔDEC) = [(1/2) × AE × DM]/ [(1/2) × EC × DM] = AE/EC....(ii)
ΔBDE and ΔDEC are on the same base DE and between the same parallels BC and DE.
ar(ΔBDE) = ar(ΔDEC)....(iii)
From (i), (ii) and (iii),
AD/DB = AE/EC

Hence proved.

**OR**

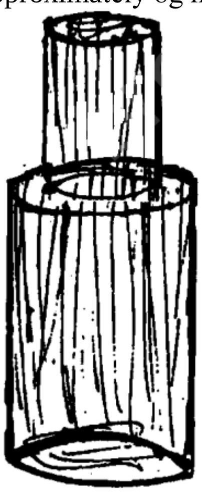
Two tangents TP and TQ are drawn to a circle with a centre O from an external point T. Prove that ∠PTQ = 2∠OPQ.

**Solution:**
Given,




Let ∠PTQ = θ
We know that TP = TQ
Thus, TPQ is an isosceles triangle.
∠TPQ = ∠TQP = 1/2 (180° - θ) = 90° - θ/2
Also, ∠OPT = 90°
∠OPQ = ∠OPT - ∠TPQ
= 90° - (90° - θ/2)
= θ/2
= 1/2 ∠PTQ
∠PTQ = 2∠OPQ

**27.** A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm3 of iron has approximately 8g mass. (use π = 3.14)

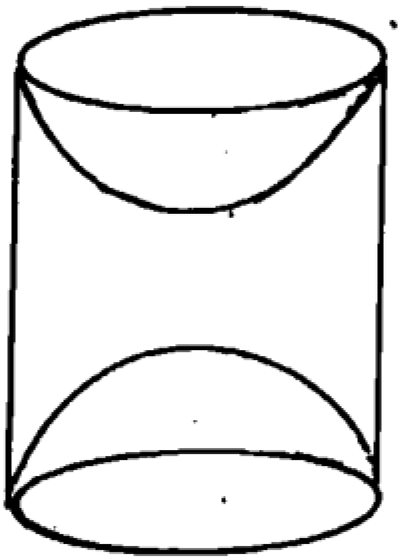


**Solution:**
Given,
Radius of larger cylinder = R = 12 cm
Height of the larger cylinder = 220 cm
Radius of smaller cylinder = r = 8 cm
Height of the smaller cylinder = h = 60 cm
Volume of pole = Volume of larger cylinder + Volume of smaller cylinder
= $\pi R^2 H + \pi r^2 h$
= π[12 × 12 × 220 + 8 × 8 × 60]
= 3.14 (144 × 220 + 64 × 60]
= 3.14 × 35520
= 111532.8 cm3
Mass of 1 cm3 of iron = 8 g (given)

The mass of 111532.8 cm3 = 111532.8 × 8
= 892262.4 g
= 892.262 kg

**OR**

A wooden article was made by scooping out a hemisphere from each end of a solid cylinder as shown in the figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm, find the total surface area of the article.



**Solution:**
Given,
Radius of hemispherical part = Radius of cylindrical part = r = 3.5 m
Height of cylinder = h = 10 cm
Total surface area of the article = CSA of cylinder + CSA of two hemispheres
$= 2\pi rh + 2 \times 2\pi r^2$
$= 2\pi[3.5 \times 10 + 2 \times 3.5 \times 3.5]$
$= 2\pi[35 + 24.5]$
$= 2 \times (22/7) \times 59.5$
$= 44 \times 8.5$
$= 374$ cm$^2$

**28.** A survey regarding the heights (in cm) of 51 girls of Class X of a school was conducted and the following data was obtained.

| Height (in cm) | Number of girls |
|---|---|
| Less than 140 | 4 |
| Less than 145 | 11 |
| Less than 150 | 29 |

| | |
|---|---|
| Less than 155 | 40 |
| Less than 160 | 46 |
| Less than 165 | 51 |

Find the median height.

**Solution:**
The given data can be tabulated as below:

| Height (in cm) | Number of girls (fi) | Cumulative frequency |
|---|---|---|
| 0 - 140 | 4 | 4 |
| 140 - 145 | 7 | 11 |
| 145 - 150 | 18 | 29 |
| 150 - 155 | 11 | 40 |
| 155 - 160 | 6 | 46 |
| 160 - 165 | 5 | 51 |

n/2 = 51/2 = 25.5
The cumulative frequency greater than 25.5 is 29 which lies in the class 145 - 150
l = 145
cf = 11
f = 18
h = 5
Median = l + {[(n/2) - cf]/ f} × h
= 145 + [(25.5 - 11)/18] × 5
= 145 + (72.5/18)
= 145 + 4.03
= 149.03
Hence, the median height is 149.03 cm.

**OR**

The following data gives the information on the observed lifetime (in hours) of 225 electrical components:

| Lifetime (in hours) | 0 - 20 | 20 - 40 | 40 - 60 | 60 - 80 | 80 - 100 | 100 - 120 |
|---|---|---|---|---|---|---|
| Frequency | 10 | 35 | 52 | 61 | 38 | 29 |

Determine the modal lifetimes of the components.

**Solution:**
From the given,
Maximum frequency is 61 which lies in the class interval 60 - 80.

Modal class = 60 - 80
Lower limit of the modal class = l = 60
Frequency of the modal class = $f_1$ = 61
Frequency of the class preceding the modal class = $f_0$ = 52
Frequency of the class succeeding the modal class = $f_2$ = 38
Class height = h = 20
Mode = $l + [(f_1 - f_0)/ (2f_1 - f_0 - f_2)] \times h$
= $60 + [(61 - 52)/ (2 \times 61 - 52 - 38)] \times 20$
= $60 + [9/ (122 - 90)] \times 20$
= $60 + (9/32) \times 20$
= 60 + 5.625
= 65.625
Hence, the modal lifetimes of the components is 65.625 hours.